

# Theory and Design of Reinforced Concrete Tanks

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2018



## شكر ودعاء

جريل الشكر لكل من حرص على اقتناء هذا الكتاب  
ودعاء الى الله  
ان يتمتع بما حواه من علم وان يعتمد المولى برحمته  
مولمه الاستاذ الدكتور / محمد هلال

سامح هلال



## **P R E F A C E**

This new revised edition includes the following additions

- 1) Determination of fixing and connecting moments in fixed and continuous surfaces of revolution. Chapter IV
- 2) The application of the data given in the previous article appears in the design of the Inze tank given in Chapter VI
- 3) The design of circular flat plates with overhanging cantilevers and eventually central holes under different load conditions is shown in Chapter VI in a numerical example of a circular water tower 300 m<sup>3</sup> capacity
- 4) Bunkers and Silos being a natural continuation to tanks and containers are dealt with in Chapter IX.
- 5) The statistical behavior of pump rooms is similar to that of under ground tanks the discussion of some examples is given in Chapter X.
- 6) Temperature stresses in walls of tanks bunkers, silos and pump rooms are shown in Chapter XII.

The author hopes that these additions open new scopes in the theory and design of reinforced concrete tanks and containers and may be of benefit to both graduate and undergraduate students in the faculties of engineering and practicing structural engineers.

January 1976

**M. HILAL**



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## INTRODUCTION

Reinforced concrete and eventually prestressed concrete are generally the most convenient materials for liquid tanks and containers

Due to the internal pressure of the liquid stored in such structures the walls and floors are mainly subject to tensile forces bending moments and eccentric tension which cause in most cases critical tensile stresses on the surface of the different elements facing the liquid. If such elements are designed according to the general principles adopted in ordinary reinforced concrete cracks will be developed and the liquid contained in the tank has the possibility to penetrate under its hydrostatic pressure through the cracks and cause rusting of the steel reinforcement. Therefore special provisions must be taken to prevent the formation of such cracks. Such provisions generally lead to an increased thickness of the walls towards their foot and at their other corners. If the effect of this increase is not considered it may lead to serious defects so that a thorough investigation is absolutely essential.

Porous concrete or concretes containing honey combing or badly executed joints lead to the same possibility of rusting with all its ill effects to the structure. Therefore dense water tight concrete is one of the essential requirements of liquid tanks and containers the necessary provisions required in the careful design of the mix and in the execution of the structure must be taken.

The protection of the finished concrete structure by convenient plastering painting or casing as well as its thorough curing must be carefully studied.

The previous investigation gives some points showing that liquid containers are delicate structures and need due to their intensive use in structural engineering special care and knowledge in the design execution and protection.

The present work is concerned with the theory and design of liquid tanks and containers giving the necessary provisions aiming to satisfy the final goal of structural engineering by designing safely economically and efficiently. It is designed to adapt the teaching requirements in our universities and higher schools for undergraduate and advanced studies. It gives the practicing designer a simple scientific and basic reference not only in liquid containers but also in many other fields such as surfaces of revolution high towers circular beams rectangular and circular flat plates deep beams beams on elastic foundation pyramid roofs etc as they are needed in design of tanks and containers. The statistical behavior of bunkers and silos is similar to that of tanks for this reason it has been decided to show their design in a separate chapter of this edition.

In preparing this book, it was taken for granted that the reader has a thorough knowledge of the strength of materials and fundamentals of reinforced concrete that is usually covered by our faculties of engineering. It lays thereon the foundation of a thorough understanding of the basic principles of reinforced concrete and eventually prestressed concrete tanks presented and developed in a simple systematic clear manner

One of the main aims of this book is to show that careful scientific designs based on sound theoretical basis do not need necessarily complicated calculations and, in most cases an attempt has been made to simplify complicated derivations as much as possible and to give simple practical relations carefully derived and easy to remember for other wise complicated problems. In complicated cases for which solutions could not be derived without going beyond the limit of the usual standard in structural engineering studies or in cases which need tedious or lengthy calculations the final results only were given and where possible simplified by design tables and curves

Using these methods of simplifying the presentation the author was able to discuss a big number of problems in a detailed manner facing and proposing solutions for the different difficulties that may arise in the design.

In this manner a complete treatment of the design fundamentals is given and fully discussed and illustrated with numerical examples and full constructional details showing the engineer how to attack structural problems with confidence.

The book includes the following main chapters

Chapter I gives a short account on the production of dense water tight concrete.

Chapter II deals with the design of sections under different kinds of stresses. The effect of shrinkage on symmetrical sections is also included. In order to have sufficient safety against cracking the final stresses in the different elements are generally low so that the elastic theory which is taken as a basis for the design is justified.

The design of circular tanks with sliding hinged fixed and continuous base are shown in Chapter III. In this chapter the theory of Lamé for thick cylinders and the theory of Reissner for determining the internal forces in circular tank walls fixed at the base and subject to hydrostatic pressure are given. The design of such tanks can be made by a very quick and simple manner if the simplified methods proposed by the author are used. The tables published by the American Portland Cement Association for circular tanks and circular plates are also included.

Chapter IV is devoted to the roofs and floors of circular tanks. It includes the membrane theory of thin surfaces of revolution and the internal forces in circular flat plates.

Prestressed concrete may give a convenient economic solution for big circular tanks. For this purpose the fundamentals of circumferential, longitudinal and dome prestressing as may be used for liquid containers are given in Chapter V.

In order to show the application of the theories of circular tanks, circular plates, domes and cones, three detailed examples are illustrated in Chapter VI. The first example shows the design of a paste container and the other two show different types of water towers.

Chapter VII gives a thorough investigation of rectangular tanks calculated according to the approximate strip method and according to the mathematical theory of flat plates for which purpose the internal forces in flat rectangular plates supported on three and four sides subject to uniform and triangular loads as determined by Czerny for different conditions at the supports are given.

Tanks directly built on the ground are dealt with in Chapter VIII. The theory of beams on elastic foundations and its application to some tank problems are shown in two numerical examples.

The design of bunkers and silos according to the classic theories and the new researches is dealt with in Chapter IX.

The behavior of pump stations is similar to empty underground tanks. Some examples are shown in Chapter X.

Some complementary designs required for tank problems showing stress distribution, internal forces and design of deep beams, pyramid roofs and temperature stresses in walls are given in Chapter XI.



## I PRODUCTION OF WATER-TIGHT CONCRETE

Dense concrete, free from cracks or honey combing is the main requirement for water-tightness. Porous concrete having cracks on the liquid side allow the liquid in the tank, under its hydrostatic pressure to penetrate through the concrete and cause rusting of the steel reinforcement leading to all its serious effects on the structure. Dense water-tight concrete can be achieved through careful selection of aggregates suitable granular composition use of low water cement ratio, sufficient cement content and thorough mixing, compaction and curing.

We give, in the following, a short account about the main factors affecting the density and water-tightness of concrete.

### I 1 COMPOSITION, PLACING AND COMPACTION

Dense concrete can be produced if the voids are reduced to a minimum such a provision can be attained through the choice of a convenient mix composed of fine aggregates (smaller than 5 mm), medium aggregates (between 5 mm and 10 mm) and coarse aggregates (over 10 mm). The maximum grain size is to be according to the thickness of the element in which it is used and preferably not more than 30 mm in reinforced concrete water structures.

In normal cases, the cement content in the mix is generally 350 kg per cubic meter finished concrete, in small tanks and in cases of low stresses, the cement dose may be reduced to 300 kgs/m<sup>3</sup>. Richer doses with a maximum of 400 kgs/m<sup>3</sup> may be used for big under-ground tanks in wet medium. The use of high cement doses in dry weather under

normal conditions is not recommended because the shrinkage tensile stresses causing cracks in the concrete increase with the increase of the cement content

It is recommended to use the least possible amount of mixing water giving good plastic concrete. The water-cement ratio to be specified depends on the method of compaction - by hand or by mechanical vibration - and on the nature of the concrete constituents, in this respect figures based on a slump test are recommended. Excess of mixing water is to be avoided as it leads to porous concrete due to the evaporation of the surplus water not needed for the chemical action, and increases the shrinkage strains

In big tanks, the use of mechanical mixers with automatic water control is essential

To produce dense concrete, good compaction is necessary as it compensates for the possible gaps in the granulometric composition of the aggregates. The use of surface and immersion vibrators gives satisfactory results

## I 2 ADMIXTURES

Some admixtures have a mechanical effect on concrete while others have a chemical effect. Admixtures having a mechanical lubricating effect increase the workability of concrete mixes, thus allowing a reduction in the water content which in turn results in an increase in the strength and water-tightness of concrete ( e g baraplast and air entraining agents ) Admixtures having chemical effects on the concrete mix are to be used only when tests prove that they have no ill effect on the concrete or the steel throughout their lifetime

Other admixtures help to seal the pores in the concrete, their presence is to be considered only as an addition to the water - tightness attained by the above mentioned steps and not in any way as a replacement

### I 3 CURING

Concrete undergoes a volume change during hardening it shrinks in dry weather and swells under water. Shrinkage causes tensile stresses in the concrete. If such stresses are developed and act on fresh concrete of low strength, they cause shrinkage cracks. It is absolutely essential to prevent such stresses from being developed until the concrete has gained sufficient strength to resist them. This can be done by intensive curing of fresh concrete ( keeping it continuously wet ) starting immediately after the final setting of the concrete and for a minimum period of 15 days.

### I 4 SURFACE TREATMENT, PAINTS AND CASINGS

The most effective surface treatment is cement mortar plaster composed of 600 to 650 kgs cement per cubic meter sand and applied by the cement gun. The thickness may be chosen 1.5 to 2.0 cms. It is recommended to apply such a plaster on side facing the liquid after filling the tank with water for 7 days. The surface should be thoroughly cleaned by wire brushes before the application of the cement gun. In this manner, the preliminary cracks which may appear after the first filling of the tank will be sealed by the plaster. Moreover the plaster will not be subject to a big part of the plastic strains due to water pressure.

Paints ( e.g. barafuate, baranormal, bituminous paints etc) may also be used either directly on the concrete surface or on the cement plaster. The use of special paints whose object is to close the surface pores ( such as glass paints, plastic paints, watertight casings lining with metallic sheets - e.g. stainless steel or water tight tiling ) may be of advantage.

Any material for water-tightness either as admixture or surface treatment must not be used unless it is proved by experiments to be suitable for the purpose.

## II DESIGN OF SECTIONS

### II 1 NOTATIONS

$\sigma_{c28}$	=	Compressive strength of standard concrete cube after 28 days
$\sigma_t$	=	Axial tensile strength of concrete
$\sigma_{tb}$	=	Bending strength of concrete
$\sigma_{co}$	=	Allowable axial compressive stress of concrete
$\sigma_{cb}$	=	Eventually $\sigma_c$ = allowable compressive bending stress of concrete
$\sigma_t$	=	Tensile stress on concrete
$\sigma_{tb}$	=	Max tensile concrete stress in bending
$\sigma_s$	=	Allowable stress in steel
N	=	Normal force acting on a section
T	=	Tensile force acting on a section
A	=	Area of a section ( general )
$A_c$	=	Area of concrete section
$A_s$	=	Area of steel reinforcement
E	=	Modulus of elasticity -( general )
$E_c$	=	Modulus of elasticity of concrete
$E_s$	=	Modulus of elasticity of steel
n	=	$E_s / E_c$ = modular ratio
$\epsilon$	=	Strain ( general )
$\epsilon_c$	=	Strain of concrete
$\epsilon_s$	=	Strain of steel
$\epsilon_{sh}$	=	Strain due to shrinkage of concrete
$\epsilon_{cr}$	=	Strain due to creep of concrete

- $b$  = Breadth of a rectangular section  
 $t$  = Total depth of a section  
 $d$  = Theoretical depth of a section = distance between center of gravity of tension steel and outside fiber of compression zone  
 $\mu$  =  $A_s / A_c$  = Ratio of tension steel reinforcement in a section  
 =  $A_s / b t$  in case of axial forces acting on a rectangular section  
 =  $A_s / b d$  in case of bending moments acting on a rectangular section  
 $y_o$  = Distance of c g axis from outside fiber in tension zone  
 $A_v$  =  $A_c + n A_s$  = virtual area of a reinforced concrete section  
 $Z$  = Section modulus, for a rectangular section  $Z = b t^2 / 6$   
 $Z_v$  = Section modulus of the virtual section  
 $I$  = Moment of inertia of a section, for a rectangular section  
 $I = b t^3 / 12$   
 $I_v$  = Moment of inertia of the virtual section  
 $M$  = Bending moment acting on a section  
 $M_s$  = Bending moment about tension steel =  $N e_s$   
 $M_f$  = Fixing moment  
 $e$  =  $M / N$  = eccentricity of normal force  $N$  from c g axis  
 $e_s$  = eccentricity of normal force  $N$  from tension steel  
 $k_1$  = Coefficient for determining the theoretical depth  $d$  of a reinforced concrete section subject to bending moment  
 $d = k_1 \sqrt{M / b}$  or eccentric forces  $d = k_1 \sqrt{M_s / b}$   
 $k_2$  = Coefficient for determining the area of the tension steel  $A_s$  in a reinforced concrete section subject to bending moment  
 $A_s = M / k_2 d$  or eccentric forces  $A_s = \frac{M_s}{k_2 d} \pm \frac{N}{\sigma_s}$   
 ( + ) for eccentric tension and ( - ) for eccentric compression  
 $\alpha$  =  $A'_s / A_s$  = ratio of compression steel to tension steel

## II 2 REQUIREMENTS AND ALLOWABLE STRESSES

Sections of liquid containers must be so designed that no cracks in concrete are allowed in the fibers facing the liquid, because if such cracks are allowed the liquid in the container will penetrate through these cracks & cause rusting of the steel reinforcement which must be prevented by all possible means

In order to satisfy this requirement the concrete dimensions must be chosen so that the tensile stresses in concrete- if they take place on the liquid side - are smaller than its tensile strength

But as the tensile strength of concrete cannot be guaranteed and because the above mentioned provision does not include any factor of safety for the tensile strength of concrete, the tension steel reinforcement must be designed to carry all the tensile stresses i e the concrete in tension is neglected - Stage II -

The axial tensile strength of concrete  $\sigma_t$  may be assumed a factor of the prism strength according to the relation

$$\sigma_t = 0.7 \text{ to } 1 \sqrt{\sigma_{cp}}$$

The building Research Institute in Egypt gives for  $\sigma_t$  the following relation

$$\sigma_t = 0.03 \sigma_{c28} + 8 \text{ kg/cm}^2$$

The tensile bending strength of concrete,  $\sigma_{tb}$  is double as much as its tensile strength  $\sigma_t$  i e  $\sigma_{tb} = 2\sigma_t = 1.4 \text{ to } 2 \sqrt{\sigma_{cp}}$  The average value to be used in the design is generally not bigger than  $3/4$  this value

$$\sigma_{tb} = 1.0 \text{ to } 1.5 \sqrt{\sigma_{cp}}$$

The code of practice for the use of reinforced concrete in buildings gives

" In structures where no cracks in tension are allowed, the permissible tensile stress of concrete may be taken

$\bar{\sigma}_t = \sigma_{co}/4$  for axial tension and  $\bar{\sigma}_{tb} = \sigma_{cb}/4$  for tension in bending  
For normal water structures, the values of the design stresses in concrete and steel can accordingly be determined

Values of  $\bar{\sigma}_t$  &  $\bar{\sigma}_{tb}$  recommended for design purposes are given in the following table

Values of  $\bar{\sigma}_t$  &  $\bar{\sigma}_{tb}$  for different concrete mixes

Case of Design	cement dose kg/m <sup>3</sup>	cube strength kg/cm <sup>2</sup>	tensile stress of concrete kg/cm <sup>2</sup>	
			Axial tension $\bar{\sigma}_t$	Bending $\bar{\sigma}_{tb}$
Shrinkage is not taken into consideration	300	160	10	15
	350	200	12	<u>18</u>
	400	250	15	20
Shrinkage is taken into consideration	300	160	14	18
	350	200	<u>16</u>	20
	400	250	20	25

The allowable stresses in steel subject to tensile stresses can be chosen as follows

For normal mild steel	$\sigma_s = 1400 \text{ kg/cm}^2$
For deformed high grade steel	$\sigma_s = 2000 \text{ kg/cm}^2$
For deformed cold twisted steel	$\sigma_s = 2200 \text{ kg/cm}^2$
On condition that the cube strength of concrete	$\sigma_{c28} > 200 \text{ kg/cm}^2$
In case $\sigma_{c28} < 200 \text{ kg/cm}^2$	$\sigma_s = 1200 \text{ kg/cm}^2$

### II 3 SECTIONS SUBJECT TO AXIAL TENSION

#### II 3 1 Stability Design

The steel alone must be sufficient to resist all the tensile force T acting on the section

$$\text{i e } A_s = T/\sigma_s \quad (1)$$

### II 3 2 Safety Against Cracking

The tensile stress in concrete  $\sigma_t$  must be smaller than its tensile strength  $\tilde{\sigma}_t$

$$i.e., \sigma_t = \frac{T}{A_c + n A_s} < \tilde{\sigma}_t \quad (2)$$

$A_c$  = The area of the required concrete section

$A_s$  = Area of steel reinforcement determined from equation (1)

$n$  = Modular ratio = 10 ( no cracks in concrete Stage I )

### II 3 3 Effect of Shrinkage

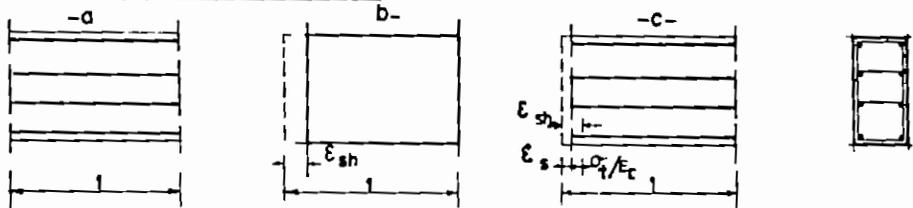


Fig II-1

The effect of shrinkage in a symmetrically reinforced element can be determined as follows ( Fig II 1 )

Figure (a) represents a symmetrically reinforced concrete block of unit length. If the bars are left out as in figure (b), shrinkage will shorten the concrete block a distance =  $\epsilon_{sh}$ . The presence of the steel bars prevents some of the shortening of the concrete and the block shortens  $\epsilon_s < \epsilon_{sh}$  only i.e. the steel shortens a distance  $\epsilon_s$  and accordingly is subject to compressive stresses  $\sigma_s$ , while the concrete will elongate a distance =  $\epsilon_{sh} - \epsilon_s$  and accordingly will be subject to tensile stresses  $\sigma_t$ .

The element being not subject to any external forces, the sum of the stresses on the section must be equal to zero

The stresses in steel & concrete can be calculated in the following manner

$$\text{Strain due to shrinkage} = \epsilon_{sh}$$

$$\text{Final strain in steel} = \epsilon_s = \sigma_s / E_s$$

$$\text{Final strain in concrete} = \epsilon_c = \epsilon_{sh} - \frac{\sigma_t}{E_c}$$

Both materials are subject to the same final strain  $\epsilon_s = \epsilon_c$

$$\text{or } \frac{\sigma_s}{E_s} = \epsilon_{sh} - \frac{\sigma_t}{E_c}$$

$$\text{or } \sigma_s = \epsilon_{sh} E_s - n \sigma_t \quad (3)$$

As no external forces act on the element, then

$$\sigma_t A_c - \sigma_s A_s = 0 \quad (4)$$

Equations (3) & (4) give

$$\sigma_t A_c - (\epsilon_{sh} E_s - n \sigma_t) A_s = 0$$

$$\text{or } \sigma_t = \frac{\epsilon_{sh} E_s A_s}{A_c + n A_s} \quad \text{tension} \quad (5a)$$

Equation (4) gives

$$\sigma_s = \sigma_t \frac{A_c}{A_s} \quad \text{1 s}$$

$$\sigma_s = \frac{\epsilon_{sh} E_s A_c}{A_c + n A_s} \quad \text{compression} \quad (5b)$$

In this manner the tensile stress in concrete  $\sigma_t$  due to a tensile force  $T$  and shrinkage is given by

$$\sigma_t = \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s} \quad (6)$$

in which

$\epsilon_{sh}$  = Strain due to shrinkage = 0.0002 to 0.0003 (0.2 to 0.3 mm/m)

$E_s$  = Modulus of elasticity of steel = 2100 t/cm<sup>2</sup>

$n$  = Modular ratio in stage I = 10

The area of steel  $A_s$  and the thickness  $t$  of a rectangular section of breadth  $b = 1$  m can be determined as follows

$$A_s = T / \sigma_s \quad \text{and} \quad A_c = b \quad t = 100 \text{ t}$$

Introducing these values in equation (6), we get

$$\sigma_t = \frac{T + \epsilon_{sh} E_s T / \sigma_s}{100 t + n T / \sigma_s} \quad \text{or} \quad t = \frac{\epsilon_{sh} E_s + \sigma_s - n \sigma_t}{100 \sigma_s \sigma_t} T$$

Assuming further that

$$\epsilon_{sh} = 0.00025 \quad \sigma_s = 1400 \text{ kg/cm}^2 \quad \sigma_t = 16 \text{ kg/cm}^2,$$

$$\text{then} \quad \underline{t = 0.8 T} \quad (7)$$

t in cms for T in tons/m

For this thickness, the area of steel can be given as

$$A_s = \frac{T}{\sigma_s} = \frac{t}{0.8 \sigma_s} \quad \text{for } \sigma_s = 1.4 \text{ t/cm}^2, \quad \text{we get}$$

$$\underline{A_s = 0.9 t} \quad (8)$$

$A_s$  in  $\text{cm}^2$  for t in cms i.e.  $\mu = 0.9$  of section

#### Example 1

To illustrate the effect of shrinkage assume

$$T = 40 \text{ t/m}, \quad \epsilon_{sh} = 0.00025 \quad E_s = 2100 \text{ t/cm}^2 \quad n = 10 \quad b = 100 \text{ cm}$$

$$\& \sigma_s = 1400 \text{ kg/cm}^2, \quad \text{then}$$

the thickness  $t = 0.8 T = 0.8 \times 40 = 32 \text{ cms}$

reinforcement  $A_s = 0.9 t = 0.9 \times 32 = 28.8 \text{ cm}^2$  or

$$A_s = T / \sigma_s = 40 / 1.4 = 28.5 \text{ cm}^2$$

chosen 14  $\phi$  16 mm/m ( $A_s = 28 \text{ cm}^2$ )

7  $\phi$  16 mm/m on each side

$$\text{and } \sigma_t = \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{40000 + 0.00025 \times 2100000 \times 28}{100 \times 32 + 10 \times 28}$$

$$= \frac{40000 + 14700}{3480} = \frac{54700}{3480} = 15.7 \text{ kg/cm}^2 \text{ tension! } < 16 \text{ kg/cm}^2$$

This example shows that the effect of shrinkage on the tensile stresses of concrete is about 37% of the effect of the direct tensile force T

Taking the effect of shrinkage into consideration, it is possible to prove that the concrete tensile stress  $\sigma_t$  and respectively the possibility of cracking increases with decreasing steel stress as follows

It has been proved that 
$$\sigma_t = \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s} \quad \text{but } A_s = T/\sigma_s$$

$$\text{then } \sigma_t = \frac{\sigma_s + \epsilon_{sh} E_s}{A_c \sigma_s + n T} T$$

Assuming for example  $T = 40 \text{ t/m}$  then  $t = 0.8 \times 40 = 32 \text{ cms}$  and  $A_c = 100 \times 32 = 3200 \text{ cm}^2$ , with  $\epsilon_{sh} = 0.00025$  &  $E_s = 2100 \text{ t/cm}^2$

we get

$$\sigma_t = \frac{\sigma_s + 0.00025 \times 2100000}{3200 \sigma_s + 10 \times 40000} \times 40000 = \frac{40 \sigma_s + 21000}{32 \sigma_s + 400}$$

Taking  $\sigma_s = 700$  to  $\infty \text{ kg/cm}^2$ , the corresponding  $\sigma_t$  will be as follows

$\sigma_s$	700	800	1000	1200	1400	1600	1800	2000	$\infty \text{ kg/cm}^2$
$\sigma_t$	18.6	17.9	17.0	16.3	15.6	15.4	15.1	14.8	13.33 $\text{kg/cm}^2$

N.B. for  $\sigma_s = \infty$ ,  $A_s = 0$  i.e. plain concrete wall

This means that if the steel stress is reduced from 1400 to 700  $\text{kg/cm}^2$ , the concrete tensile stress is increased from 15.6 to 18.6  $\text{kg/cm}^2$  which is about 20% increase

From the previous investigation, one can see that the lower the allowable stress in steel, the bigger the amount of the reinforcement and the sooner the concrete will crack. From this point of view it is desirable to use higher allowable steel stress, and it is recommended to choose  $\sigma_s > 1400 \text{ kg/cm}^2$

The use of lower steel stress  $\sigma_s$  in order to reduce the tensile strains in concrete has now no meaning so long as the section is designed such that  $\sigma_t < \bar{\sigma}_t$

#### II 4 SECTIONS SUBJECT TO SIMPLE BENDING

If the tension side of the section is not facing the liquid, the section is designed as ordinary reinforced concrete without any special precautions. If the tension is on the liquid side, it must have :

- a) Adequate resistance against cracking and
- b) Adequate strength

In order to satisfy condition (a) the section may be designed as plain concrete with the stress  $\sigma_{tb} = M/Z$

In case of rectangular sections  $Z = b t^2 / 6$  &  $\sigma_{tb} = 6 M / b t^2$

For normal conditions  $\sigma_{tb} = 18 \text{ kg/cm}^2$  &  $b = 1 \text{ m}$ , thus,

$$t = \sqrt{M / 3} \quad (9) \quad t \text{ in cms for } M \text{ in kgm}$$

In order to satisfy condition (b) proceed according to normal principles of reinforced concrete design as follows

For the value of  $t$  determined according to equation (9), calculate the value of  $k_1$  from the relation

$$d = k_1 \sqrt{M} \quad \text{where } d = t - 2.5 \text{ to } 4 \text{ cms}$$

For this value of  $k_1$  and the corresponding stress in steel (e.g.  $\sigma_s = 1400 \text{ kg/cm}^2$  &  $\alpha = 0$ ), determine  $k_2$ ; then

$$A_s = M / k_2 d$$

The tension steel  $A_s$  can however be taken as a factor of the concrete section determined according to equation (9) as follows :

$$t = \sqrt{M / 3} \quad \text{or} \quad t^2 = M / 3 \quad \text{or} \quad M = 3 t^2$$

The tension steel :

$$A_s = M / k_2 d \quad \text{where } k_2 \approx 1300 \text{ kg/cm}^2 \text{ for } \sigma_s = 1400 \text{ kg/cm}^2$$

Assuming further that  $d = 0.9 t$  we get :

---

\* Values of  $k_1$  and  $k_2$  for different  $\sigma_c$ ,  $\sigma_s$  and  $\alpha$  are given in text-books of reinforced concrete design

$$A_s = \frac{3 t^2 \times 100}{1300 \times 0.9 t} = 0.26 t$$

$A_s$  in  $\text{cm}^2$  for  $t$  in  $\text{cms}$

therefore

$$\underline{A_s = 0.26 t} \quad \text{or} \quad \underline{A_s = 0.26 \% \text{ of } A_c} \quad (10)$$

### Example 2 Simple Bending

Rectangular section subject to  $M = 6000 \text{ kgm}$  with tension on liquid side. Determine  $t$  and  $A_s$ . Assume  $b = 1 \text{ m}$ ,  $\sigma_{tb} = 18 \text{ kg/cm}^2$  and  $\sigma_s = 1400 \text{ kg/cm}^2$

### Solution

For safety against cracking (condition a) the depth of the section  $t$  is first to be determined according to equation (9)

$$\text{thus, } t = \sqrt{M / 3} = \sqrt{6000 / 3} = 45 \text{ cms}$$

Having determined the thickness  $t$ , the steel reinforcement can be calculated to satisfy condition b (adequate strength) according to stage II as follows

$$t = 45 \text{ cms} \quad \text{then } d = 41 \text{ cms} \quad \text{and}$$

$$41 = k_1 \sqrt{6000} \quad \text{i.e. } k_1 = 0.53$$

$$\text{for } \sigma_s = 1400 \text{ kg/cm}^2, \quad n = 15, \quad \alpha = 0 \quad \text{we get}$$

$$\sigma_c = 31.5 \text{ kg/cm}^2 \quad \& \quad k_2 = 1282 \text{ kg/cm}^2 \quad \text{so that}$$

$$A_s = \frac{6000}{1282 \times 0.41} = 11.4 \text{ cm}^2 \text{ chosen } 6 \phi 16 \text{ mm/m}$$

According to equation (10)

$$A_s = 0.26 \times 45 = 11.7 \text{ cm}^2 \text{ chosen } 6 \phi 16 \text{ mm/m}$$

### II 5 SECTIONS SUBJECT TO ECCENTRIC TENSION OR COMPRESSION

If the resultant stress on the liquid side is compression, the section is to be designed as ordinary reinforced concrete. But if the resultant stress on the liquid side is tension, the section must have a) adequate resistance to cracking and (b) adequate strength

To satisfy condition (a), the section may be designed as plain

concrete such that :

$$\frac{M}{Z} \pm \frac{N}{A} \leq \bar{\sigma}_{tb} \quad (11)$$

For rectangular sections

$$\frac{6 M}{b t^2} \pm \frac{N}{b t} \leq \bar{\sigma}_{tb} \quad (12)$$

As a good approximation take

$$t = \sqrt{M / 3} \pm 1.5 \text{ to } 2.5 \text{ cms} \quad (13)$$

t in cms for M in kgm

Positive sign for eccentric tension and negative sign for eccentric compression. The amount of increase or decrease depends on the magnitude of N in proportion to M.

To satisfy condition (b) calculate the area of steel reinforcement required for the section as an ordinary reinforced concrete section with breadth  $b = 1 \text{ m}$  and depth  $d = t - 2.5 \text{ to } 4 \text{ cms}$ .

### Example 3      Eccentric Tension

Rectangular section subject to  $M = 6000 \text{ kgm}$  and  $N = 8000 \text{ kgf}$  (tension) with the tensile stresses on liquid side. Determine t and  $A_s$ .

Assume  $b = 1 \text{ m}$ ,  $\sigma_{tb} = 18 \text{ kg/cm}^2$  and  $\sigma_s = 1400 \text{ kg/cm}^2$ .

To satisfy condition (a) - safety against cracking

$$\begin{aligned} t &= \sqrt{M / 3} + 2 \text{ cms} \\ &= \sqrt{6000 / 3} + 2 = 47 \text{ cms} \end{aligned}$$

Check of the tensile stress in concrete

$$\sigma_{tb} = \frac{6 M}{b t^2} + \frac{N}{b t} = \frac{6 \times 6000}{47^2} + \frac{8000}{100 \times 47} = 16.3 + 1.7 = 18 \text{ kg/cm}^2$$

To satisfy condition (b) adequate strength :

$$e = \frac{M}{N} = \frac{6000}{8000} = 0.75 \text{ m}$$

$$e_B = e - \frac{t}{2} + 0.04 = 0.75 - 0.235 + 0.04 = 0.555 \text{ m} < e$$

$$M_B = N e_B = 8000 \times 0.555 = 4440 \text{ kg/m} < M$$

$$d = k_1 \sqrt{M_B} \quad \text{or} \quad 43 = k_1 \sqrt{4440} \quad \text{i.e.} \quad k_1 = 0.645$$

For  $\sigma_s = 1400 \text{ kg/cm}^2$ ,  $\alpha = 0$  and  $n = 15$  we get :

$$\sigma_c = 24 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1300 \text{ kg/cm}^2 \quad \text{so that}$$

$$A_s = \frac{M_B}{k_2 d} + \frac{N}{\sigma_s} = \frac{4440}{1300 \times 0.43} + \frac{8000}{1400} = 7.95 + 5.7 = 13.65 \text{ cm}^2$$

7  $\phi$  16 mm/m

#### Example 4 Eccentric Compression

Same section subject to  $M = 6000 \text{ kgm}$  and  $N = 8000 \text{ kg}$  ( comp )

To satisfy condition (a) safety against cracking

$$t = \sqrt{1/3} - 2.0 \text{ cms} = \sqrt{6000/3} - 2 = 45.0 - 2 = 43 \text{ cms}$$

Check of tensile stress in concrete

$$\sigma_{tb} = \frac{6 \times 6000}{(43)^2} - \frac{8000}{100 \times 43} = 19.5 - 1.86 = 17.64 \text{ kg/cm}^2$$

To satisfy condition (b) adequate strength

$$e = \frac{M}{N} = \frac{6000}{8000} = 0.75 \text{ cms}$$

$$e_B = e + \frac{t}{2} - 0.04 = 0.75 + 0.215 - 0.04 = 0.925 \text{ m} > e$$

$$M_B = N e_B = 8000 \times 0.925 = 7400 \text{ kg m} > M$$

$$d = k_1 \sqrt{M_B} \quad \text{or} \quad 39 = k_1 \sqrt{7400} \quad \text{i.e.} \quad k_1 = 0.455$$

For  $\sigma_s = 1400 \text{ kg/cm}^2$ ,  $\alpha = 0$  and  $n = 15$  we get

$$\sigma_c = 38 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1264 \text{ kg/cm}^2 \quad \text{so that}$$

$$A_s = \frac{M_B}{k_2 d} + \frac{N}{\sigma_s} = \frac{7400}{1264 \times 0.39} + \frac{8000}{1400} = 15.57 = 9.3 \text{ cm}^2$$

7  $\phi$  13 mm/m

It is however recommended to begin with the choice of the wall thickness satisfying condition (a) (Stage I) and then to determine the

steel reinforcements satisfying condition (b) (Stage II) as in previous examples

The effect of the steel reinforcements on the value of  $\sigma_t$  may be taken into consideration as can be seen from the following example

Example 5 (Fig II 2)

Assume that in example 3 given before for case of eccentric tension, the max thickness available is 44 cms only, then

$$t = 44 \text{ cms} \quad e = 0.75 \text{ ms} \quad e_s = 0.75 - 0.22 + 0.04 = 0.57 \text{ m} < e$$

$$M_s = 8000 \times 0.57 = 4560 \text{ kgm} < M$$

$$d = k_1 \sqrt{M_s} \quad \text{or} \quad 40 = k_1 \sqrt{4560} \quad \text{i.e.} \quad k_1 = 0.593$$

$$\text{For } \sigma_s = 1400 \text{ kg/cm}^2 \quad \alpha = 0 \quad \text{and} \quad n = 15 \quad \text{we get}$$

$$\sigma_c = 27.5 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1294 \text{ kg/cm}^2 \quad \text{so that}$$

$$A_s = \frac{4560}{1294 \times 0.4} + \frac{8000}{1400} = 8.8 + 5.7 = 14.5 \text{ cm}^2 \quad 8 \phi 16 \text{ mm/m}$$

i.e. actual  $A_s = 16 \text{ cm}^2$

Check of max tensile stress in concrete  $\sigma_{tb}$

$$A_v = A_c + n A_s$$

$$= 100 \times 44 + 10 \times 16 = 4560 \text{ cm}^2$$

$$y_o = \frac{100 \times 44^2}{2} + 10 \times 16 \times 4 / 4560$$

$$= 21.37 \text{ cms}$$

$$I_v = \frac{100 \times 44^3}{12} + 4400 \times 0.63^2 + 10 \times 16 \times 17.37^2 = 743021 \text{ cm}^4$$

$$Z_v = \frac{I_v}{y_o} = \frac{743021}{21.37} = 34774 \text{ cm}^3$$

$$\sigma_{tb} = \frac{N}{A_v} + \frac{M}{Z_v} = \frac{8000}{4560} + \frac{6000 \times 1000}{34774} = 1.76 + 17.24$$

$$= 19 \text{ kg/cm}^2$$

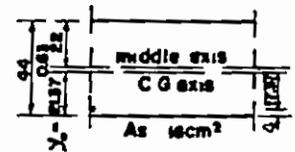


Fig II-2

### III DESIGN OF CIRCULAR TANKS

#### III 1 FUNDAMENTAL TYPES OF JOINTS OF WALLS

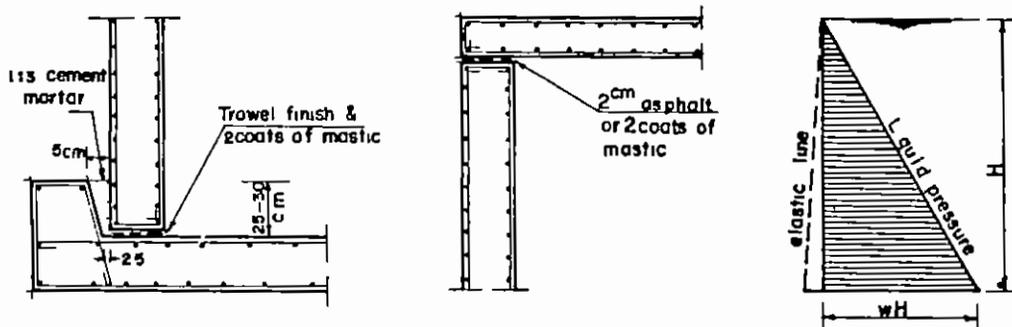
##### III 1 1 Introduction

In this section, our study will be limited to circular liquid containers, other forms will be dealt with later

The structural behavior of circular cylindrical tank walls, subject to the action of hydrostatic pressure varies according to the type of joint between wall and other elements (base and roof if any) There are three main types of joints free (or sliding), fixed and hinged Any case in practice can be analysed by combination of these cases

##### III 1 2 Free Joint (Sliding Joint) ( Fig III 1 )

No restraint for motion of wall due to liquid pressure



Sliding base

Free top

Liquid pressure & elastic line

Fig III-1

For this type of joint the elastic line of the wall is a straight line and the wall resists the liquid pressure by ring action ( i e by horizontal strips only ) With respect to base no indeterminate stresses are created

to ensure water tightness in the joint a copper plate may be placed to join the wall and floor neoprene plates may also be used

### III 1 3 Fixed Joint (continuous Joint)

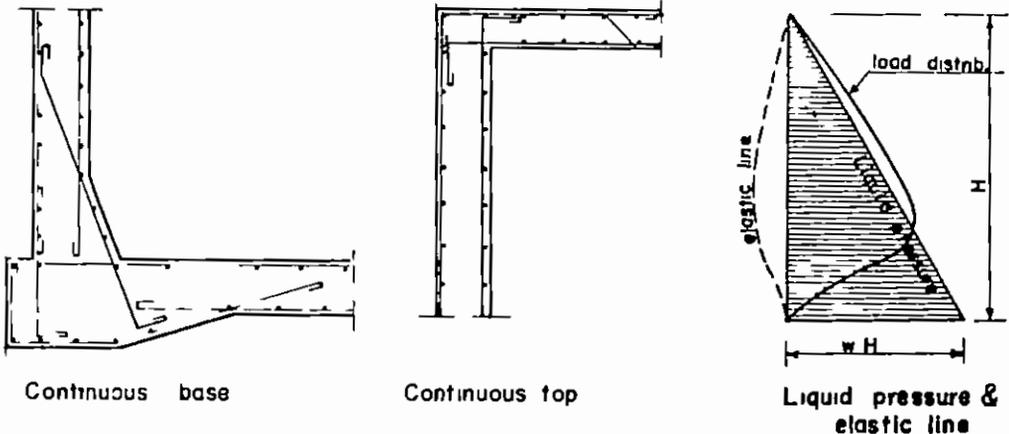


FIG III-2

In this case no allowance for motion or rotation is allowed for wall base (or top) wall will carry liquid pressure partly by riling and partly by cantilever action (combined resistance by both vertical and horizontal strips) There is a connecting moment between wall and base To obtain the required fixation vertical reinforcement extends across the joint as shown Good bond qualities are obtained by the following procedure

After the concrete is placed and has stiffened sufficiently but is not thoroughly hardened - about 6 hours - clean the joint surface with a pressure water jet Then cover the joint and keep it continuously wet Just before new concrete is placed, flush the old surface with 1 2 portland cement mortar Vibrate the new concrete and keep it moist for several days

### III 1 4 Hinged Joint ( Fig III 3 )

It may sometimes be desirable to avoid transmitting moment between wall and base , in such cases we use a hinged joint which allows for rotation as shown in (a) and (b)

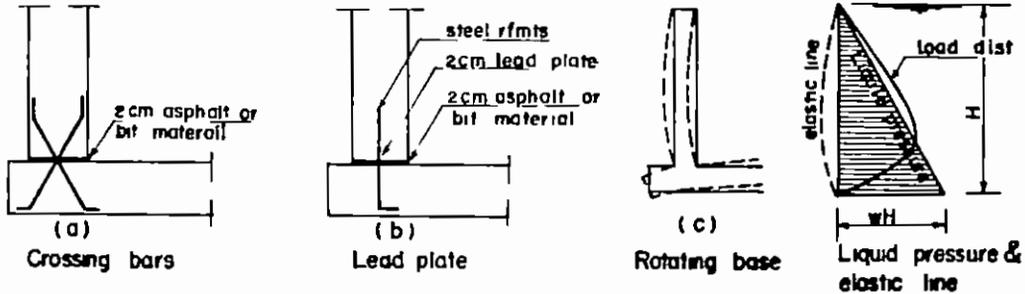


Fig III-3

A wall rigidly connected to the base may however be considered hinged if the soil underneath is liable to rotate as shown in case c

III 2 DESIGN OF WALLS WITH SLIDING BASE ( Fig III 4 )

Liquid pressure  $p_x$  at any depth  $x$  is given by

$$p_x = w x \quad \text{and} \quad p_{max} = w H \quad \text{in which}$$

$$w = \text{weight of liquid} / m^3$$

For a wall with sliding base the full water pressure will be resisted horizontally by ring action, thus

$$2 T_x = p_x D = 2 p_x R \quad \text{or} \quad T_x = p_x R \tag{14}$$

If the thickness of the wall  $t$  is small in proportion to the radius  $R$ , the ring tension  $T_x$  may be assumed as uniformly distributed over the cross-section which can be designed as shown in II 2, i e

$$T_x = p_x R = w x R$$

$$A_s = T / \sigma_s$$

$$\sigma_t = \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s}$$

For  $\sigma_s = 1400 \text{ kg/cm}^2$ ,

$\sigma_t = 16 \text{ kg/cm}^2$ ,

$\epsilon_{sh} = 0.00025$ ,

$E_s = 2100 \text{ t/cm}^2$  and

$n = 10$ ,

we get:

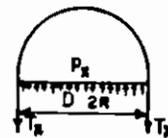
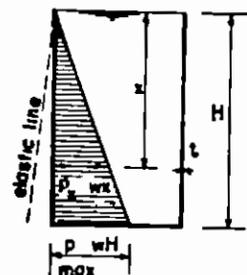


Fig III-4

$$t \text{ cms} = 0.8 T \text{ tons/m}$$

$$A_s \text{ cm}^2 = 0.9 t \text{ cms}$$

In deep circular tanks, with the wall rigidly connected to the floor, the liquid pressure will be mainly resisted by ring action. Due to the fixation of the wall to the floor, the horizontal displacement of the wall at its foot cannot be fully developed and the pressure resisted by ring action will decrease to zero at the point of fixation. The small part of the liquid pressure at the foot of the wall that has not been resisted in the horizontal direction by ring action will be resisted in the vertical direction by cantilever action creating bending moments in the wall. In this case, the max ring tension  $T_{\max}$  takes place at 0.8 to 0.9 H and is given by

$$T_{\max} = 0.80 \text{ to } 0.90 w H R$$

The fixing moment at the base of the wall may be estimated from the relation

$$M_f = \frac{w H D t_{\max}}{7.5 \text{ to } 8}$$

which will be proved later

In both equations, bigger values are to be chosen for deeper tanks. The mentioned bending moments affect only a small part generally not exceeding  $H/10$  at the foot of the wall.

Fig III-5 shows the ring tension and cantilever moments of a deep circular tank subject to liquid pressure.

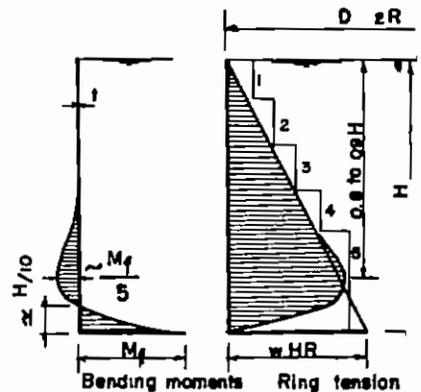


Fig III-5

Example :

Deep water tank  $H = 12 \text{ ms}$   
and  $D = 8 \text{ ms}$  Wall fixed to floor Max  
ring tension at  $0.9 H$  is given by

$$T_{\max} = 0.9 \text{ w H R} = 0.9 \times 12 \times 4 = 43.2 \text{ tons}$$

$$\text{Required thickness } t = 0.8 T = 0.8 \times 43.2 = 35 \text{ cms}$$

As the wall is mainly subject to ring tension which increases with the depth, the section of the wall will be chosen trapezoidal with a minimum thickness at the top of 20 cms and a maximum thickness at the bottom  $t_{\max}$  given by

$$t_{\max} = 0.8 \text{ w H R} = 0.8 \times 12 \times 4 = 38 \text{ cms} \quad \text{chosen } 40 \text{ cms}$$

Thickness available at position of max ring tension (0.9 H) is given by  $20 + 0.9 \times 20 = 38 \text{ cms} > 35 \text{ cms}$

Ring reinforcements at different depths is given in the following table

Strip No	1 0-2m	2 2-4m	3 4-6m	4 6-8m	5 8-12m from top
$P_{\max}$	2	4	6	8	10 $t / m^2$
$T = P_{\max} R$	8	16	24	32	40 $t / m$
Total $A_s = T/1.4$	5.7	11.4	17.1	22.8	28.5 $cm^2$
Steel on each side = $A_s/2$	2.85	5.7	8.55	11.4	14.3 $cm^2$
Reinforcement	6 $\phi$ 8	7 $\phi$ 10	7 $\phi$ 13	3.5 $\phi$ 16 + 3.5 $\phi$ 13	7 $\phi$ 16 $mm/m$

The fixing moment at the base is given by 
$$M_f = \frac{1 \times 12 \times 8 \times 0.4}{7.5} = 5.1 \text{ m t}$$

Max thickness of wall required to resist this moment can be determined from equation (9) as follows :

$$t_{\max} = \sqrt{M_f / 3} = \sqrt{5100 / 3} = 41 \text{ cm}$$

i e the maximum chosen thickness of 40 cms is convenient  
 Vertical tension steel on the inside surface of the wall at its base  
 can be determined from equation (10), thus

$$A_s = 0.26 t = 0.26 \times 41 = 10.7 \text{ cm}^2 \quad \text{chosen } 6 \phi 16 (12 \text{ cm}^2)$$

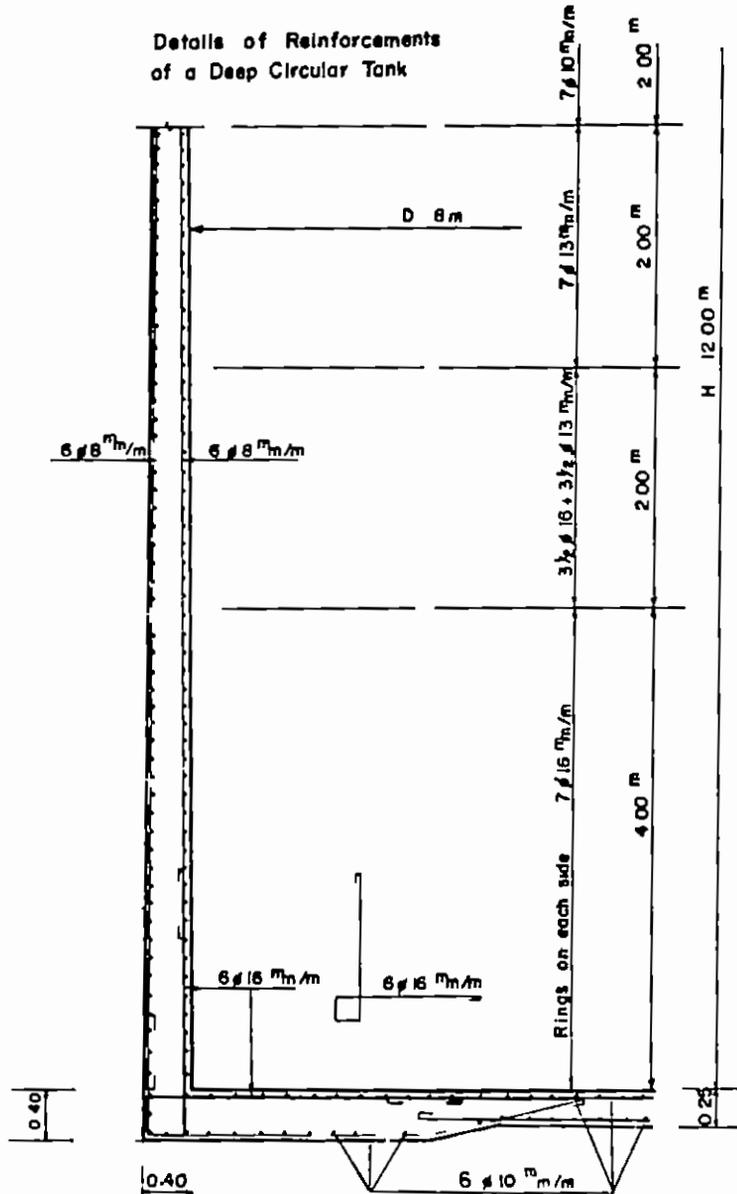


Fig III-6

Other vertical reinforcements necessary to bind the rings and to resist shrinkage stresses and eventual field bending moments may be chosen  $> 20\%$  of max ring reinforcements and not less than  $5\phi 8$  mm/m

$$\text{In our case } M_{\max}^+ \approx M_p/5 = 5100/5 = 1020 \text{ kg m}$$

wall thickness at position of max field moment  $\approx 33$  cms

$$A_s = \frac{1020}{1300 \times 30} = 2.6 \text{ cm}^2$$

20 % of max rings (  $7 \phi 16$  mm/m )  $= 14 \times 0.2 = 2.8 \text{ cm}^2$

Choose  $6 \phi 8$  mm/m on outside and inside surfaces of wall The details of reinforcements in the lower part of the wall are shown in figure III 6

### III 3 THEORY OF THICK CYLINDERS

If the thickness of the wall  $t$  of a circular tank is not small in proportion to the radius  $R$ , the stress  $\sigma_t$  due to  $T$  will not be uniformly distributed over the cross-section Lamé has solved this problem in 1852 as follows Fig III 7

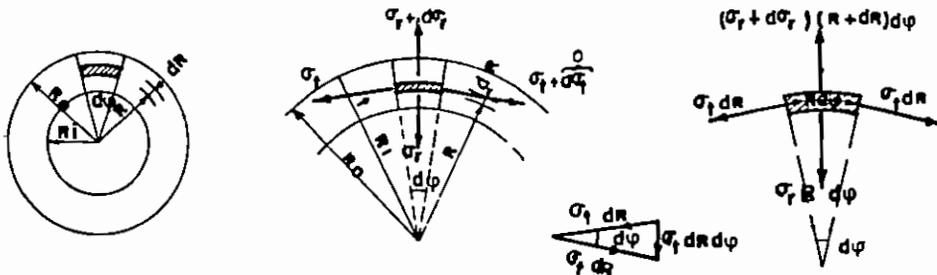


Fig III-7

The stress distribution  $\sigma_t$  on the cross-section of a thick wall of a circular tank subject to hydrostatic pressure  $p$  can be determined if we consider the equilibrium of an element of a tank enclosing an angle  $d\phi$  as shown Assume

$R$  = radius of any fiber inside the wall

$R_1$  = radius of inner face of wall

$R_0$  = radius of outer face of wall

$\sigma_r$  &  $\epsilon_r$  = radial stress and strain

$\sigma_t$  &  $\epsilon_t$  = tangential stress and strain

$\nu = \frac{1}{m} = \text{Poisson's ratio} = \frac{1}{2} + \frac{1}{6}$  for concrete

Due to symmetry in shape and loading we have  
 in tangential direction :  $d\sigma_t = 0$  and  
 in radial direction the shearing force  $Q = 0$

Due to equilibrium in radial direction, we get the following relation

$$(\sigma_r + d\sigma_r) (R + dR) d\phi - \sigma_r R d\phi - \sigma_t dR d\phi = 0$$

$$\sigma_r R d\phi + d\sigma_r R d\phi + \sigma_r dR d\phi + d\sigma_r dR d\phi - \sigma_t R d\phi - \sigma_t dR d\phi = 0$$

Reducing by  $d\phi$  and neglecting  $d\sigma_r dR$  being a small value of the second degree we get

$$d\sigma_r R + \sigma_r dR = \sigma_t dR \quad \text{1 e}$$

$$\sigma_t = \frac{d(\sigma_r R)}{dR} \quad (15)$$

According to theory of elasticity, one can express the relation between stresses and strains in the following manner

$$\left. \begin{aligned} \epsilon_r &= \frac{1}{E} (\sigma_r - \nu \sigma_t) \\ \epsilon_t &= \frac{1}{E} (\sigma_t - \nu \sigma_r) \end{aligned} \right\} \quad (16)$$

These two equations give

$$\left. \begin{aligned} \sigma_r &= \frac{E}{1 - \nu^2} (\epsilon_r + \nu \epsilon_t) \\ \sigma_t &= \frac{E}{1 - \nu^2} (\epsilon_t + \nu \epsilon_r) \end{aligned} \right\} \quad (17)$$

Due to the deformation of the wall of the tank caused by the hydrostatic pressure, the radius  $R$  will be increased by the radial displacement

y i e it will be  $(R + y)$  Therefore the strain in the radial direction  $\epsilon_r$  is given by

$$\epsilon_r = \frac{dy}{dR} \quad (18)$$

We have further

Circumference of tank before deformation	=	$2 \pi R$
Circumference of tank after deformation	=	$2 \pi (R + y)$
Increase in length of circumference	=	$2 \pi y$
Strain in tangential direction		

$$\begin{aligned} \epsilon_t &= \frac{2 \pi y}{2 \pi R} & \text{or} \\ \epsilon_t &= y / R \end{aligned} \quad (19)$$

Substituting the values of  $\epsilon_r$  and  $\epsilon_t$  as given by equations 18 & 19 in equations 17, we get :

$$\begin{aligned} \sigma_r &= \frac{E}{1 - \nu^2} \left( -\frac{dy}{dR} + \nu \frac{y}{R} \right) \\ \sigma_t &= \frac{E}{1 - \nu^2} \left( \frac{y}{R} + \nu \frac{dy}{dR} \right) \end{aligned} \quad (20)$$

Introducing these values in equation 15, we get

$$\begin{aligned} \frac{y}{R} + \nu \frac{dy}{dR} &= \frac{d}{dR} \left( R \frac{dy}{dR} + \nu y \right) & \text{or} \\ \frac{y}{R} + \nu \frac{dy}{dR} &= \frac{dy}{dR} + R \frac{d^2 y}{dR^2} + \nu \frac{dy}{dR} & \text{i e} \end{aligned}$$

$$R^2 \frac{d^2 y}{dR^2} + R \frac{dy}{dR} - y = 0 \quad (21)$$

The solution of this differential equation is

$$y = C_1 R + \frac{C_2}{R} \quad (22)$$

in which

$C_1$  and  $C_2$  are the integration constants Differentiating equation 22 with respect to R we get

$$\frac{dy}{dR} = C_1 - \frac{C_2}{R^2} \quad \text{and} \quad \frac{d^2y}{dR^2} = \frac{2C_2}{R^3}$$

Substituting these values in the elements of equation 21, we get

$$R^2 \frac{2C_2}{R^3} + R \left( C_1 - \frac{C_2}{R^2} \right) - \left( C_1 R + \frac{C_2}{R} \right) = \frac{2C_2}{R} + R C_1 - \frac{C_2}{R} - C_1 R - \frac{C_2}{R} = 0$$

which means that the solution is correct

Determination of  $C_1$  &  $C_2$  Fig III 8

1) For  $R = R_1$   $\sigma_r = -p$

ii) For  $R = R_0$   $\sigma_r = 0$  (no outside pressure)

therefore

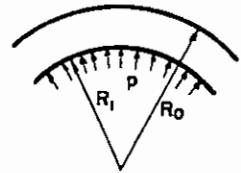


Fig III-8

$$\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2} \left( \frac{dy}{dR} + \nu \frac{y}{R} \right) = \frac{E}{1-\nu^2} \left( C_1 - \frac{C_2}{R^2} + \nu C_1 + \nu \frac{C_2}{R^2} \right) \\ &= \frac{E}{1-\nu^2} \left[ C_1 (1+\nu) - \frac{C_2}{R^2} (1-\nu) \right] \end{aligned}$$

Because of condition ii, we have

$$0 = C_1 (1+\nu) - \frac{C_2}{R_0^2} (1-\nu) \quad \text{and}$$

Because of condition i we have

$$-p = \frac{E}{1-\nu^2} \left[ C_1 (1+\nu) - \frac{C_2}{R_1^2} (1-\nu) \right]$$

These two equations give

$$C_1 (1+\nu) \left( 1 - \frac{R_1^2}{R_0^2} \right) = p \frac{1-\nu^2}{E} \frac{R_1^2}{R_0^2} \quad \text{or}$$

$$C_1 = p \frac{1-\nu}{E} \frac{R_1^2}{R_0^2 - R_1^2} \quad (23)$$

$$\text{and} \quad C_2 = p \frac{1+\nu}{E} \frac{R_0^2 R_1^2}{R_0^2 - R_1^2} \quad (24)$$

Therefore

$$\begin{aligned}\sigma_t &= \frac{E}{1-\nu^2} \left( \frac{Y}{R} + \nu \frac{dY}{dR} \right) = \frac{E}{1-\nu^2} \left[ C_1 + \frac{C_2}{R^2} + \nu \left( C_1 - \frac{C_2}{R^2} \right) \right] \\ &= \frac{E}{1-\nu^2} \left[ C_1 (1+\nu) + \frac{C_2}{R^2} (1-\nu) \right] \\ &= \frac{E}{1-\nu^2} \left[ p \frac{1-\nu}{E} \frac{R_1^2}{R_0^2 - R_1^2} (1+\nu) + \frac{p}{R^2} \frac{1+\nu}{E} \frac{R_0^2 - R_1^2}{R_0^2 - R_1^2} (1-\nu) \right]\end{aligned}$$

or

$$\sigma_t = p \frac{R_1^2}{R_0^2 - R_1^2} \left( 1 + \frac{R_0^2}{R^2} \right) \quad (25) \text{ Hyperbola}$$

which gives the formula of Lamé for stress distribution on the cross-section of thick cylinders subject to internal pressure  $p$ . Therefore the stress at the inner surface of the tank

$$R = R_1 \text{ is } \sigma_{t1} = p \frac{R_0^2 + R_1^2}{R_0^2 - R_1^2} \quad (25a)$$

and at the outer surface :  $R = R_0$

$$\sigma_{t0} = p \frac{2 R_1^2}{R_0^2 - R_1^2} \quad (25b)$$

Assuming  $R_0 = \lambda R_1$  we get

$$\sigma_{t1} = p \frac{\lambda^2 + 1}{\lambda^2 - 1}, \quad \sigma_{t0} = p \frac{2}{\lambda^2 - 1} \quad \text{and} \quad \frac{\sigma_{t1}}{\sigma_{t0}} = \frac{\lambda^2 + 1}{2}$$

$$\text{For } \lambda = 1 \quad \frac{\sigma_{t1}}{\sigma_{t0}} = 1$$

$$\lambda = 1.2 \quad " = 1.22 \text{ difference not big}$$

$$\lambda = 2 \quad " = 2.5$$

The stresses are graphically represented in figure III-9

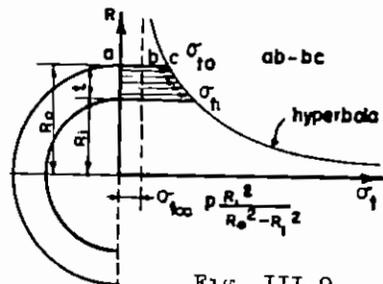


Fig III-9

For  $R = \infty$   $\sigma_{t\infty} = p \frac{R_1^2}{R_0^2 - R_1^2} = 1/2 \sigma_{t0}$

This means that the stresses are bigger at the inside surface of the tank. In reinforced concrete circular walls with one mesh, the ring reinforcement is to be placed nearer to the inner surface of the wall and in walls with double meshes the inner rings may be bigger than the outer ones.

In reinforced concrete circular tanks  $R_0 < 1.2 R_1$  ( i.e.  $\lambda < 1.2$  ) which means that the error by use of cylinder formula (  $\sigma_t = T/t$  ) is less than 10 %

For very thin cylinders  $R_1 \approx R_0 \approx R$ , the formula of Lamé gives the same result as the cylinder formula

$$\sigma_t = p \frac{R_1^2}{\underbrace{(R_0 + R_1)}_{2 R_1} \underbrace{(R_0 - R_1)}_t} \left( 1 + \frac{R_0^2}{R_1^2} \right) = p \frac{R_1^2}{2 R_1 t} \times 2 = p \frac{R_1}{t} = \frac{T}{t}$$

Whereas in relatively thick plain concrete circular walls, the average tensile stresses calculated according to the cylinder formula may give serious not allowed errors

III 4 TANK WALLS TIED TO FLOOR

a

III 4 1 Theory of Reissner and Lewy Fig III 10

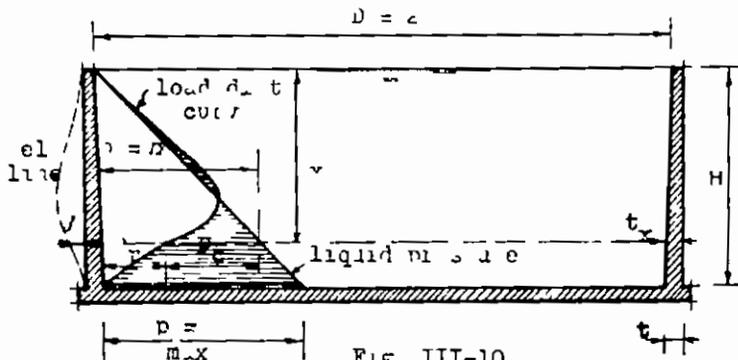


Fig III-10

The liquid pressure  $p_x$  at any depth  $x$  will be counteracted by the combined resistance of rings and cantilevers, thus

$$P_x = w x = P_r + P_c \quad (26) \quad \text{in which}$$

$P_r$  = the part of the liquid pressure resisted in the horizontal direction by ring action

$P_c$  = the part of the liquid pressure resisted in the vertical direction by cantilever action

The value of  $P_r$  can be determined as follows :

$$\text{Strain in tangential direction} \quad \epsilon_t = \frac{y}{R} \quad (\text{refer to equation 19})$$

$$\text{Stress in tangential direction} \quad \sigma_t = \epsilon_t E = \frac{y}{R} E \quad (27)$$

$$\text{Ring tension} \quad T = \sigma_t t_x = \frac{y}{R} t_x E \quad (28)$$

$$\text{Pressure resisted by ring action} \quad P_r = \frac{T}{R} = \frac{y}{R^2} t_x E \quad (29)$$

This equation means that if we know the equation of the elastic line of the wall, we can find  $P_r$  and respectively  $P_c$ . The equation of the elastic line can be determined as follows :

It is known from the general theory of elasticity that the relation between the external force  $q$ , the shearing force  $Q$ , the bending moment  $M$  and the deflection  $y$  are

$$\frac{d^2 y}{dx^2} = -M / EI, \quad \frac{dM}{dx} = Q, \quad \frac{dQ}{dx} = -q \quad \& \quad \frac{d^4 y}{dx^4} = q / EI \quad (30)$$

in which  $q$  = force acting on wall in radial direction  
( Fig III 11)

$$q = w x R d\varphi - \sigma_t t_x d\varphi \quad \text{but} \quad \sigma_t = \frac{y}{R} E \quad \text{then}$$

$$q = w x R d\varphi - \frac{y}{R} E t_x d\varphi = (w x R - \frac{y}{R} t_x E) d\varphi \quad \text{but}$$

$$I = \frac{R d\varphi t_x^3}{12}$$

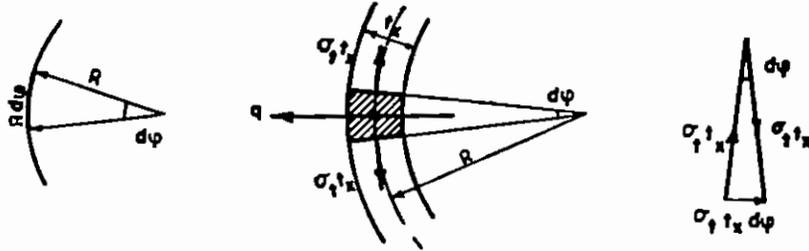


Fig III-11

Substituting these values in the differential equation of the elastic line - 30 - we get

$$E \frac{R d\phi t_x^3}{12} \frac{d^4 y}{dx^4} = (w x R - \frac{\Sigma t_x E}{R}) d\phi \quad \text{or}$$

$$\frac{R^2 t_x^2}{12} \frac{d^4 y}{dx^4} = \frac{w R^2}{E t_x} x - y$$

Assuming a tank with constant wall thickness  $t$  and putting

$$K = \frac{12 H^4}{R^2 t^2} \quad n = 4 \sqrt{\frac{K}{4}} \quad \text{and} \quad \xi = \frac{x}{H} \quad (31) \quad \text{we get}$$

$$\frac{H^4}{4 n^4} \frac{d^4 y}{dx^4} + y - \frac{w R^2}{E t} x = 0 \quad (32)$$

This equation gives the final form of the differential equation of the elastic line. Its solution gives the value of  $y$  and respectively  $p_r$  and  $p_c$  or the ring tension  $T$  and the cantilever moment  $M$ .

The solution of the differential equation 32 is given by

$$y = \frac{w R^2}{E t} x + \left. \begin{aligned} & (A_1 e^{n\xi} + A_2 e^{-n\xi}) \cos n\xi + \\ & (B_1 e^{n\xi} + B_2 e^{-n\xi}) \sin n\xi \end{aligned} \right\} (33)$$

in which  $A_1$ ,  $A_2$ ,  $B_1$  &  $B_2$  are 4 integration constants to be determined from the conditions at the supports. The derivatives of  $y$  are given by

$$\begin{aligned} \frac{H}{n} \frac{dy}{dx} &= (A_1 e^{n\xi} - A_2 e^{-n\xi}) \cos n\xi - (A_1 e^{n\xi} + A_2 e^{-n\xi}) \sin n\xi \\ &+ (B_1 e^{n\xi} - B_2 e^{-n\xi}) \sin n\xi + (B_1 e^{n\xi} + B_2 e^{-n\xi}) \cos n\xi \\ &+ \frac{w R^2 H}{E t n} \end{aligned}$$

$$\frac{H^2}{2 n^2} \frac{d^2 y}{dx^2} = - (A_1 e^{n\xi} - A_2 e^{-n\xi}) \sin n\xi + (B_1 e^{n\xi} - B_2 e^{-n\xi}) \cos n\xi$$

$$\begin{aligned} \frac{H^3}{2 n^3} \frac{d^3 y}{dx^3} &= - (A_1 e^{n\xi} + A_2 e^{-n\xi}) \sin n\xi - (A_1 e^{n\xi} - A_2 e^{-n\xi}) \cos n\xi \\ &+ (B_1 e^{n\xi} + B_2 e^{-n\xi}) \cos n\xi - (B_1 e^{n\xi} - B_2 e^{-n\xi}) \sin n\xi \end{aligned}$$

$$\begin{aligned} \frac{H^4}{4 n^4} \frac{d^4 y}{dx^4} &= - (A_1 e^{n\xi} + A_2 e^{-n\xi}) \cos n\xi - (B_1 e^{n\xi} + B_2 e^{-n\xi}) \sin n\xi \\ &= - \left( y - w \frac{R^2}{E t} x \right) \end{aligned}$$

which means that the solution is correct

The integration constants for an open tank fixed at the base can be determined from the following conditions

$$\text{a) at base } x = H \quad y = 0 \quad \frac{dy}{dx} = 0$$

$$\text{b) at top } x = 0 \quad M = Q = 0, \quad \frac{d^2 y}{dx^2} = \frac{d^3 y}{dx^3} = 0$$

$$\text{From condition b) } M \propto \frac{d^2 y}{dx^2} = 0 \quad \text{or} \quad B_1 - B_2 = 0$$

and

$$Q \propto \frac{d^3 y}{dx^3} = 0 \quad \text{or} \quad (A_1 - A_2) + (B_1 + B_2) = 0$$

From these relations we get  $B_1 = B_2$  and  $A_1 = A_2 - 2 B_1$ , thus

$$y = \left[ A_1 e^{n\xi} + (A_1 - 2 B_1) e^{-n\xi} \right] \cos n\xi + B_1 (e^{n\xi} + e^{-n\xi}) \sin n\xi + \frac{w R^2}{E t} x, \quad \text{or}$$

$$y = A_1 (e^{n\xi} + e^{-n\xi}) \cos n\xi + B_1 \left[ (e^{n\xi} + e^{-n\xi}) \sin n\xi - 2 e^{-n\xi} \cos n\xi \right] + \frac{w R^2}{E t} x$$

and

$$\begin{aligned} \frac{H}{n} \frac{dy}{dx} = & A_1 \left[ (e^{n\xi} - e^{-n\xi}) \cos n\xi - (e^{n\xi} + e^{-n\xi}) \sin n\xi \right] \\ & + B_1 \left[ (e^{n\xi} - e^{-n\xi}) \sin n\xi + (e^{n\xi} + e^{-n\xi}) \cos n\xi \right. \\ & \left. + 2 e^{-n\xi} \cos n\xi + e^{-n\xi} \sin n\xi \right] + \frac{w R^2 H}{E t n} \end{aligned}$$

at the base  $x = H$ ,  $\xi = 1$ ,  $y = 0$ ,  $\frac{dy}{dx} = 0$  and  $e^{-n} = e^{-n\xi} = \text{small value}$

$$\text{Assuming } \frac{1}{R} = 1 \quad \frac{1}{t} > 10 \quad n = \sqrt[4]{3 \times 1 \times 100} = 4.2$$

$$e^{-4.2} = 0.018, \text{ neglected!}$$

$$y = 0 = A_1 e^n \cos n + B_1 e^n \sin n + \frac{w R^2 H}{E t} \quad \text{and}$$

$$\frac{H}{n} \frac{dy}{dx} = 0 = A_1 e^n (\cos n - \sin n) + B_1 e^n (\sin n + \cos n) + \frac{w R^2 H}{E t n}$$

The solution of these two equations gives

$$\left. \begin{aligned} A_1 = - \frac{w R^2 H}{E t e^n} \left( \cos n + \sin n - \frac{\sin n}{n} \right) \\ B_1 = + \frac{w R^2 H}{E t e^n} \left( \cos n - \sin n - \frac{\cos n}{n} \right) \end{aligned} \right\} \quad (34)$$

Substituting these constants in the equation of  $y$ , we can find the ring tension and respectively the load distribution

The second derivative of  $y$  gives the bending moment because

$$\frac{d^2 y}{dx^2} = - \frac{M}{EI} \quad \text{The moment at the foot can be determined from the relation}$$

$$\frac{H^2}{2n^2} \frac{d^2 y}{dx^2} = - A_1 e^{n\xi} \sin n\xi + B_1 e^{n\xi} \cos n\xi$$

For  $x = H$ , we get

$$\begin{aligned} f &= \frac{2n^2}{H^2} EI e^{n\xi} (A_1 \sin n - B_1 \cos n) \\ &= \frac{2n^2}{H^2} EI \left[ - \frac{wR^2 H}{Et} (\cos n + \sin n - \frac{\sin n}{n}) \sin n \right] \\ &\quad - \left[ \frac{wR^2 H}{Et} (\cos n - \sin n - \frac{\cos n}{n}) \cos n \right] \quad \text{or} \end{aligned}$$

$$\begin{aligned} I_f &= -2n^2 I \frac{wR^2}{tH} (\cos n \sin n + \sin^2 n - \frac{\sin^2 n}{n} \\ &\quad + \cos^2 n - \sin n \cos n - \frac{\cos^2 n}{n}) \quad \text{or} \end{aligned}$$

$$-I_f = -2n^2 I \frac{wR^2}{tH} \left(1 - \frac{1}{n}\right) \quad \text{but } I = \frac{t^3}{12} \quad \text{then}$$

$$-I_f = -2n^2 \frac{t^3}{12} \frac{wR^2}{tH} \frac{n-1}{n} \quad \text{or}$$

$$-I_f = - \frac{wR^2 t^2}{12H} 2n(n-1) \quad (35)$$

Calculating the max shearing force at the base from the relation

$$Q = -EI \frac{d^3 y}{dx^3}, \quad \text{we get}$$

$$Q_{\max} = \frac{wR^2 t^2}{12H^2} 2n^2 (2n-1) \quad (36)$$

In the same way other cases can be calculated according to their edge conditions e g

i) the edge conditions for an open tank hinged at the base are

$$\text{at base } x = H \quad y = 0 \quad \text{and} \quad M = 0 \quad \text{or} \quad \frac{d^2 y}{dx^2} = 0$$

$$\text{at top } x = 0 \quad M = 0 \quad \& \quad Q = 0 \quad \text{or} \quad \frac{d^2 y}{dx^2} = 0 \quad \text{and} \quad \frac{d^3 y}{dx^3} = 0$$

ii) the edge conditions for a wall fixed at top & bottom are

$$\text{at base } x = H \quad y = 0 \quad \text{and} \quad \frac{dy}{dx} = 0$$

$$\text{at top } x = 0 \quad y = 0 \quad \text{and} \quad \frac{dy}{dx} = 0$$

The shape of the elastic line, load distribution curve and the bending moment diagram for the case of an open tank fixed at the base are as shown in Fig III 12

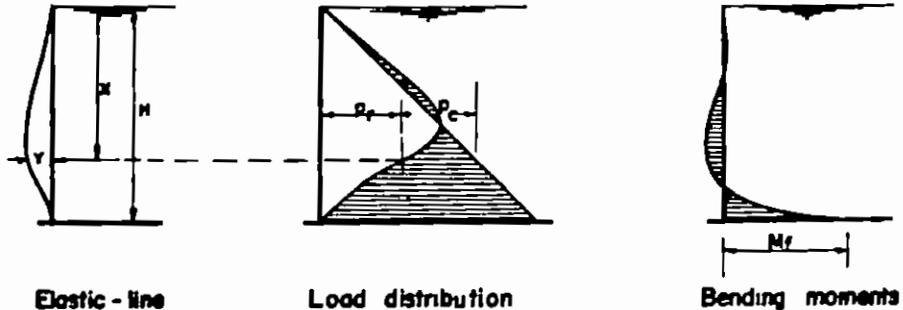


Fig III-12

If the wall were fixed at its upper edge the main ordinates of the diagrams will be practically not affected, and a small fixing moment - shown dotted - will be created at the top

When examining the load distribution curve, we notice that the pressure at the bottom of the wall is resisted by the cantilever action and as the deflection at the base by full fixation is equal to zero, therefore the load resisted horizontally by ring action must be equal to zero. In the upper portion of the wall  $p_c$  assumes negative values. If no such load of opposite sign were acting near the top of the wall, the cantilevers should there receive their greatest deflec-

tion, but obviously this cannot be the case owing to the restraint afforded by the rings, and in consequence of the load being zero at the water level the deflection for the upper most ring must be nil

### Dimensioning

The maximum bending moment at the foot of the wall causes tensile stresses on the water side. In order not to have cracks causing rusting of the steel the working tensile bending stress in concrete must not exceed its tensile strength i.e. for a rectangular section with breadth  $b = 1m$   $\sigma_{tb} = 6 M_f / t^2$

Substituting for  $M_f$  the value given in equation 35, we get

$$\sigma_{tb} = w \frac{R^2}{H} \sqrt{n(n-1)} \quad \text{or}$$

$$n = 0.5 + \sqrt{\frac{\sigma_{tb} H}{R^2 w} + 0.25} \quad (37)$$

Knowing that  $n = \sqrt[4]{\frac{K}{4}}$  where  $K = \frac{12 H^4}{R^2 t^2}$  then

$$t^2 = \frac{12 H^4}{R^2 K} = 12 \frac{H^4}{R^2 4 n^4} = \frac{3 H^4}{R^2 n^4} \quad \text{or}$$

$$t = 1.73 \frac{H^2}{R n^2} \quad (38)$$

Reissner & Lewy found that all tanks with equivalent values of  $K$  (or  $n$ ) have similar load distribution curves. Accordingly they gave a series of curves for load distribution and respectively the ring tension as well as the cantilever moments in walls of open circular tanks fixed to floor and having rectangular or triangular section as a factor of  $K$  or  $n$  in the following manner (Figure III 13)

Ring tension  $T = C_0 w H R = C_0 p R$  where  $C_0 = \gamma \frac{K}{\lambda} \frac{t}{x}$

Bending moment  $M = \beta w R^2 t^2 / 12 H$  where  $\beta = \frac{d^2 y}{d t^2} \frac{K}{\lambda} \frac{t^3}{x^3}$

in which  $\lambda = \frac{12 H^5}{E t^3} w$  and  $\xi = \frac{x}{H}$

In order to use these curves proceed as follows

- 1) Determine the value of  $n$  from equation 37, namely

$$n = 0.5 + \sqrt{\frac{\sigma_{tb} H}{R^2 w} + 0.25} \quad \text{in which}$$

$\sigma_{tb} = 18 \text{ kg/cm}^2$  for normal cases and  $w = 0.001 \text{ kg/cm}^3$  for water

- 2) Calculate the thickness at the foot of the wall from equation 38 which gives

$$t = 1.73 \frac{H^2}{R n^2}$$

then choose a convenient profile for the wall

- 3) To determine the ring tension in the wall draw for the calculated value of ( $n$ ) a vertical line to meet the projection line for interpolation in a point - a -, through - a - draw a horizontal line to meet the maxima curve in - b - This last point - b - gives the apex of the distribution curve if the wall were rectangular For trapezoidal walls, linear interpolation is sufficient Having drawn the load distribution curve, the ring tension in every section can be determined from the relation :

$$T = C_o p R$$

- 4) To draw the bending moment diagram, it is recommended to determine the maximum fixing moment  $M_f$  from equation 35, namely

$$M_f = - \frac{w R^2 t^2}{12 H} 2 n (n - 1)$$

In the same way as given under (3) determine the apex of the positive part of the bending moment diagram for a rectangular wall If the wall were trapezoidal multiply the maximum positive value by  $(t_x / t)^3$  where  $t_x$  is the thickness of the wall at position of

maximum positive moment. The curves show that the cantilever action increases for diminishing values of  $K$  ( or  $n$  ) i.e. the smaller the height, the bigger the radius and the stiffer the wall. By  $R = \infty$ ,  $K = 0$  ( the limiting case of a straight cantilever ) the water pressure is resisted totally by cantilever action.

### Example

It is required to design the wall of a cylindrical reinforced concrete open water tank, 5.0 ms deep and 20 ms diameter. Assume the wall to be fixed to the floor.

### Solution

$$H = 5.00 \text{ ms} \quad R = 10.00 \text{ ms} \quad \text{Assume } \sigma_{tb} = 18 \text{ kg/cm}^2 \\ \& \quad w = 0.001 \text{ kg/cm}^2$$

$$n = 0.5 + \sqrt{\frac{\sigma_{tb} H}{R^2 w} + 0.25} = 0.5 + \sqrt{\frac{18 \times 5.00}{1000^2 \times 0.001} + 0.25} = 3.53$$

$$t = 1.73 \frac{H^2}{R n^2} = 1.73 \frac{5.00^2}{1000 \times 3.53^2} \quad \text{or} \quad t = 35 \text{ cms}$$

Choose thickness at top 20 cms and at bottom 35 cms

From the curves of Reissner ( Fig III 13 ), we find

for  $n = 3.53$ ,  $\xi = 0.5$  and  $C_0 = 0.425$  at  $T_{\max}$ , thus

$$T_{\max} = C_0 w H R = 0.425 \times 1 \times 5 \times 10 = 21.25 \text{ tons/m}$$

acting at 2.5 ms from top surface

The max ring reinforcement is given by :

$$\max A_B = \frac{T_{\max}}{\sigma_B} = \frac{21.25}{1.4} = 15.2 \text{ cm}^2/\text{m}$$

chosen  $12 \checkmark \phi 13 \text{ mm/m}$  ( $A_B = 15.9 \text{ cm}^2$ ) on both sides i.e.  $6 \phi 13 \text{ mm/m}$

on each side. Thickness of wall at position of max ring tension is given by :

$$t_x = \frac{20 + 35}{2} = 27.5 \text{ cms}$$

The max tensile stress in concrete taking shrinkage into consideration is given by :

$$\sigma_t = \frac{T_{max} + \epsilon_{sh} E_s A_s}{A_c + n A_s} \quad \text{in which}$$

$$\epsilon_{sh} = 0.25 \text{ mm/m} \quad E_s = 2100 \text{ t/cm}^2 \quad \& \quad n = 10 \quad \text{then}$$

$$\sigma_t = \frac{21250 + 0.00025 \times 2100000 \times 15.9}{27.5 \times 100 + 10 \times 15.9} = \frac{21250 + 8350}{2909} = 10.2 \text{ kg/cm}^2$$

The max fixing moment is given by

$$M = \frac{w R^2 t^2}{12 H} \cdot 2n(n-1) = - \frac{1 \times 10^2 \times 35^2}{12 \times 5} \times 2 \times 3.53$$

$$(3.53 - 1) = -3.67 \text{ m t}$$

Then calculating the necessary vertical reinforcement in the wall of a tank, the effect of the compression due to the own weight of the wall may be neglected without affecting the safety of the wall as it causes a reduction of the thickness of the concrete in sections subject to tensile stresses on the water side and reduces the area of the tension reinforcement which can be determined at the foot of the wall as follows

$$t = 35 \text{ cms} \quad \text{therefore} \quad d = 35 - 3 = 32 \text{ cms} \quad \text{and}$$

$$32 = k_1 \sqrt{3670} \quad \text{i e} \quad k_1 = 0.53$$

$$\text{Assuming } \sigma_s = 1400 \text{ kg/cm}^2, \quad n = 15 \quad \text{and} \quad \alpha = 0 \quad \text{we get}$$

$$\sigma_c = 31 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1284 \text{ kg/cm}^2 \quad \text{so that}$$

$$A_s = \frac{M_f}{k_2 d} = \frac{3670}{1284 \times 32} = 9.0 \text{ cm}^2 \quad \text{chosen } 7 \phi 13 \text{ mm/m}$$

According to equation 10, we get

$$A_s = 0.26 t = 0.26 \times 35 = 9.1 \text{ cm}^2 \sim \text{the same result !}$$

For max positive bending moment, we get for a wall of constant thickness

$\beta = 5.5$  and lies at  $\xi = 0.6$  where  $t_x = 29$  cms, therefore

$$+M_{\max} = \beta \frac{w R^2 t^2}{12 H} \left( \frac{t_x}{t} \right)^3 = 5.5 \times \frac{1 \times 10^2 \times 0.35^2}{12 \times 5} \times \left( \frac{29}{35} \right)^3$$

$$= 0.64 \text{ m t}$$

$$26 = k_1 \sqrt{640} \quad k_1 = 0.975 \quad \sigma_c \text{ low, } k_2 = 1300 \text{ kg/cm}^2$$

$$A_B = \frac{640}{1300 \times 26} = 1.9 \text{ cm}^2 \quad \text{chosen } 5 \phi 8 \text{ mm/m}$$

It is however possible to construct the wall with constant thickness of 25 cms and provide it with a haunch 10 x 40 cms at base. In this case the effect of the haunch on the ring tension and the positive bending moments may be neglected and determined in the following manner

For  $t = 25$  cms

$$K = \frac{12 H^4}{R^2 t^2} = \frac{12 \times 5^4}{10^2 \times 0.25^2} = 1200 \quad \text{and} \quad n = \sqrt[4]{K/4} = \sqrt[4]{1200/4} = 4.16$$

For max ring tension, the curves of Reissner give

$$C_0 = 0.48 \quad \& \quad \xi = 0.545$$

$$T_{\max} = C_0 w H R = 0.48 \times 5 \times 10 = 24 \text{ tons}$$

$$A_B \max = 24/1.4 = 17.2 \text{ cm}^2/\text{m}$$

chosen 7  $\phi$  13 mm/m on each side (18.5 cm<sup>2</sup>)

$$\sigma_t = \frac{24000 + 0.00025 \times 2100000 \times 18.5}{100 \times 25 + 10 \times 18.5}$$

$$= \frac{24000 + 9750}{2680} = 12.6 \text{ kg/cm}^2$$

For max positive bending moment

we get

For  $n = 4.16$   $\beta = 7.1$  and

$$+M_{\max} = \beta \frac{w R^2 t^2}{12 H}$$

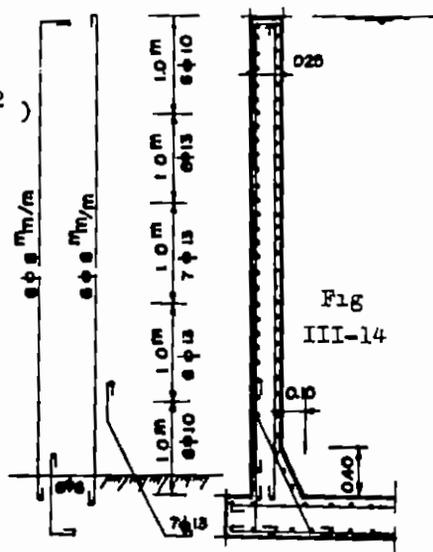


Fig  
III-14

$$M_{\max} = 7.1 \times \frac{1 \times 10^2 \times 0.25^2}{12 \times 5} = 0.74 \text{ m t}$$

$$A_s = \frac{740}{1300 \times 0.22} = 2.6 \text{ cm}^2 \quad \text{chosen } 6 \phi 8 \text{ mm/m}$$

The details of reinforcement are shown in Fig III 14

III 4 2 Simplified Methods for Determining the Fixing Moment, the Shearing Force and the Thickness of the Wall at the base

It has been proved in equation 38 that  $t = 1.73 \frac{H^2}{R n^2}$  from which one can determine the value of  $n$ , thus

$$n = \frac{1.315 H}{\sqrt{R t}} \quad (39)$$

The max fixing moment is given according to equation 35 by the relation

$$M_f = - \frac{w R^2 t^2}{12 H} 2 n (n - 1)$$

Substituting for  $t$  the value given in equation 38, we get

$$\begin{aligned} M_f &= - \frac{w R^2 t^2}{12 H} \left( 1.73 \frac{H^2}{R n^2} \right)^2 2 n (n - 1) \\ &= - \frac{w H (2 R) t}{6.95} \frac{n - 1}{n} \end{aligned}$$

$$\text{Assuming } C_1 = \frac{6.95}{1 - \frac{1}{n}},$$

$$\text{we get } M_f = - \frac{w H D t}{C_1} \quad (40)$$

The shearing force at the base of the wall is given according to equation 36 by the relation :

$$Q_{\max} = \frac{w R^2 t^2}{12 H^2} 2 n^2 (2 n - 1)$$

$$\text{But according to equation 38 } t^2 = \frac{3 H^4}{R^2 n^4}$$

Substituting this value of  $t^2$  in the equation of  $Q_{\max}$ , we get :

$$Q_{\max} = \frac{w R^2}{6 H^2} \frac{3 H^4}{R^2 n^4} n^2 (2n - 1) = \frac{w H^2}{2} \frac{2n - 1}{n^2}$$

Assuming  $\frac{2n - 1}{n^2} = C_2$  and substituting, we get :

$$Q_{\max} = C_2 \frac{w H^2}{2} \quad (41)$$

It is further known that the bending tensile stress at the foot of the wall is given by  $\sigma_{tb} = \frac{6 M_f}{t^2}$

Assuming  $\sigma_{tb} = 18 \text{ kg/cm}^2 = 180 \text{ t/m}^2$  and substituting for  $M_f$  the value given in equation 40, we get :

$$180 = \frac{6 w H D t}{C_1 t^2} = \frac{12 w H R}{C_1 t} \quad \text{or} \quad t = \frac{w H R}{15 C_1}$$

Assuming further  $\frac{1}{15 C_1} = C_3'$  then  $t = C_3' w H R$

In order to get  $t$  in cms for  $w$  in  $\text{t/m}^3$  (equals 1 for water),  $H$  and  $R$  in meters, we should have :

$$t = C_3 w H R \quad (42)$$

in which  $C_3 = 100 C_3' = \frac{100}{15 C_1} = \frac{6.66}{C_1}$

The values of  $M_f$ ,  $Q_{\max}$  and  $t$  at the base of a wall fixed to the floor as given by equations 40, 41 and 42 depend on the value of  $n$  which in turn depends on the magnitude of the thickness  $t$ . In this manner a method of trial and error is to be applied i.e. we have first to assume a convenient value for  $t$ , then determine the corresponding value of  $n$  according to equation 39. Applying equation 42, it is possible to check the assumed value of  $t$ .

It is however possible to avoid these trials in special cases e.g. water tanks in the following manner. Equation 37 gives :

$$n = 0.5 + \sqrt{\frac{\sigma_{tb} H}{R^2 w} + 0.25}$$

Assuming  $\sigma_{tb} = 180 \text{ t/m}^2$  and  $w = 1 \text{ t/m}^3$ , then

$$n = 0.5 + \sqrt{180 \frac{H}{R^2} + 0.25} \text{ which means that}$$

$$n \text{ is a factor of } \frac{H}{R^2} \text{ or } n \propto \frac{H}{R^2}$$

The following table and curves fig III 15 give the values of  $C_1$ ,  $C_2$  &  $C_3$  as factors of  $n$  in general or as factors of  $H/R^2$  for the special cases in which  $w = 1 \text{ t/m}^3$  ( e g water tanks )

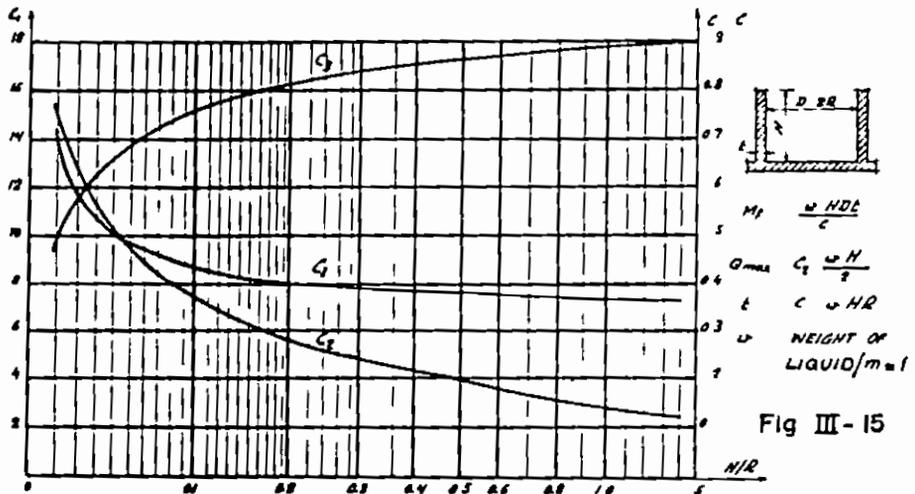


Fig III-15

$H/R^2$	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0.1	0.08	0.06	0.04	0.02	0.01
$n$	16.4	15.2	13.9	12.5	10.9	9.00	6.50	4.77	4.32	3.82	3.23	2.46	1.93
$C_1$	7.40	7.44	7.49	7.55	7.65	7.82	8.22	8.80	9.04	9.4	10.1	11.7	14.4
$C_2$	118	127	138	154	175	211	284	375	409	455	525	646	769
$C_3$	900	895	890	882	870	852	812	756	736	708	663	570	462

Circular tanks may be divided into 3 categories

- a) Deep tanks with big depths, small diameters and thin walls where

n is bigger than 8 The convenient cross section for the wall is trapezoidal with a min thickness at the top of 20 - 25 cms and a max thickness at the bottom

$$t_{\max} = 0.8 \text{ to } 0.85 \text{ w H R}$$

The max ring tension

$$T_{\max} = 0.8 \text{ to } 0.90 \text{ w H R at } 0.8 \text{ to } 0.90 \text{ H from top}$$

The max fixing moment at base

$$M_f = - \frac{w H D t_{\max}}{7.5 \text{ to } 8}$$

The max field moment is about

$$M_{\max} \approx M_f / 5$$

b) Shallow tanks with small depths big diameters and thick walls, n is smaller than ca 2.5 A small part of the water pressure is resisted by ring tension and the wall behaves mainly as a simple cantilever

c) Medium tanks which lie between the previous two categories The convenient section for the wall is rectangular and provided with a haunch at the base

The max ring tension

$$T_{\max} = 0.5 \text{ to } 0.75 \text{ w H R at } 0.5 \text{ to } 0.75 \text{ H from top}$$

$$\begin{aligned} \text{The corresponding thickness of wall } t^{\text{cms}} &= 0.8 T \text{ tons} \\ &= 0.4 \text{ to } 0.6 \text{ w H R} \end{aligned}$$

$$\text{The max thickness at base } t_{\max} = 0.8 \text{ to } 0.7 \text{ w H R}$$

$$\text{The max fixing moment at base } : M_f = - \frac{w H D t_{\max}}{8 \text{ to } 10}$$

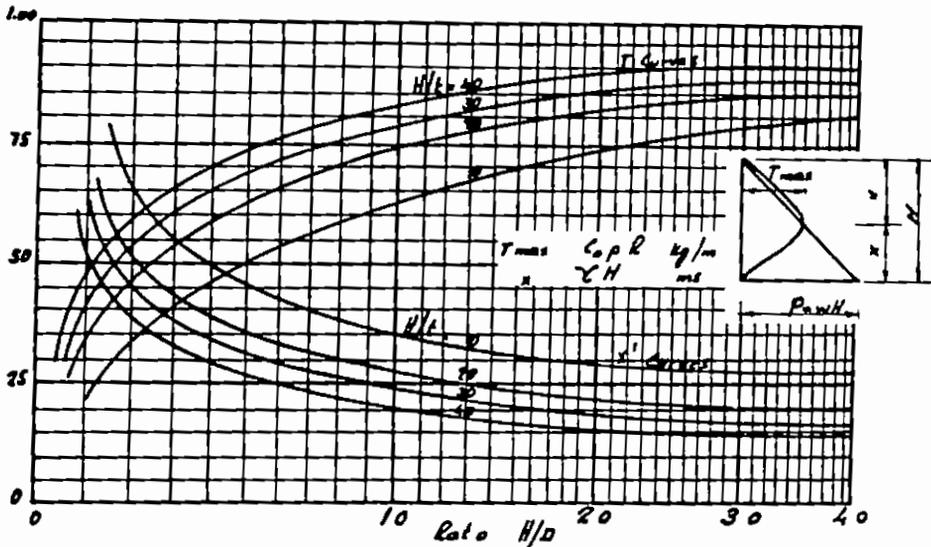
$$\text{The max field moment } : M_{\max} \approx - M_f / 5$$

The max ring tension and its position in rectangular walls of circular tanks fixed at the base and subject to liquid pressure have

been given by W S Gray in his book " Reinforced Concrete Water Towers Bunkers, Silos and Gantries " in the following curves (Fig III 16 ):

VALUES OF MAX BENDING TENSION  
& ITS POSITION

Fig III-16



Applications :

We show in the following examples, the direct application of the given simplified method for determining the internal forces in open circular water tanks with walls fixed at base

- 1) Depth  $H = 10^m$  , Diameter  $D = 10^m$  ,  $w = 1 \text{ t/m}^3$   
 $H / R^2 = 10 / 25 = 0.4$

the corresponding  $n = 9$  i.e. case of deep tank

According to table  $C_1 = 7.8$   $C_2 = 0.211$   $C_3 = 0.855$

Wall is chosen trapezoidal : 20 cms thick at its top end, the thickness at the base is given by :

$$t_{\max} = C_3 w H R = 0.855 \times 1 \times 10 \times 5 = 42.75 \text{ cms chosen } 42 \text{ cm}$$

$$M_r = \frac{w H D t_{\max}}{C_1} = \frac{1 \times 10 \times 10 \times 42}{7.8} = 5.40 \text{ m t,}$$

$$M_{\max} = M_r / 5 = 1.08 \text{ m t}$$

$$Q_{\max} = C_2 \frac{w H^2}{2} = 0.211 \times \frac{1 \times 10^2}{2} = 10.5 \text{ t}$$

According to Gray Fig III 16

Average thickness of wall  $t = 31 \text{ cms}$

$$\frac{H}{t} = \frac{10}{0.31} = 32, \quad \frac{H}{D} = \frac{10}{10} = 1, \quad C_0 = 0.8, \quad C = 225$$

$$T_{\max} = C_0 w H R = 0.8 \times 1 \times 10 \times 5 = 40 \text{ t}$$

$$x' = C H = 0.225 \times 10 = 2.25 \text{ ms from base}$$

2) Depth  $H = 5 \text{ m}$ , diameter  $D = 20 \text{ m}$   $w = 1 \text{ t/m}^3$

For section at base  $\frac{H}{R^2} = \frac{5}{10^2} = 0.05$  medium tank

According to table  $C_1 = 9.7$   $C_3 = 0.685$

$$t_{\max} = C_3 w H R = 0.685 \times 1 \times 5 \times 10 = 34.25 \text{ cms chosen } 35 \text{ cms}$$

$$M_{\max} = - \frac{w H D t_{\max}}{C_1} = - \frac{1 \times 5 \times 20 \times 35}{9.7} = -36 \text{ m t}$$

$$M_{\max} = \frac{M_{\max}}{5} = \frac{36}{5} = 7.2 \text{ m t}$$

The wall will be chosen of constant thickness 25 cms and provided with a haunch at the base of  $10 \times 40 \text{ cms}$ . The max ring tension and its position can be determined according to curves of Gray, thus

$$\frac{H}{t} = \frac{5.00}{0.25} = 20, \quad \frac{H}{D} = \frac{5}{20} = 0.25, \quad C_0 = 0.51, \quad C = 0.47$$

$$T_{\max} = C_0 w H R = 0.51 \times 1 \times 5 \times 10 = 25.5 \text{ tons at } 2.35 \text{ ms from base}$$

The shear at the base for a wall 25 cms thick can be calculated as follows

$$n = \frac{1.315 H}{\sqrt{R t}} = \frac{1.315 \times 5}{\sqrt{10 \times 25}} = 4.16$$

The corresponding value of  $C_2$  is given by 0.425 and

$$Q_{\max} = C_2 \frac{w H^2}{2} = 0.425 \times \frac{5^2}{2} = 5.3 \text{ tons}$$

The use of this method for determining the internal forces in walls of circular tanks is very convenient, because when one is used to it, he can easily estimate the coefficients  $C_1$  required for determining the fixing moment,  $C_3$  required for determining the max thickness of the wall,  $C_0$  required for determining the max ring tension without appreciable errors which may affect the design as can be seen from the following example

3) It is required to design the wall of an open circular tank 7 ms deep and 15 ms diameter Make a quick estimate and check the results

a) Estimate

This tank is a medium one, its diameter  $\approx$  twice its depth then the max ring tension may be estimated by

$$T_{\max} = 0.65 w H R = 0.65 \times 1 \times 7 \times 7.5 = 34 \text{ ton}$$

Wall thickness ( equation 7 )

$$t = 0.8 T_{\max} = 0.8 \times 34 = 27 \text{ cms}$$

Max ring reinforcements

$$A_{s_{\max}} = \frac{T_{\max}}{\sigma_s} = \frac{34}{1.4} = 24.5 \text{ cms} \quad 6 \phi 16 \text{ mm/m on each side}$$

Max thickness at base

$$t_{\max} = 0.8 w H R = 0.8 \times 1 \times 7 \times 7.5 = 42 \text{ cms}$$

The wall is chosen 27 cms and provided with a haunch 15 x 50 cms at the base

The fixing moment

$$M_f = - \frac{w H D t_{\max}}{8} = - \frac{1 \times 7 \times 15 \times 42}{8} = - 550 \text{ m t}$$

Thickness required to resist this moment safely

$$t_{\max} = \sqrt{M_f / 3} = \sqrt{5500 / 3} = 42.7 \text{ cms}$$

ie chosen thickness of 42 cms is convenient

Vertical tension steel required at base

$$A_s = \frac{M}{k_2 d} = \frac{5500}{1300 \times 385} = 11 \text{ cm}^2 \quad 6 \phi 16/m$$

Max field moment

$$M_{\text{max}} = M_f / 5 = 5500 / 5 = 1100 \text{ kgm}$$

Vertical reinforcement required

$$A_s = \frac{M_{\text{max}}}{k_2 d} = \frac{1100}{1300 \times 24} = 3.50 \text{ cm}^2 \quad 7 \phi 8 \text{ mm/m}$$

Check

$$\text{Section at base} \quad \frac{H}{R^2} = \frac{7}{7 \times 5^2} = 0.125$$

$$\text{According to table} \quad C_3 = 0.77$$

$$t_{\text{max}} = C_3 \times w \times H \times R = 0.77 \times 1 \times 7 \times 7.5 = 40.5 \text{ cm}$$

Chosen thickness of 42 cms is convenient

The corresponding value of n is given by

$$n = \frac{1.315 H}{\sqrt{R t_{\text{max}}}} = \frac{1.315 \times 7}{\sqrt{7.5 \times 42}} = 5.20 \text{ giving } C_1 = 8.65$$

So that

$$M_f = \frac{-w H D t}{8.65} = -\frac{7 \times 15 \times 42}{8.65} = 5.10 \text{ mt}$$

The thickness required to resist this moment is given by

$$t_{\text{max}} = \sqrt{\frac{5100}{3}} = 41 \text{ cms} < 42 \text{ cms}$$

According to Gray : ( t = 27 cms )

$$\frac{H}{t} = \frac{7}{27} = 0.26, \quad \frac{H}{D} = \frac{7}{15} = 0.465, \quad C_0 = 0.65, \quad C = 34$$

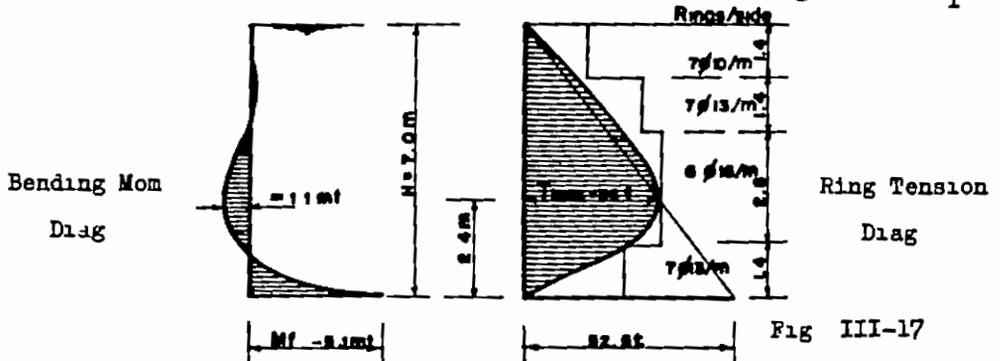
$$T_{\text{max}} = C_0 \times w \times H \times R = 0.65 \times 1 \times 7 \times 7.5 = 34 \text{ t} \quad \text{at}$$

$$x' = C \times H = 0.34 \times 7 = 2.4 \text{ ms} \quad \text{from base}$$

The internal forces are shown in figure III 17

The details of reinforcements are similar to those shown in figure III 14

If the wall of the tank were free at top and hinged at bottom, the maximum ring tension will be increased by ca 10% in deep tanks and 20 to 25 % in medium tanks and its position moves a small distance towards the base of the wall. In this case the fixing moment  $M_f$



will be equal to zero and the field moment will be increased to ca  $M_f / 3$ . The distribution of the ring tension and cantilever moment will have the form shown in fig III 19

#### III 4 3 - Tables of the Portland Cement Association

The American Portland Cement Association has published the following series of tables<sup>2</sup> giving the ring tension and cantilever moments in rectangular walls of cylindrical tanks free at top and fixed or hinged at bottom for different cases of loading. The tables include further a lot of data very useful in the design of circular tanks.

#### Illustrative Example

The use of the "Portland Cement Association" tables will be explained in the following example.

Figure III 18 shows an open cylindrical reinforced concrete water tank 5 ms deep and 20 ms diameter. It is required to determine the internal forces for the following cases :

<sup>2</sup> "Circular Concrete Tanks without Prestressing" Concrete Information of the Portland Cement Association ST - 57 - 1

- 1) Bottom edge sliding
- 2) " hinged
- 3) " fixed

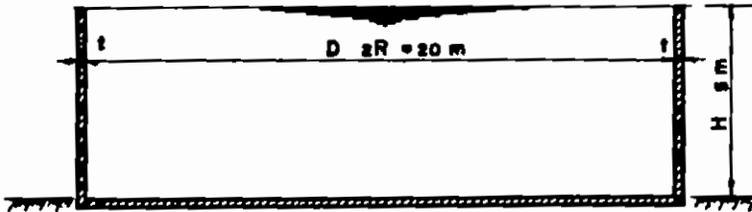


Fig III-18

1) Top Edge Free and Bottom Edge Sliding

$$H = 5.00 \text{ ms} \quad \text{and} \quad R = 10 \text{ ms}$$

The maximum wall thickness can be determined according to equation 7 from the relation

$$t_{\max} = 0.8 w H R$$

thus

$$t_{\max} = 0.8 \times 1 \times 5 \times 10 = 40 \text{ cms}$$

As the water pressure increases with the depth, and is fully resisted by ring action, it is convenient to choose a trapezoidal wall with a minimum thickness  $t = 20$  cms at top and a maximum thickness of 40 cms at bottom

Table 1 Ring Tension in Cylindrical Tank Wall with Free Top & Sliding Base  $T = x R$

Distance from top edge $x$	0	0.2H	0.4H	0.6H	0.8H	1.0H
Ring Tension $T$ t/m	0	10 000	20 000	30 000	40 000	50 000

2) Top Edge Free Bottom Edge Hinged

As the max ring tension and bending moment take place at the middle part of the wall, it is recommended to choose a wall of constant

thickness with

$$\underline{t \approx 0.5 H R} \quad (26)$$

In our case  $t = 0.5 \times 5 \times 10 = 25 \text{ cms}$

Therefore 
$$\frac{H^2}{D t} = \frac{5^2}{20 \times 0.25} = 5$$

Ring tension per meter height will be computed by multiplying  $p_{\max}^R$  by the coefficients for  $H^2 / Dt = 5.0$  taken from table II,  $p_{\max}^R = 5 \times 10 = 50 \text{ t/m}$

Bending moments in the vertical wall strips  $1^m$  wide are computed by multiplying the coefficient given in table VIII by

$$p_{\max} H^2 = 5 \times 5^2 = 125 \text{ mt/m}$$

It has to be noted that

- \* The coefficients are given such that point 0 OH denotes the top and point 1 OH the base of the wall
- \* The positive ring forces are tensile while the negative are compressive
- \* Positive bending moments cause tensile stresses in the outer surface and compressive stresses in the inner surface of the wall

Table 2 Ring Tension & Bending Moments in Cylindrical Tank Wall with Free Top and Hinged Base

Point		0 0H	0 1H	0 2H	0 3H	0 4H	0 5H	0 6H	0 7H	0 8H	0 9H
Ring Ten- sion	Coef II	-	+	+	+	+	+	+	+	+	+
	T t/m	0 4	5 70	11 75	17 8	23 45	28 1	30 85	30 5	25 1	14 7
B M	Coef VIII	00	+ 00	+ 0001	+ 0006	+ 0016	+ 0034	+ 0057	+ 008	+ 0094	+ 0078
	M mt/m	00	+ 00	+ 013	+ 075	+ 20	+ 425	+ 712	+ 1 00	+ 1 172	+ 975

Reaction at base ( coefficient table XVI )

$$Q_{\max} = 0.121 \cdot w \cdot H^2 = 0.121 \times 1 \times 25 = 3.0 \text{ t/m'}$$

The max ring tension in the wall  $T_{\max} = 30.85$  tons and acts at  $0.6d = 3.00$  ms from the top edge. The required ring reinforcement at  $x = 0.6H$  is given by

$$A_s = \frac{T}{\sigma_s} = \frac{30850}{1400} = 22 \text{ cms} \quad 16 \phi 13 \text{ mm/m ( 8 } \phi 13 \text{ mm/m on each side) } 21.2 \text{ cm}^2$$

The tensile stress in concrete, including shrinkage, is given by

$$\begin{aligned} \sigma_t &= \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{30850 + 0.00025 \times 2100000 \times 21.2}{100 \times 25 \times 10 \times 21.2} \\ &= \frac{30850 + 11100}{2500 + 212} = \frac{41950}{2712} = 15.5 \text{ kg/cm}^2 \quad \text{safe} \end{aligned}$$

The max moment  $M_{\max} = 1172$  kgm and acts at  $x = 0.8H = 4$  ms from top

$$\begin{aligned} d &= t - 3 = 25 - 3 = 22 \text{ cms} \quad d = k_1 \sqrt{M} \quad \text{or} \\ 22 &= k_1 \sqrt{1172} \quad \text{i.e.} \quad k_1 = 0.643 \end{aligned}$$

For  $\sigma_s = 1400 \text{ kg/cm}^2$ ,  $\alpha = 0$ ,  $\sigma_c = 25 \text{ kg/cm}^2$  &  $k_2 = 1300$  and

$$A_s = \frac{M}{k_2 d} = \frac{1172}{1300 \times 22} = 4.1 \text{ cm}^2 \quad \text{choose } 6 \phi 10 \text{ mm/m}$$

### 3) Top Edge Free, Bottom Edge Fixed

In this case, the water pressure will be resisted by ring tension in the horizontal direction and cantilever action in the vertical direction. The max ring tension and max positive moment will be smaller than in case 2, wall hinged at base, while relatively big fixing moments will be induced at the fixed bottom edge of the wall.

For this reason, it is recommended to choose for this case a wall of constant thickness  $t = 25$  cms stiffened with a haunch ( ca  $10 \times 40$  cms) at its foot

Due to the existence of the haunch it is recommended to determine the fixing moment and respectively the thickness of the wall at its foot either by method of Reissner or by the simplified method given before while the ring tension and positive moments can be determined from the P C A tables neglecting the effect of the haunch. Thus,

For section at base Refer to example 2 in the simplified methods

$$A_s = \frac{3600}{1284 \times 32} = 8.8 \text{ cm}^2 \quad 7 \phi 13 \text{ mm}$$

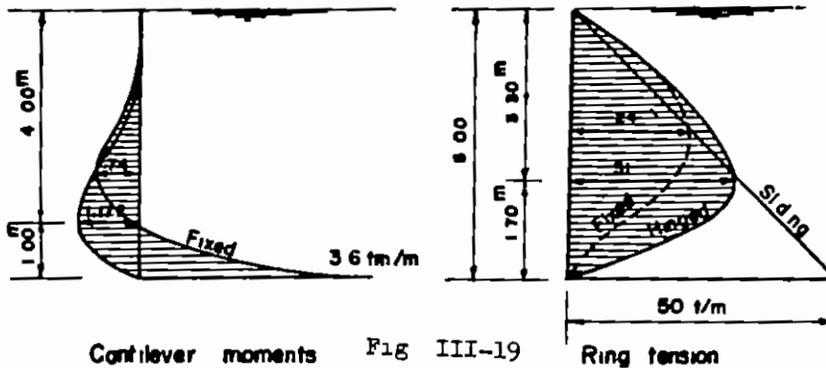
The ring tension per meter height will be computed by multiplying  $P_{\max} R = 5 \times 10 = 50 \text{ t/m}$  by the coefficients for  $H^2 / D t$  taken from table 1. The positive bending moments in the vertical wall strip 1 m wide are computed by multiplying the coefficient given in table VII by  $P_{\max} H^2 = 5 \times 5^2 = 125 \text{ mt/m}$

The ring tension and the cantilever moments at the different depths of the wall are given in table 3 in which all the values are determined from the P C A tables for  $H^2 / D t = 5$  with the exception of the fixing moment at the foot of the wall which equals  $-3.6 \text{ mt}$  as given in example 2 page 45

Table 3 Ring Tension & Bending Moments in Cylindrical Tank Wall with Free Top & Fixed Base

Point		0 0H	0 1H	0 2H	0 3H	0 4H	0 5H	0 6H	0 7H	0 8H	0 9H
Ring Ten	Coef. I	+ 025	+ 137	+ 245	+ 346	+ 428	+ 477	+ 469	+ 398	+ 259	+ 092
	T t/m	+ 1 25	+ 6 85	+ 12 25	+ 17 3	+ 21 4	+ 23 85	+ 23 45	+ 19 9	+ 12 9	+ 4 6
B M	Coef. II	+ 00	+ 0002	+ 0008	+ 0016	+ 0029	+ 0046	+ 0059	+ 006	+ 003	+ 006
	M mt/m	00	+ 025	+ 10	+ 20	+ 36	+ 58	+ 74	+ 74	+ 35	- 72

The shear at base of wall equals  $p \cdot H \times \text{coef}$  of 0.213 for  $H/D \cdot t = 5$  taken from table XVI i.e.  $Q_{\max} = 0.213 \times 5 \times 5 = 5.33 \text{ t/m}$



The ring tension and cantilever moments for the 3 cases are shown in figure III 19

In cases of tank walls monolithically cast with the floor supported on clayey soils liable to unknown rotations, it is recommended to design the section at the foot of the wall for the max possible negative bending moment  $M = -3600 \text{ kgm}$  - case of total fixation the max positive bending moment  $M_t = 1172 \text{ kgm}$  and the max ring tension  $T_{\max} = 31 \text{ t}$  are to be taken from the case of hinged base i.e. we make the design for the hatched values shown in figure. The tension in the floor due to cantilever action  $Q_{\max} = 5.33 \text{ t/m}$

### III 5 Tank Walls Continuous with Roof or Floor :

When the bottom of the wall and the edge of the floor slab are made continuous as shown in figure III 20, the elastic deformation of the floor slab tends to rotate the lower point of the wall and introduces a moment  $M$  in the corner. The procedure for obtaining  $M$ , is so much like moment distribution applied to continuous frames, only there is no carry over moments. The data in tables numbers XVIII and XIX of the P C A, are stiffnesses which denote moments required to cause a unit rotation at the edge of the wall and the floor slab. Only relative values of stiffness are required in this appli-

cation The example shown in fig III 20 gives a good explanation for the problem The main steps for the solution are

1) Determine the fixing moment at the bottom of the wall and at the edge of the floor, assuming joint between wall and floor totally fixed The two fixing moments are generally not equal

2) The difference between the two fixing moments will be distributed between wall and floor by moment distribution method

according to the relative stiffness and distribution factors of both wall and floor

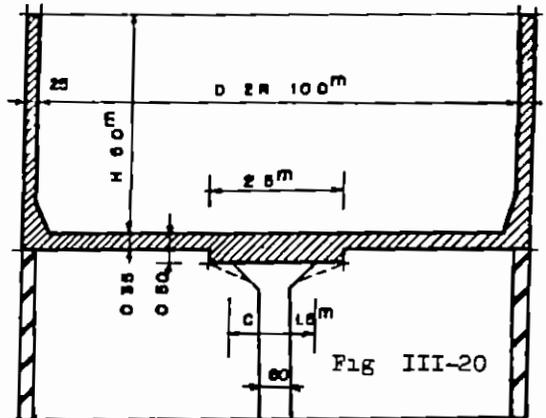
3) Find out the effects of the distributed moments on both and add it to the solution of fixed joint

4) Determine the concrete dimensions and steel reinforcements such that there is adequate strength and sufficient safety against cracking

#### Example

A reinforced concrete water tank 10 ms diameter and 5 0 ms deep is supported on a cylindrical wall at its outside edge & on a central column at the centre as shown in fig III 20 The wall is free at its top edge and continuous with the floor slab, at its bottom edge The column capital is 1 5 m diameter, and the drop panel, 50 cms thick, is 2 5 ms diameter

It is required to clarify the effect of continuity between wall and floor on both of them



### Step 1 Fixed End Moments of Wall & Floor Slab

The fixed end moment of the wall is computed by multiplying the coeff given in table VII for the corresponding  $H^2/D t$  at  $1.0 H$  by

$$P_{\max} H^2 = 5 \times 5^2 = 125$$

$$\frac{H^2}{D t} = \frac{5^2}{10 \times 25} = 10 \quad \text{coeff ( table VII ) at } 1.0 H = - 0.0122$$

$$M = - 0.0122 \times 125 = - 1.53 \text{ mt}$$

The radial moments of the floor slab are effected by the drop panel, but since the ratio of the panel area to the total slab area is about  $1/20$ , its effect on the loads and moments may be neglected and the coefficients given in table XIII, XIV & XV can be used

load/m<sup>2</sup> floor slab = weight of water + own weight

$$p = 5.0 + 0.35 \times 2.5 = 5.875 \text{ t/m}^2$$

The radial fixed end moment equals the coefficient of 0.049 from table XIII for  $c/D = 1.5/10 = 0.15$  at point  $1.0 R$  multiplied by  $p R^2$

$$i.e. \quad = - 0.049 \times 5.875 \times 5^2 = - 7.2 \text{ mt}$$

### Step 2 The Connecting Moment

The fixed end moments of the wall ( - 1.53 mt ) and floor ( - 7.2 mt ) being not equal the difference will be distributed between wall and floor slab according to their relative stiffness, thus

The relative stiffness of the wall equals  $t^3 / H = 25^3 / 500 = 31.25$  multiplied by coefficient taken from table XVIII for  $H^2 / D t = 10$   
i.e.  $31.25 \times 1.01 = 31.56$

The relative stiffness of the floor slab equals  $t^3 / R = 35^3 / 500 = 85.75$  multiplied by coefficient taken from table XIX for  $c/D = 1.5/10 = 0.15$

$$i.e. \quad 85.75 \times 0.332 = 28.47$$

The distribution factors are therefore :

$$\text{for wall} : \frac{31\ 56}{31\ 56 + 28\ 47} = \frac{31\ 56}{60\ 03} = 0\ 523$$

$$\text{for floor} : \frac{28\ 47}{31\ 56 + 28\ 47} = \frac{28\ 47}{60\ 03} = 0\ 477$$

The final connecting moment will be determined by moment distribution as follows ( fig III 21 )

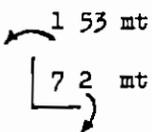
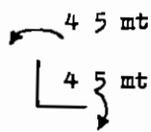
	Wall	Floor	Fixed end moments	Connecting moments
Distribution Factor	0 523	0 477		
Fixed end oment	+1 53	-7 20		
Distributed Moment	<u>+2 97</u>	<u>+2 70</u>		
	+4 50	-4 50		

Fig III 21

The quick damping of the bending moments makes the simple moment distribution shown sufficient for determining the connecting moment in the corner

### Step 3 Ring Tension & Cantilever oment of wall

Ring tension per meter height & cantilever moment for 1m strip will be calculated for a wall with fixed base, subject to triangular water pressure plus the effect of an induced moment equal to - 2 97

a) Ring tension due to triangular water pressure will be computed by multiplying coefficients from table I for  $(H^2 / Dt) = 10$  by

$$P_{\max} R = 5 \times 5 = 25$$

b) Ring tension due to an induced moment of - 2 97 mt will be computed by multiplying coefficient from table VI for  $H^2 / Dt = 10$  by

$$M R / H^2 = - \frac{2\ 97 \times 5}{5^2} = 0\ 594$$

c) Cantilever moment due to triangular water pressure will be computed by multiplying coefficient from table VII for  $H^2 / Dt = 10$  by

$$P_{\max} H^2 = 5 \times 5^2 = 125$$

d) Cantilever moment due to an induced moment of - 2 97 mt will be

computed by multiplying coefficient from table XI for  $H^2 / Dt = 10$   
by  $M = - 2 97 \text{ mt}$

The final value will be as given in the following table and figure  
III 22 :

P o i n t	Ring Tension					Cantilever Moment				
	Coeff Table I	Coeff Table VI	T t/m due to	T t/m due to	T total in t/m	Coeff Table VII	Coeff Table XI	M mt/m due to	M mt/m due to	M total in mt/t
	Δ load	moment	load	moment		load	moment	Δ load	moment	
0 H	-	+	-	-	-					
	0 011	0 21	0 265	0 124	0 389	000	0 000	0 000	0 000	0 000
1H	+	-	+	+	+					
	0 098	0 23	2 46	0 136	2 596	000	0 000	0 000	0 000	0 000
2H	+	-	+	+	+		-	+	+	+
	0 208	0 64	5 20	0 38	5 58	000	0 002	0 000	0 006	0 006
3H	+	-	+	+	+	+	-	+	+	+
	0 323	0 94	8 10	0 56	8 66	0001	0 009	0 013	0 027	0 040
4H	+	-	+	+	+	+	-	+	+	+
	0 437	0 73	11 00	0 43	11 43	0004	0 028	0 050	0 084	0 134
5H	+	+	+	-	+	+	-	+	+	+
	0 542	0 82	13 55	0 49	13 06	0007	0 053	0 088	0 159	0 247
6H	+	+	+	-	+	+	-	+	+	+
	0 608	4 79	15 20	2 84	12 36	0019	0 067	0 238	0 200	0 438
7H	+	+	+	-	+	+	-	+	+	+
	0 589	11 63	14 73	6 82	7 91	0029	0 031	0 363	0 093	0 456
8H	+	+	+	-	-	+	+	+	-	-
	0 440	19 48	11 00	11 55	0 55	0028	0 123	0 350	0 369	0 019
9H	+	+	+	-	-	-	+	-	-	-
	0 179	20 87	4 47	12 40	7 93	0012	0 467	0 150	1 400	1 550
H	0	0	0	0	0	0122	1 000	1 530	2 970	4 500



III 22 The radial moment around the central column is relatively high, and the actual value is only = 0.7 the theoretical  $0.7(16 + 4.3) = 14$  mt. It is however recommended in this case to enlarge the column capital as shown by the dotted line fig III 23 so that

at 0.1 H      0.15 H      0.2 H      0.25 H  
thickness 75 cm    66.66 cm    58.33 cm    50 cm

Point	15R	20R	25P	30R	40R	50R	60R	70P	80R	90R	1.0R
Coeff T XIII Fixed, $10^{-2}$	-	-	-	+	+	+	+	+	+	-	-
	10.9	5.21	2.00	0.02	2.20	2.93	2.69	1.69	0.06	2.16	4.9
Coeff Table YV, at edge	-	-	-	-	+	+	+	+	+	+	+
	1.59	9.30	7.45	2.80	0.78	3.23	5.10	6.63	7.90	9.00	1.0
Radial moment Fixed Edge	-	-	-	+	+	+	+	+	+	-	-
	16.0	7.67	2.04	0.03	3.23	4.31	3.95	2.48	0.09	3.18	7.2
Radial moment M at edge	-	-	-	-	+	+	+	+	+	+	+
	4.30	2.51	1.47	0.76	0.21	0.87	1.38	1.79	2.14	2.43	2.7
Total Radial Moment t/m	-	-	-	-	+	+	+	+	+	+	-
	14.0	8.50	3.50	0.73	3.44	5.18	5.33	4.27	2.23	0.75	4.5
Total Radial Mom/Segment	-	-	-	-	+	+	+	+	+	+	-
	3.05	2.04	1.10	0.22	1.38	2.59	3.20	3.00	1.78	0.68	4.5

The tangential moments are computed by selecting coefficients for  $c/D = 0.1$ , from tables XIII & XV and multiplying them by  $w R^2 = 147$  kgm/m ( for fixed edge ), and by  $M = 2.7$  mt/m ( for moment at edge )

The total tangential bending moments are plotted in figure III 24. The effective depth within the drop panel is 46 centimeters instead of 32 cms in the rest of the floor slab, and if the moments

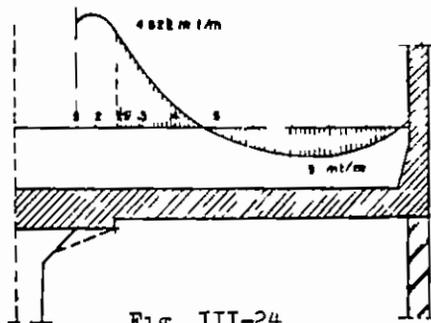


Fig III-24

in that region are reduced in the ratio 32/46 it is seen that the critical moment, 4 822 occurs at the edge of the drop panel

Point	15R	20R	25R	30R	40R	50R	60R	70R	80R	90R	1 0R
Coeff T XIII Fixed, $10^{-2}x$	-	-	-	-	-	+	+	+	+	-	-
	2 18	2 84	2 43	1 77	0 51	0 31	0 80	0 86	0 57	0 06	0 98
Coeff Table XV, M at edge	-	-	-	-	-	-	+	+	+	+	+
	319	472	463	404	251	100	035	157	263	363	451
Tang Moment Fixed edge	-	-	-	-	-	+	+	+	+	-	-
	3 21	4 18	3 57	2 60	0 75	0 47	1 18	1 27	0 84	0 09	1 44
Tang Moment M at edge	-	-	-	-	-	-	+	+	+	+	+
	0 86	1 27	1 25	1 09	0 68	0 27	0 10	0 42	0 71	0 98	1 22
Total Tang Moment mt/m	-	-	-	-	-	+	+	+	+	+	-
	4 07	5 45	4 82	3 69	1 43	0 20	1 28	1 69	1 55	0 89	0 22

Step 5 Design of the Different Elements :

- a) Wall Max ring tension :  $T_{max} = 13 06 t$  at  $0.5 H t = 25 cm$   
 max ring reinforcements ( $\sigma_s = 1400 kg/cm^2$ )

$$A_s = \frac{T}{\sigma_s} = \frac{13060}{1400} = 9.3 cm^2 \quad 6 \phi 10 /m \text{ on each side } (9.4 cm^2)$$

max tensile stress in concrete ( including shrinkage )

Assuming  $\epsilon_{sh} = 0.00025$ ,  $E_s = 2100 000 kg/cm^2$

$$\begin{aligned} \sigma_t &= \frac{T + \epsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{13060 + 0.00025 \times 2100000 \times 9.4}{100 \times 25 + 10 \times 9.4} \\ &= \frac{13060 + 5740}{2594} = 7.27 kg/cm^2 \end{aligned}$$

The max positive moment in wall = 456 kgm at  $0.7 H$  from top

Vertical reinforcement on outside surface :

$$A_s = \frac{456}{1300 \times 22} = 1.6 cm^2 \text{ chosen } 5 \phi 8 \text{ mm (min)}$$

Section at foot of wall : max negative moment  $M_{max} = 4500 kgm$

$$t = \sqrt{\frac{4500}{3}} = 38.7 \text{ cms chosen } 40 \text{ cms}$$

$$37 = k_1 \sqrt{4500} \quad k_1 = 0.553 \text{ for } \sigma_s = 1400 \text{ kg/cm}^2 \quad k_2 = 1280$$

$$A_s = \frac{4500}{1280 \times 37} = 9.5 \text{ cm}^2 \text{ chosen } , \quad \phi \quad 13 \text{ mm/m}$$

at top end of haunch  $M = 1550 \text{ kgm}$

$$A_s = \frac{1550}{1300 \times 22} = 5.44 \text{ cm}^2 \text{ chosen } 7 \quad \phi \quad 13 \text{ mm/m} \quad 0 \text{ K}$$

### b) Floor Slab

The column load is determined by multiplying coefficients taken from table (VII) for  $c/D = 0.15$  by  $p R^2$ , for edge fixed, and by  $M = 2.7 \text{ mt/m}$ , for induced moment

$$\text{Col load when edge is fixed} \quad 1.007 \times 5.375 \times 5^2 = 147 \text{ t}$$

$$\text{Col load due to induce moment at edge} \quad 9.29 \times 2.7 = \underline{34 \text{ t}}$$

$$\text{Total} \quad 181 \text{ t}$$

Diam of critical section for shear around theoretical capital is  $1.50 + 0.50 = 2.00 \text{ ms}$

$$\text{Length of this section is given by} \quad \pi \times 2.00 = 6.28 \text{ ms}$$

$$\text{Load on area within the section} \quad 5.875 \times \pi \times 1^2 = 18.5 \text{ t}$$

$$\text{Shear stress } \tau = \frac{Q}{87 b d} = \frac{181000 - 18500}{87 \times 628 \times 47} = \frac{162500}{25600} = 6.35 \text{ kg/cm}^2$$

Diam of critical section for shear around drop panel =  $250 + 35 = 285 \text{ cm}$

$$\text{Length of this section} = \pi \times 2.85 = 9 \text{ ms}$$

$$\text{Load on area within the section} \quad 5.875 \times \pi \times 1.425^2 = 37.5 \text{ t}$$

$$\text{Shear stress } \tau = \frac{Q}{87 b d} = \frac{181000 - 37500}{87 \times 900 \times 32} = \frac{143500}{25000} = 5.85 \text{ kg/cm}^2$$

$$\begin{aligned} \text{Shear at edge of wall} &= p \pi R^2 - \text{column load} \\ &= 5.875 \times 3.14 \times 5^2 - 181 = 460 - 181 = 269 \text{ t} \end{aligned}$$

$$\text{Length of this section} = 3.14 \times 10 = 31.4 \text{ ms}$$

$$\text{Shear stresses } \tau = \frac{Q}{87 b d} = \frac{269000}{87 \times 3140 \times 32} = 3.10 \text{ kg/cm}^2$$

The horiz. reaction of the wall  $Q = 7.4 \text{ t/m}$  will be resisted by ring reinforcement at the foot of the wall

$$A_s = \frac{7.4 \times 5}{1.4} = 24.5 \text{ cm}^2 \quad 10 \text{ } \phi \text{ } 19 \text{ mm}$$

The max positive radial moment  $M_{\text{max}} = 5.33 \text{ mt}$   $t = 35 \text{ cm}$

$$32 = k_1 \sqrt{5330} \quad k_1 = 0.44 \quad \sigma_c = 40 \text{ kg/cm}^2 \quad k_2 = 1240$$

$$A_s = \frac{5330}{1240 \times 32} = 13.4 \text{ cm}^2 \quad 10 \text{ } \phi \text{ } 13 \text{ mm/m}$$

Max - ve B M at wall surface  $M_- = 3500 \text{ kgm}$

$$\sigma_t = \frac{6 \times 350000}{100 \times 35^2} = 17 \text{ kg/cm}^2$$

$$32 = k_1 \sqrt{3500} \quad k_1 = 0.54 \quad \sigma_c = 30 \quad k_2 = 1280$$

$$A_s = \frac{3500}{1280 \times 32} = 8.6 \text{ cm}^2 \quad \text{chosen } 7 \text{ } \phi \text{ } 13 \text{ mm/m}$$

Top radial reinforcements at middle

$M_{\text{max}} = 14 \text{ mt}$   $t = 66.66 \text{ cms}$   $\text{at } r = 75 \text{ cms}$

$$\sigma_t = \frac{6 \times 14 \times 10^5}{100 \times 66.66^2} = 19 \text{ kg/cm}^2$$

$$A_s = \frac{14000}{1250 \times 63} = 18 \text{ cm}^2/\text{m} \quad \text{total } A_s = 18 \times \pi \times 1.5 = 85 \text{ cm}^2$$

$M = 8.5 \text{ mt}$   $t = 58.33 \text{ cms}$   $\text{at } r = 1.00 \text{ m}$

$$\sigma_t = \frac{6 \times 2.5 \times 10^5}{100 \times 58.33^2} = 15 \text{ kg/cm}^2$$

$$A_s = \frac{8500}{1250 \times 55} = 12.3 \text{ cm}^2/\text{m} \quad \text{total } A_s = 12.3 \times \pi \times 2 = 78 \text{ cm}^2$$

$M = 3.5 \text{ mt}$   $t = 35 \text{ cms}$   $\text{at } r = 1.25 \text{ ms}$

$$\sigma_t = 17 \text{ cms}^2 \quad A_s/n = 8.6 \text{ cm}^2 \quad \text{total } A_s = 8.6 \times \pi \times 2.5 = 67.50$$

The above calculations show that the total amount of the top radial reinforcement does not vary much in the central part of the slab around the column head so that the same number of bars may be used, at smaller distances in the middle and at bigger distances as we move outwards. Therefore

Use 32  $\phi$  19 mm arranged as shown in fig III 25 in which we use 4 types of bars of the following form

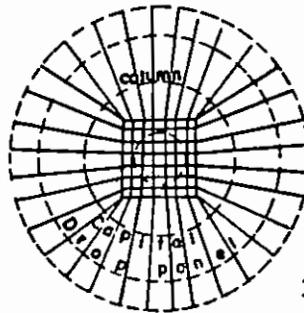
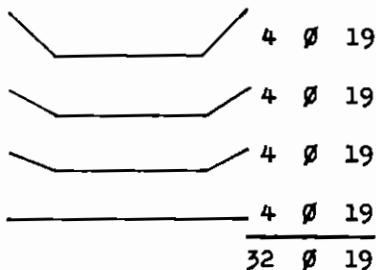


Fig III 25

Max ring reinforcements in lower

fiber at  $x = 0.75 r$  :

$$A_B = \frac{M_{\text{tang}}}{k_2 d} = \frac{1690}{1300 \times 0.3} = 4.4 \text{ cm}^2 \quad \phi 10 \text{ mm} @ 15 \text{ cm}$$

Ring reinforcements in upper fiber at  $x = 0.25 r$

$$A_B = \frac{4822}{1250 \times 0.3} = 13 \text{ cm}^2 \quad \phi 16 \text{ mm} @ 15 \text{ cms}$$

Ring reinforcements in upper fiber at  $x = 0.35 r$

$$A_B = \frac{2750}{1250 \times 0.3} = 7.35 \text{ cm}^2 \quad \phi 13 \text{ mm} @ 15 \text{ cms}$$

c) Central Column :  $P = 181 \text{ t}$ ,  $\phi = 60 \text{ cms}$ ,  $A_B = 12 \phi 16 \text{ mm}$

$$P = \sigma_c (A_c + n A_B) \text{ or } 181000 = \sigma_c (2850 + 15 \times 24) = 24 \text{ cms}$$

$$\text{or } \sigma_c = 56 \text{ kg/cm}^2 \quad 0 \text{ K}$$

The details of reinforcements are shown in fig III 26

It has to be noted that in the given details, bent up bars have been avoided because the shear stresses are low (max value =  $6.35 \text{ kg/cm}^2$ ), moreover, such bars in walls are liable to move from their position during concreting operations and the laying of straight bars in circular elements is easier.



IV ROOFS AND FLOORS OF CIRCULAR TANKS

IV 1 INTRODUCTION

Roofs and floors of circular tanks may be of the beam and slab type as shown in figures IV - 1 a and b

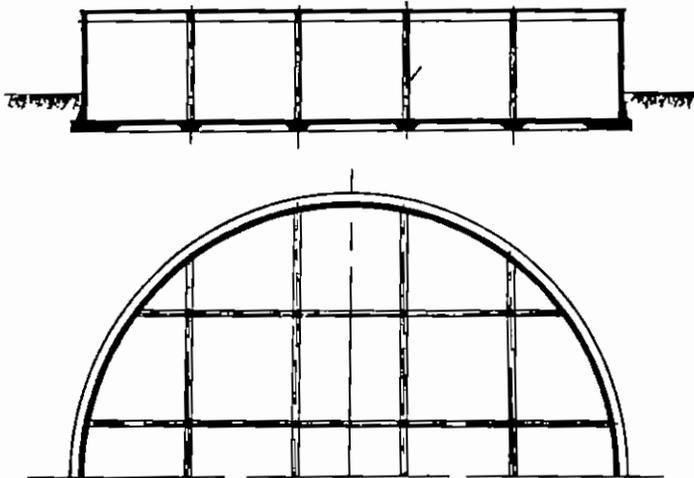


Fig IV-1a

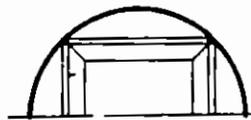
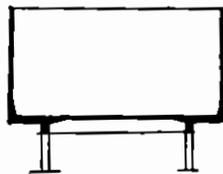


Fig IV-1b

or of the flat slab type as shown in figure IV- 2

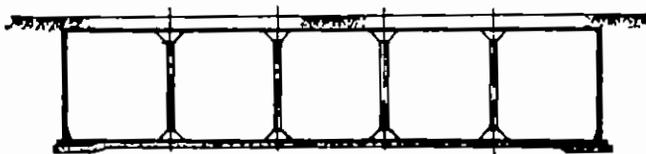


Fig IV-2

The determination of the internal forces in such systems includes no special difficulty as they follow the general rules of the theory and design of reinforced concrete

They may also be composed of one circular slab with or without central support The internal forces in these two types are given in

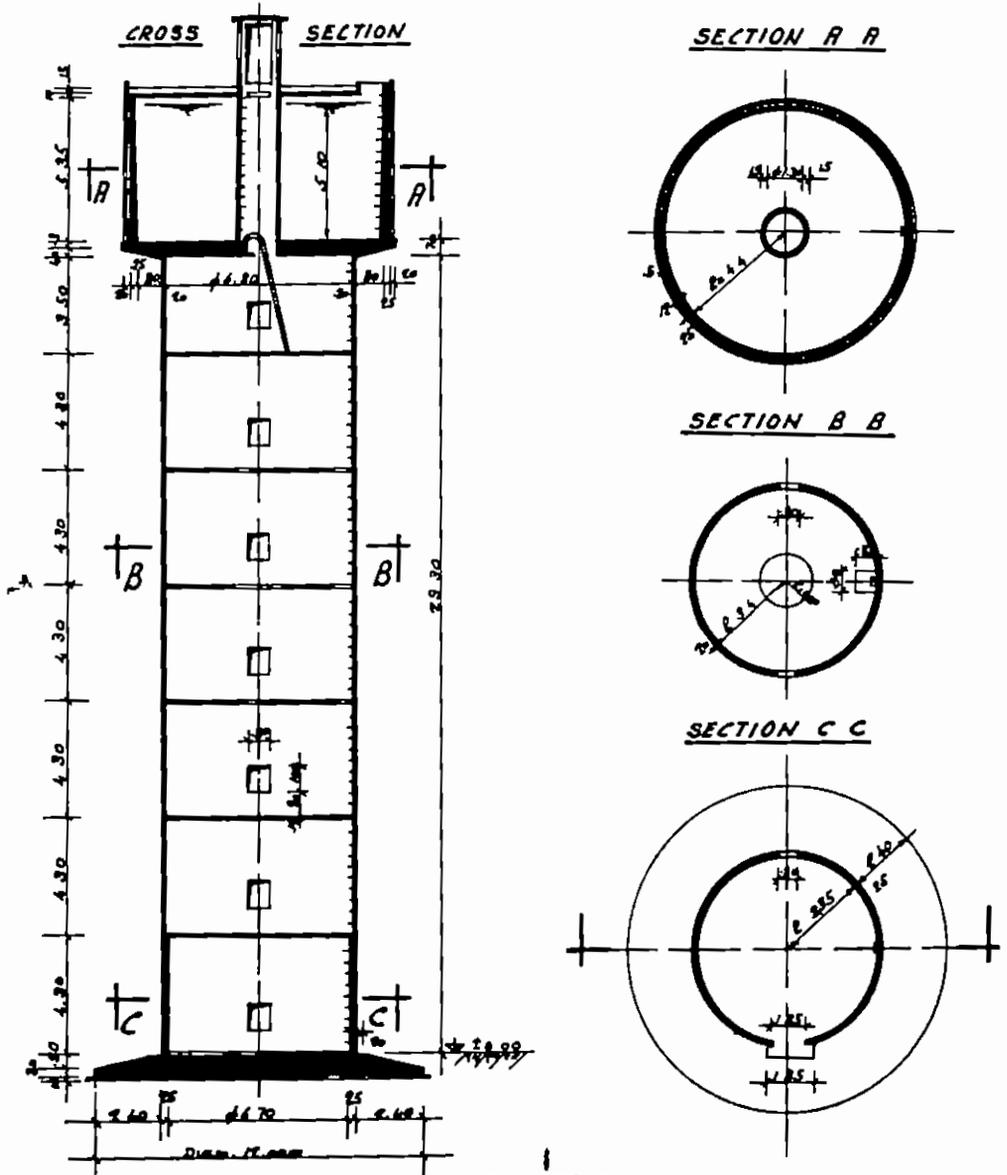


Fig. IV-3a

the table of the P C A The method of application is shown in the numerical example just explained Circular slabs may however be used in circular tanks under different conditions of supports as shown in figures IV-3 a & b Figure a gives a water tower in which circular slabs with and without cantilevers and central holes are used for the roof and the floor of the tank, the intermediate floors of the tower and the foundation Figure b shows a circular tank in which the roof circular slabs are supported on circular beams and columns arranged along radial axes The determination of the internal forces in such

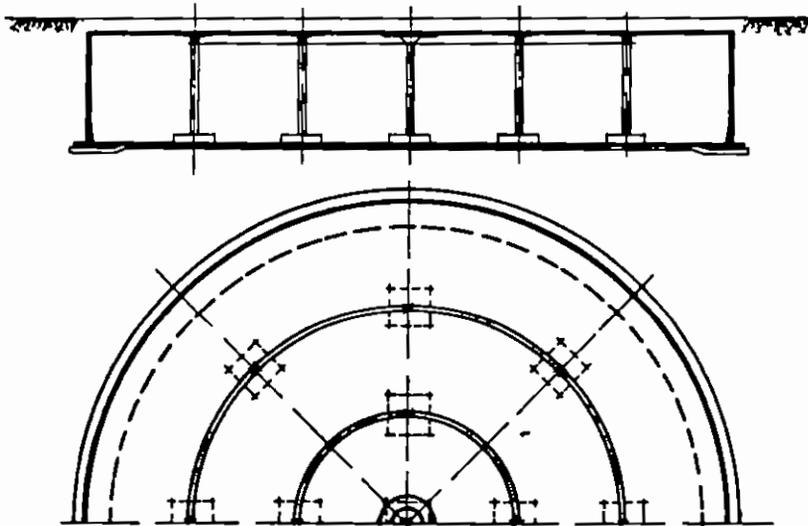


Fig IV-3b

cases are given in article IV-2

In many cases, the choice of a reinforced concrete surface of revolution - for example a dome or a cone - results in an ultimate saving in cost even when the greater expenses of the



#### IV-2 INTERNAL FORCES IN CIRCULAR FLAT PLATES

It will be assumed that

- $p$  = Uniform load per unit area  
 $P$  = Total load, concentrated load or live load per unit length  
 $t$  = Plate thickness  
 $E$  = Modulus of elasticity ( generally = 210 000 kg/cm<sup>2</sup> for concrete )  
 $\nu$  =  $1/m$  = Poisson's ratio For concrete, it varies between 1/10 and 1/5 It is generally assumed in the following equal to 1/6 Some neglected its effect and assume it equal to zero  
 $D = \frac{E t^3}{12 (1 - \nu^2)}$  = Flexural rigidity of a plate  
 $\delta$  = Deflection of the middle surface of the plate  
 $\varphi$  = Slope angle ' ' " " '  
 $A$  = Reaction at the support  
 $Q_r$  = Radial shearing force  
 $M_r$  = Radial bending moment  
 $M_t$  = Tangential bending moment  
 $a$  = Radius of plate from its middle axis to the support  
       = Span of a circular strip of a continuous plate  
 $b = \beta a$  = radius of a specified circle on the plate  
 $r = \rho a$  = radius of any circle on the plate

#### Notes

All logarithms appearing in the following equations<sup>■</sup> are natural ( i e to the base  $e$  )

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#### Reference :

Beton-Kalender 1952 Volume II Page 271 Published by  
 Wilhelm Ernst & Sohn Berlin Munchen Düsseldorf

1) Solid Circular Plate Subjected to a Uniformly Distributed Load

(Fig IV 6)

Load  $p = \text{constant}$ Total load  $P = p \pi a^2$ 

$$Q_r = - \frac{P}{2 \pi a} \rho$$

$$A = \frac{P}{2 \pi a}$$

a) Fixed Edge

$$\delta = \frac{P a^2}{64 \pi D} (1 - \rho^2)^2$$

$$\varphi = \frac{P a}{16 \pi D} \rho (1 - \rho^2)$$

$$M_r = \frac{P}{16 \pi} [1 + \nu - (3 + \nu)\rho^2]$$

$$M_t = \frac{P}{16 \pi} [1 + \nu - (1 + 3\nu)\rho^2]$$

b) Simply Supported Edge

$$\delta = \frac{P a^2}{64 \pi D} (1 - \rho^2) \left( \frac{2 + \nu}{1 + \nu} - \rho^2 \right)$$

$$\varphi = \frac{P a}{16 \pi D} \rho \left( \frac{2 + \nu}{1 + \nu} - \rho^2 \right)$$

$$M_r = \frac{P}{16 \pi} (3 + \nu) (1 - \rho^2)$$

$$M_t = \frac{P}{16 \pi} [3 + \nu - (1 + 3\nu)\rho^2]$$

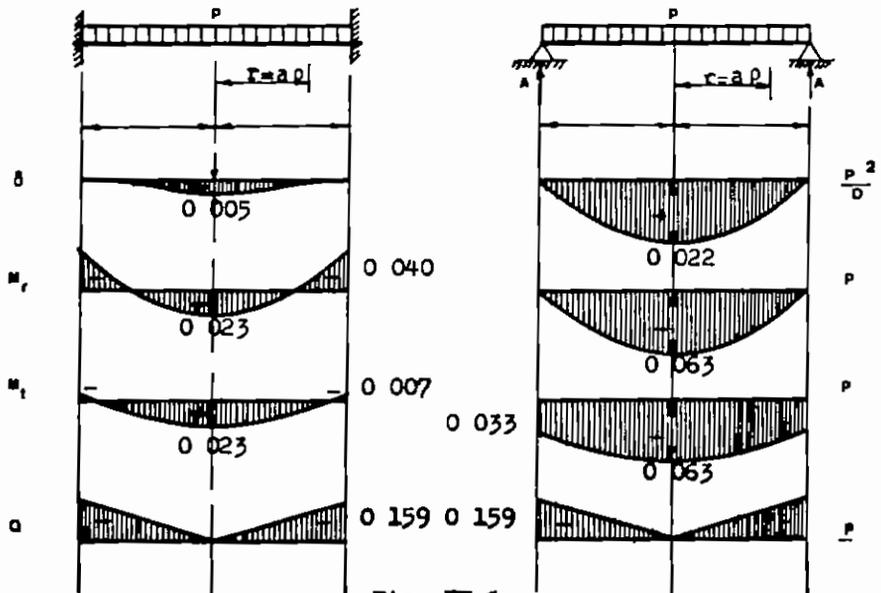


Fig IV-6

2) Solid Circular Plate Subjected to a Concentrated Load P at Center  
(Fig IV 7)

$$Q_r = -\frac{P}{2\pi a} \frac{1}{\rho} \quad \text{for } \rho > \beta, \quad Q_r = 0 \quad \text{for } \rho = 0, \quad A = \frac{P}{2\pi a}$$

a) Fixed Edge

$$\delta = \frac{P a^2}{16\pi D} (1 - \rho^2 + 2\rho^2 \ln \rho) \qquad \varphi = -\frac{Pa}{4\pi D} \rho \ln \rho$$

$$\left. \begin{aligned} M_r &= -\frac{P}{4\pi} [1 + (1 + \nu) \ln \rho] \\ M_t &= -\frac{P}{4\pi} [\nu + (1 + \nu) \ln \rho] \end{aligned} \right\} \quad \text{for } \rho > \beta$$

$$M_r = M_t = -\frac{P}{4\pi} (1 + \nu) \ln \beta \quad \text{for } \rho = 0$$

b) Simply Supported Edge

$$\delta = \frac{P a^2}{16\pi D} \left[ \frac{3 + \nu}{1 + \nu} (1 - \rho^2) + 2\rho^2 \ln \rho \right] \qquad \varphi = \frac{Pa}{4\pi D} \rho \left( \frac{1}{1 + \nu} - \ln \rho \right)$$

$$\left. \begin{aligned} M_r &= -\frac{P}{4\pi} (1 + \nu) \ln \rho \\ M_t &= \frac{P}{4\pi} [1 - \nu - (1 + \nu) \ln \rho] \end{aligned} \right\} \quad \text{for } \rho > \beta$$

$$M_r = M_t = \frac{P}{4\pi} [1 - (1 + \nu) \ln \beta] \quad \text{for } \rho = 0$$

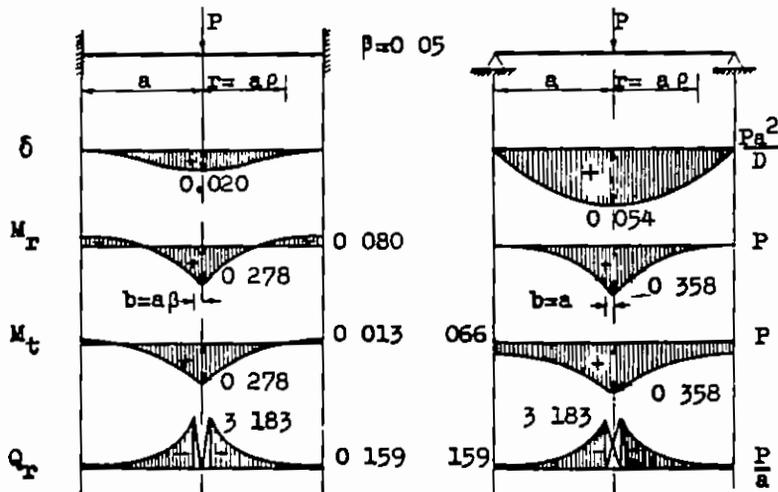


Fig IV-7

3) Solid Circular Plate Subjected to Partial Uniform Load at the Middle

(Fig IV 8)

$$\text{Total load } P = p \pi b^2 = p \pi a^2 \beta^2 \quad \Delta = P / 2 \pi a$$

$$Q_r = - \frac{P}{2 \pi a \beta^2} \rho \quad \text{for} \quad 0 < \rho < \beta$$

$$Q_r = - \frac{P}{2 \pi a} \frac{1}{\rho} \quad \text{for} \quad \beta < \rho < 1$$

a) Fixed Edgei) Inner loaded part  $0 < \rho < \beta$ 

$$\delta = \frac{P a^2}{64 \pi D} \left[ 4 - 3 \beta^2 + 4 \beta^2 \ln \beta - 2 (\beta^2 - 4 \ln \beta) \rho^2 + \frac{\rho^4}{\beta^2} \right]$$

$$\varphi = \frac{P a}{16 \pi D} \rho \left( \beta^2 - 4 \ln \beta - \frac{\rho^2}{\beta^2} \right)$$

$$M_r = \frac{P}{16 \pi} \left[ (1 + \nu) (\beta^2 - 4 \ln \beta) - \frac{3 + \nu}{\beta^2} \rho^2 \right]$$

$$M_t = \frac{P}{16 \pi} \left[ (1 + \nu) (\beta^2 - 4 \ln \beta) - \frac{1 + 3\nu}{\beta^2} \rho^2 \right]$$

ii) Outer unloaded part  $\beta < \rho < 1$ 

$$\delta = \frac{P a^2}{32 \pi D} \left[ (2 - \beta^2) (1 - \rho^2) + 2 (\beta^2 + 2 \rho^2) \ln \rho \right]$$

$$\varphi = \frac{P a}{16 \pi D} \rho \left[ \beta^2 \left( 1 - \frac{1}{\rho^2} \right) - 4 \ln \rho \right]$$

$$M_r = \frac{P}{16 \pi} \left[ -4 + (1 + \nu) \frac{\beta^2}{\rho^2} + (1 + \nu) (\beta^2 - 4 \ln \rho) \right]$$

$$M_t = \frac{P}{16 \pi} \left[ -4 \nu - (1 - \nu) \frac{\beta^2}{\rho^2} + (1 + \nu) (\beta^2 - 4 \ln \rho) \right]$$

b) Simply Supported Edge  $0 < \rho < \beta$ i) Inner loaded part

$$\delta = \frac{P a^2}{64 \pi D} \frac{1}{1 + \nu} \left\{ 4 (3 + \nu) - (7 + 3 \nu) \beta^2 + 4 (1 + \nu) \beta^2 \ln \beta - 2 \left[ 4 - (1 - \nu) \beta^2 - 4 (1 + \nu) \ln \beta \right] \rho^2 + \frac{1 + \nu}{\beta^2} \rho^4 \right\}$$

$$\varphi = \frac{Pa}{16 \pi D} \frac{\rho}{1+\nu} \left[ 4 - (1-\nu)\rho^2 - (1+\nu) \ln \rho - \frac{1+\nu}{\rho^2} \rho^2 \right]$$

$$M_r = \frac{P}{16 \pi} \left[ 4 - (1-\nu)\rho^2 - 4(1+\nu) \ln \rho - \frac{3+\nu}{\rho^2} \rho^2 \right]$$

$$M_t = \frac{P}{16 \pi} \left[ 4 - (1-\nu)\rho^2 - 4(1+\nu) \ln \rho - \frac{1+3\nu}{\rho^2} \rho^2 \right]$$

11) Outer unloaded part

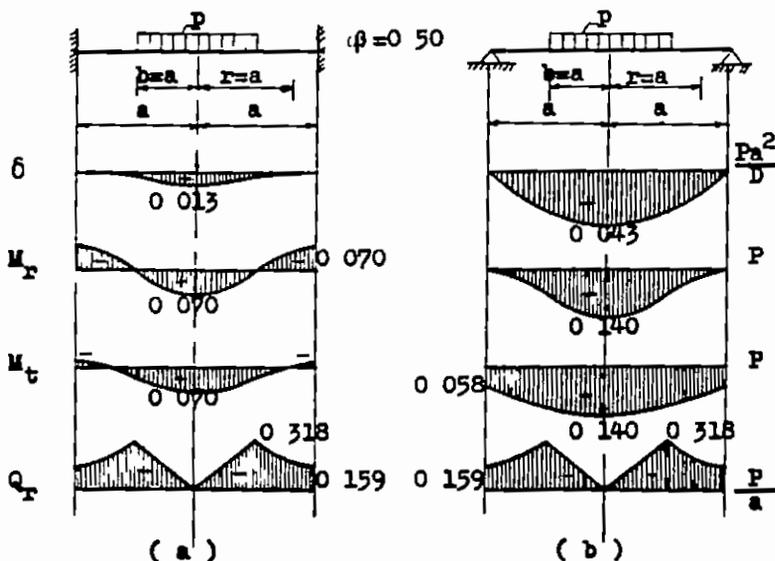
$$\beta \leq \rho < 1$$

$$\delta = \frac{Pa^2}{32 \pi D} \frac{1}{1+\nu} \left\{ \left[ 2(3+\nu) - (1-\nu)\rho^2 \right] (1-\rho^2) + 2(1+\nu)\rho^2 \ln \rho + 4(1+\nu)\rho^2 \ln \rho \right\}$$

$$\varphi = \frac{Pa}{16 \pi D} \frac{1}{1+\nu} \left\{ \left[ 4 - (1-\nu)\rho^2 \right] \rho - (1+\nu) \frac{\rho^2}{\rho} - 4(1+\nu)\rho \ln \rho \right\}$$

$$M_r = \frac{P}{16 \pi} \left[ (1-\nu)\rho^2 \left( \frac{1}{\rho^2} - 1 \right) - 4(1+\nu) \ln \rho \right]$$

$$M_t = \frac{P}{16 \pi} \left\{ (1-\nu) \left[ 4 - \rho^2 \left( \frac{1}{\rho^2} + 1 \right) \right] - 4(1+\nu) \ln \rho \right\}$$



Solid Circular Plate Subjected to Partial Uniform Load at the Middle

Fig IV-8

4) Solid Circular Plate Loaded by a Ring Load

(Fig IV-9)

$$\begin{array}{lll} \text{Ring load} & P/m & A = P \beta \\ Q_r = 0 & \text{for } 0 < \rho < \beta & Q_r = -\frac{P\beta}{\rho} \quad \text{for } \beta < \rho < 1 \end{array}$$

a) Fixed Edge1) Inner part  $0 < \rho \leq \beta$ 

$$\delta = \frac{P a^3}{8 D} \beta \left[ 1 - \beta^2 + 2 \beta^2 \ln \beta + (1 + \beta^2 + 2 \ln \beta) \rho^2 \right]$$

$$\varphi = -\frac{P a^2}{8 D} \beta \rho (1 - \beta^2 + 2 \ln \beta)$$

$$r_r = r_t = -\frac{P a}{4} \beta (1 + \nu) (1 - \beta^2 + 2 \ln \beta)$$

1.1) Outer part  $\beta < \rho < 1$ 

$$\delta = \frac{P a^3}{8 D} \beta \left[ (1 + \beta^2) (1 - \rho^2) + 2 (\beta^2 + \rho^2) \ln \rho \right]$$

$$\varphi = \frac{P a^2}{4 D} \beta \rho \left[ \beta^2 \left(1 - \frac{1}{\rho^2}\right) - 2 \ln \rho \right]$$

$$M_r = -\frac{P a}{4} \beta \left[ 2 - (1 - \nu) \frac{\rho^2}{\beta^2} - (1 + \nu) (\beta^2 - 2 \ln \rho) \right]$$

$$r_t = -\frac{P a}{4} \beta \left[ 2\nu + (1 + \nu) \frac{\rho^2}{\beta^2} - (1 + \nu) (\beta^2 - 2 \ln \rho) \right]$$

b) Simply Supported Edge1) Inner part  $0 < \rho < \beta$ 

$$\delta = \frac{P a^3}{8 D} \frac{\beta}{1 + \nu} \left\{ (3 + \nu) (1 - \beta^2) + 2 (1 + \nu) \beta^2 \ln \beta - [(1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta] \rho^2 \right\}$$

$$\varphi = \frac{P a^2}{4 D} \frac{\beta \rho}{1 + \nu} \left[ (1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta \right]$$

$$M_r = M_t = \frac{P a}{4} \beta \left[ (1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta \right]$$

ii) Outer part

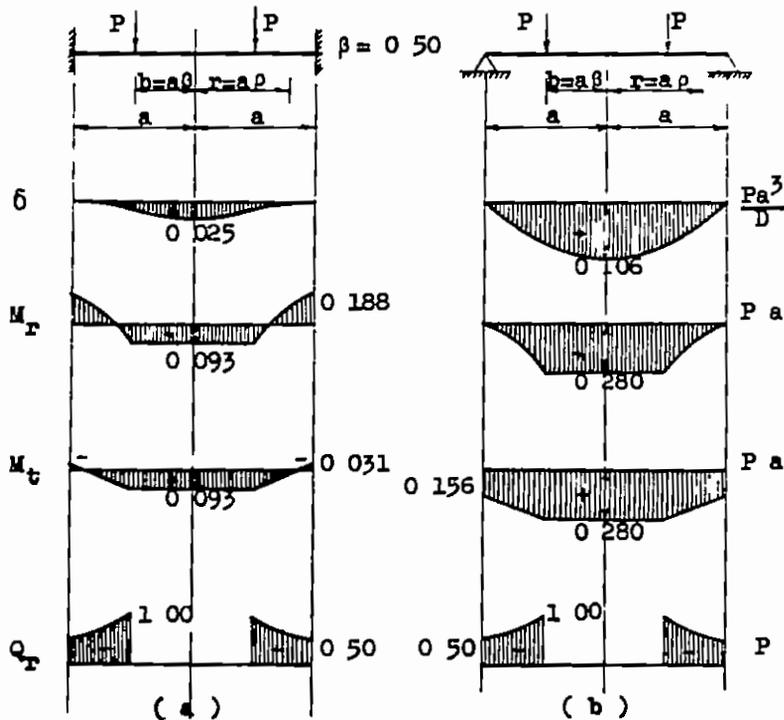
$$\beta < \rho < 1$$

$$\delta = \frac{P a^3}{8 D} \frac{\beta}{1 + \nu} \left\{ \left[ 3 + \nu - (1 - \nu) \beta^2 \right] (1 - \rho^2) + 2 (1 + \nu) (\beta^2 + \rho^2) \ln \rho \right\}$$

$$\varphi = \frac{P a^2}{4 D} \frac{\beta \rho}{1 + \nu} \left[ 2 - (1 - \nu) \beta^2 - (1 + \nu) \frac{\beta^2}{\rho^2} - 2 (1 + \nu) \ln \rho \right]$$

$$r_r = \frac{P a}{4} \beta \left[ (1 - \nu) \beta^2 \left( \frac{1}{\rho^2} - 1 \right) - 2 (1 + \nu) \ln \rho \right]$$

$$L_t = \frac{P a}{4} \beta \left\{ (1 - \nu) \left[ 2 - \beta^2 \left( \frac{1}{\rho^2} + 1 \right) \right] - 2 (1 + \nu) \ln \rho \right\}$$



Solid Circular Plate Loaded by a Ring Load

Fig. IV-9

5) Solid Circular Plate Subjected to an Intermediate Radial Ring Moment

(Fig IV-10)

$$M \frac{\text{kgm}}{\text{m}}$$

$$Q_r = 0$$

$$A = 0$$

a) Fixed Edge1) Inner part

$$0 \leq \rho < \beta$$

$$\delta = -\frac{M a^2}{4 D} \left[ 2\beta^2 \ln \beta + (1 - \beta^2) \rho^2 \right] \quad \psi = \frac{M a}{2 D} (1 - \beta^2) \rho$$

$$M_r = M_t = \frac{M}{2} (1 + \nu) (1 - \beta^2)$$

11) Outer part

$$\beta < \rho < 1$$

$$\delta = -\frac{M a^2}{4 D} \beta^2 (1 - \rho^2 + 2 \ln \rho) \quad \psi = \frac{M a}{2 D} \beta^2 \left( \frac{1}{\rho} - \rho \right)$$

$$M_r = -\frac{M \beta^2}{2} \left[ 1 + \nu + (1 - \nu) \frac{1}{\rho^2} \right] \quad t = -\frac{M \beta^2}{2} \left[ 1 + \nu - (1 - \nu) \frac{1}{\rho^2} \right]$$

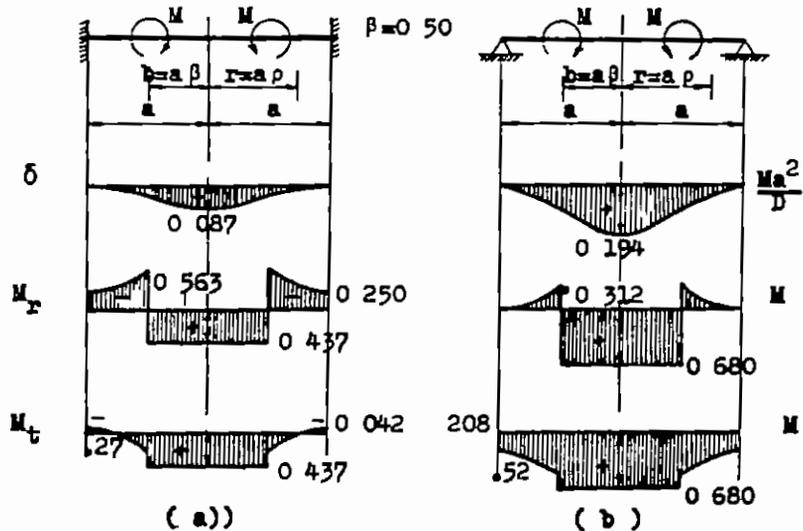


Fig. IV-10

b) Simply Supported Edge1) Inner part

$$0 < \rho < \beta$$

$$\delta = \frac{M a^2}{4 D} \frac{1}{1 + \nu} \left\{ 2 \beta^2 \left[ 1 - (1 - \nu) \ln \beta \right] - \left[ 1 + \nu + (1 - \nu) \beta^2 \right] \rho^2 \right\}$$

$$\varphi = \frac{M a}{2 D} \frac{1}{1 + \nu} \rho \left[ 1 + \nu + (1 - \nu) \beta^2 \right]$$

$$M_r = M_t = \frac{M}{2} \left[ 1 + \nu + (1 - \nu) \beta^2 \right]$$

ii) Outer part

$$\delta = \frac{M a^2}{4 D} \frac{\beta^2}{1 + \nu} \left[ (1 - \nu) (1 - \rho^2) - 2 (1 + \nu) \ln \rho \right]$$

$$\varphi = \frac{M a}{2 D} \frac{\beta^2}{1 + \nu} \left[ (1 - \nu) \rho + (1 + \nu) \frac{1}{\rho} \right]$$

$$M_r = \frac{M}{2} (1 - \nu) \beta^2 \left( 1 - \frac{1}{\rho^2} \right) \quad M_t = \frac{1}{2} (1 - \nu) \beta^2 \left( 1 - \frac{1}{\rho^2} \right)$$

6) Simply Supported Solid Circular Plate Subjected to Radial Ring Moments at its Edges

Fig IV-11

Radial moment  $M \frac{\text{kgm}}{\text{m}}$ 

$$\delta = \frac{M a^2}{2 D (1 + \nu)} (1 - \rho^2)$$

$$\varphi = \frac{M a}{D (1 + \nu)} \rho$$

$$M_r = M_t = M$$

$$r = 0 \quad A = 0$$

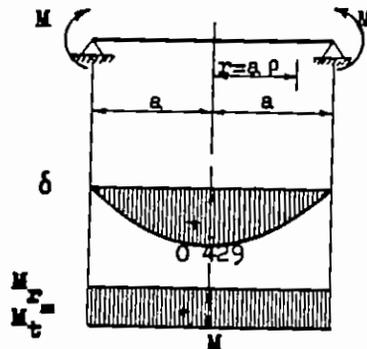


Fig. IV-11

7) Circular Plate with a Circular Hole at the Center and Subjected to a Uniformly Distributed Load  $p/m^2$  (Fig IV-12)

$$A = \frac{p a}{2} (1 - \beta^2) \quad \text{for } \beta < 1 \quad A = \frac{p a}{2} (\beta^2 - 1) \quad \text{for } \beta > 1$$

$$Q_r = -\frac{p a}{2} \left( \rho - \frac{\beta^2}{\rho} \right)$$

a) Fixed Edge

assuming  $k_1 = \beta^2 \frac{(1 - \nu)\beta^2 + (1 + \nu)(1 + 4\beta^2 \ln \beta)}{1 - \nu + (1 + \nu)\beta^2}$  then

$$\delta = \frac{p a^4}{64 D} \left[ -1 + 2(1 - k_1 - 2\beta^2)(1 - \rho^2) + \rho^4 - 4k_1 \ln \rho - 8\beta^2 \rho^2 \ln \rho \right]$$

$$\varphi = \frac{p a^3}{16 D} \left[ (1 - k_1) \rho - \rho^3 + k_1 \frac{1}{\rho} + 4\beta^2 \rho \ln \rho \right]$$

$$M_r = \frac{p a^2}{16} \left[ (1 + \nu)(1 - k_1) + 4\beta^2 - (3 + \nu)\rho^2 - (1 - \nu)k_1 \frac{1}{\rho^2} + 4(1 + \nu)\beta^2 \ln \rho \right]$$

$$M_t = \frac{p a^2}{16} \left[ (1 + \nu)(1 - k_1) + 4\nu\beta^2 - (1 + 3\nu)\rho^2 + (1 - \nu)k_1 \frac{1}{\rho^2} + 4(1 + \nu)\beta^2 \ln \rho \right]$$

b) Simply Supported Edge

assuming  $k_2 = \beta^2 \left[ 3 + \nu + 4(1 + \nu) \frac{\beta^2}{1 - \beta^2} \ln \beta \right]$  then

$$\delta = \frac{p a^4}{64 D} \left\{ \frac{2}{1 + \nu} \left[ (3 + \nu)(1 - 2\beta^2) + k_2 \right] (1 - \rho^2) - (1 - \rho^4) - \frac{4}{1 - \nu} k_2 \ln \rho - 8\beta^2 \rho^2 \ln \rho \right\}$$

$$\varphi = \frac{p a^3}{16 D} \left[ \frac{1}{1 + \nu} (3 + \nu - 4\beta^2 + k_2) \rho - \rho^3 + \frac{k_2}{1 - \nu} \frac{1}{\rho} + 4\beta^2 \rho \ln \rho \right]$$

$$M_r = \frac{p a^2}{16} \left[ (3 + \nu)(1 - \rho^2) + k_2 \left(1 - \frac{1}{\rho^2}\right) + 4(1 + \nu)\beta^2 \ln \rho \right]$$

$$M_t = \frac{p a^2}{16} \left[ 2(1 - \nu)(1 - 2\beta^2) + (1 + 3\nu)(1 - \rho^2) + k_2 \left(1 + \frac{1}{\rho^2}\right) + 4(1 + \nu)\beta^2 \ln \rho \right]$$

Circular Plate with a Circular Hole at the Center  
 Subjected to  
 a Uniformly Distributed Load  $p / m^2$

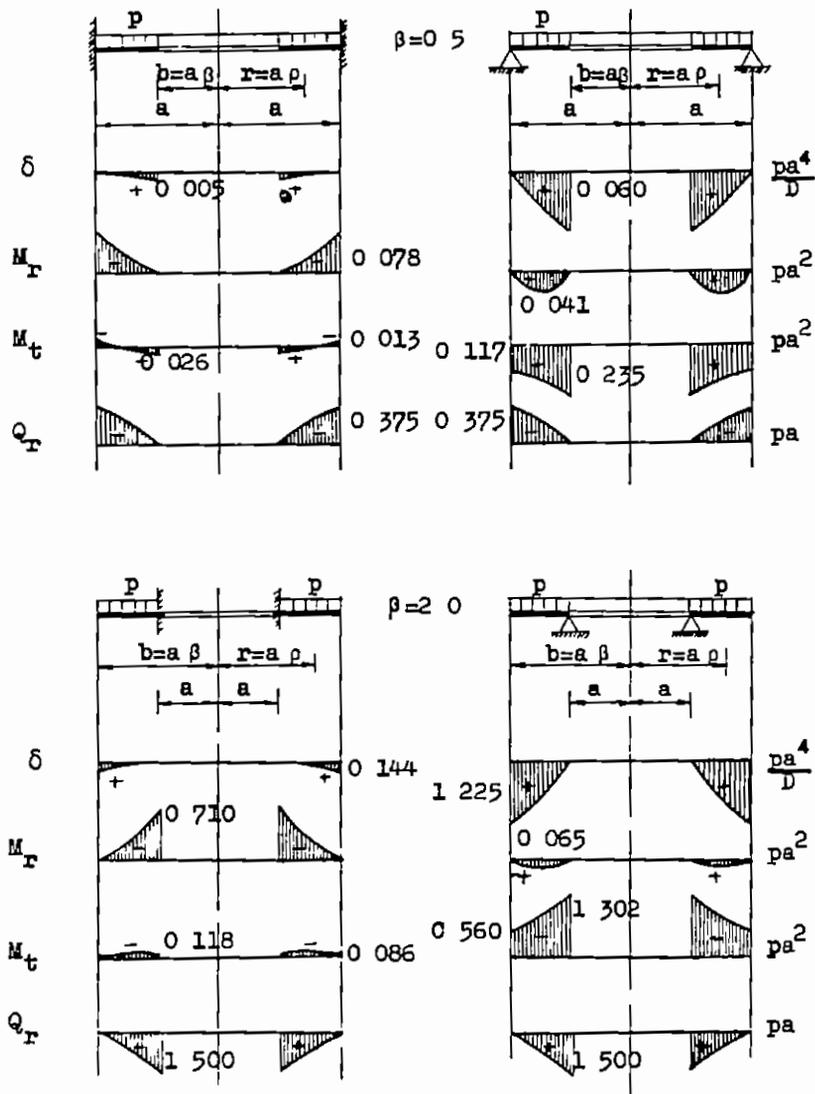


Fig IV-12

8) Circular Plate with a Circular Hole at the Center and Subjected to a Ring Load along the Inner Edge  $\rho = \beta$

(Fig IV-13 )

Load  $P / m$

$$Q_r = -P \beta \frac{1}{\rho}$$

$A = P \beta$

for  $\beta > 1$

$A = -P \beta$

for  $\beta > 1$

a) Fixed Edge

Assuming  $k_3 = \beta^2 \frac{1 + (1 + \nu) \ln \beta}{1 - \nu + (1 + \nu) \beta^2}$  then

$$\delta = \frac{P a^3}{8 D} \beta \left[ (1 + 2 k_3) (1 - \rho^2) + 4 k_3 \ln \rho + 2 \rho^2 \ln \rho \right]$$

$$\varphi = \frac{P a^2}{2 D} \beta \left[ k_3 \left( \rho - \frac{1}{\rho} \right) - \rho \ln \rho \right]$$

$$M_r = \frac{P a}{2} \beta \left[ -1 + (1 + \nu) k_3 + (1 - \nu) k_3 \frac{1}{\rho^2} - (1 + \nu) \ln \rho \right]$$

$$M_t = \frac{P a}{2} \beta \left[ -\nu + (1 + \nu) k_3 - (1 - \nu) k_3 \frac{1}{\rho^2} - (1 + \nu) \ln \rho \right]$$

b) Simply Supported Edge

Assuming  $k_4 = (1 + \nu) \frac{\beta^2}{1 - \beta^2} \ln \beta$  then

$$\delta = \frac{P a^3}{8 D} \beta \left[ \frac{3 + \nu - 2 k_4}{1 + \nu} (1 - \rho^2) + \frac{4 k_4}{1 - \nu} \ln \rho + 2 \rho^2 \ln \rho \right]$$

$$\varphi = \frac{P a^2}{2 D} \beta \left[ \frac{1 - k_4}{1 + \nu} \rho - \frac{k_4}{1 - \nu} \frac{1}{\rho} - \rho \ln \rho \right]$$

$$M_r = \frac{P a}{2} \beta \left[ k_4 \left( \frac{1}{\rho^2} - 1 \right) - (1 + \nu) \ln \rho \right]$$

$$M_t = \frac{P a}{2} \beta \left[ 1 - \nu - k_4 \left( \frac{1}{\rho^2} + 1 \right) - (1 + \nu) \ln \rho \right]$$

**Circular Plate with a Circular Hole at the Center  
Subjected to  
a Ring Load along the Inner Ring =**

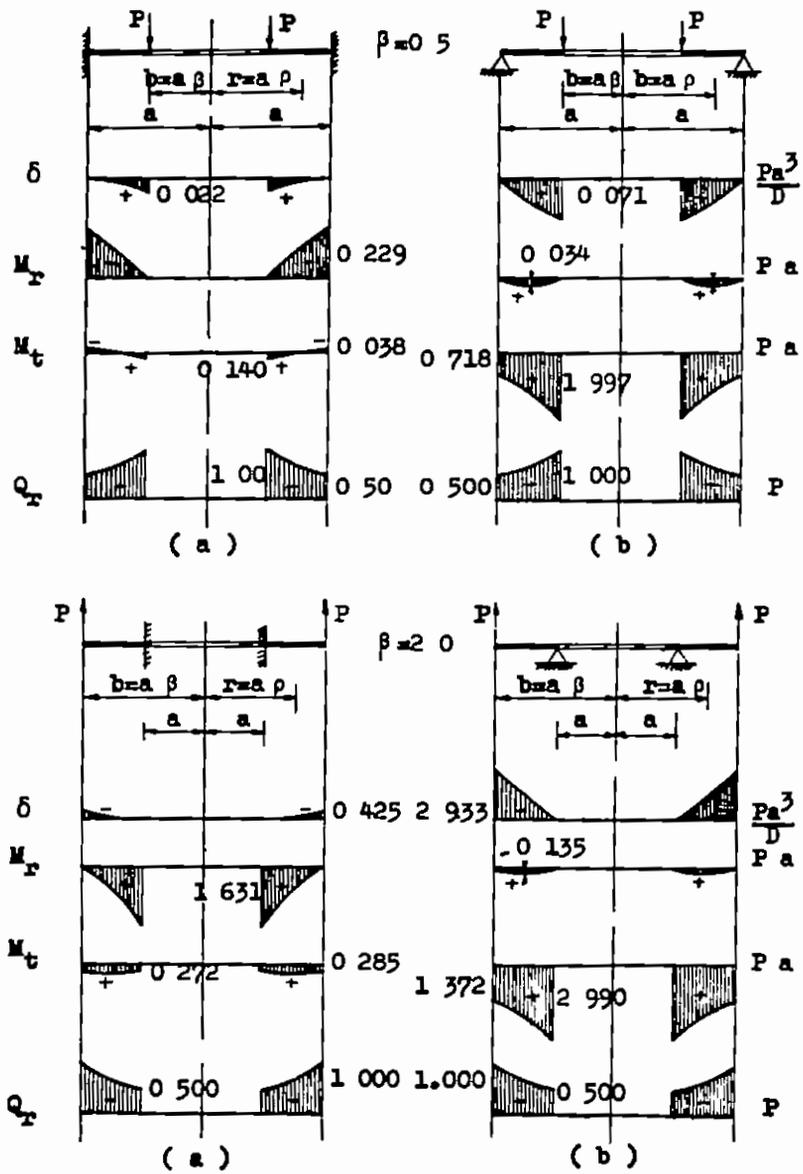


Fig IV-13

9) Circular Plate with a Circular Hole at the Center and Subjected to Radial Ring Moments on the Inner Edge  $\rho = \beta$

(Fig IV-14)

$$M \frac{kcm}{m} \quad Q_r = 0 \quad A = 0$$

a) Fixed Edge

Assuming  $k_5 = \frac{\beta^2}{1 - \nu + (1 + \nu) \beta^2}$  then

$$\delta = \frac{M a^2}{2 D} k_5 (-1 + \rho^2 - 2 \ln \rho) \quad \varphi = \frac{1 a}{D} k_5 \left( \frac{1}{\rho} - \rho \right)$$

$$M_r = -M k_5 \left[ 1 + \nu + (1 - \nu) \frac{1}{\rho^2} \right] \quad M_t = -1 k_5 \left[ 1 + \nu - (1 - \nu) \frac{1}{\rho^2} \right]$$

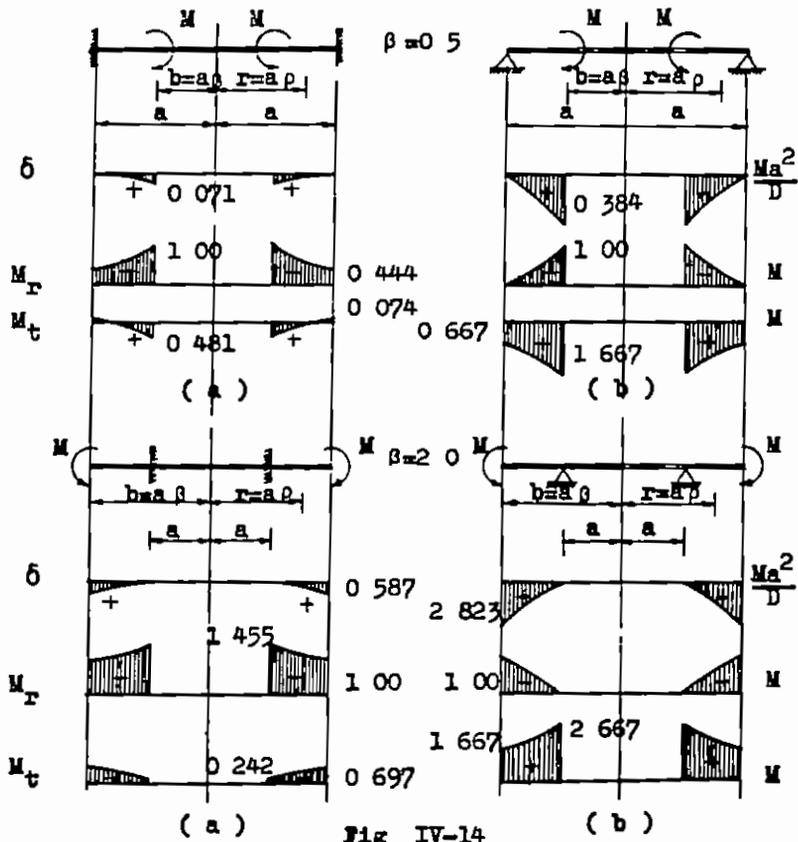


Fig IV-14

b) Simply Supported Edge

Assuming  $k_6 = \frac{\beta^2}{1 - \beta^2}$  then

$$\delta = \frac{M a^2}{2 D} \frac{k_6}{1 + \nu} \left( 1 - \rho^2 - 2 \frac{1 + \nu}{1 - \nu} \ln \rho \right)$$

$$\varphi = \frac{M a}{D} \frac{k_6}{1 + \nu} \left( \rho + \frac{1 + \nu}{1 - \nu} \frac{1}{\rho} \right)$$

$$M_r = M k_6 \left( 1 - \frac{1}{\rho^2} \right)$$

$$M_t = M k_6 \left( 1 + \frac{1}{\rho^2} \right)$$

10) Simply Supported Circular Plate with a Circular Hole at the Center and Subjected to Radial Ring Moments along the Supported Edge

Assume radial moment  $M$  in kg/m and  $k_7 = \frac{1}{1 - \beta^2}$  then (Fig IV-15)

$$\delta = \frac{M a^2}{2 D} \frac{k_7}{1 + \nu} \left( 1 - \rho^2 - 2 \frac{1 + \nu}{1 - \nu} \beta^2 \ln \rho \right)$$

$$\varphi = \frac{M a}{D} \frac{k_7}{1 + \nu} \left( \rho + \frac{1 + \nu}{1 - \nu} \beta^2 \frac{1}{\rho} \right)$$

$$M_r = M \beta^2 k_7 \left( \frac{1}{\beta^2} - \frac{1}{\rho^2} \right)$$

$$Q_r = 0$$

$$A = 0$$

$$M_t = M \beta^2 k_7 \left( \frac{1}{\beta^2} + \frac{1}{\rho^2} \right)$$

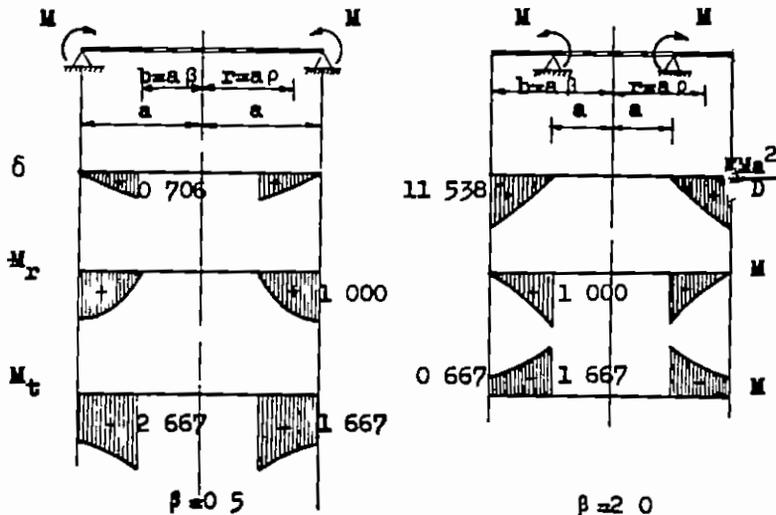


Fig IV-15

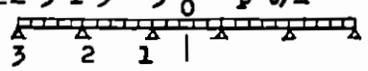
II RADIAL MOMENTS ( $M_r$ ), TANGENTIAL MOMENTS ( $M_t$ ) AND REACTIONS OF CIRCULAR FLAT PLATES CONTINUOUS OVER RING SUPPORTS AT EQUAL DISTANCES (a)

a) <u>Three equal spans</u>					b) <u>Four equal spans</u>				
$v = 1/6$					$p = 0, \quad M_r = M_t = \infty \quad v = 1/6$				
BENDING MOMENTS					BENDING MOMENTS				
p	Case 1		Case 2		p	Case 1		Case 2	
	$M_r$	$M_t$	$M_r$	$M_t$		$M_r$	$M_t$	$M_r$	$M_t$
1 5	0	+ 0188	- 0835	- 0139	2 0	0	+ 0129	- 0811	- 0135
1 3	+ 0578	+ 0271	- 0132	+ 0049	1 6	+ 0847	+ 0179	+ 0346	+ 0093
1 1	+ 0889	+ 0225	+ 0330	+ 0119	1 5	+ 0848	+ 0127	+ 0444	+ 0086
1 0	+ 0925	+ 0151	+ 0455	+ 0100	1 4	+ 0748	+ 0053	+ 0449	+ 0057
0 9	+ 0875	+ 0042	+ 0506	+ 0068	1 0	- 1069	- 0314	- 0769	- 0172
0 7	+ 0411	- 0267	+ 0300	- 0097	0 6	- 0008	+ 0080	+ 0179	+ 0109
0 5	- 0851	- 0591	- 0552	- 0291	0 5	+ 0137	+ 0110	+ 0284	+ 0099
0 3	- 0535	- 0444	- 0235	- 0141	0 4	+ 0225	+ 0104	+ 0322	+ 0044
0 1	- 0376	- 0336	- 0077	- 0066	0 1	- 0035	- 0310	- 0244	- 0676
0 0	- 0357	- 0357	- 0057	- 0057	Mult	$p a^2$	$p a^2$	$p a^2$	$p a^2$
Mult	$p a^2$	$p a^2$	$p a^2$	$p a^2$	REACTIONS				
REACTIONS					REACTIONS				
Sup	Case 1		Case 2		Sup	Case 1		Case 2	
2	+ 0 3618		+ 0 4535		2	+0 3655		+0 5612	
1	+ 1 1642		+ 0 8898		1	+1 2001		+0 9131	
0					0	+0 4330		+0 6709	
Mult	p a		p a		Mult	p a	$p a^2$	p a	$p a^2$

RADIAL MOMENTS ( $M_r$ ), TANGENTIAL MOMENTS ( $M_t$ ) AND REACTIONS OF CIRCULAR FLAT PLATES CONTINUOUS OVER RING SUPPORTS AT EQUAL DISTANCES ( a )

**c) Five equal spans**

$\rho = 2 \ 5 \ 1 \ 5 \ 5 \ 0 \ p \ t/m$

Case 1 

Case 2 

$v = 1/6$

**BENDING MOMENTS**

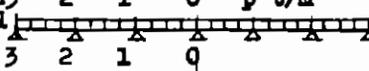
$\rho$	Case 1		Case 2	
	$M_r$	$M_t$	$M_r$	$M_t$
2 5	0	+ 0098	- 0787	- 0131
2 1	+ 0818	+ 0159	+ 0348	+ 0076
2 0	+ 0799	+ 0118	+ 0425	+ 0075
1 9	+ 0676	+ 0058	+ 0403	+ 0051
1 5	- 1130	- 0254	- 0875	- 0155
1 1	+ 0132	+ 0082	+ 0302	+ 0115
1 0	+ 0289	+ 0094	+ 0432	+ 0107
0 9	+ 0375	+ 0077	+ 0488	+ 0069
0 5	- 0446	- 0186	- 0537	- 0277
0 1	+ 0029	+ 0039	- 0062	- 0052
0 0	+ 0048	+ 0048	- 0043	- 0043
ult	$p a^2$	$p a^2$	$p a^2$	$p a^2$

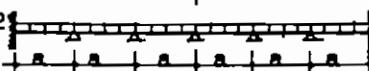
**REACTIONS**

Sup	Case 1	Case 2
3	+ 0 3675	+ 0 4594
2	+ 1 2065	+ 1 0255
1	+ 0 7931	+ 0 8765
ult	$p a$	$p a$

**d) Six equal spans**

$\rho = 3 \ 2 \ 1 \ 0 \ p \ t/m$

Case 1 

Case 2 

$\rho = 0, \quad M_r = M_t = -\infty \quad v = 1/6$

**BENDING MOMENTS**

$\rho$	Case 1		Case 2	
	$M_r$	$M_t$	$M_r$	$M_t$
3 0	0	+ 0081	- 0799	- 0133
2 6	+ 0814	+ 0153	+ 0354	+ 0076
2 5	+ 0789	+ 0118	+ 0426	+ 0073
2 4	+ 0659	+ 0066	+ 0401	+ 0052
2 0	- 1106	- 0232	- 0852	- 0149
1 6	+ 0164	+ 0062	+ 0321	+ 0089
1 5	+ 0296	+ 0069	+ 0422	+ 0084
1 4	+ 0340	+ 0058	+ 0433	+ 0057
1 0	- 0660	- 0121	- 0753	- 0164
0 6	+ 0247	+ 0120	+ 0189	+ 0110
0 5	+ 0338	+ 0095	+ 0292	+ 0098
0 4	+ 0357	+ 0022	+ 0327	+ 0040
0 1	- 0320	- 0809	- 0255	- 0695
ult	$p a^2$	$p a^2$	$p a^2$	$p a^2$

**REACTIONS**

Sup	Case 1	Case 2
3	+0 3725	+0 4675
2	+1 1500	+0 9733
1	+0 9622	+1 0420
0	+0 7573	+0 6834
ult	$p a$	$p a$

### IV-3 MEMBRANE FORCES IN SURFACES OF REVOLUTION <sup>⊠</sup>

It is assumed here that the thickness of the shell is so small that it can be considered as a membrane which can resist meridian and ring forces, in the plane of the surface only i.e. the bending moments due to fixation at supports, unsymmetrical loading and similar effects are neglected

#### IV-3 1 Notations

It will be assumed that ( Fig IV-16 )

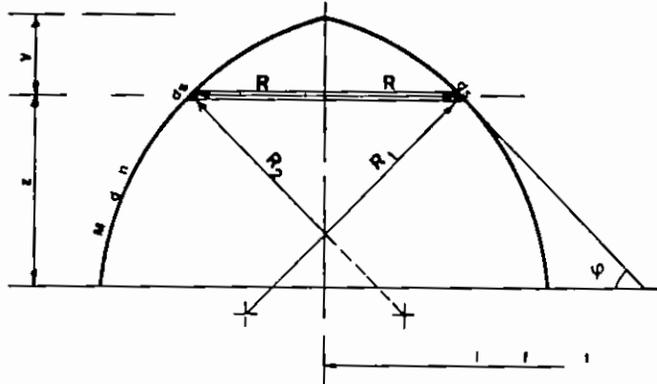


Fig IV-16

$R$  = radius normal to axis of revolution of any circular ring at any plane  $z - z$

$R_1$  = radius of curvature of meridian

$R_2$  = cross radius curvature along the normal - to axis of rotation

$N_\phi$  (evt  $T_1$ ) = resultant meridian force per unit length of circumference

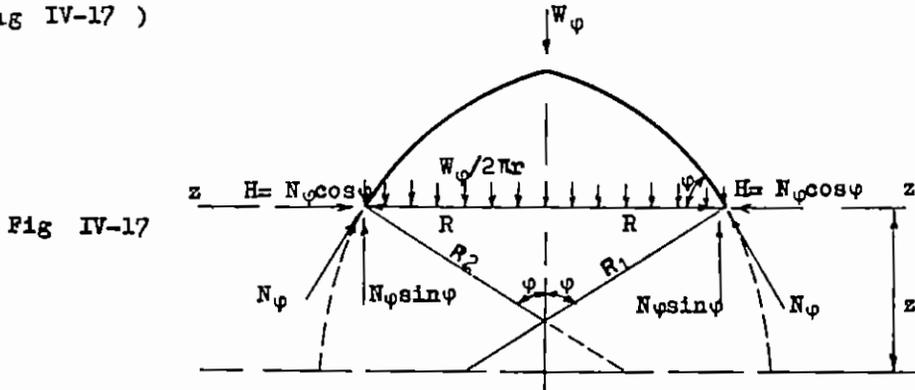
$N_\theta$  (evt  $T_2$ ) = resultant ring force per unit length of meridian

$H$  = horizontal thrust of shell per unit length of circumference

$W_\phi$  = Sum of vertical forces above  $z - z$  (expressed through the angle  $\phi$ )

### IV-3 2 The Meridian Force

In order to have equilibrium at any horizontal section  $z - z$ , the vertical component of the meridian forces  $N_\varphi$  must be equal to the vertical load above  $z - z$  per meter run circumference. Hence we get ( Fig IV-17 )



$$l_\varphi / 2 \pi R = N_\varphi \sin \varphi \quad \text{or}$$

$$l_\varphi = \frac{w_\varphi / 2 \pi R \sin \varphi}{\sin \varphi} \quad (43)$$

But  $r = R_2 \sin \varphi$ , so that  $N_\varphi$  can also be given in the form

$$N_\varphi = \frac{l_\varphi / 2 \pi R_2 \sin^2 \varphi}{\sin^2 \varphi} \quad (44)$$

The horizontal thrust  $H$  per unit length of circumference is

$$H = \frac{l_\varphi / 2 \pi R \tan \varphi}{\tan \varphi} = N \cos \varphi \quad (45)$$

### IV-3 3 The Ring Force

Assuming that the radial component of the external loads per square meter surface is  $p_r$  and considering the equilibrium of the external and internal forces normal to the surface, one can prove that

$$\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} = p_r \quad (46)$$

For a spherical surface  $R_1 = R_2 = a$  and

$$\frac{N_\varphi}{a} + \frac{N_\theta}{a} = p_r a \quad (47)$$



$$N_{\varphi} = \xi \frac{a}{1 + \cos \varphi} = \xi \frac{a^2}{a + z} \quad (53)$$

$$N_{\theta} = \xi a \left( \cos \varphi - \frac{1}{1 + \cos \varphi} \right) = \xi \left( z - \frac{a^2}{a + z} \right) \quad (54)$$

The internal forces due to a vertical load  $\underline{p/m^2}$  horizontal is shown in figure IV-20

$$V_{\varphi} = p \pi a^2 \sin^2 \varphi \quad (55)$$

$$H = p a \frac{\cos \varphi}{2} = p z/2 \quad (56)$$

$$N_{\varphi} = p a/2 = \text{constant}$$

$$N_{\theta} = p \frac{a \cos 2\varphi}{2} = \frac{p}{2a} (2z^2 - a^2) \quad (57)$$

The internal forces due to a liquid pressure as that shown in Fig IV-21 is given by

Assuming  $w = \text{weight} / m^3$  liquid, then

$$v_r = w (h_1 + a - a \cos \varphi) \quad (58)$$

$$N_{\varphi} = -w \frac{a}{6} \left[ 3 h_1 + a \frac{1 - \cos \varphi}{1 + \cos \varphi} (1 + 2 \cos \varphi) \right] \quad (59)$$

$$N_{\theta} = +w \frac{a}{6} \left[ 3 h_1 + a \frac{1 - \cos \varphi}{1 + \cos \varphi} (5 + 4 \cos \varphi) \right] \quad (60)$$

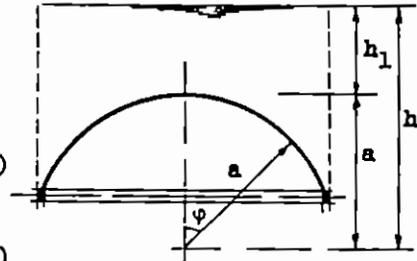


Fig IV-21

#### b) Conical Shells

The surface area of a conical shell (fig IV-22) is given by :

$$A = 2 \pi R s / 2 \quad (61)$$

The meridian force is

$$N_{\theta} = V_{\varphi} / 2 \pi y \cos \varphi \quad (62)$$

The ring force is

$$N_{\varphi} = p_r y \cos \varphi / \sin^2 \varphi \quad (63)$$

Internal forces due to a vertical dead load  $g/m^2$  surface are :

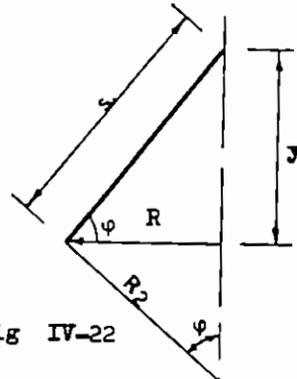


Fig IV-22

$$N_s = g y / 2 \sin^2 \varphi \quad (64) \quad N_\theta = g R^2 / y \quad (65)$$

Internal forces due to a vertical load  $p/m^2$  horizontal are

$$N_s = p y \cos \varphi / 2 \sin^2 \varphi \quad (66) \quad N_\theta = p y \cos^3 \varphi / \sin^2 \varphi \quad (67)$$

The internal forces in the conical floor of an Inzet tank (fig IV-23) are

Assuming  $w = \text{weight} / m^3$  liquid, then

$$p_r = w (h_2 - s \sin \varphi) \quad (68)$$

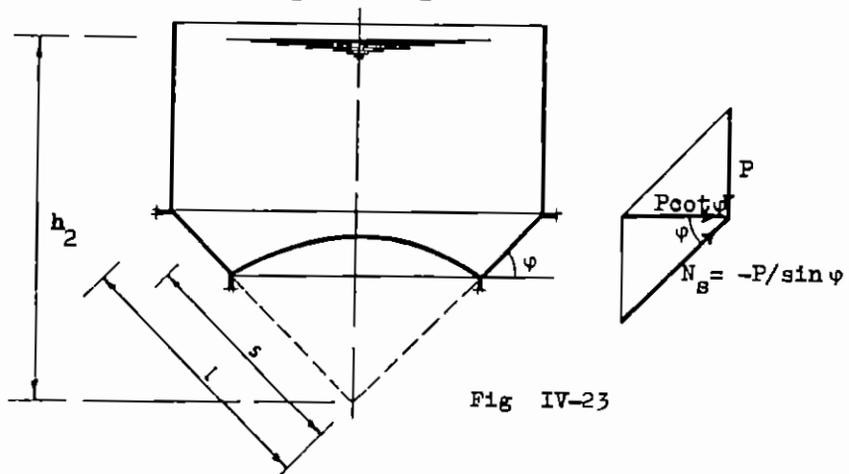


Fig IV-23

$$N_\theta = w s \left( \frac{h_2}{\sin \varphi} - s \right) \cos \varphi \quad (69)$$

$$N_s = w \frac{\cot \varphi}{6 s} \left[ 2 (l^3 - s^3) \sin \varphi - 3 h_2 (l^2 - s^2) \right] - \frac{p l}{s \sin \varphi} \quad (70)$$

where  $p$  is the weight of the cylindrical wall of the tank and the eventual loads that may be applied to it

Since the cone can absorb forces only in the direction of its generators, a ring must be provided at the foot of the wall. It receives a radial load  $P \cot \varphi$  producing the hoop force

$$P l \cos \varphi \cot \varphi \quad (71)$$

Where the conical and spherical parts of the bottom meet, a ring must be provided which resists the difference of the horizontal components of the meridian forces in the cone and in the dome

### IV-3 5 Edge Forces and Transition Curve

It is easy to prove that the upper zones of domes are subject to compressive ring forces, while the lower zones are subject to tensile ring forces. In case, the dome, or cone, does not end with a vertical tangent, the horizontal thrust  $H$  at the foot must be resisted by a tension ring. Fig IV-24

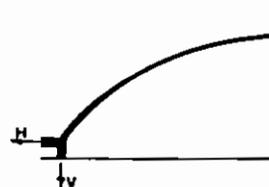


Fig IV-24

On the other hand, the meridian forces in domes and conical roofs due to vertical dead and live loads are always compressive giving relatively low stresses

In conical shells and flat spherical domes, bending moments will be developed due to the big difference between the high tensile stresses in the footing and the compressive stresses or low tensile stresses in the adjacent zones of the shell. The bigger the difference in the strains between the ring and the adjacent zone the higher will be the bending moments. The shape and magnitude of the bending moments at the footing can be estimated according to the values given in figure IV-25

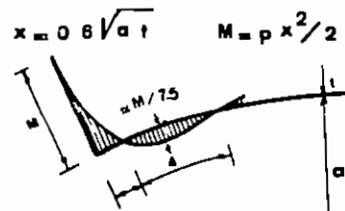


Fig IV-25

As the bending moments are due to the sudden change from high tensile stresses in the footing to low tensile or even compressive stresses in the dome, they can be avoided if the shape of the meridian is changed in a convenient manner. This change can be done by a transition curve (fig IV-26) which when well chosen, gives a relief to the stresses at the footing. It is recommended to make the change gradual from footing to shell

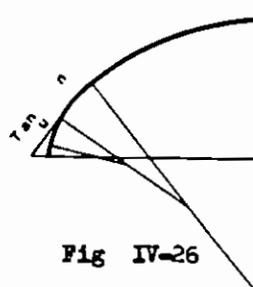


Fig IV-26

In order to decrease the stresses due to

the forces at the footings, it is recommended to increase the thickness of the shell in the region of the transition curve

#### IV-3-6 Fixed and Continuous Surfaces of Revolution

The bending moments shown in figure IV-25 give only a rough estimate, they may however be used in cases where the expected values of the bending moments are small

Surfaces of revolution in tanks are generally continuous with each other e.g. the Inze tank shown in figure IV-4. The connecting moments at the joints can be determined by the moment distribution method if the fixed end moments and the relative stiffness of the elements meeting at a joint are known

We give in the following<sup>2</sup>, the fixing moments and the stiffness factor of some simple cases generally met with in tank problems

##### 1) Cylindrical wall of variable thickness (Fig. IV-27)

$$\text{Assuming } k_1 = \frac{\sqrt[4]{3} (1-\nu^2)}{\sqrt{R t_2}} \quad \text{and} \quad \nu = 1/6$$

$$\text{then } k_1 = \frac{1.3008}{\sqrt{R t_2}} \quad \text{and}$$

the fixing moment  $\bar{M}$  is

$$\bar{M} = \frac{E t_2}{2 R^2 k_1^2} \left( \Delta r_0 + \frac{\theta_0}{k_1} \right)$$

and the stiffness  $S$  is

$$S = 2 t_2^3 k_1$$

For water pressure

$$\Delta r_0 = \frac{w R^2}{E t_2} H \quad \text{and} \quad \theta_0 = - \frac{w R^2 (y_2 - H)}{E t_2 y_2}$$

Hence

$$\bar{M} = \frac{p}{2 k_1^2} \left( 1 + \frac{1}{y_2 k_1} - \frac{1}{H k_1} \right)$$

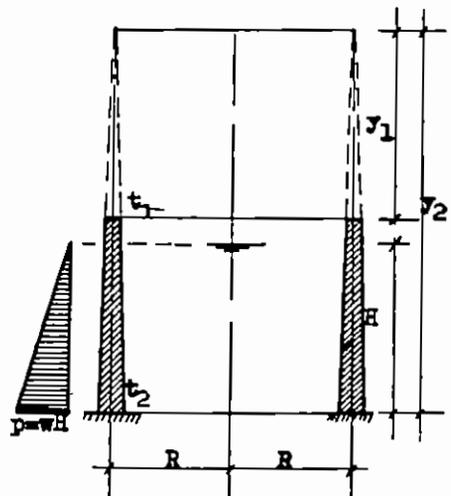


Fig. IV-27

For a wall of constant thickness ( $t_1 = t_2 = t$ ) subject to hydrostatic pressure

$$\bar{M} = \frac{P}{2 k_1^2} \left( 1 - \frac{1}{k_1 H} \right)$$

2) Spherical dome of variable thickness ( Fig IV-28 )

Assume  $k_2 = \sqrt{\frac{a}{t}} \sqrt{3(1-\nu^2)}$

and  $\nu = 1/6$  then

$$k_2 = 1.3068 \sqrt{\frac{a}{t}}$$

$$\bar{M} = - \frac{Et}{2 k_2^2} \left( \frac{\Delta r_0}{\sin \varphi} - \frac{a}{k_2} \theta_0 \right)$$

$$S = 2 t^3 k_2 / a$$

For dead load  $g t/m^2$

$$\Delta r_0 = \frac{S_0 a^2}{Et t_0} \left[ \frac{1+\nu}{\sin^2 \varphi} \left( t_0 - t \cos \varphi + \frac{t-t_0}{\varphi} \sin \varphi \right) - t \cos \varphi \right]$$

$$\theta_0 = \frac{S_0 a}{Et t_0} \left\{ \frac{t-t_0}{\varphi} \cos \varphi - (2+\nu) t \sin \varphi + \frac{(1+\nu)(t-t_0)}{t \varphi \sin^2 \varphi} x \right. \\ \left. \left[ t_0 - t \cos \varphi + \frac{t-t_0}{\varphi} \sin \varphi - \frac{t}{1+\nu} \cos \varphi \sin^2 \varphi \right] \right\}$$

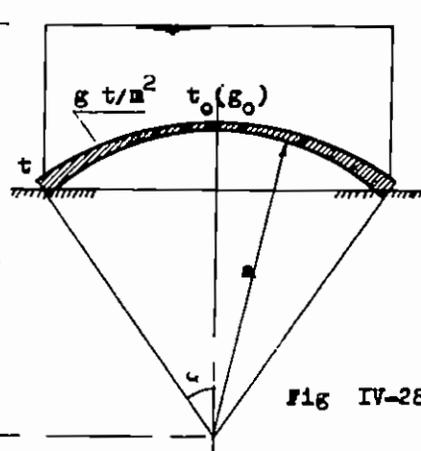
For water pressure ( $w = 10 t/m^3$ )

$$\Delta r_0 = - \frac{w a^3}{Et} \left[ \frac{(1-\nu)h}{2a} - \cos \varphi + \frac{1+\nu}{3} \left( \cos \varphi + \frac{1}{1+\cos \varphi} \right) \right] \sin \varphi$$

$$\theta_0 = \frac{w a^2}{Et} \left\{ \sin \varphi + \frac{t-t_0}{t \varphi} (1+\nu) \left[ \frac{h}{2a} - \frac{1}{3} \left( \cos \varphi + \frac{1}{1+\cos \varphi} \right) \right] \right. \\ \left. - \frac{t-t_0}{t \varphi} \left( \frac{h}{a} - \cos \varphi \right) \right\}$$

For a dome of constant thickness  $t_0 = t$

$$\bar{M} = \frac{Et}{2 k_2^2} \left( \frac{\Delta r_0}{\sin \varphi} + \frac{a}{k_2} \theta_0 \right)$$



Case of dead load  $g \text{ t/m}^2$  surface

$$\Delta r_0 = \frac{g a^2}{E t} \left( \frac{1+v}{1+\cos\phi} - \cos\phi \right) \sin\phi$$

$$\theta_0 = - \frac{g a}{E t} (2+v) \sin\phi$$

Case of live load  $p \text{ t/m}^2$  horizontal

$$\Delta r_0 = \frac{p a^2}{E t} \left( \frac{1+v}{2} - \cos^2\phi \right) \sin\phi$$

$$\theta_0 = - \frac{p a}{E t} (3+v) \sin\phi \cos\phi$$

Case of water pressure

$$\Delta r_0 = - \frac{w a^3}{E t} \left[ \frac{(1-v)h}{2a} - \cos\phi + \frac{1+v}{3} \left( \cos\phi + \frac{1}{1+\cos\phi} \right) \right] \sin\phi$$

$$\theta_0 = \frac{w a^2}{E t} \sin\phi$$

3) Conical surface of constant thickness (Fig IV-29)

$$\text{Assume } k_3 = \sqrt{\frac{\tan\phi}{t}} \sqrt{3(1-v^2)} \quad \text{and } v = 1/6 \quad \text{then}$$

$$k_3 = 1.3068 \sqrt{\tan\phi / t}$$

$$s = \frac{t \sin\phi}{2 s^2 k_3^2 \cos^2\phi} (\Delta r_0 - \frac{\sin\phi}{a_3} \theta_0)$$

$$s = 2 t^3 k_3$$

Case of dead load  $g \text{ t/m}^2$  surface

$$\Delta r_0 = \frac{g s^2}{E t} \left[ 1 - \frac{v}{2 \cos^2\phi} \left( 1 - \frac{l^2}{s^2} \right) \right] \cos^2\phi \cot\phi$$

$$\theta_0 = \frac{g s}{E t} \left[ \frac{1}{2} \left( 1 - \frac{l^2}{s^2} \right) + v - (2+v) \cos^2\phi \right] \frac{\cot\phi}{\sin\phi}$$

Case of a concentrated load  $P/m$

$$\Delta r_0 = \frac{P l}{E t} \cot\phi \quad \text{and}$$

$$\theta_0 = - \frac{P l}{E t s} \frac{\cos\phi}{\sin^2\phi}$$

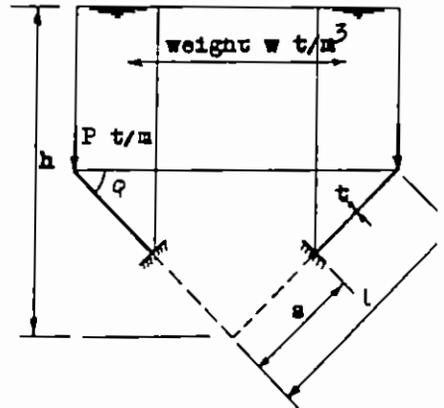


Fig IV-29

Case of water pressure

$$\Delta r_0 = \frac{w s}{E t} \left\{ s \left( \frac{h}{\sin \psi} - s \right) + \frac{v}{2 s} \left[ \frac{h}{\sin \psi} (l^2 - s^2) - \frac{2}{3} (l^3 - s^3) \right] \right\} \cos^2 \psi$$

$$\theta_0 = \frac{w}{E t} \left[ 3 s^2 \sin \psi - 2 h s - \frac{h}{2 s} (l^2 - s^2) + \frac{\sin \psi}{3 s} (l^3 - s^3) \right] \cot^2 \psi$$

4) Ring Beams

At joints of different continuous surfaces of revolution (e.g. a cone and a dome) there exists a ring beam which resists a part of the distributed moment according to its relative stiffness

The stiffness of a circular ring beam of radius  $R$ , breadth  $b$  and total depth  $t$  is given by

$$S = b (1 - \nu^2) \frac{t^3}{R^2} \quad \text{for } \nu = 1/6 \quad \text{then}$$

$$S = 0.972 b t^3 / R^2$$

IV-3-7 Circular Beams

Circular beams loaded and supported normal to their plane (Fig. IV-30) are dealt with in detail

in text books on theory of elasticity. We give in the following, the internal forces in a circular beam subjected to uniform load  $p/m$  and supported symmetrically on  $n$  columns. Thus:

$$2 \psi_0 = 2 \pi / n \quad \text{i.e.} \quad \psi_0 = \pi / n \quad (72)$$

Reaction at any column

$$V = 2 \pi R p / n \quad (73)$$

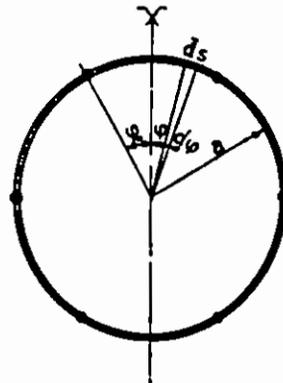


Fig. IV-30

Max shearing force to the right or left of any support

$$Q_{\max} = \pm \pi R p/n \quad (74)$$

The bending moment  $M$ , the torsional moment  $M_t$  and the shearing force  $Q$  in any section at an angle  $\psi$  from the center line between two successive supports are given by

$$M = R^2 p \left( \frac{\cos \psi}{n \sin \phi_0} - 1 \right) \quad (75)$$

$$M_t = R^2 p \left( \frac{\pi \sin \psi}{n \sin \phi_0} - \psi \right) \quad (76)$$

$$Q = - R p \psi \quad (77)$$

We give in the following table, the reactions, the maximum shearing forces, bending moments and torsional moments in a circular beam of radius  $R$  supported symmetrically on  $n$  columns and subjected to a total uniformly distributed load  $P$  where

$$P = 2 \pi R p$$

Number of Cols	Load on each Column	Max Shearing Force	Max Bending Moment		Max Torsional Moment	Angle bet axis of Col & sec of max $M_t$
			at C L of span	Over C L of Columns		
$n$	$V$	$Q_{\max}$	$M (+)$	$M (-)$	$M_t$	Deg
4	$P/4$	$P/8$	0176 P R	0053 P R	0053 P R	19° 21'
6	$P/6$	$P/12$	0075 P R	0148 P R	0015 P R	12° 44'
8	$P/8$	$P/16$	0042 P R	0083 P R	0006 P R	9° 33'
12	$P/12$	$P/24$	0019 P R	0037 P R	0002 P R	6° 21'

## V P R E S T R E S S E D C I R C U L A R T A N K S

### V 1 I N T R O D U C T I O N

The design of prestressed structures is based on a knowledge of the fundamental principles of prestressed concrete. This is as true for the design of tanks as for beams and slabs. Before analysing the stresses in a prestressed tank it may be of advantage to give the most important basic principles of prestressed concrete as may be required for design of circular tanks.

Prestressed concrete denotes concrete in which effective internal stresses are induced generally by the use of high tensile steel. This operation is done in such a way as to completely eliminate or at least to effectively reduce tensile stresses under the action of working loads together with the provision of an ample factor of safety against cracking or collapse.

The most effective use of prestressing can only be obtained if the concrete and steel are of very high quality. The higher the crushing strength of concrete and the tensile strength of steel, the greater the effectiveness with which the prestress can be utilised.

High quality concrete can be achieved through careful selection of aggregates, suitable granular composition, use of low water-cement ratio, sufficient cement content and thorough mixing, compaction and curing.

The minimum cement content in the concrete mix is  $350 \text{ kg/m}^3$  of finished concrete, the minimum crushing strength is  $300 \text{ kg/cm}^2$  after 28 days. The allowable compressive stress may be assumed one third of

the crushing strength The modulus of elasticity  $E_c$  may be assumed equal to  $300 \text{ t/cm}^2$

The steel to be used for prestressing is generally hard drawn wires of an ultimate strength not less than  $15 \text{ t/cm}^2$  and having diameters varying between 3 and 7 mm

The permissible stress in the prestressing steel shall be based on the ultimate strength and the 0.2 % proof stress as follows :

The maximum tensile stress at transfer  $\sigma_{s0}$  shall not exceed 80 % of the proof stress or 65 % of the ultimate tensile strength which ever is the lesser Accordingly, for hard drawn wires 16 5/14 having an ultimate tensile strength of  $16.5 \text{ t/cm}^2$  and a proof stress of  $14.0 \text{ t/cm}^2$  the maximum allowable tensile stress  $\sigma_{s0} = 10 \text{ t/cm}^2$

The modulus of elasticity  $E_s$  is generally equal to  $2000 \text{ t/cm}^2$  The modular ratio  $n = E_s/E_c$  varies between 6 and 7

Due to shrinkage, creep etc the prestress is reduced by 15 to 20 % of its initial value, so that the final prestress  $\sigma_{s00}$  will be equal to 0.80 to 0.85  $\sigma_{s0}$

Stresses in prestressed elements under working loads may be computed by the elastic theory both at transfer with full prestress ( $\sigma_s = \sigma_{s0}$ ) and no live loads and, under final conditions after losses have taken place ( $\sigma_s = \sigma_{s00}$ ) and full live loads

Prestressing of circular tanks is made in order to eliminate the tensile stresses created by the hydrostatic pressure In most prestressed circular structures, prestress is applied both circumferentially and longitudinally, the circumferential prestress being circular and the longitudinal prestress actually linear It is however easier to use circumferential prestressing only This is possible in walls with freely sliding edges

## V-2 CIRCUMFERENTIAL PRESTRESSING

Prestressed concrete circular tanks<sup>■</sup> are generally constructed by winding prestressing wires around the walls using a special winding machine. The normal method of prestressing consists of the following process. First, the walls of the tank are built of either concrete or pneumatic mortar, mortar being generally used if the walls are less than 12 cms thick. Often, the walls are poured in alternative vertical slices keyed together. After the concrete walls have attained sufficient strength they are prestressed circumferentially by a self propelled machine, which winds the wire around the walls in a continuous operation, stressing it and spacing it at the same time.

After the circumferential prestressing is completed for each layer, a coat of pneumatic mortar is placed around the tank for protection. Two or more layers of prestressing are used for large tanks (Fig V 1)

Vertical prestressing for the tanks can be applied using any system of linear prestressing, whichever may be the most economic.

Circumferential prestress in tanks is designed to resist ring tension produced by liquid pressure.

Consider one half of a thin horizontal slice of a tank as a free

body, fig V 2. Under the action of the initial prestress  $F_0$  in the steel, the total compression  $C$  in the concrete is equal to  $F_0$ , causing an initial compressive stress in the concrete equal to  $\sigma'_0 = F_0 / A_c$ . After the losses due to shrinkage and creep are developed the pres-

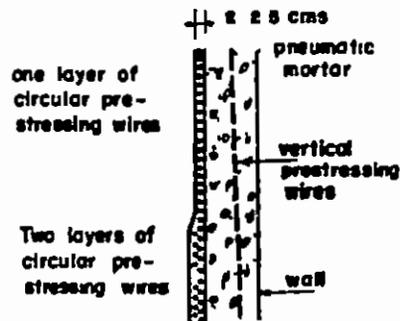


Fig V-1

■ Lin " Prestressed Concrete Structures "

stress will be reduced to  $F_{\infty} = 0.8 - 0.85 F_0$  and the corresponding compressive stress will be  $\sigma_c = F_{\infty} / 2 A_c$

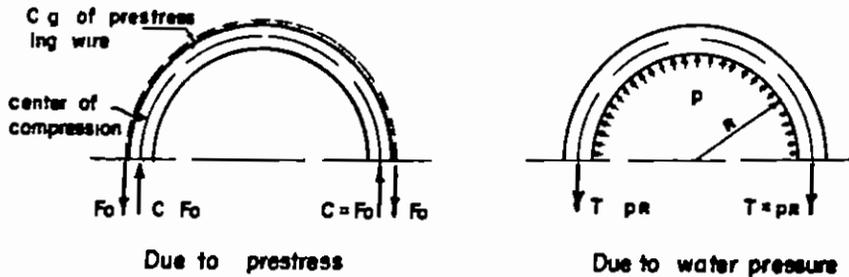


Fig V-2

With the application of the internal liquid pressure, the steel and concrete act together, and the stress can be obtained by the usual elastic theory, thus  $\sigma_c = p R / A_v$ , where

- $p$  = internal pressure intensity
- $R$  = internal radius of the tank
- $A_v$  = the virtual area =  $A_c + n A_s$

The resultant stress in the concrete under the effective prestress  $F_{\infty}$  and the internal pressure  $p$  is therefore

$$\sigma_c = - F_{\infty} / A_c + p R / A_v \quad (78)$$

If a coating of concrete or mortar is added after the application of prestress, then the area  $A_c$  under prestress may be the core area while the  $A_c$  sustaining the liquid pressure may include the additional coating

The general practice has been to provide a slight residual compression in the concrete under the working pressure. This residual compression serves as a margin of safety in addition to whatever tension that may be taken by the concrete.

Since the serviceability of a tank is impaired as soon as the concrete begins to crack, it is of utmost importance that an adequate

margin of safety to be provided against any tensile stresses. A factor of safety  $s = 1.25$  is recommended. This is accomplished by the following procedure of design.

Equation 78 may now be given in the form

$$0 = -F_{\infty} / A_c + s p R / A_v \quad (79)$$

at the same time, in order to limit the maximum compression in concrete to  $\sigma_c$ , we have

$$A_c = -F_o / \sigma_c \quad (80)$$

Substituting this value in equation 79 and noting that

$$A_v = A_c + n A_s,$$

$$F_{\infty} = \sigma_{s\infty} A_s \quad \text{and}$$

$$F_o = \sigma_{s0} A_s$$

we have

$$-\frac{\sigma_{s\infty} A_s}{\sigma_{s0} A_s / \sigma_c} + \frac{s p R}{(\sigma_{s0} A_s / \sigma_c) + n A_s} = 0$$

Solving for  $A_s$ , we get :

$$A_s = \frac{s p R}{\sigma_{s\infty} (1 + n \sigma_c / \sigma_{s0})} \quad (81a)$$

For normal cases  $n = 7$  and  $\sigma_{s\infty} / \sigma_{s0} = 0.8 \approx 0.85$ , then :

$$A_s = \frac{s p R}{\sigma_{s\infty} + 6 \sigma_c} \quad (81b)$$

For  $s = 1.25$ ,  $\sigma_{s\infty} = 8 \text{ t/cm}^2$  and  $\sigma_c = 100 \text{ kg/cm}^2 = 0.1 \text{ t/cm}^2$

We get :

$$\underline{A_s = 0.145 p R} \quad (81c)$$

in which  $A_s$  in  $\text{cm}^2$  for  $p$  in  $\text{t/m}^2$  and  $R$  in  $\text{ms}$

We have further :

$$A_c = F_o / \sigma_c = \sigma_{so} A_s / \sigma_c$$

and for  $\sigma_{so} = 10 \text{ t/cm}^2$ , we get

$$A_c = 10 A_s / 0.1 = 100 A_s$$

but  $A_c = 100 \text{ t}$  then  $t \text{ cms} = A_s \text{ cm}^2$  i.e.

$$\underline{A_s = t = 0.145 \text{ p R}} \quad (82)$$

If it is required to limit  $\sigma_c$  to  $80 \text{ kg/cm}^2$  only, the amount of the prestressing steel will not be affected while the thickness will be increased to

$$\underline{t = 0.18 \text{ p R}} \quad (83)$$

Example :

Determine max  $t$  and  $A_s$  required for a prestressed open circular slurry tank 10 ms diameter and 12 ms deep with sliding base. Assume  $w = 25 \text{ t/m}^3$ ,  $\sigma_{so} = 10 \text{ t/cm}^2$ ,  $\sigma_c = 100 \text{ kg/cm}^2$ ,  $P_{\max} = 2.5 \times 12 = 30 \text{ t/m}^2$ ,  $t = A_s = 0.145 \text{ P R}$  or

$$t \text{ cms} = A_s \text{ cm}^2 = 0.145 \times 30 \times 5 = \underline{22}$$

### V-3 VERTICAL PRESTRESSING

Due to hydrostatic pressure the walls of circular tanks are subject to ring tension in the horizontal direction and cantilever moments in the vertical direction in case they are hinged or fixed to the floor.

It is evident that circumferential prestressing will also cause ring forces and cantilever moments in a sense opposite to that of hydrostatic pressure and that they exist by themselves when the tank is empty and act jointly with the ring forces and moments produced by liquid pressure when the tank is full.

In order to reinforce the walls against these moments, vertical

prestressing may be applied. If vertical prestress is axially applied to the concrete, only direct compressive stress is produced and the solution is simple. If the vertical tendons are bent or curved, the vertical prestress produces radial components which, in turn, influence the circumferential prestress and the analysis can become quite complicated.

Let us investigate the effect of circumferential prestressing on the vertical moments. If the circumferential prestress varies triangularly from zero at the top to a maximum at the bottom, its effect is equal but opposite to the application of an equivalent liquid pressure. If the circumferential prestress is constant throughout the entire height of the wall, it is the same as the application of an equivalent gaseous pressure. For both cases, the P C A tables\* or any similar ones are available for the computation of vertical moments.

Vertical prestressing should be designed to stand the stresses produced by various possible combinations of the following forces:

- 1) The vertical weight of the roof and the walls themselves.
- 2) The vertical moments produced by the applied circumferential prestress.
- 3) The vertical moments produced by internal liquid pressure.

It must be noted that the maximum stresses in the concrete usually exist when the tank is empty, because then the circumferential prestress would have its full effect. When the tank is filled the liquid pressure tends to counterbalance the effect of circumferential prestress and the vertical moments are smaller. Since it is convenient to use the same amount of vertical prestress throughout the entire height of the wall, the amount will be controlled by the point of maximum moment. By properly locating the vertical tendons to resist such moment, a most economical design can be obtained. However, efforts are

---

\* Refer to P. C. A. tables of circular tanks.

seldom made to do so and the amount of prestress as well as the location of the tendons is generally determined empirically rather than by any logical method of design

In order to avoid complications in the position of the vertical prestressing cables due to the varying sign of the bending moments - as in case of circular tanks with fixed base - the walls may be made hinged to the floor at their foot. The vertical prestressing cables may then be located nearer to the inside surface of the wall at its bottom end where the maximum moments occur. At the upper end, the cables may be anchored at the center of gravity of the concrete section as shown in figure V-3

a) Pressure on wall due to circumferential prestress

b) Corresponding  $\delta$  and  $D$  for a wall hinged at base

c) Longitudinal and circumferential prestressing wires

d) Location of long prestressing wires

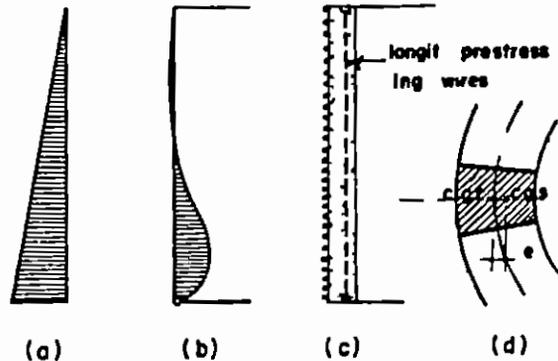


Fig V-3

In order to determine the magnitude of the prestressing force, one can proceed as follows

- 1) The magnitude of the maximum bending moment due to circumferential prestressing can be determined from table VIII of the  $\text{PCA}$  tables
- 2) The corresponding stress in the section is then computed
- 3) Assuming the eccentricity of the longitudinal prestressing steel at position of maximum bending moment is  $e$ , compute the stresses in

the section due to a vertical prestressing force  $F = 1$  acting in the center of gravity of the steel ( c g s )

4) In order to get zero stress in the inner surface of the tank due to the combined action of the longitudinal prestressing force and bending moments due to the circumferential prestressing, (case of full prestressing), divide the maximum tensile stress computed under 2 by the maximum compressive stress computed under 3

The stresses in the section under the different combinations of the forces acting will be shown in the example given in V 5

#### V-4 DOME PRESTRESSING

In the following, only the general principles and practice of dome prestressing especially as applied to tank roofs will be mentioned. Generally speaking, for domes with diameter greater than 30 ms, the economy of prestressing should be seriously considered. Domes for tanks up to 75 ms in diameter have been constructed.

The dome roof itself is made of concrete or pneumatic mortar with thickness varying from 6 to 15 cms. For domes of large diameter, variable thicknesses may be employed and thicknesses greater than 15 cms are used for the lower portion. Before concreting the dome, some erection bars are compressed around the base of the dome. After the hardening of the shell concrete, wires are prestressed around it. During this operation the dome shell rises from its forms as it is compressed thus simplifying the careful procedure of decentering required for non-prestressed domes.

The conventional method of dome prestressing consists of prestressing the edge ring to induce sufficient compressive stresses to counteract the tensile stresses set up in the ring under the maximum live and dead loads. With this process, it is usually possible to raise the dome from its false work, since only the dead load is actually

acting on the dome

The determination of the horizontal thrust and the corresponding prestressing follows the fundamentals given in chapter IV

Consider a spherical dome carrying loads symmetrical about the axis of rotation (fig V 4) If the total load is  $W$ , the vertical reaction per meter of length along the edge member will be

$$V = \frac{W}{2 \pi R}$$

and the horizontal thrust per meter is

$$H = V \cot \varphi$$

Assuming that this horizontal thrust

to be entirely supplied by the prestressing force  $F$  acting in ring tension, then ,

$$F = H R = V R \cot \varphi \quad \text{1 e}$$

$$F = W \cot \varphi / 2 \pi \quad (84)$$

The effective prestressing force  $F$  having been determined, the cross-sectional area of the ring concrete can be determined from the relation

$$A_c = N / \sigma_c \quad (85)$$

where  $N$  = the net compressive force acting on the section at transfer and  $\sigma_c$  = the allowable compressive stress in concrete

It is desirable to keep  $\sigma_c$  at a relatively low value say about  $0.2 \sigma_{cp}$  and not greater than  $50 \text{ kg/cm}^2$

#### V-5 Example

In order to show the way of design of prestressed domes and circular tanks and the possible economy that can be attained we will

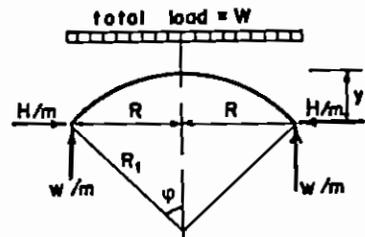
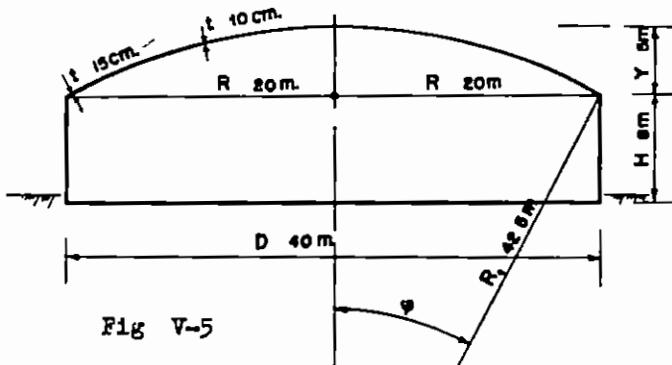


Fig V-4

design a covered circular tank once in reinforced concrete and then in prestressed concrete as shown in the following example

It is required to design a circular water tank 40 ms internal diameter and 8 ms deep with the walls hinged to the floor if the tank is covered by a 5 ms high spherical dome as shown in fig V 5



The design is to be first made using normal reinforced concrete having a max allowable tensile stress of  $\sigma_t = 20 \text{ kg/cm}^2$  and mild steel with an allowable stress of  $\sigma_s = 1400 \text{ kg/cm}^2$ . The effect of shrinkage is to be considered with

assuming  $E_s = 2000 \text{ t/cm}^2$  and  $\epsilon_{sh} = 0.25 \text{ mm/m}$   
and  $n = 10$

Then the design is to be repeated using prestressed concrete for the following data

Concrete  $\sigma_{c28} = 300 \text{ kg/cm}^2$ , allowable compressive stress  $\sigma_c = 100 \text{ kg/cm}^2$

Prestressing wires 16 5/14  $\sigma_{s0} = 10 \text{ t/cm}^2$ , losses 20%  $\sigma_{sm} = 8 \text{ t/cm}^2$

$$n = E_s / E_c = 7 \text{ load factor } s = 1.25$$

#### a) Design of dome and walls in reinforced concrete

For the roof dome which is a part of a hemisphere, we have :

$$\text{Radius } a = \frac{R^2 + y^2}{2y} = \frac{20^2 + 5^2}{2 \times 5} = \frac{425}{10} = 42.5 \text{ m}$$

$$\text{Surface area } A = 2 \pi a y = 2 \pi \times 42.5 \times 5 = 1335 \text{ m}^2$$

Loads :

Own weight ( assumed 10 cms thick )	g	= 250 kg/m <sup>2</sup>
Superimposed dead and live loads	p	= 150
	total	= <u>400 kg/m<sup>2</sup></u>
Total roof own weight $W_g = 0.25 \times 1335$		= 334 ton
Total roof superimposed loads $W_p = 0.15 \times 1335$		= 200 ' <u></u>
	total W =	<u>534 ton</u>

$$\text{Total load / m' wall } V = \frac{i}{2 \pi R} = \frac{534}{2 \times 3.14 \times 20} = 4.25 \text{ t/m}$$

$$\text{The horizontal thrust/m H} = V \cot \phi = 4.25 \times \frac{37.5}{20} = 8.0 \text{ "}$$

$$\text{The ring tension } T = H R = 8 \times 20 = 160 \text{ t}$$

$$\text{The ring reinforcement } A_s = T / \sigma_s = 160 / 1.4 = 114 \text{ cm}^2$$

chosen 2,  $\phi$  25 mm

As a preliminary estimate for the wall assume

$$\text{The max ring tension } T = 0.5 p_{\max} R = 0.5 \times 8 \times 20 = 80 \text{ t/m}$$

$$\text{Required wall thickness } t = 0.75 T = 0.75 \times 80 = 60 \text{ cms}$$

According to P C A table II, we est

$$\text{for } H^2 / D t = 8^2 / 40 \times 0.6 = 2.67,$$

the maximum ring tension :

$$T_{\max} = 0.5 H R = 0.5 \times 8 \times 20 = 80 \text{ ton at } 0.6 H \text{ from top i.e. the assumed thickness of 60 cms is acceptable}$$

The required ring reinforcement at 0.6 H is given by :

$$A_s \text{ max} = T_{\max} / \sigma_s = 80 / 1.4 = 57 \text{ cm}^2$$

chosen 8  $\phi$  22 mm/m on each surface giving  $A_s = 60 \text{ cm}^2$

The max tensile stress in concrete is given by :

$$\sigma_{t \text{ max}} = \frac{T_{\max} + E_{sh} E_s A_s}{A_c + n A_s}$$

$$= \frac{80000 + 0.00025 \times 2000000 \times 60}{100 \times 60 + 10 \times 60} = 16.6 \text{ kg/cm}^2 < 20$$

b) Design of dome and wall in prestressed concrete

For the roof dome, only the foot ring, will be prestressed thus the final prestressing force required to counterbalance the max ring tension  $T_{\max}$  of 160 t is according to equation 84 given by

$$F_{\infty} = r \cot \varphi / 2 \pi = T_{\max} = 160 \text{ t}$$

The required prestressing steel

$$A_s = F_{\infty} / \sigma_s = 160/8 = \underline{20 \text{ cm}^2}$$

which is less than 20% of the amount required in case of ordinary reinforced concrete

The max compressive stress in the foot ring will take place at transfer with a prestressing force  $F_0$  and a ring force  $T_0$  due to own weight of dome. Thus,

$$F_0 = F_{\infty} / 0.8 = 160/0.8 = 200 \text{ t}$$

$$\text{and } T_0 = T_{\max} \cdot r_s / r = 160 \times 334 / 534 = 100 \text{ t}$$

The net compressive force acting on the foot ring

$$N = F_0 - T_0 = 200 - 100 = 100 \text{ t}$$

Assuming the max allowable compressive stress in concrete = 50 kg/cm<sup>2</sup> then the area of concrete section required for the foot ring is given by

$$A_c = N / \sigma_c = 100000/50 = \underline{2000 \text{ cm}^2}$$

A foot ring 70 x 30 cms will be chosen

Prestressed circular tanks require a wall thickness much smaller than that required for reinforced concrete tanks and therefore the pressure resisted by ring action  $P_r$  is much bigger. As a first estimate, assume :

$$\text{max } P_r = 0.7 \cdot w \cdot H = 0.7 \times 1 \times 8 = 5.6 \text{ t/m}^2$$

The thickness required for the wall can be estimated according to equation 101 from the relation

$$t = 0.145 p_r R = 0.145 \times 5.6 \times 20 = 16.5 \text{ cms}$$

A wall 17 cms thick will be chosen

According to P C A table II, we get :

$$\text{for } H^2 / D t = 8^2 \quad 40 \times 0.17 = 9.4$$

The maximum ring tension

$$T_{\max} = p R = 0.713 w H R = 0.713 \times 1 \times 8 \times 20 = 114 \text{ t}$$

at 0.7 H from top

The maximum area of circumferential prestressing steel at 0.7 H from top is, according to equation 100 b, given by

$$A_s = \frac{s p R}{\sigma_s + 6 \sigma_o} = \frac{1.25 \times 114}{8 + 6 \times 0.1} = 16.6 \text{ cm}^2$$

This value of the area of the steel, as well as the wall thickness can be directly determined from equation 101 thus,

$$A_s = t = 0.145 p R = 0.145 T_{\max} = 0.145 \times 114 \quad \text{t.e.}$$

$$A_s = 16.5 \text{ cm}^2 \text{ \& } t = 16.5 \text{ cm}$$

which means that the chosen thickness of 17 cms is acceptable

The residual stress in the concrete under the effective prestress  $F_{\infty}$  and the internal pressure at position of maximum ring tension can be calculated according to equation 78 as follows

$$\sigma_c = - \frac{F_{\infty}}{A_c} + \frac{p R}{A_v} = - \frac{16.5 \times 8000}{100 \times 17} + \frac{114000}{100 \times 17 + 7 \times 16.5} = -77.5 + 6$$

$$\text{or } \underline{\sigma_c = -14.5 \text{ kg/cm}^2}$$

which provides a margin of safety of 25 % up to zero compression in concrete

The cantilever moment due to the hydrostatic pressure can be calculated from the P C A table VIII, thus

For  $H^2 / D t = 9.4$ ,  $M_{\max} = 0.005 w H^3 = 0.005 \times 1 \times 8^3 = 2.56 \text{ mt}$  acting at  $0.8 H$  from top and causes tensile stresses on the outer face of the wall

The initial and final maximum cantilever moments due to the prestressing steel ( with tension on the inside face of the wall ) can be determined as follows

$16.5 \text{ cm}^2$  prestressing circumferential wires tensioned to an initial stress of  $10 \text{ t/cm}^2$  will cause a ring compression of

$$C = A_s \sigma_s = 16.5 \times 10 = 165 \text{ t}$$

The corresponding intensity of radial compression will be

$$P_o = C / R = 165 / 20.17 = 8.2 \text{ t/m}^2$$

This pressure causes a cantilever moment equal to

$$M_o = 2.56 \times 8.2 / 0.713 w H = 2.56 \times 8.2 / 0.713 \times 1 \times 8 = 3.68 \text{ mt}$$

For a final prestress of  $8 \text{ t/cm}^2$  this value reduces to

$$M_{oo} = M_o \times 8 / 10 = 3.68 \times 8 / 10 = 2.94 \text{ mt}$$

The circumferential prestressing will counterbalance the internal pressure in case of full tank but in case of empty tank, the tensile stresses due to the cantilever moments induced by the prestressing force can be counterbalanced by vertical prestressing wires placed in the tension zone - near the inside surface - at a distance of say 5 cms from it

The magnitude of the vertical prestressing force can be determined by dividing the magnitude of the maximum tensile stress due to the moment caused prestressing -  $3.68 \text{ mt}$  - by the magnitude of the maximum compressive stress caused by a prestressing vertical force of 1 ton acting at 5 cms from the inside face of the wall, thus, max tensile stress

**Initial and Final Stresses  
in a Prestressed Circular Tank  
under Different Cases of Loading**

Conditions	Initial stress in $\text{kg/cm}^2$		Final Stresses in $\text{kg/cm}^2$	
	Inside	Outside	Inside	Outside
A) Weight of Roof 4250/1700	- 2 50	- 2 50	- 2 50	- 2 50
B) Weight of Wall $0.17 \times 2400 = 410 \text{ kg/m}^2$ $410 \times 0.8 \times 8 / 1700$	- 1 54	- 1 54	- 1 54	- 1 54
C) Axial Compression of Vertical Prestress Initial 5000 / 1700 Final 45400 / 1700	- 34 10	- 34 10	- 27 30	- 27 30
D) Eccentricity of Vertical Prestress Initial $6 \times 58000 \times 3.5/100 \times 17^2$ Final $6 \times 45400 \times 3.5/100 \times 17^2$	- 42 20	+ 42 20	- 32 50	+ 33 80
E) Vertical moment due to Circum- ferential Prestress Initial $6 \times 3.68 \times 10^5 / 100 \times 17^2$ Final $6 \times 2.94 \times 10^5 / 100 \times 17^2$	+ 76 5	- 76 5	+ 61 20	- 61 20
Total for Tank Empty	- 3 84	- 72 44	- 3 04	- 58 74
F) Vertical Moment due to Hydrostatic Pressure $6 \times 2.56 \times 10^5 / 100 \times 17^2$	- 53 00	+ 53 00	- 53 00	+ 53 00
Total for Tank Full	- 56 84	- 19 44	- 56 04	- 5 74

due to  $M_0 = 3.68 \text{ mt}$  is given by :

$$\sigma = 6 M / b t^2 = 6 \times 3.68 \times 10^5 / 100 \times 17^2 = 76.5 \text{ kg/cm}^2$$

Maximum compressive stress due to a vertical prestressing force of 10 ton acting at 5 cms from the inside face ( $e = 3.5 \text{ cm}$ ) is given by

$$\begin{aligned} \sigma &= -F/A - 6 F e / b t^2 = -1000/1700 - 6 \times 1000 \times 3.5 / 100 \times 17^2 \\ &= -0.59 - 0.73 = -1.32 \text{ kg/cm}^2 \end{aligned}$$

Vertical initial prestressing force is therefore given by :

$$F_0 = 76.5 / 1.32 = 58 \text{ t}$$

The corresponding final vertical prestressing force is

$$F = 0.8 F_0 = 0.8 \times 58 = 46.4 \text{ t}$$

The stresses for both the inside and outside fibers under both initial and final conditions at position of maximum cantilever moment are computed and listed as in the table

It is seen that the wall is subject to compressive stresses under all load stages

The details of reinforcements are shown in figure V 6

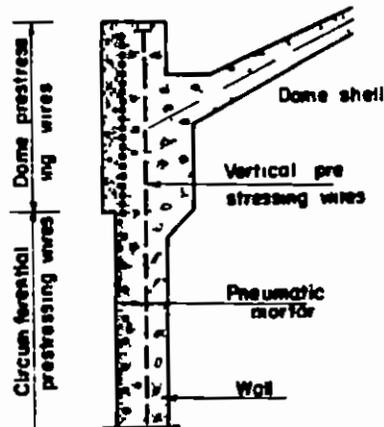


Fig V-6

#### V 6 ECO NOMIC PROPORTIONS OF PRESTRESSED CIRCULAR TANKS

The ratio of diameter to height is of some importance with regard to the cost of circular cylindrical prestressed concrete tanks. Favourable dimensional proportions are given in the following table Fig V 7 which has been published by Preload Engineers, New York, on the basis of many year's experience

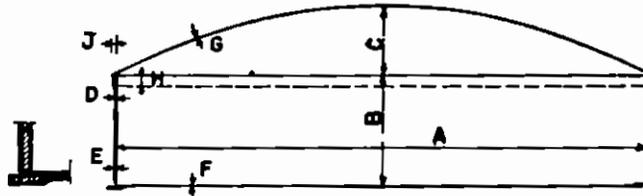


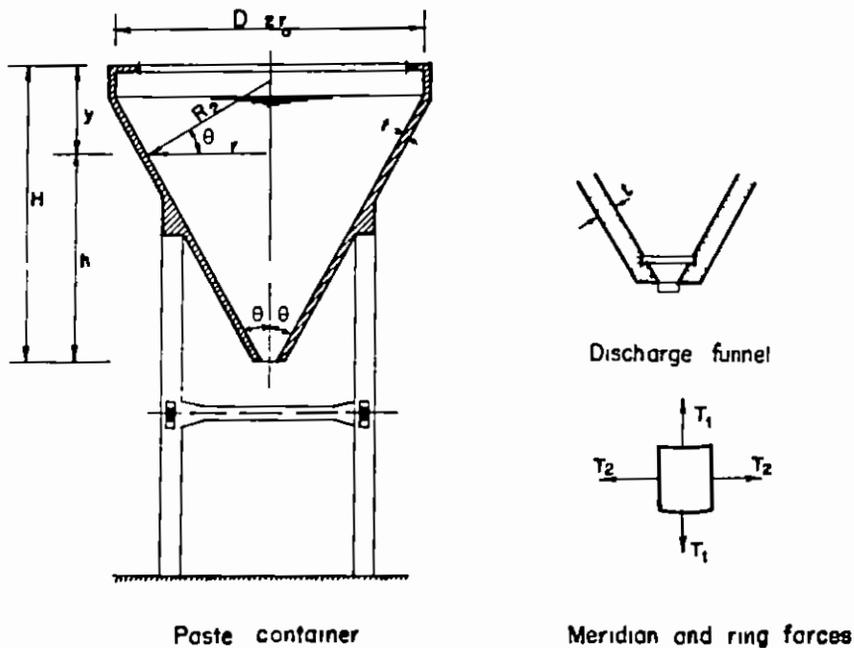
Fig V-7

Capacity in m <sup>3</sup>	Dimensions in m								
	A	B	C	D	E	F	G	h	J
378	12 50	3 50	1 56	12	12	05	05	20	15
945	16 00	4 30	2 11	12	12	05	05	22	15
1840	21 35	5 35	2 67	12	12	05	05	30	17
2835	24 40	6 10	3 05	12	15	05	05	36	19
3780	26 95	6 70	3 36	12	18	05	05	38	22
5670	30 00	7 30	3 86	12	23	05	05	42	25
7550	33 85	8 55	4 23	12	24	05	06	48	27
9450	36 40	15	4 75	22	26	05	06	51	30
18 00	46 00	11 45	5 75	22	44	05	10	60	38
37800	57 00	14 50	7 24	22	74	05	11	80	49
Economic Proportions in J S A      B A = 1    4									

The wall thickness is probably rather too small for normal conditions, the increase in thickness from top to bottom should be noted. The thickness of the base slab has in all cases been given as 5 cm because in the U.S.A. the base is usually constructed as a thin but closely reinforced gunite layer.

VI EXAMPLES OF CIRCULAR TANKS

As typical examples showing the use of surfaces of revolution in tanks, we give the following three structures

VI 1 - PASTE CONTAINER\* Fig VI 1

Data

Fig VI-1

$w$  = weight of paste per cubic meter

$r = h \tan \theta$  = horizontal radius of any section

$R_2 = \frac{r}{\cos \theta} = \frac{h \tan \theta}{\cos \theta}$  = cross radius of curvature

\* The notations used in this example differ from those used in the membrane theory. All equations being derived from the first principles, it has been found that changing the notations is not essential.

The internal forces in the wall of the container due to weight of paste can be determined as follows

a) Ring Forces Due to Paste

For determining the ring tension  $T_2$  in any section, we apply the fundamental equation 46 using the given notations, thus

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = p_r \quad \text{where}$$

$R_1$  = radius of curvature of meridian =  $\infty$

$p_r$  = component of load of paste normal to surface of container assuming angle of friction between paste and reinforcement concrete = 0 thus

$$p_r = w y$$

Therefore

$$T_2 = R_2 p_r = \frac{w y h \tan \theta}{\cos \theta} \quad \text{tension} \quad (86)$$

There :

$$h = (H - y)$$

The depth  $y$  at which the ring force  $T_2$  is maximum can be determined from the condition

$$\frac{dT_2}{dy} = 0 \quad \text{but}$$

$$\frac{dT_2}{dy} = w \frac{\tan \theta}{\cos \theta} \frac{dy (H - y)}{dy} = w \frac{\tan \theta}{\cos \theta} (H - 2y)$$

Therefore, for maximum ring tension, we must have

$$w \frac{\tan \theta}{\cos \theta} (H - 2y) = 0$$

or

$$y = H/2$$

and

$$\max T_2 = w \frac{\tan \theta}{\cos \theta} = w \frac{H^2}{4} \quad (87)$$

b) Meridian Forces Due to Paste

When determining the meridian force  $T_1$ , two separate cases have to be considered, namely above and below the ring beam as follows

1) Above the Ring Beam      Fig VI 2

If we equate the weight of the paste above the horizontal section at depth  $y$  (hatched volume) to the vertical component of the compressive meridian force  $T_1$ , we get Fig 2

$$T_1 2 \pi r \cos \theta = w \frac{b y^2}{2} 2 \pi r_1$$

in which

$$r = (H - y) \tan \theta$$

$$b = y \tan \theta \quad \text{and}$$

$$\begin{aligned} r_1 &= \text{radius of center of gravity of hatched volume} \\ &= (H - \frac{2}{3} y) \tan \theta \end{aligned}$$

Therefore

$$T_1 2 \pi (H - y) \tan \theta \cos \theta = w \pi y^2 (H - \frac{2}{3} y) \tan^2 \theta$$

or

$$T_1 = \frac{w}{6} \frac{3H - 2y}{H - y} y^2 \frac{\tan \theta}{\cos \theta} \text{ compression} \quad (88)$$

11) Below the Ring Beam      Fig VI 3

In this case, the weight of the paste acting on the lower conical part (hatched volume) will be hung to the wall of the container causing meridian tensile stresses which can be treated in a similar way as follows (Fig VI 3)

$$T_1 2 \pi r \cos \theta = w \left[ \pi r^2 y + \frac{1}{3} \pi r^2 h \right]$$

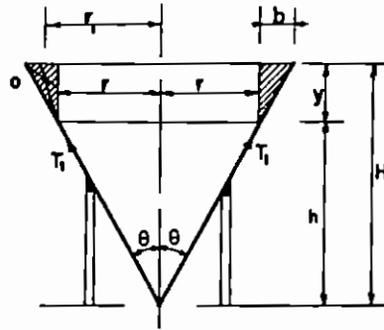


Fig VI-2

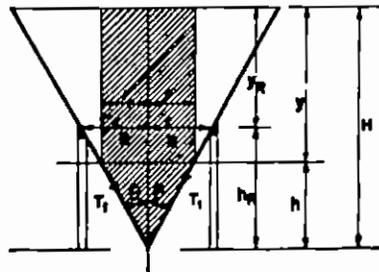


Fig VI-3

Substituting  $r = (H - y) \tan \theta$  and  $h = H - y$ , we get

$$T_1 = 2\pi (H - y) \tan \theta \cos \theta = w \left[ \pi (H - y)^2 \tan^2 \theta \left( y + \frac{1}{3} \right) + \pi (H - y)^2 \tan^2 (H - y) \right]$$

or

$$T_1 = \frac{w}{6} (H - y) (H + 2y) \frac{\tan \theta}{\cos \theta} \quad \text{tension} \quad (89)$$

The depth  $y$  at which the tensile meridian force below the ring beam is max can be determined from the condition  $\frac{dT_1}{dy} = 0$  but

$$\frac{dT_1}{dy} = \frac{w}{6} \frac{\tan \theta}{\cos \theta} (H - 4y)$$

For max meridian tension  $T_1$   $y = \frac{H}{4}$  and

$$\max T_1 = \frac{3}{16} w H^2 \frac{\tan \theta}{\cos \theta} \quad (90)$$

provided the ring beam is at a distance from the apex greater than  $3/4 H$ . If not, the max  $T_{1R}$  occurs at the ring beam and can be determined according to equation 90 from the relation

$$T_{1R} = \frac{w}{6} (H - y_R) (H + 2y_R) \frac{\tan \theta}{\cos \theta} \quad (90b) \quad \text{in which}$$

$y_R > \frac{H}{4}$  is the distance of the ring beam from the surface of the container

#### c) Internal Forces in Ring Beam Due to Paste

The ring beam is subject to a compressive force  $C_R$  determined by (Fig VI 4)

$$C_R = H_R r_R = T_1 \sin \theta r_R \quad \text{and} \quad T_1 = T_1^i + T_1^n \quad \text{in which}$$

$T_1^i$  is the compressive meridian force just above the middle of the ring beam and

$T_1^n$  is the tensile meridian force just below the middle of the ring beam

#### d) Internal Forces Due to Own Weight of Container

The ring and meridian forces due to the own weight of the

container are usually small compared with those caused by the weight of the past, they can however be calculated as follows (Fig VI 5)

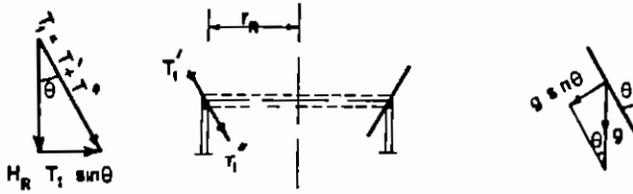


Fig VI-4

Fig VI-5

Assuming  $g = \text{own weight of tank} / \text{m}^2$ , the ring force can be determined from the relation

$$T_2 = R_2 Z \quad \text{in which}$$

$$R_2 = \frac{h \tan \theta}{\cos \theta} \text{ and } Z = g \sin \theta, \quad \text{therefore,}$$

$$\underline{T_2 = g h \tan^2 \theta} \quad (91)$$

which means that  $T_2$  is proportional to  $h$ , so that

$T_2$  is maximum at the top where  $h$  is maximum and equals  $H$ , thus,

$$\max T_2 = g H \tan^2 \theta \quad (91a)$$

The meridian compressive force  $T_1$  above the ring beam can be calculated as follows (Fig VI 6)

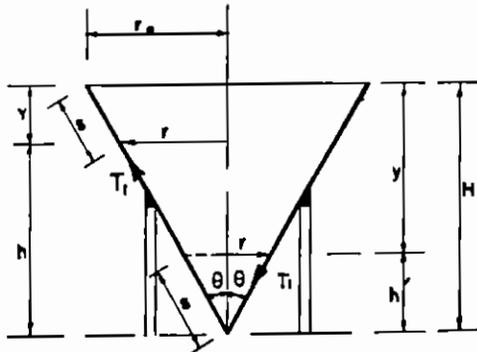


Fig VI-6

$$T_1 \cdot 2 \pi r \cos \theta = g \left( 2 \pi r_0 \frac{H}{2 \cos \theta} - 2 \pi r \frac{h}{2 \cos \theta} \right)$$

$$\text{but } r = h \tan \theta \quad \text{and} \quad r_0 = H \tan \theta$$

so that

$$T_1 h \tan \theta \cos \theta = \frac{g}{2 \cos \theta} ( H^2 \tan \theta - h^2 \tan \theta ) \quad \text{or}$$

$$T_1 = \frac{g}{2 \cos^2 \theta} \left. \begin{array}{l} \frac{H^2 - h^2}{h} \\ y \frac{2H - y}{h} \end{array} \right\} \quad \text{or} \quad (92)$$

The meridional tensile force  $T_1$  below the ring beam can be calculated from the relation

$$2 \pi r T_1 \cos \theta = g 2 \pi r \frac{h}{2 \cos \theta} \quad \text{or}$$

$$T_1 = \frac{g h}{2 \cos^2 \theta} \quad (93)$$

For the special case  $\theta = 30^\circ$  we have

$$\sin \theta = 1/2 \quad \cos \theta = \sqrt{3}/2 \quad \tan \theta = 1/\sqrt{3}$$

$$\text{and} \quad \cos^2 \theta = 3/4 \quad \tan^2 \theta = 1/3$$

If we assume further that the ring beam is arranged at mid height i.e.  $y = H/2$ , the equations giving the internal forces can be much simplified as shown in table given on page 121

#### e) Numerical Example

The design of a paste container of the form shown in fig VI 1 having  $D = 10$  ms,  $\theta = 30^\circ$ , supported symmetrically at mid-height on six columns and used for mixing paste weighing 2 tons per cubic meter can accordingly be done as follows :

$$D = 10 \text{ ms} \quad r_0 = 5 \text{ ms} \quad \theta = 30^\circ \quad w = 2 \text{ t/m}^3$$

$$\text{Therefore} \quad H = r_0 / \tan \theta = 5 \sqrt{3} = 8.66 \text{ ms}$$

$$\text{For the ring beam} \quad r_R = 25 \text{ ms} \quad y_R = h_R = \frac{H}{2} = 4.33 \text{ ms}$$

# INTERNAL FORCES IN PASTE CONTAINERS



	Any angle have $T_c$		H within comp force also c ring & air		At mid air from the force below may be air	
	Due to paste $w/m^2$	Due to air weight $g/m^2$	Due to paste $w/m^2$	Due to air weight $g/m^2$	Due to paste $w/m^2$	Due to air weight $g/m^2$
<b>GENERAL</b>	(86) $\frac{w}{2} h \frac{\tan^2 \theta}{\cos \theta}$	(91) $g h \frac{\tan^2 \theta}{\cos \theta}$ (91a) $\max g h \frac{\tan^2 \theta}{\cos \theta}$ at top	(88) $\frac{w}{2} \frac{2H \sin^2 \theta}{h} \frac{\tan^2 \theta}{\cos \theta}$	(92) $\frac{g}{2} \frac{2H \sin^2 \theta}{h} \frac{\tan^2 \theta}{\cos \theta}$	(89) $\frac{w}{2} h (H + 2H) \frac{\tan^2 \theta}{\cos \theta}$ (90) $\max \frac{w}{2} h (H + 2H) \frac{\tan^2 \theta}{\cos \theta}$ at $2/3 H$	(93) $\frac{g}{2} h \frac{\tan^2 \theta}{\cos \theta}$
	at any depth $y = h - \frac{H}{3}$	(86a) $\frac{w}{3} w' y h$	(91b) $\frac{g}{3} h$	(88a) $\frac{w}{3} \frac{2H \sin^2 \theta}{h} \frac{\tan^2 \theta}{\cos \theta}$	(92a) $\frac{g}{3} \frac{2H \sin^2 \theta}{h} \frac{\tan^2 \theta}{\cos \theta}$	(89a) $\frac{w}{3} h (H + 2H) \frac{\tan^2 \theta}{\cos \theta}$
at mid height $y = h - \frac{H}{2}$	(87) $\frac{w}{6} \frac{H^2}{\cos^2 \theta}$	(91c) $\frac{g}{6} \frac{H}{\cos^2 \theta}$	(88b) $\frac{w}{6} \frac{H^2}{\cos^2 \theta}$	(92b) $\frac{g}{6} H$	(89b) $\frac{w}{6} \frac{H^2}{\cos^2 \theta}$	(93b) $\frac{g}{6} \frac{H}{\cos^2 \theta}$
$y = 0$ $= H/4$ $= H/4$ $= 3H/4$ Multiplication $H$						

Note: Figures between brackets give the number of the equations

The container wall will be assumed 20 cms thick, so that

$$g = 0.2 \times 2.5 = 0.5 \text{ t/m}^2$$

Max ring force  $T_2$  max at  $y = H/2$

Due to pressure of paste ( equation 87 )

$$T_2 = \frac{w H^2}{6} = 2 \times \frac{8.66^2}{6} = 25.00 \text{ t/m}$$

Due to own weight ( equation 91c )

$$T_2 = g \frac{H}{6} = 0.5 \times \frac{8.66}{6} = 0.72 \text{ t/m}$$

$$\text{Total } T_2 \text{ max} = 25.72 \text{ t/m}$$

Meridian force  $T_1$  max above ring beam

Due to pressure of paste ( equation 88b )

$$T_1 = \frac{w H^2}{9} = \frac{2 \times 8.66^2}{9} = 16.7 \text{ t/m}$$

Due to own weight ( equation 92b )

$$T_1 = g H = 0.5 \times 8.66 = 4.33 \text{ t/m}$$

Total compressive meridian force above ring beam

$$\text{Total } T_1' \text{ max} = 16.7 + 4.33 = 21.03 \text{ t/m}$$

Meridian force  $T_1$  max below ring beam

Due to pressure of paste ( equation 89b )

$$T_1 = \frac{w H^2}{9} = \frac{2 \times 8.66^2}{9} = 16.7 \text{ t/m}$$

Due to own weight ( equation 93b )

$$T_1 = g \frac{H}{3} = \frac{0.5 \times 8.66}{3} = 1.44 \text{ t/m}$$

Total tensile meridian force below ring beam

$$\text{Total } T_1'' \text{ max} = 16.7 + 1.44 = 18.14 \text{ t/m}$$

Total meridian force above and below ring beam

$$T_{1R} = T_1' + T_1'' = 21.03 + 18.14 = 39.17 \text{ t/m}$$

$$\begin{aligned} \text{Vertical load on ring beam } P_R &= T_{1R} \cos \theta \\ &= 39.17 \times \frac{\sqrt{3}}{2} = 34.00 \text{ t/m} \end{aligned}$$

$$\begin{aligned} \text{Horizontal load on ring beam } H_R &= T_{1R} \sin \theta \\ &= 39.17 \times 1/2 = 19.59 \text{ t/m} \end{aligned}$$

$$\begin{aligned} \text{Compressive force in ring beam } C_R &= H_R r_R \\ &= 19.59 \times 2.5 = 49.00 \text{ t} \end{aligned}$$

Having determined the maximum values of the internal forces in the container, one can design its different elements as follows :

Design of container wall slab       $t = 20 \text{ cms}$ 

$$\text{Max ring tensile force } T_2 \text{ max} = 25.72 \text{ t/m}$$

$$\text{Max meridian tensile force } T_1'' \text{ max} = 18.14 \text{ t/m}$$

$$\text{Max meridian compressive force } T_1' \text{ max} = 21.03 \text{ t/m}$$

$$\text{Max ring reinforcement at ring beam } A_s = \frac{T_{2\text{max}}}{\sigma_s} = \frac{25.72}{1.4} = 17.6 \text{ cm}^2$$

$$\text{Choose } 7 \phi 13 \text{ mm/m on each side } A_s = 18.5 \text{ cm}^2$$

Assuming shrinkage strain  $\epsilon_{sh} = 0.25 \text{ mm/m}$  &  $E_s = 2100 \text{ t/cm}^2$ , the max ring tensile stress in concrete  $\sigma_t$  is given by :

$$\begin{aligned} \sigma_t &= \frac{T_2 \text{ max} + \epsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{25720 + 0.00025 \times 2100000 \times 18.5}{100 \times 20 + 10 \times 18.5} \\ &= \frac{25720 + 9730}{2185} = 16.4 \text{ kg/cm}^2 \end{aligned}$$

Max longitudinal reinforcement below ring beam

$$\begin{aligned} A_s &= \frac{T_1'' \text{ max}}{\sigma_s} = \frac{18.14}{1.4} = 13 \text{ cm}^2 \text{ choose } 9 \phi 10 \text{ mm/m on each side} \\ &A_s = 14.2 \text{ cm}^2 \end{aligned}$$

Max longitudinal tensile stress in concrete  $\sigma_t'$  is given by ;

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$$\sigma_t = \frac{T_{l \max}^{II} + E_{sh} E_s A_s}{A_c + n A_s} = \frac{18140 + 0.00025 \times 2100000 \times 14.2}{10 \times 20 + 10 \times 14.2}$$

$$= \frac{18140 + 7460}{2142} = 12.0 \text{ kg/cm}^2$$

Max longitudinal compressive stress in concrete  $\sigma_c$  above ring beam is

$$\sigma_c = \frac{T_{l \max}^I}{A_c} = \frac{21030}{100 \times 20} = 10.57 \text{ kg/cm}^2$$

The given investigation shows that the chosen 20 cms thickness for the wall is convenient as all the critical stresses are within the allowable limits

#### Design of Ring Beam

Radius of ring beam  $r_R = 2.5$  m, perimeter  $2 \pi r_R = 15.7$  m. The ring beam being symmetrically supported on six columns and the load including own weight

$$P_R = 35 \text{ t/m} \quad \text{then}$$

the total load on the ring beam  $P = 2 \pi r_R P_R = 15.7 \times 35 = 550$  ton

According to table, page 96, we get

$$\text{Load on each column} = \frac{550}{6} = 91.7 \text{ ton}$$

$$\text{Max shearing force} = \frac{550}{12} = 45.85 \text{ ton}$$

$$\begin{aligned} \text{Max bending moment at center} &= +0.00750 P r_R \\ &= +0.00750 \times 550 \times 2.5 \\ &= 10.3 \text{ mt} \end{aligned}$$

$$\begin{aligned} \text{Max bending moment over support} &= -0.01482 P r_R \\ &= -0.01482 \times 550 \times 2.5 \\ &= 20.4 \text{ mt} \end{aligned}$$

$$\begin{aligned} \text{Max torsional moment} &= 0.00151 P r_R \\ &= 0.00151 \times 550 \times 2.5 \\ &= 2.06 \text{ mt} \end{aligned}$$

acting at a central angle of  $12^\circ 44'$  from the center line of any support

Axial compression

$$C_R = 50 \text{ tons}$$

This beam will be monolithically cast with the container wall, its depth being bigger than its span, it will behave as a deep circular beam of irregular cross-section. To simplify the calculation of the required steel reinforcement, its cross-section will be assumed rectangular with breadth  $b = 50$  cms and depth  $t = 87$  cms as shown in fig VI-7. The stresses in the beam will be checked for the following critical internal forces

Max  $M = 20.4 \text{ mt}$  ,  $N = 50 \text{ t}$  for normal stresses  
 max  $Q = 45.85 \text{ mt}$  for shear stresses  
 max  $M_t = 2.06 \text{ mt}$  for torsional shear stresses

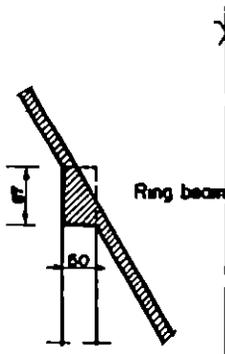


Fig VI-7

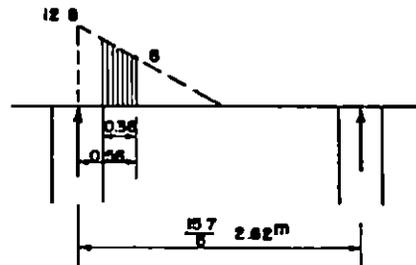


Fig VI-8

### Normal Stresses

$$\text{Max } M = 20.4 \quad N = C_R = 50 \text{ t comp}$$

$$\text{Eccentricity } e = \frac{M}{N} = \frac{20.4}{50} = 0.408 \text{ ms}$$

Eccentricity to tension steel  $e_s$

$$e_s = e + \frac{t}{2} - \text{cover} = 40.8 + 43.5 - 5 = 79.3 \text{ cms}$$

Moment about tension steel  $M_s$

$$M_s = N e_s = 50 \times 0.793 = 39.65 \text{ mt}$$

$$\frac{e_s}{d} = \frac{79.3}{82} = 0.97$$

a case of medium eccentricity

Reinforcement assumed ( about 2/3 % of section ) 5  $\phi$  19, top and 5  $\phi$  19 bottom

Percentage of tension or compression steel :

$$\mu = \mu' = \frac{A_s}{b d} = \frac{14.2}{50 \times 82} \times 100 = 0.346 \%$$

$$\sigma_c = C_1 \frac{M_s}{b d^2} = 4 \times \frac{39.05 \times 10^5}{50 \times 82^2} = 47 \text{ kg/cm}^2$$

$$\sigma_s = C_2 \frac{M_s}{b d^2} = 55 \times \frac{39.65 \times 10^5}{50 \times 82^2} = 645 \text{ kg/cm}^2$$

Note  $C_1$  &  $C_2$  are coefficients depending on  $\mu$ ,  $\mu'$  and  $\frac{e_s}{d}$  and are extracted from the design curves of reinforced concrete

### Shear stresses

$$Q_{max} = 45.85 \text{ t}, \quad V = 50 \text{ t}$$

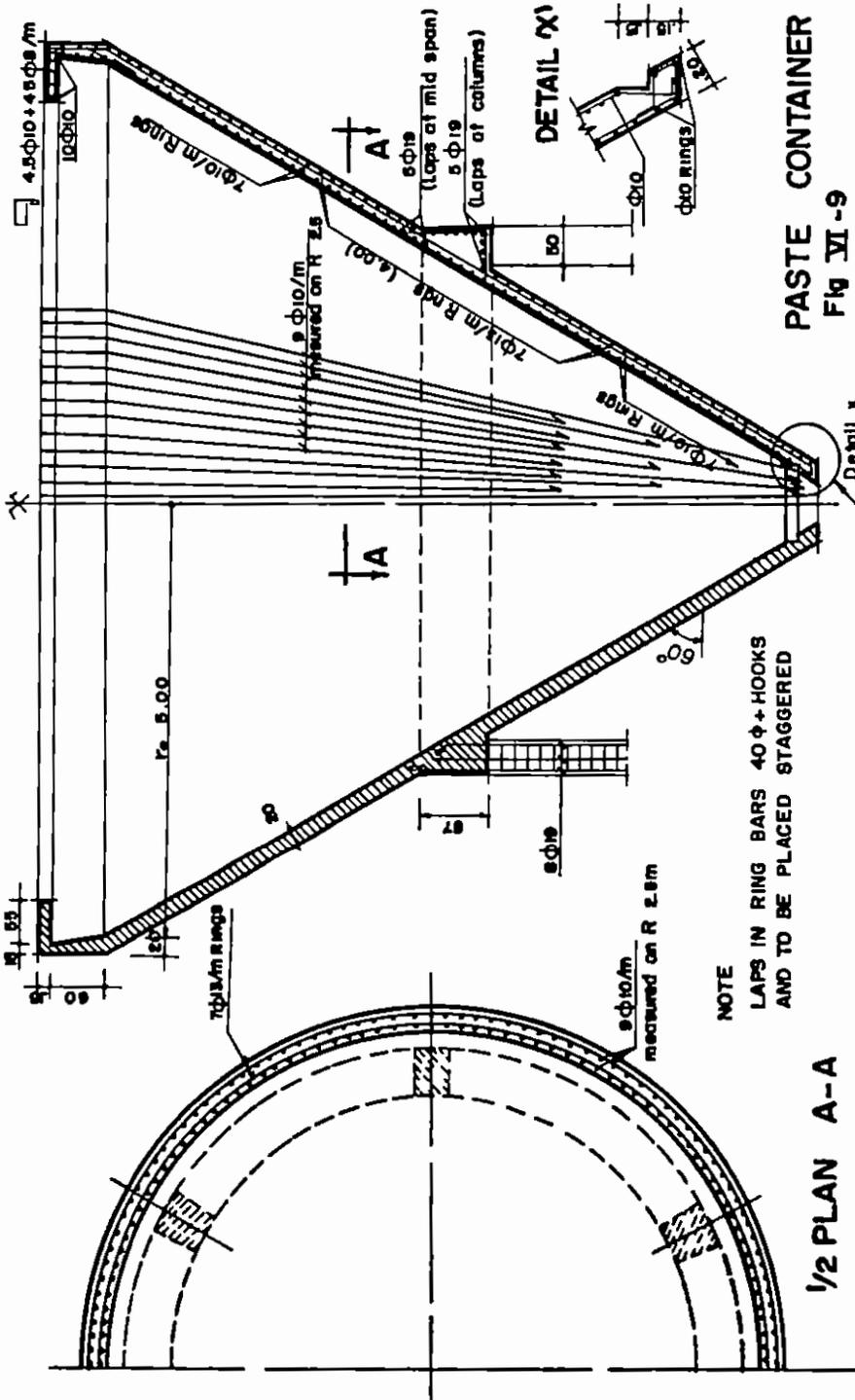
$$\tau_{max} = \frac{Q_{max}}{0.87 b d} = \frac{45850}{0.87 \times 50 \times 82} = 12.8 \text{ kg/cm}^2$$

Assuming the allowable shear stress in reinforced concrete = 8 kg/cm<sup>2</sup> then the part of the beam subject to shear stresses bigger than 8 kg/cm<sup>2</sup> is about 36 cms from the face of the column (fig VI 8), this length being smaller than 3/4 the theoretical depth of the beam, there is no need for bent bars. Vertical 2 branch stirrups  $\phi$  10 mm spaced every 10 cms will be used. In addition, 5  $\phi$  13 horizontal bars will be arranged on each side of the beam (fig VI 9)

### Torsional Shear Stresses

$$M_t = 2.06 \text{ mt} \quad b = 50 \text{ cms} \quad t = 87 \text{ cms}$$

$$\psi = 3 + \frac{2.6}{t/b + 0.45} = 3 + \frac{2.6}{87/50 + 0.45} = 4.19$$



PASTE CONTAINER  
Fig VI -9

NOTE  
LAPS IN RING BARS 40φ + HOOKS  
AND TO BE PLACED STAGGERED

1/2 PLAN A-A

The torsional shear stress is therefore given by

$$\tau_t = \psi \frac{M_t}{b^2 t} = 4.19 \times \frac{2.06 \times 10^5}{50^2 \times 87} = 4 \text{ kg/cm}^2$$

The stress being low, no additional provisions are necessary. The details of reinforcements are shown in fig VI 9

## VI 2 AN INZE WATER TANK OF CAPACITY 850 m<sup>3</sup> ( Fig VI 10 )

Capacity

Cylinder	$\pi \times 6^2 \times 6.5$	= 735 m <sup>3</sup>	
+	Cone	$\pi \times 2/3 (6^2 + 4^2 + 6 \times 4)$	= 159 m <sup>3</sup> 894 m <sup>3</sup>
-	Internal cylinder	$\pi \times 0.75^2 \times 7.4$	= 13 m <sup>3</sup>
-	Bottom Dome	$\frac{\pi \times 1.1}{6} (3 \times 4^2 + 1 \times 1^2)$	= 28 m <sup>3</sup> 41 m <sup>3</sup>
			capacity = 853 m <sup>3</sup>

### 1) Design of roof dome

For the roof dome which is a part of a hemi-sphere, we have

$$\text{Radius } a = \frac{R^2 + y^2}{2y} = \frac{6^2 + 1.4^2}{2 \times 1.4} = 13.5 \text{ m}, \quad \sin \psi = \frac{6}{13.5} = 0.445$$

$$\cos \psi = \frac{1.2}{13.5} = 0.89$$

$$\text{Area } A = 2\pi a y = 2\pi \times 13.5 \times 1.4 = 118.5 \text{ m}^2$$

$$\tan \psi = \frac{6}{1.2} = 0.50$$

Loads on roof

Own weight ( ~ 8 cms ) = 200 kg/m<sup>2</sup>

Superimposed dead and live loads = 150 "

total g = 350 " ( lantern neglected )

Total roof load W = 350 x 118.5 = 41500 kgs

Vertical load/meter run of cylindrical wall

$$P_1 = \frac{W}{2\pi R} = \frac{41500}{2\pi \times 6} = 1100 \text{ kg/m}$$

INZE WATER TOWER  
CAPACITY 850 M<sup>3</sup>

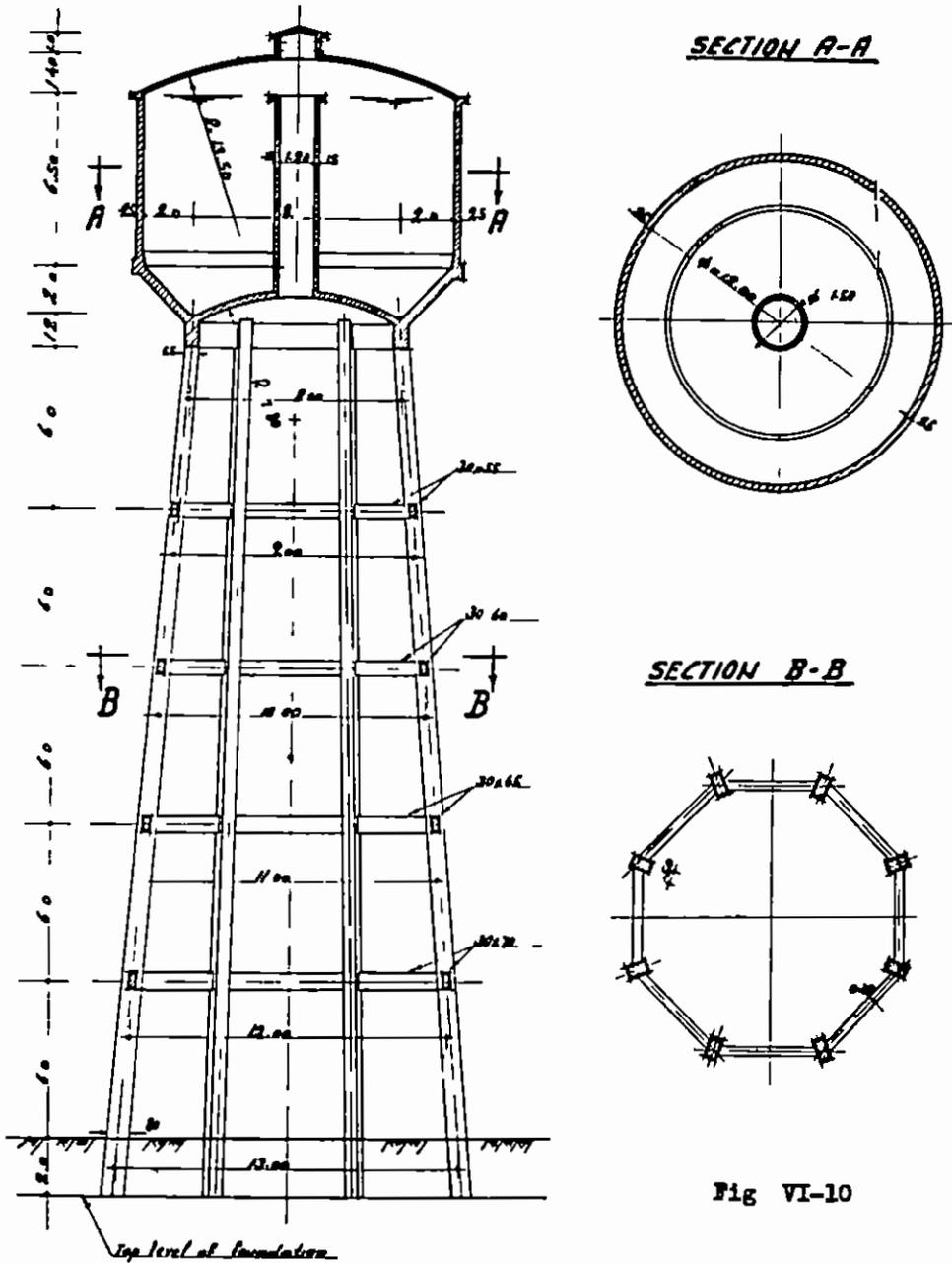


Fig VI-10

The outward horizontal thrust

$$H_1 = \frac{P_1}{\tan \phi} = \frac{1100}{0.5} = 2200 \text{ kg/m}$$

The force  $H_1$  must be resisted by a tension ring. The tension in the ring

$$T_1 = H_1 R = 2200 \times 6 = 13200 \text{ kg}$$

The steel required in the ring

$$A_s = \frac{T_1}{\sigma_s} = \frac{13200}{1400} = 9.4 \text{ cm}^2 \text{ chosen } 8 \phi 13 \text{ ( } 10.3 \text{ cm}^2 \text{ )}$$

Assuming the average cross section of the ring beam 15 x 50 cms the tensile stress in concrete is

$$\sigma_t = \frac{T_1}{A_c + n A_s} = \frac{13200}{15 \times 50 + 10 \times 10.3} = \frac{13200}{833} = 15.5 \text{ kg/cm}^2$$

Stress in dome shell

$$\text{meridian force } I_\phi = \frac{r a}{1 + \cos \phi}$$

$$\text{Crown } \phi = 0, \cos \phi = 1, I_\phi = \frac{r a}{2} = \frac{350 \times 13.5}{2} = 2350 \text{ kg/m compression}$$

$$\text{Foot ring } \cos \phi = 0.89, I_\phi = \frac{r a}{1.89} = \frac{350 \times 13.5}{1.89} = 2480 \text{ kg/m compression}$$

$$\text{or } N_\phi = \frac{P_1}{\sin \phi} = 1100 \times \frac{13.5}{6} = 2430 \text{ kg/m compression}$$

$$\text{Ring force } N_\theta = r a \left( \cos \phi - \frac{1}{1 + \cos \phi} \right)$$

$$\text{Crown } \phi = 0, \cos \phi = 1, N_\theta = \frac{r a}{2} = 350 \times \frac{13.5}{2} = 2350 \text{ kg/m compression}$$

$$\text{Foot ring } \cos \phi = 0.89, N_\theta = 0.36 r a = 36 \times 350 \times 13.5 = 1730 \text{ kg/m compression}$$

The max compressive stress in concrete

$$\text{Crown } \sigma_c = \frac{2350}{100 \times 8} = 2.93 \text{ kg/cm}^2$$

Foot-ring:  $\sigma_c = 2480/100 \times 15 = 1.58 \text{ kg/cm}^2$  meridian  
 and  $= 1730/100 \times 15 = 1.15$  " ring

The compressive stress in concrete of dome shell is very low and a very thin shell might be used. An 8 cm shell is easier to execute and gives better isolation and insulation.

The foot ring being subjected to high tensile stresses ( $15.5 \text{ kg/cm}^2$ ) and strains while the dome shell at the same ring is subjected to low compressive stresses ( $1.15 \text{ kg/cm}^2$ ) and strains, bending moments will take place at the junction between the shell and the foot ring along the meridian. It is thus advisable to increase the thickness of the shell at the footing for a length  $x = 0.6 \sqrt{at}$  as shown in fig VI-11.

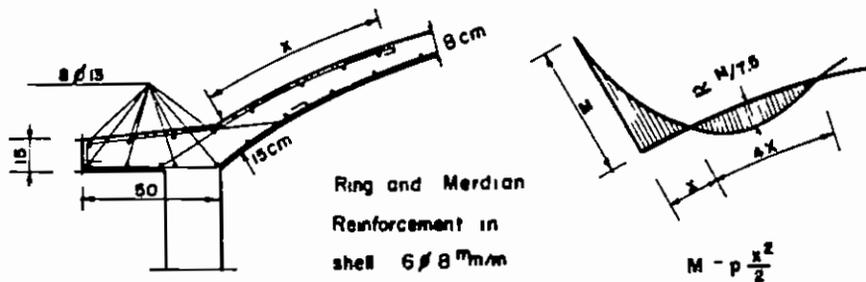


Fig VI-11

Assuming the thickness of the dome shell at footing is equal to 15 cms, we get

$$x = 0.6 \sqrt{at} = 0.6 \sqrt{13.5 \times 0.15} = 0.85 \text{ ms}$$

The fixing moment is therefore approximately given by

$$M = p x^2 / 2 = 350 \times 0.85^2 / 2 = 125 \text{ kgm}$$

In spite of the small value of the fixing moment, the recommended gradual increase of the shell at the footing and reinforcing it on both surfaces is a good practice.

#### Design of Tank :

The cylindrical wall, the conical floor and the spherical

bottom dome of the tank are rigidly connected together and may be considered as continuous in which case, the statically indeterminate connecting moments can be determined by the moment distribution method as follows ( fig VI 12 )

Assume the cylindrical wall to be fixed at a, the conical floor to be fixed at a and b and the bottom spherical dome to be fixed at b and determine the corresponding fixing moments due to the acting loads and forces namely water pressure, own weight, temperature changes etc

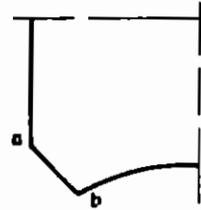


Fig VI-12

The difference between the fixing moments at a and at b will be distributed between any two adjacent elements according to their relative stiffness and the corresponding distribution factors

In order to simplify the calculations and at the same time to have ample safety, one can proceed as follows

#### Design of Cylindrical Wall :

The ring tension field moments and shear at base can be determined for the case of wall with hinged base, while the bending moment at the bottom of the wall may be determined for the case of wall with fixed base. According to these assumptions the thickness of the wall will be governed by the maximum ring tension which can be estimated by using the simplified method shown in III 4 2. Having chosen the thickness, the internal forces in the wall and the corresponding design can be determined with the help of the P C A tables. Thus

$$T_{\max} = 0.75 w H R = 0.75 \times 1 \times 6.5 \times 6 = 29 \text{ t for } \frac{D}{R} = \frac{12}{6.5} = 1.85$$

$$\text{Therefore } t = 0.8 \times 29 = 23.2 \text{ cms}$$

The wall will be chosen of constant thickness equal to 25 cms. According to the P C A tables we have :

$$\frac{H^2}{D t} = \frac{6.5^2}{12 \times 0.25} = 14 \quad \text{and}$$

for a wall with hinged base we get :

According to table II  $T_{\max} = 0.761 w H R$  at  $0.7 H$  or

$$T_{\max} = 0.761 \times 1 \times 6.5 \times 6 = 30 \text{ t/m acting at } \sim 4.5 \text{ ms from top}$$

According to table VIII  $M_{\max} = +0.0033 w H^3$  at  $0.9 H$  or

$$M_{\max} = +0.0033 \times 1 \times 6.5^3 = 0.91 \text{ m t}$$

According to table XVI the shear at base is given by

$$V_{\max} = 0.073 w H^2 = 0.073 \times 1 \times 6.5^2 = 3.1 \text{ t} \quad \text{and}$$

for a wall with fixed base we get

The maximum fixing moment, according to table VII is given by

$$V_{\max} = -0.09 w H^3 = -0.009 \times 1 \times 6.5^3 = -2.5 \text{ m t}$$

Further the wall is subject to compressive stresses due to the loads from the roof and its own weight

Thrust at top edge

$$P_1 = P_1 = 1.1 \text{ t/m}$$

Thrust at bottom edge

$$N_2 = N_1 + \text{own weight of wall}$$

$$\text{or } N_2 = 1.1 + 0.25 \times 6.5 \times 2.5$$

$$= 3.15 \text{ t/m}$$

Accordingly, the internal forces for which the wall is designed are as follows (fig VI 13)

Design of section of max ring tension

$$T_{\max} = 30 \text{ t/m}$$

$$\text{Max ring reinforcement, } A_s = \frac{T}{\sigma_s} = \frac{30}{1.4} = 21.5 \text{ cm}^2$$

$$\text{chosen } 6 \phi 16 \text{ mm/m on each face ( } A_s = 24 \text{ cm}^2 \text{ )}$$

$$\text{Thickness of wall } t = 0.8 T = 0.8 \times 30 = 24 \text{ cms chosen } 25 \text{ cms}$$

The ring reinforcements will be reduced in the upper sections of the wall as shown in fig VI 13

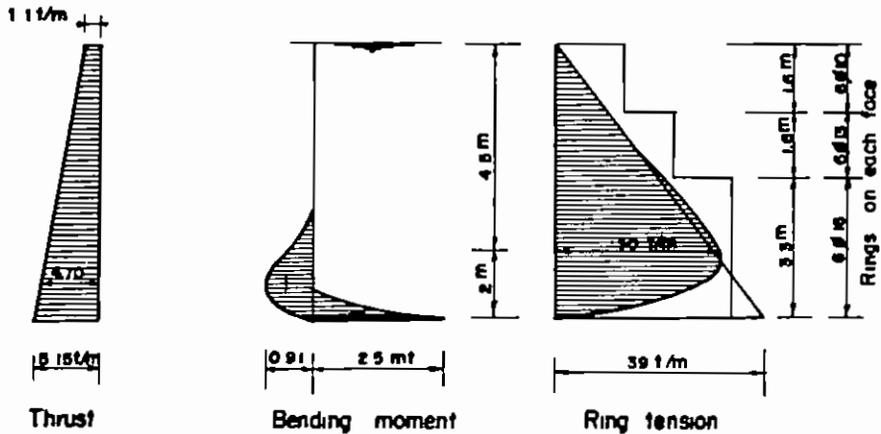


Fig VI-13

For upper 1.6 m

$$P_{max} = 1.6 \times 6 = 9.6 \text{ t/r}$$

$$A_s = \frac{9.6}{1.4} = 6.8 \text{ cm}^2$$

Use 6  $\phi$  13/m on each face

For second 1.6 m

$$P_{max} = 3.2 \times 6 = 19.2 \text{ t/r}$$

$$A_s = \frac{19.2}{1.4} = 13.6$$

Use 6  $\phi$  13/m on each face

Section of max positive moment

$$P_{max} = 10 \text{ kgm} \quad N = 4700 \text{ kgs compression} \quad t = 23 \text{ cms}$$

$$e = \frac{10}{N} = \frac{910}{4700} = 0.193 \text{ ms} \quad e_s = e + \frac{t}{2} - \text{cover} = 19.3 + 12.5 - 3 = 28.8 \text{ cms}$$

$$M_s = N e_s = 4700 \times 0.288 = 1350 \text{ kgm}$$

$$d = k_1 \sqrt{M_s} \quad \text{or} \quad 22 = k_1 \sqrt{1350} \quad \text{from which } k_1 = 0.6$$

For  $\sigma_s = 1400 \text{ kg/cm}^2$   $\sigma_c = 26 \text{ kg/cm}^2$  and  $k_2 = 1300$ , therefore

$$A_s = \frac{M_s}{k_2 d} - \frac{N}{\sigma_s} = \frac{1350}{1300 \times 0.22} - \frac{4700}{1400} = 4.75 - 3.35 = 1.4 \text{ cm}^2$$

The minimum vertical reinforcement is 20 % of the rings i.e.

$$0.2 \times 12 = 2.4 \text{ cm}^2$$

The vertical reinforcements will be chosen  $5 \phi 8 \text{ mm/m}$  on each face

### Section at base of wall

$M = 2500 \text{ kgm}$ ,  $N = 5150 \text{ kg}$  (compression)

The tension being on the water side then

$$t = \sqrt{f_3} - 2 \text{ cms} = \sqrt{2500/3} - 2 = 27 \text{ cms}$$

The wall at its bottom edge will be provided by a haunch  $10 \times 40 \text{ cms}$

so that  $t_{\max} = 35 \text{ cms}$  and  $d = 32 \text{ cms}$ , therefore

$$e = \frac{M}{N} = \frac{2500}{5150} = 0.485 \text{ ms} \quad e_s = e - \frac{t}{2} + \text{cover} = 48.5 + \frac{35}{2} - 3$$

$$= 63 \text{ cms}$$

$$M'_s = e_s = 5150 \times 63 = 3270 \text{ kgm} \quad \text{and}$$

$$32 = k_1 \sqrt{2500} \quad \text{or} \quad k_1 = 0.56$$

$$\text{for } \sigma_s = 1400 \text{ kg/cm}^2 \quad \sigma_c = 30 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1280$$

$$A_s = \frac{M}{k_2 d} - \frac{N}{\sigma_s} = \frac{3250}{1280 \times 0.32} - \frac{5150}{1400} = 7.95 - 3.65 = + 4.30 \text{ cm}^2$$

chosen  $6 \phi 10 \text{ mm/m}$

### 3 Design of Ring Beam at Foot of Wall

The load of the roof and cylindrical wall ( $N_2$ ) will cause meridian compressive forces  $N_{s1}$  in

the conical floor. But  $N_{s1}$  is not

colinear with  $N_2$ , thus, in order to

have equilibrium the horizontal

thrust  $H$  is created at joint a, its

magnitude can be determined from

the triangle of forces shown in

figure VI-14. In order to be able

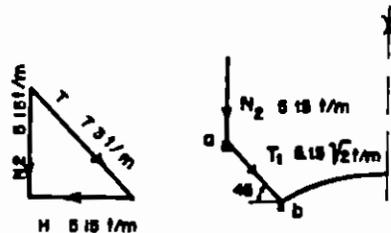


Fig VI-14

to resist this horizontal force  $H$ , a ring beam can be arranged at joint a. This beam has, in addition, to resist the horizontal shear  $V_{\max}$  at the base of the wall. So that the total horizontal force  $H_2$  acting on the ring beam is given by

$$H_2 = 5.15 + 3.1 = 8.25 \text{ t/m}$$

The corresponding ring tension  $T$  is given by :

$$T = H_2 \times R = 8.25 \times 6 = 49.5 \text{ t}$$

Required ring reinforcement

$$A_s = T / \sigma_s = 49.5 / 1.4 = 35.3 \text{ cm}^2$$

Chosen 10  $\phi 22$  ( $A_s = 38 \text{ cm}^2$ )

The details of reinforcements are shown in figure VI 15

#### 4 Design of Conical Part of the Tank floor

The conical floor is subjected to meridional compressive forces  $N_s$  and ring tensile forces  $H_\theta$ . Their values can be determined according to the fundamental principles given in chapter I 3 + 6 as follows

The vertical component of the meridional forces at any section is equal to the vertical forces per meter acting at that section, thus,

The meridional force at the upper edge of the cone is given by

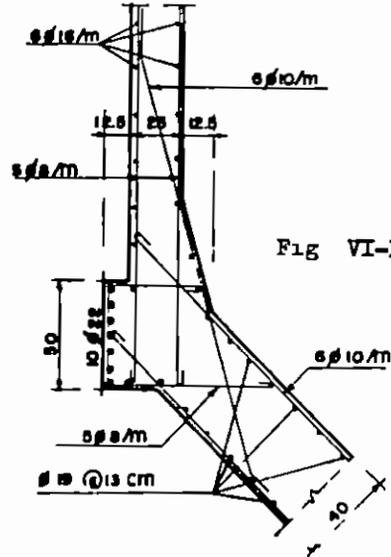
$$N_s = \frac{W / m'}{\sin \phi} = 5.15 \times \sqrt{2} = 7.3 \text{ t/m}$$

at mid-height of cone ( fig VI 16 )

Depth of water = 7.5 m horizontal radius  $R = 5$  m

$$\text{Weight of roof and wall} = \frac{5.15 \times \pi \times 12}{\pi \times 10} = 6.18 \text{ t/m}$$

Assuming thickness of cone 40 cms ( $1 \text{ t/m}^2$ )



Weight of upper half of cone

$$\pi \times \frac{12+10}{2} \times 1\sqrt{2} \times \frac{1}{\pi \times 10} = \frac{22\sqrt{2}}{20}$$

$$= 1.70 \text{ t/m}$$

Weight of water on upper half of cone

$$\pi \times (6^2 - 5^2) \times 7 \times \frac{1}{\pi \times 10} = \frac{11 \times 7}{10}$$

$$= 7.70 \text{ t/m}$$

$$\text{Total load } 7 = 6.18 + 1.70 + 7.70$$

$$= 15.58 \text{ t/m}$$

$$\text{Meridian force } N_s = \frac{7}{\sin \varphi} = 15.58 \sqrt{2}$$

$$= 22 \text{ t/m}$$

At foot of cone

$$\text{Weight of roof and wall} = \frac{5.15 \times \pi \times 12}{\pi \times 8}$$

$$= 7.72 \text{ t/m}$$

$$\text{Weight of cone} \pi \times \frac{12+8}{2} \times 2\sqrt{2} \times \frac{1}{\pi \times 8} = 3.57 \text{ t/m}$$

$$\text{Weight of water } \pi (6^2 - 4^2) \times 7.5 \times \frac{1}{\pi \times 8} = 18.75 \text{ t/m}$$

$$\text{Total load } 1 = 7.72 + 3.57 + 18.75 = 30.02 \text{ t/m}$$

$$\text{Meridian force } N_s = 30.02 \sqrt{2} = 42.6 \text{ t/m}$$

Ring force  $N_\theta$  can be determined from the relation :

$$N_\theta = R_2 p_r \quad \text{in which } p_r = \text{water pressure} + g \cos \varphi$$

$$\text{at upper edge of cone : } R_2 = 6\sqrt{2} = 8.5 \text{ ms, } p_r = 6.5 + 1/\sqrt{2}$$

$$= 7.207 \text{ t/m}^2 \text{ i.e.}$$

$$N_\theta = 8.5 \times 7.207 = 61.3 \text{ t/m}$$

$$\text{at mid-height of cone : } R_2 = 5\sqrt{2} = 7.07 \text{ ms, } p_r = 7.5 + 1/\sqrt{2}$$

$$= 8.207 \text{ t/m}^2 \text{ i.e.}$$

$$N_\theta = 7.07 \times 8.207 = 58.1 \text{ t/m}$$

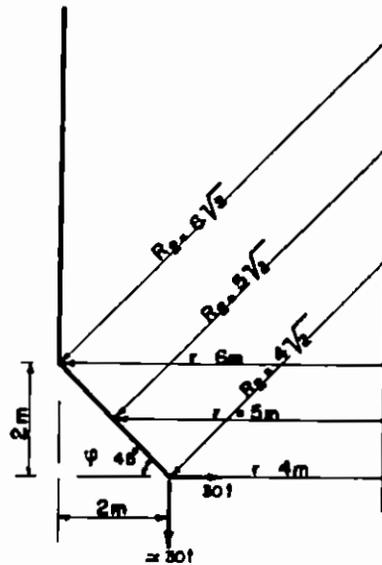


Fig VI-16

at foot of cone  $R_2 = 4\sqrt{2} = 5.657 \text{ ms}$ ,  $p_r = 8.5 + 1/\sqrt{2}$   
 $= 9.207 \text{ t/m}^2$  i.e.

$$N_\theta = 5.66 \times 9.207 = 52.2 \text{ t/m}$$

Accordingly, the ring reinforcements and the tensile stresses in concrete are given by

$$A_s = \frac{N_\theta}{\sigma_s} = \frac{N_\theta}{1.4} \quad \text{and} \quad \sigma_t = \frac{N_\theta}{A_c + n A_s} = \frac{N_\theta}{4000 + 10 A_s}$$

Section	top	Mid-height	Foot
Ring force $N_\theta$	61.3 t/m	58.1 t/m	52.2 t/m
Area of steel $A_s$	43.6 cm <sup>2</sup>	41.5 cm <sup>2</sup>	37.2 cm <sup>2</sup>
Reinf. on each face	$\Phi 19 \text{ mm} @ 13 \text{ cm}$	$\Phi 19 \text{ mm} @ 13 \text{ cm}$	$\Phi 19 \text{ mm} @ 16 \text{ cm}$
Tensile stress $\sigma_t$	14.5 kg/cm <sup>2</sup>	14.0 kg/cm <sup>2</sup>	12.5 kg/cm <sup>2</sup>

The longitudinal reinforcement is chosen 20% of the ring reinforcement i.e.  $+4 \text{ cm}^2$  or  $6 \Phi 10 \text{ mm/m}$  on each face

The stresses in the cone in the direction of the meridian will be calculated for the meridian force plus the flexing moment of the wall and the spherical floor. Thus

Stresses due to meridian force

at top edge  $N = N_\theta = 7.3 \text{ t}$  compression  $L = 2.5 \text{ m}$

$$\sigma_c = - \frac{7300}{100 \times 40} \pm \frac{6 \times 2500 \times 00}{100 \times 40^2} = -1.82 \pm 9.38 = -11.2 \text{ kg/cm}^2 \text{ comp} \\ + 7.56 \text{ " tension}$$

The stresses at the bottom edge can be calculated when the moment is determined

5 Design of Domed Floor (f.g VI )

$$\text{Radius } a = \frac{r^2 + y^2}{2y} = \frac{4^2 + 11^2}{2 \times 11} = 7.8 \text{ m}$$

$$\text{Area } A = 2 \pi a y_2 = 2 \pi \times 7.8 \times 11 = 540 \text{ m}^2$$

Weight of shaft is neglected and the water is assumed to cover the whole surface of the dome

The dome is assumed 15 cms thick at crown and 30 cms at the foot ring

The internal forces at the crown and the foot of the dome will be calculated for the water pressure and the own weight of the dome assuming that it has an average thickness of about 23 cms

The dome is calculated according to the membrane theory given in chapter IV 3 + a, then the fixing moment at its foot is estimated according to data given in figure IV-25 Thus

$$\text{Own weight at crown} = 0.15 \times 2500 = 375 \text{ kg/m}^2, \text{ water} = 7400 \text{ kg/m}^2$$

$$\text{Own weight at footing} = 0.30 \times 2500 = 750 \text{ kg/m}^2 \quad \text{water} = 8500 \text{ kg/m}^2$$

$$\text{Average weight of dome} = \frac{0.375 + 0.75}{2} \times 54 = 30.4 \text{ t}$$

$$\text{Total weight of water} = \pi \times 4^2 \times 8.5 = 28 \quad = 398.6 \text{ t}$$

$$\text{total} = 429 \text{ t}$$

Meridian and ring forces at crown

$$N_\phi = N_\theta = p a / 2$$

$$N_\phi = N_\theta = \frac{(7400 + 375) \times 7.8}{2} = 30.3 \text{ t/m compression}$$

Meridian and ring force at footing :

$$\text{Meridian force } N_\phi = \frac{429}{\pi \times 8} \times \frac{7.8}{4} = 33 \text{ t/m compression}$$

$$\text{Ring force } N_\theta = a p_r - N_\phi = 7.8 p_r - 33 \quad \text{but}$$

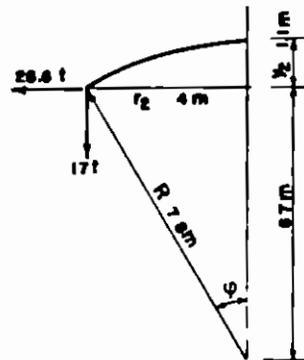


Fig VI-17

$$P_r = \text{water pressure} + g \cos = 8.5 + 0.75 \times \frac{6.7}{7.8} = 9.142 \text{ t/m}^2$$

$$\text{So that } N_\theta = 7.8 \times 9.142 - 33 = 38 \text{ t/m} \quad \text{compression}$$

The bending moments at the lower ring beam can be determined according to Markus - refer to pages (92-94) - as follows

i) Fixing end moment and stiffness of conical floor

Referring to figure I/ - 29, we have

$$h = 12.5 \text{ m} \quad s = 5.60 \text{ m} \quad l = 8.49 \text{ m} \quad t = 0.40 \quad \varphi = 45$$

$$g = 0.4 \times 2.5 = 1.00 \text{ t/m}^2 \quad P = 5.15 \text{ t/m}$$

Therefore

$$k_3 = 1.3068 \quad \sqrt{1/5 \times 0.40} = 0.87 \quad \text{and}$$

$$\begin{aligned} E \Delta r_0 &= \frac{1.00 \times 5.66^2}{0.40} \left[ 1 - \frac{0.167}{2 \times 0.5} \left( 1 - \frac{8.49^2}{5.66^2} \right) \right] \times 0.5 \times 1.0 \\ &+ 0.167 \times \frac{5.15 \times 8.49}{0.4} \times 1.0 \\ &+ \frac{1.00 \times 5.66}{0.40} \times 5.66 \left( \frac{12.50}{0.707} - 5.60 \right) + \frac{0.167}{2 \times 5.60} \left[ \frac{12.50}{0.707} (8.49^2 - 5.66^2) \right. \\ &\quad \left. - \frac{2}{3} (8.49^3 - 5.66^3) \right] \times 0.5 \\ &= 48.6 + 18.5 + 524.0 = 591.1 \end{aligned}$$

$$\begin{aligned} E \theta_0 &= \frac{1.00 \times 5.66}{0.40} \left[ \frac{1}{2} \left( 1 - \frac{8.49^2}{5.66^2} \right) + 0.167 - 2 \times 1.67 \times 0.5 \right] \frac{1}{0.707} \\ &- \frac{5.15 \times 8.49}{0.4 \times 5.66} \times \frac{0.707}{0.50} \\ &+ \frac{1.00}{0.40} \left[ 3 \times 5.66^2 \times 0.707 - 2 \times 12.5 \times 5.66 - \frac{12.50}{2 \times 5.66} (8.49^2 - 5.66^2) \right. \\ &\quad \left. + \frac{0.707}{3 \times 5.66} (8.49^3 - 5.66^3) \right] \times 1.0 \\ &= -30.7 - 27.3 - 250.0 = -308 \end{aligned}$$

hence,

The fixed end moment  $\bar{M}$  is

$$\bar{M} = \frac{0.40 \times 0.707}{2 \times 5.66^2 \times 0.87^2 \times 0.5} (592 + \frac{707}{87} \times 308) = 9.8 \text{ m t}$$

The stiffness  $S$  is

$$S = 2 \times 0.40^3 \times 0.87 = 0.1115$$

11) Fixed end moment and stiffness of floor spherical dome

Referring to figure IV - 28, we have

$$a = 7.8 \text{ m} \quad t_0 = 15 \text{ cm} \quad t = 30 \text{ cms} \quad h = 15.2 \text{ ms}$$

$$\sin \varphi = 4.0 / 7.80 = 0.5128$$

$$\sin^2 \varphi = 0.263$$

$$\varphi = 30^\circ 51' = 0.538 \text{ radians}$$

$$\cos \varphi = 0.8585$$

$$h = 8.5 + \frac{4.0}{\tan \varphi} = 15.20 \text{ ms}$$

$$g_0 = 0.15 \times 2.5 = 0.375 \text{ t/m}^2$$

$$k_2 = 1.30 \times 8 \sqrt{\frac{7.80}{0.3}} = 6.66$$

$$\Delta r_0 = \frac{0.375 \times 7.8^2}{0.30 \times 0.15} \left[ \frac{1.167}{0.263} (0.15 - 0.3 \times 0.8585) + \frac{0.30 - 0.15}{0.538} - 0.3 \times 0.8585 \right]$$

$$- \frac{1.00 \times 7.8^3}{0.30} \left[ \frac{0.833 \times 15.20}{2 \times 7.80} - 0.8585 + \frac{1.167}{3} (0.8585 + \frac{1}{1.8585}) \right] \times 0.5128$$

$$= -51 - 403 = -454$$

$$e_0 = \frac{0.375 \times 7.8}{0.30 \times 0.15} \left\{ \frac{0.30 - 0.15}{0.538} \times 0.8585 - 2.167 \times 0.30 \times 0.5128 + \frac{1.167(0.30 - 0.15)}{0.30 \times 0.538 \times 0.263} \right.$$

$$\left. \times \left[ 0.15 - 0.3 \times 0.8585 + \frac{0.30 - 0.15}{0.538} \times 0.5128 - \frac{0.30}{1.167} \times 0.8585 \times 0.263 \right] \right\}$$

$$+ \frac{1.00 \times 7.8^2}{0.30} \left\{ 0.5128 + \frac{0.30 - 0.15}{0.30 \times 0.538} \times 1.167 \left[ \frac{15.20}{2 \times 7.8} - \frac{1}{3} (0.8585 + \frac{1}{1.8585}) \right] \right.$$

$$\left. - \frac{0.30 - 0.15}{0.30 \times 0.538} \left( \frac{15.2}{7.8} - 0.8585 \right) \right\}$$

$$= 18.9 + 10.1 = +29$$

The fixed end moment  $\bar{M}$  and the stiffness are given by

$$\bar{M} = - \frac{0.30}{2 \times 6.66^2} \left( \frac{-454}{0.5128} - \frac{7.80}{6.66} \times 29 \right) = \underline{3.1} \text{ mt}$$

$$S = 2 \times \frac{0.30^3}{7.80} \times 6.66 = \underline{0.046}$$

ii) Stiffness of ring beam

$$b = 0.50 \text{ m} \quad t = 1.20 \text{ m} \quad R = 4.0 \text{ m} \quad \text{then}$$

$$S = 0.972 \times 50 \times 1.20^3 / 4.0^2 = \underline{0.0525}$$

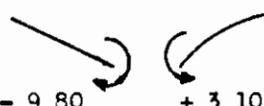
Relative stiffness

$$\text{Cone} = \frac{0.115}{0.1115 + 0.04 + 0.0525} = \frac{0.115}{0.21} = 0.53$$

$$\text{Dome} = \frac{0.046}{0.21} = 0.22$$

$$\text{Ring beam} = \frac{0.0525}{0.21} = 0.25$$

Distributed moments (Fig VI-18)

$\bar{M} = 9.80 - 3.10 = \underline{6.70} \text{ mt}$		
Moment at foot of cone = $-\underline{6.25} \text{ mt}$	-	9.80
Moment at foot of dome = $-\underline{4.47} \text{ mt}$	+	3.10
Torsional moment in ring beam	-	6.70
	+	4.47

$$6.70 \times 0.25 = \underline{1.68} \text{ mt}$$

Fig VI-18

Stresses in the spherical dome of the floor

at crown :  $t_0 = 15 \text{ cms}$   $N_\phi = N_\theta = 30.30 \text{ t/m}$

$$\sigma_c = \frac{30.300}{100 \times 15} = -20.20 \text{ kg/cm}^2$$

at footing  $t = 30$  cms  $N_{\phi} = - 33$  t/m  $M_{\phi} = 4.47$  mt/m  
 $N_{\theta} = - 38$  t/m

Compressive stress in tangential direction is

$$\sigma_c = - \frac{38000}{100 \times 30} = - 12.7 \text{ kg/cm}^2 \quad \text{compression}$$

Safety against cracking in meridian direction

$$\begin{aligned} \sigma_c &= - \frac{33000}{100 \times 30} \pm \frac{6 \times 447000}{100 \times 30^2} = - 11 \pm 29.8 \\ &= - 40.80 \text{ kg/cm}^2 \text{ comp} \\ &= + 18.80 \text{ tension} \end{aligned}$$

Neglecting the concrete in tension (stage II and  $n = 15$ ) and with

$$A_s = A'_s = 6 \text{ } \phi \text{ 10 mm/m} \quad \text{i.e.} \quad \mu = \mu' = \frac{4.70}{27 \times 100} = 0.175 \%$$

$$e = e_{\phi} / I_{\phi} = 0.13 \text{ ms} \quad e_s = 0.25 \text{ ms} \quad M_s = 8250 \text{ kgm}$$

$$s / b d^2 = 9.20 \text{ kg/cm}^2 \quad e_s / d = 0.93 \quad C_1 = 4.4 \quad C_2 = 70$$

$$\sigma_c = + 4 \times 9.2 = 40.5 \text{ kg/cm}^2 \quad \text{and} \quad \sigma_s = 70 \times 9.2 = 644 \text{ kg/cm}^2$$

Meridian stresses in core at footing

$$\begin{aligned} t &= 40 \text{ cms} & A_s = A'_s &= 9 \text{ } \phi \text{ 10 mm/m} & \mu = \mu' &= \frac{7.10}{37 \times 100} = 0.192\% \\ & & N_{\theta} &= - 42.6 \text{ t/m,} & M_{\phi} &= 6.25 \text{ mt/m} \end{aligned}$$

Safety against cracking in meridian direction

$$\begin{aligned} \sigma_c &= - \frac{42600}{100 \times 40} \pm \frac{625000}{100 \times 40^2} = - 10.7 \pm 23.4 \\ &= - 34.1 \text{ kg/cm}^2 \text{ compression} \\ &= + 12.7 \text{ " tension} \end{aligned}$$

Neglecting the concrete in tension

$$s = 0.147 \text{ ms} \quad e_s = 0.31 \text{ ms} \quad M_s = 13200 \text{ kgm}$$

$$s / b d^2 = 10 \quad e_s / d = 0.85 \quad C_1 = 3.8 \quad C_2 = 40$$

$$= 3.8 \times 10 = 38 \text{ kg/cm}^2 \quad \sigma_s = 40 \times 10 = 400 \text{ kg/cm}^2$$

actions of dome at footing

$$\text{Vertical reaction} = 429/\pi \times 8 = 17 \text{ t/m}$$

$$\text{Horizontal reaction} = 17 \times \frac{6.7}{4} = 28.6 \text{ t/m}$$

## 6 Design of Lower Circular Beam

This beam is supported symmetrically on eight columns

$$\text{The resultant vertical force on the beam} = 30 + 17 = 47 \text{ t/m} \quad \text{and}$$

$$\text{The resultant inward horizontal force} = 30 - 28.6 = 1.4 \text{ t/m} \quad \text{e}$$

$$\text{The axial compression in the beam} = 1.4 \times 4 = 5.6 \text{ t}$$

It is however recommended to choose the different elements of the tank such that the axial force in this beam is minimum

The total vertical load on the beam  $P = (47 + \text{own weight}) \pi D$

$$\text{Assuming own weight} = 1.2 \text{ t/m}, \quad \text{then } P = 46.2 \times \pi \times 8 = 1212 \text{ ton}$$

According to table we get

$$\text{Max shearing force} \quad Q_{\text{max}} = \frac{P}{16} = \frac{1212}{16} = 76 \text{ ton}$$

$$\begin{aligned} \text{Max bending moment at center of each span} &+ = 0.0416 \text{ P R} \quad \text{or} \\ \text{max} &+ = 0.0416 \times 1212 \times 4 = 20 \text{ mt} \end{aligned}$$

$$\begin{aligned} \text{Max bending moment over center line of support} &M_{\text{max}} - = 0.0827 \text{ P R} \quad \text{or} \\ M_{\text{max}} &- = 0.0827 \times 1212 \times 4 = 40 \text{ mt} \end{aligned}$$

$$\begin{aligned} \text{Max torsional moment} & \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} I_t = 0.0063 \text{ P R} \\ M_t = 0.0063 \times 1212 \times 4 = 3.1 \text{ mt} \end{array} \\ \text{due to vertical loads} & \end{aligned}$$

$$\text{Torsional moment due to unbalanced moment} = 1.68 \text{ mt}$$

$$\text{Total torsional moment} = 4.78 \text{ mt}$$

Assuming the ring beam  $50 \times 120$  cms, we get

$$\text{For middle section of any span} \quad M = 20 \text{ mt} \quad N = 5.6 \text{ t}$$

$$e = \frac{M}{N} = \frac{20}{5.6} = 3.57 \text{ ms}$$

$$e_s = 3.57 + 0.6 - 0.05 = 4.12 \text{ ms}$$

$$M_s = N e_s = 5.6 \times 4.12 = 23.1 \text{ mt}$$

$$115 = k_1 \sqrt{\frac{23100}{0.5}} \quad k_1 = 0.54, \text{ for } \sigma_s = 1400 \text{ kg/cm}^2, \alpha = 0$$

$$\sigma_c = 31 \text{ kg/cm}^2 \quad k_2 = 1285 \quad \text{and}$$

$$A_s = \frac{M_s}{k_2 d} - \frac{N}{\sigma_s} = \frac{23100}{1285 \times 1.15} - \frac{5600}{1400} = 15.6 - 4 = 11.6 \text{ cm}^2$$

which must be bigger than the minimum steel (0.25% of cross-section)

$$\text{minimum } A_s = \frac{0.25}{100} \times 50 \times 120 = 15 \text{ cm}^2 \text{ chosen } 4 \phi 22$$

For section over center line of supports  $M = 40 \text{ mt}$ ,  $N = 5.6 \text{ t}$

$$e = \frac{40}{5.6} = 7.14 \text{ ms} \quad e_s = 7.14 + 0.6 - 0.05 = 7.69 \text{ ms}$$

$$M_s = 5.6 \times 7.69 = 43 \text{ mt}$$

$$115 = k_1 \sqrt{\frac{43000}{0.5}} \quad k_1 = 0.392, \text{ for } \sigma_s = 1400 \text{ kg/cm}^2 \quad \alpha = 0$$

$$\sigma_c = 45 \text{ kg/cm}^2 \quad k_2 = 1250 \quad \text{and}$$

$$A_s = \frac{43000}{1250 \times 1.15} - \frac{5600}{1400} = 30 - 4 = 26 \text{ cm}^2 \quad \text{chosen } 7 \phi 22$$

check of tensile stresses in concrete

At supports, the bending moments are negative and the concrete on the water side is subject to tensile stresses which must be smaller than the tensile strength of concrete if tension cracks are to be avoided

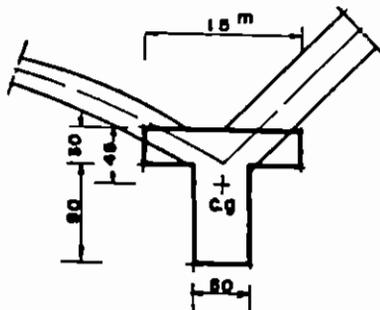


Fig  
VI-19

In order to determine the tensile stresses a part of the dome and cone slabs may be assumed acting with the web, their effect can be replaced by a horizontal flange 150 cms wide and 30 cms deep as shown in figure VI 19

$$\text{Area of concrete section } A_c = 50 \times 90 + 150 \times 30 = 9000 \text{ cm}^2$$

$$\begin{aligned} \text{Distance of c g from upper fiber } y &= \frac{50 \times 120 \times 60 + 100 \times 30 \times 15}{9000} \\ &= 45 \text{ cms} \end{aligned}$$

Moment of inertia of section

$$I = \frac{50 \times 120^3}{12} + 50 \times 120 \times 15^2 + \frac{100 \times 30^3}{12} + 100 \times 30 \times 30^2 = 4045 \times 10^5 \text{ cm}^4$$

$$\begin{aligned} \text{The tensile stress } \sigma_t &= -\frac{N}{A_c} + \frac{M y}{I} = -\frac{5600}{9000} + \frac{40 \times 10^5 \times 45}{9045 \times 10^5} \\ &= 19 \text{ kg/cm}^2 \end{aligned}$$

which can be safely allowed

**Shear Stresses** The axial compressive force of 56 t being small relative to the shearing force of 76 t, its effect on the principal diagonal tensile stresses may be neglected and therefore

$$\tau = \frac{Q}{87 \text{ bd}} = \frac{76000}{0.87 \times 50 \times 115} = 15.2 \text{ kg/cm}^2$$

It will show in the following whether this high value of  $\tau_{\max}$  needs special web reinforcements or not

Assuming that the allowed shear stress in reinforced concrete is 8 kg/cm<sup>2</sup>, then the part of the beam subject to shear stresses higher than this value is only 50 cms from the face of the column (figure VI 20 this length being smaller than 3/4 the depth of the beam, there is no need for bent bars

**Torsional shear stresses :**

The torsional moment of 1.68 mt due to the unbalanced moment is constant along the perimeter of the beam and does not cause shear stresses. Hence, the torsional moment causing shear is  $M_t = 3.1 \text{ mt}$

Knowing that  $b = 50$  cm and  $t = 120$  cm, then

$$\psi = 3 + \frac{26}{t/b + 0.45} = 3 + \frac{26}{120/50 + 0.45} = 3.91$$

The torsional shear stress is therefore given by

$$\begin{aligned} \tau_t &= \psi \frac{t}{b^2} = 3.91 \times \frac{3.1 \times 10^5}{50^2 \times 100} \\ &= 4.97 \text{ kg/cm}^2 \end{aligned}$$

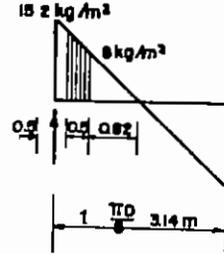


Fig VI-20

The stresses are low and no special provisions are needed

The details of reinforcements of the tank are shown in figure VI

( a & b )

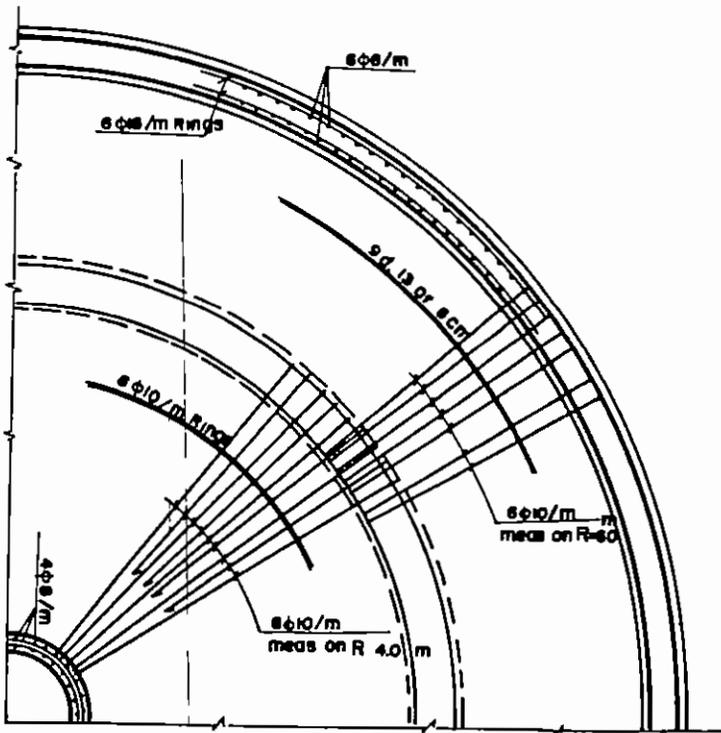


Fig VI-21a

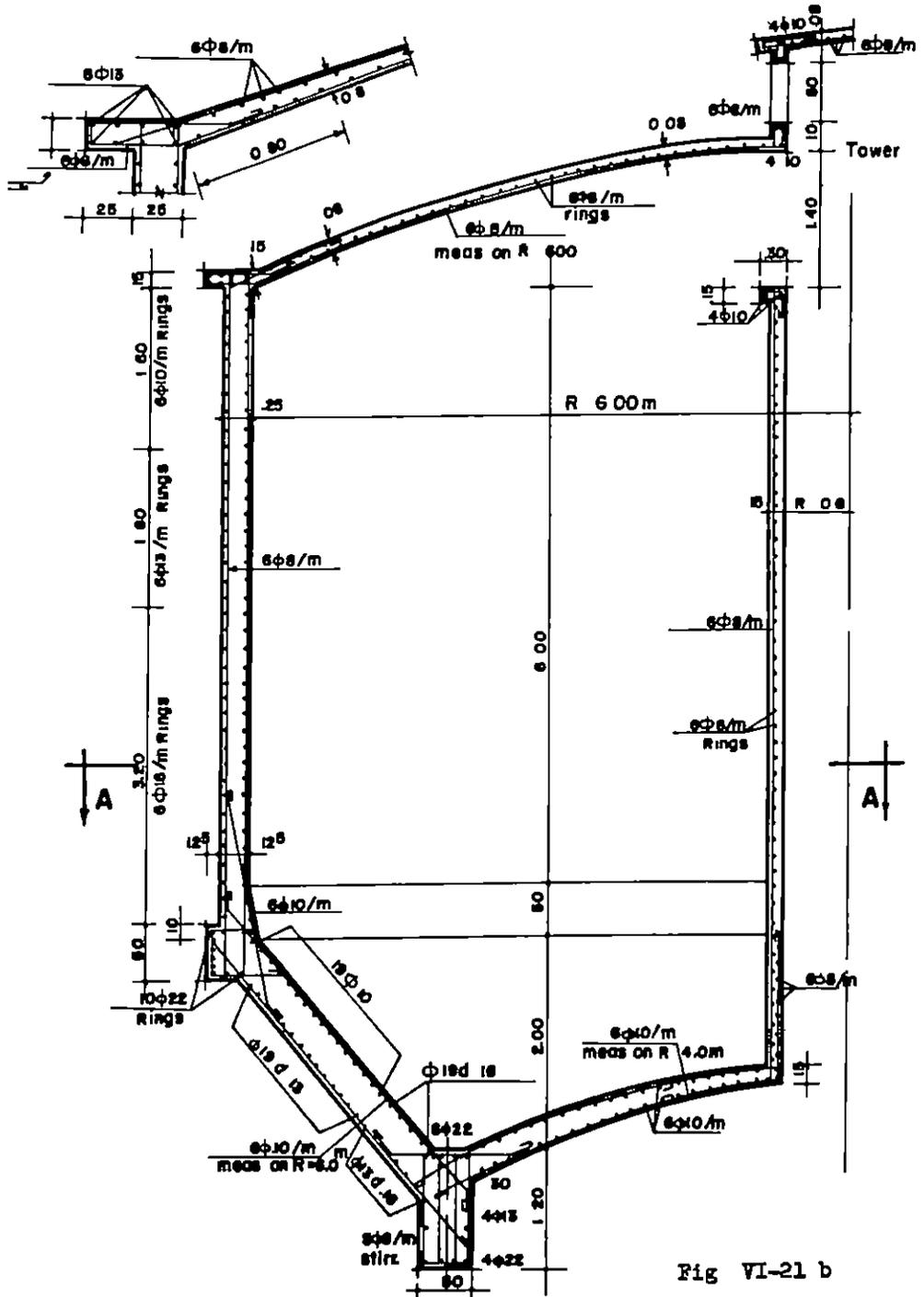


Fig VI-21 b

## 7 Design of the Supporting Tower-Vertical Loads

The supporting tower is to be treated as a space frame. We give in the following an approximate method for determining the internal forces in the columns and struts.

Vertical load of tank plus water to level 0 - 0	=	<u>1212</u>	t
" " " " " per column = 1218/8	=	152	t
Weight of water = 853 t and " " = 853/8	=	<u>106.5</u>	t
Own weight of tank to level 0 - 0 per'	=	45.5	t
Assuming own weight of column or strut = 0.75 t/m			
Weight of column between sections 0 - C			
& I - I = 0.75 x 6	=	<u>4.5</u>	t
Total dead weights per column to level I-I	=	50	t
As the length of any side of the octagonal strut			
= 0.383 D	are :		
D = the diameter of the octagon, we get			
Weight of column + strut between sections			
I - I & II - II = (6 + 0.383 x 9) 0.7	=	<u>5.6</u>	t
Total dead weights per column to level II-II	=	56.6	t
Weight of column + strut between section II-II &			
III - III = (6 + 0.383 x 10) 0.7	=	<u>6.9</u>	t
Total dead weights per column to level III-III	=	63.5	t
Weight of column + strut between sections III-III &			
IV - IV = (6 + 0.383 x 11) 0.7	=	<u>7.2</u>	t
Total dead weights per column to level IV - IV	=	70.7	t
Weight of column + strut between sections IV-IV &			
V - V = (8 + 0.383 x 12) 0.7	=	<u>8.8</u>	t
Total dead weights per column to level V - V	=	79.5	t
Therefore total dead loads + water per column			
= 79.5 + 106.5	=	186	t

Wind pressure on tank and supporting tower :

Intensity of wind pressure on tank =  $150 \text{ kg/m}^2$  horizontal. As it acts on surfaces of revolution, this pressure may be reduced to  $100 \text{ kg/m}^2$ .

The wind pressure on the supporting tower is assumed equal to  $100 \text{ kg/m}^2$  and considered as concentrated at the joints where the columns and the struts intersect.

Magnitude and position of wind horizontal forces acting on tank ( fig VI 22)

Element	Area A	$\text{m}^2$	y from 0 - 0 m	A y $\text{m}^3$
Lantern	1 0 x 1 5	= 1 5	11 5	17 3
Upper dome	12 5 x 1 5	= 18 7	10 3	129 0
Cylinder	12 5 x 6 5	= 81 0	6 25	505 0
Cone	$\frac{12 7 + 8 6}{2} \times 2$	= 21 3	2 00	42 6
Ring	8 6 x 1	= 8 6	0 50	4 3
		<u>131 1</u>		<u>698 2</u>

Total wind force =  $131 1 \times 0 1 = 13 \text{ t}$

Arm above 0 - 0 =  $698 2 / 131 1 = 5 3 \text{ m}$

Wind per meter of column or strut

$$= 0 1 \times 0 6 = 0 06 \text{ t/m}$$

Section	Wind load W
0 - 0	$8 \times 3 \times 0 06 = 1 4 \text{ t}$
I - I	$( 8 \times 6 + 9 \times 2 ) \times 0 06 = 4 0 \text{ t}$
II - II	$( 8 \times 6 + 10 \times 2 ) \times 0 06 = 4 1 \text{ t}$
III-III	$( 8 \times 6 + 11 \times 2 ) \times 0 06 = 4 2 \text{ t}$
IV - IV	$( 8 \times 6 + 12 \times 2 ) \times 0 06 = 4 3 \text{ t}$

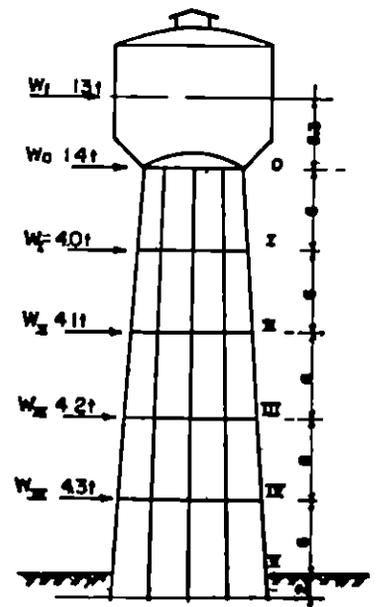


Fig VI-22

### Vertical forces due to wind

It will be assumed that the points of zero bending moments will take place at the middle of any unsupported length of column or strut

The vertical forces due to wind can be determined from the conditions :

The moment of the horizontal wind forces about any horizontal plane at mid-height between two successive struts (plane of assumed zero bending moments in columns)  $M_w$  must be equal to the couple caused by vertical wind forces in columns at the same plane. Assuming these vertical forces to be  $V_1$ ,  $V_2$  and  $V_3$ , we get (fig VI-23)

$$M_w = V_1 \times 2r + 2V_2 \times 2a \quad \text{in which}$$

$$V_2 = V_1 \times \frac{a}{r} \quad \text{and} \quad a = \frac{r}{\sqrt{2}} \quad \text{or} \quad a^2 = \frac{r^2}{2}$$

thus :

$$M_w = V_1 \times 2r + 4 \frac{V_1 a^2}{r} = 4V_1 r \quad \text{or}$$

$$V_1 = \frac{M_w}{4r} = \text{max vertical force}$$

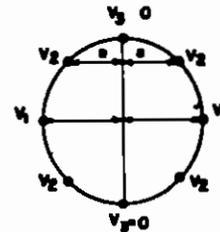


Fig VI-23

Accordingly, the max vertical forces in columns due to wind can be calculated as follows

Section	H Tons		$M_w$	$M_w$	r	$V_{max}$
0 - 0	1 4	13 0				
I - I	4 0	14 4	$13 \times 8 3 + 1 4 \times 3 = 112 2$	112 2	4 25	6 6
II - II	4 1	18 4	$14 4 \times 6 + 4 \times 3 = 98 4$	210 6	4 75	11 2
III-III	4 2	22 5	$18 4 \times 6 + 4 1 \times 3 = 122 7$	333 3	5 25	15.8
IV - IV	4 3	26 7	$22 5 \times 6 + 4 2 \times 3 = 147 6$	480 9	5 75	20 8
V - V		31 0	$26 7 \times 7 + 4 3 \times 4 = 204 1$	685 0	6 33	27 2

The horizontal forces in the columns at points of zero bending moments may be assumed equal, accordingly the bending moments and normal forces

on the columns are given by :

Column	H/Column tons	Vertical Force tons	wind force tons	max Vert force tons	Max B M m t
0 - I	$14 \frac{4}{8} = 1.80$	$152 + 4.5 = 156.5$	$\pm 6.6$	163	$1.8 \times 3 = 5.4$
I - II	$18 \frac{4}{8} = 2.30$	$156.5 + 6.6 = 163.1$	$\pm 11.2$	174	$2.3 \times 3 = 6.9$
II - III	$22 \frac{5}{8} = 2.82$	$163.1 + 6.9 = 170.0$	$\pm 15.8$	186	$2.82 \times 3 = 8.5$
III-IV	$26 \frac{7}{8} = 3.34$	$170 + 7.2 = 177.2$	$\pm 20.8$	198	$3.34 \times 3 = 10.0$
IV - V	$31 \frac{0}{8} = 3.88$	$177.2 + 8.8 = 186$	$\pm 27.2$	213	$3.88 \times 4 = 14.6$

#### Approximate Determination of Internal Forces in Struts

The internal forces in the struts vary according to the direction of wind. In some cases the struts will be subject to torsional moments which, if neglected, the internal forces can approximately be determined for the maximum horizontal forces acting on any of the sides assumed as a plane frame. The wind forces will be assumed as concentrated at the joints and points of zero bending moments lie at middle of struts or columns.

The max horizontal force acting in the plane of a side due to any of the wind loads  $W$  shown in Fig VI 22 is for octagonal towers equal to  $0.146 W$  and can be determined as follows

a) Distribute the wind load  $W$  on the different sides according to figure VI-24. Note that

$$l = D \cos 67^\circ 30' = 0.383 D$$

$$l = \frac{l}{\sqrt{2}} = 0.27 D$$

$$D' = 0.923 D$$

$$w = \frac{W}{D'} = \frac{W}{0.923 D}$$

$$= 1.08 \frac{W}{D}$$

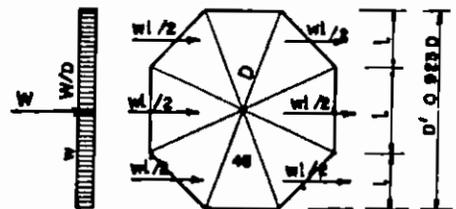


Fig VI-24

b) Determine the normal component of the wind load on the different sides (fig VI-25 )  
 Thus the normal component on the inclined side is given by

$$\frac{wl}{2} \times \cos 45^\circ = \frac{wl}{2} \times \frac{1}{\sqrt{2}} = \frac{wl}{2\sqrt{2}} = \frac{wl}{2.828} \approx \frac{wl}{2.83}$$

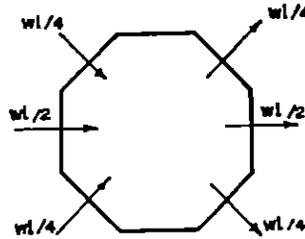


Fig VI-25

c) Distribute the wind loads given in fig VI-25 on the different joints as shown in fig VI-26

d) Determine the corresponding wind forces in the different sides by drawing the triangles of forces for joints a and b (figs VI 27- A, B and C )

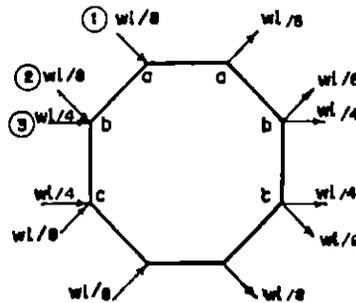
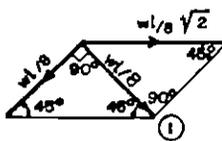
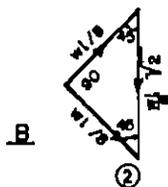


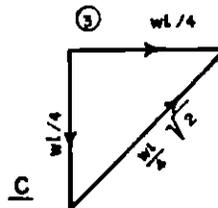
Fig VI-26



A



B



C

Fig VI-27

Accordingly, the wind force in side a - a' is given by  $\frac{wl}{8} \sqrt{2}$  at a and an equal value at a' so that the total force is

$$2 \times \frac{wl}{8} \sqrt{2} = \frac{1.08 W}{D} \times \frac{0.383 D}{4} \sqrt{2} = 0.146 W$$

The wind force in side a b is given by the components of (1) and (2) which cancel each other plus the component of (3) so that the total force is :

$$w \frac{1}{4} \sqrt{2} = 0.146 W$$

The wind force along b c due to W is equal to zero. The final forces along the different sides are shown in figure (VI 28)

Accordingly, the wind forces on the assumed plane frame will be as shown in figure VI 29, each of the forces shown is 0.146 W. The shear at points of zero bending moments at the middle of the free length of any column assumed equally resisted by the columns are also given. The max bending moment in any of the struts will be equal to the sum of the moments of the two columns just above and below it thus

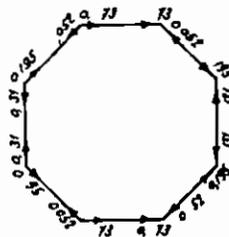


Fig VI-28

Wind forces as factor of w

The shear at points of zero bending moments at the middle of the free length of any column assumed equally resisted by the columns are also given. The max bending moment in any of the struts will be equal to the sum of the moments of the two columns just above and below it thus

$$0.146 \times 13 = 1.9t$$

$$M_I = (1.053 + 1.344) \times 3 = 2.397$$

$$\quad \times 3 = 7.2 \text{ mt}$$

$$M_{II} = (1.344 + 1.644) \times 3 = 2.988$$

$$\quad \times 3 = 9.0 \text{ mt}$$

$$M_{III} = (1.644 + 1.950) \times 3 = 3.639$$

$$\quad \times 3 = 10.9 \text{ mt}$$

$$M_{IV} = 1.195 \times 3 + 2.264 \times 4 = 5.85 + 9.05 = 14.9 \text{ mt}$$

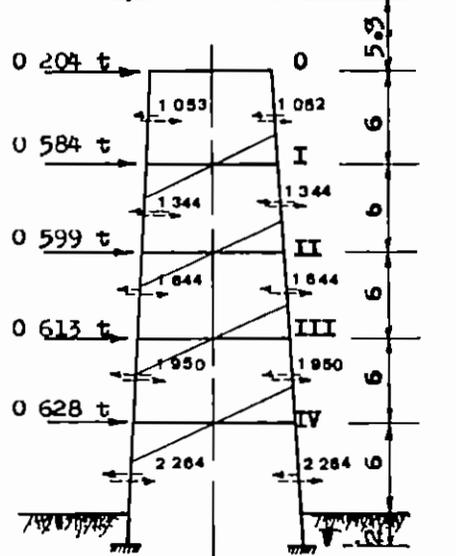


Fig VI-29

Comparing these values with the bending moments in the columns due to wind, we find that, in our case, the max bending moments in the struts at the face of the columns may be assumed equal to the bending moments at the upper ends of the columns just below them. The thrust in the struts are small and can be neglected in the design.

Following the general principles of reinforced concrete design the dimensions and reinforcements of the columns and struts ( designed with relatively low stresses ) may be assumed as follows

	Element	Normal Force H in t	Bending Moment M in m t	Concrete Dimensions cms	Reinforcement
Columns	0 - I	163	± 5 4	50 x 55	12 φ 19
	I - II	174	± 6 9	50 x 60	4 φ 22 + 8 φ 19
	II - III	186	± 8 5	50 x 65	6 φ 22 + 4 φ 19
	III - IV	198	± 10 0	50 x 70	8 φ 22 + 4 φ 19
	IV - V	213	± 14 6	50 x 75	12 φ 22
Struts	I - I	neglected	± 7 2	30 x 55	5 φ 22 top & 5 φ 22 bot
	II - II		± 9 0	30 x 60	6 φ 22 top & 6 φ 22 bot
	III - III	"	± 10 0	30 x 65	6 φ 22 top & 6 φ 22 bot
	IV - IV	'	± 14 9	30 x 70	7 φ 22 top & 7 φ 22 bot

The details of reinforcements of the supporting tower are shown in figure VI 30



### VI-3 Water Tower of Figure IV-3a

In this example, the detailed design of the circular plates of the floor of the elevated tank and the foundation of the tower will be shown

#### 1) The floor of the tank

The floor of the tank under consideration is a hollow circular plate with an overhanging cantilever ring subject to uniform and concentrated loads as well as to edge moments as shown in fig VI-31

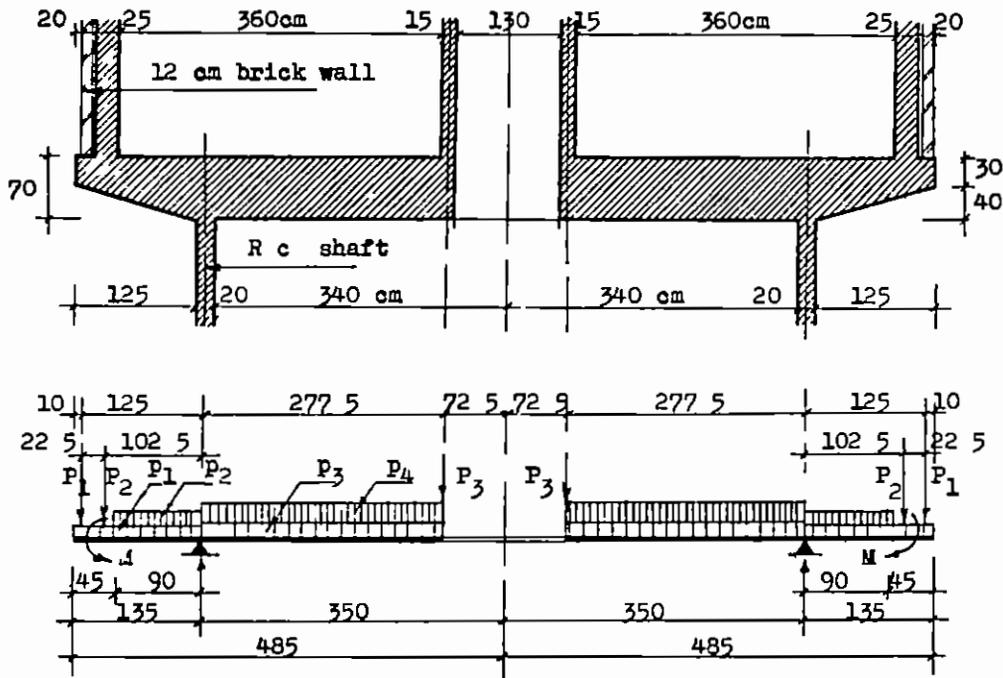


Fig VI-31

It will be considered as simply supported on the 20 cms thick reinforced concrete shaft

Data :

Depth of tank	= 5.35 m
Depth of water	= 5.1 m
Weight of R.C	= $2.5 \text{ t/m}^3$
Weight of brick walls	= $1.8 \text{ t/m}^3$
Superimposed load on roof slab	= $400 \text{ kg/m}^2$

a) Loads

Brick wall	$P_1 = 1.15 \text{ t/m}$	
Roof + R.C outside wall	$P_2 = 3.20 \text{ t/m}$	
Roof + internal shaft	$P_3 = 5.65 \text{ t/m}$	
Own weight of cantilever ring	$P_1 = \frac{0.3 + 0.7}{2} \times 2.5 = 1.25 \text{ t/m}^2$	
Weight of water	$P_2 = P_4 = 5.10 \text{ t/m}^2$	
Own weight of intern slab	$P_3 = 0.7 \times 2.5 = 1.75 \text{ t/m}^2$	

Due to the big rigidity of the tank floor and in order to simplify the calculations, the wall shall be considered as fixed to the floor

The bending moment transmitted from wall  $M = 1.40 \text{ m t/m}$

Radial reaction of wall at base  $= 3.45 \text{ t/m}$

The bending moment transmitted from the intermediate shaft to the floor is small and can be neglected

b) Bending Moments

A simply supported circular slab with overhanging cantilevers is not a statically determinate problem. In order to determine the internal forces in the cantilever ring and the hollow ring slab each shall be solved for complete fixation at the support, then, the unbalanced fixing moment will be distributed according to the stiffness of each element in the following manner :

1) The cantilever ring

The cantilever ring shall be solved according to the equations given in IV-2 for the following five cases of loading and the projection shall be considered equal to the loaded length in each case as shown in figure VI-32. It will be assumed that the Poisson's ratio for concrete :

$$\nu = 1/6 = 0.167$$

Case 1  $P = 1.15 \text{ t}$

According to 8 - a , page 80

$$\rho = \frac{b}{a} = \frac{4.75}{3.5} = 1.357$$

$$\beta^2 = 1.84 \quad \ln \beta = 0.305 \quad \text{so that}$$

$$k_3 = \beta^2 \frac{1 + (1 + \nu) \ln \beta}{1 - \nu + (1 + \nu) \beta^2} = 1.84 \times \frac{1 + 1.167 \times 0.305}{1 - 0.167 + 1.167 \times 1.84} = 0.836$$

At the fixed edge  $\rho = 1$  and the radial moment is given by :

$$M_r = - \frac{P a}{2} \beta \left[ -1 + (1 + \nu) k_3 + (1 - \nu) \left[ \frac{1}{\rho^2} - (1 + \nu) \ln \rho \right] \right]$$

$$= - \frac{1.15 \times 3.5}{2} \times 1.357 ( -1 + 1.167 \times 0.836 + 0.833 \times 0.836 ) = 1.84 \text{ mt/m}$$

at the free edge  $\rho = \beta$  and  $M_r = 0$

The tangential moments  $M_t$  are :

$$\rho = 1, \quad M_t = - \frac{P a}{2} \beta \left[ -\nu + (1 + \nu) k_3 - (1 - \nu) k_3 \right] \\ = - \frac{1.15 \times 3.5}{2} \times 1.357 ( -0.167 + 1.167 \times 0.836 - 0.833 \times 0.836 ) = -0.30 \text{ mt}$$

$$\rho = \beta \quad M_t = - \frac{P a}{2} \beta \left[ -\nu + (1 + \nu) k_3 - (1 - \nu) \frac{k_3}{\beta^2} - (1 + \nu) \ln \beta \right] \\ = - \frac{1.15 \times 3.5}{2} \times 1.357 ( -0.167 + 1.167 \times 0.836 - 0.833 \times \frac{0.836}{1.84} - 1.167 \times 0.305 ) = -0.199 \text{ mt}$$

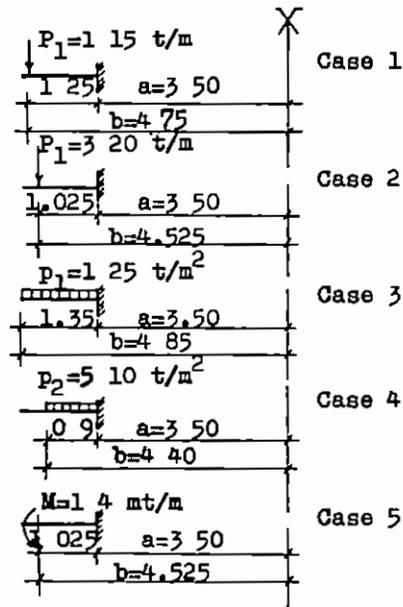


Fig VI-32

Case 2

$$P = 3.2 \text{ t}$$

( 8 - a , page 80 )

$$\beta = \frac{4.525}{3.5} = 1.293 \quad \beta^2 = 1.67 \quad \text{and} \quad \ln \beta = 0.256 \quad k_3 = 0.779$$

$$\rho = 1, \quad M_r = -4.04 \text{ mt} \quad \text{and} \quad \rho = \beta, \quad M_r = 0$$

$$\rho = 1, \quad M_t = -0.67 \text{ mt} \quad \text{and} \quad \rho = \beta \quad M_t = -0.398 \text{ mt}$$

Case 3

$$p = 1.25 \text{ t/m}^2$$

( 7 - a , page 78 )

$$\beta = \frac{4.85}{3.50} = 1.386 \quad \beta^2 = 1.92 \quad \text{and} \quad \ln \beta = 0.326 \quad \text{so that}$$

$$k_1 = \beta^2 \frac{(1-\nu)\beta^2 + (1+\nu)(1+4\beta^2 \ln \beta)}{1-\nu + (1+\nu)\beta^2}$$

$$= 1.92 \times \frac{0.833 \times 1.92 + 1.167(1 + 4 \times 1.92 \times 0.326)}{1 - 0.167 + 1.167 \times 1.92} = 3.55$$

$$\begin{aligned} \rho = 1, \quad M_r &= \frac{p a^2}{16} \left[ (1+\nu)(1-k_1) + 4\beta^2 - (3+\nu) - (1-\nu)k_1 \right] \\ &= \frac{1.25 \times 3.50^2}{16} \left[ 1.167(1-3.55) + 4 \times 1.92 - 3.167 - 0.833 \times 3.55 \right] = -1.36 \end{aligned}$$

$$\rho = \beta, \quad M_r = 0$$

$$\begin{aligned} \rho = 1, \quad M_t &= \frac{p a^2}{16} \left[ (1+\nu)(1-k_1) + 4\nu\beta^2 - (1+3\nu) + (1-\nu)k_1 \right] \\ &= \frac{1.25 \times 3.5^2}{16} \left[ 1.167(1-3.55) + 4 \times 0.167 \times 1.92 + 1.5 \right. \\ &\quad \left. + 0.833 \times 3.55 \right] = -0.22 \end{aligned}$$

$$\begin{aligned} \rho = \beta, \quad M_t &= \frac{p a^2}{16} \left[ (1+\nu)(1-k_1) + 4\nu\beta^2 - (1+3\nu) \frac{k_1}{\beta^2} + \right. \\ &\quad \left. + 4(1+\nu)\beta^2 \ln \beta \right] \\ &= \frac{1.25 \times 3.5^2}{16} \left[ 1.167(1-3.55) + 4 \times 0.167 \times 1.92 - 1.5 \right. \\ &\quad \left. \times 1.92 + 0.833 \times \frac{3.55}{1.92} + 4 \times 1.167 \times 1.92 \times \right. \\ &\quad \left. \times 0.326 \right] = -0.106 \end{aligned}$$

Case 4  $p = 5.1 \text{ t/m}^2$  (7 - a, page 78)

$$\beta = \frac{4.40}{3.5} = 1.257 \quad \beta^2 = 1.58 \quad \text{and} \quad \ln \beta = 0.228 \quad k_1 = 2.46$$

$$p = 1, \quad M_r = -2.35 \text{ mt} \quad " \quad p = \beta \quad M_r = 0$$

$$p = 1, \quad M_t = -0.39 \text{ mt} \quad " \quad p = \beta \quad M_t = -0.156 \text{ mt}$$

Case 5  $M = 1.40 \text{ mt/m}$  (9 - a, page 82)

$$\beta = \frac{4.525}{3.50} = 1.293 \quad \beta^2 = 1.67 \quad \text{so that}$$

$$k_5 = \frac{\beta^2}{1 - \nu + (1 + \nu)\beta^2} = \frac{1.67}{0.833 + 1.167 \times 1.67} = 0.6$$

$$p = 1, \quad M_r = -Mk_5(1 + \nu + 1 - \nu) = -2Mk_5 = -2 \times 1.4 \times 0.6 \\ = -1.68 \text{ mt}$$

$$p = \beta, \quad M_r = -Mk_5 \left[ 1 + \nu + (1 - \nu) \frac{1}{\beta^2} \right] = -1.4 \times 0.6 \left( 1.167 + \frac{0.833}{1.67} \right) \\ = -1.40 \text{ mt}$$

$$p = 1, \quad M_t = -Mk_5 \left[ 1 + \nu - (1 - \nu) \right] = M_r \nu = -1.68 \times 0.167 \\ = -0.28 \text{ mt}$$

$$p = \beta, \quad M_t = -Mk_5 \left[ 1 + \nu - (1 - \nu) \frac{1}{\beta^2} \right] = -1.4 \times 0.6 \left( 1.167 - \frac{0.833}{1.67} \right) \\ = -0.56 \text{ mt}$$

ii) The hollow circular slab

The hollow circular slab shall be solved for the two cases of loading shown in figure VI-33

Case 6  $P = 5.65 \text{ t/m}$  (8-a page 80)

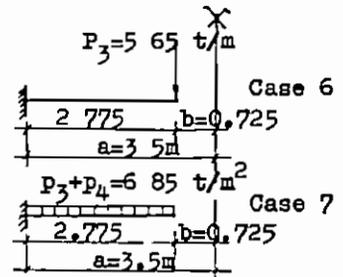
$$\beta = \frac{0.725}{3.50} = 0.207 \quad \beta^2 = 0.0428$$

$$\ln \beta = -1.573$$

so that

$$k_3 = \beta^2 \frac{1 + (1 + \nu) \ln \beta}{1 - \nu + (1 + \nu) \beta^2}$$

$$= 0.0428 \frac{1 + 1.167 \times -1.573}{0.833 + 1.167 \times 0.0428} = 0.0405$$



$$\rho = 1, \quad M_T = \frac{P a}{2} \beta \left[ -1 + (1 + \nu) k_3 + (1 - \nu) k_3 \right] \quad \text{Fig VI-33}$$

$$= \frac{5.65 \times 3.50}{2} \times 0.207 \left( -1 - 1.167 \times 0.0405 - 0.833 \times 0.0405 \right) = -2.22 \text{ m t}$$

$$\rho = \beta, \quad M_T = 0 \quad \text{and} \quad \rho = \frac{1}{2}, \quad M_T = -0.77 \text{ m t}$$

$$\rho = 1, \quad M_t = \frac{P a}{2} \beta \left[ -\nu + (1 + \nu) k_3 - (1 - \nu) k_3 \right]$$

$$= \frac{5.65 \times 3.5}{2} \times 0.207 \left( -0.167 - 1.167 \times 0.0405 + 0.833 \times 0.0405 \right) = -0.37 \text{ m t}$$

$$\rho = \beta, \quad M_t = \frac{P a}{2} \beta \left[ -\nu + (1 + \nu) k_3 - (1 - \nu) \frac{k_3}{\beta^2} - (1 + \nu) \ln \beta \right]$$

$$= \frac{5.65 \times 3.5}{2} \times 0.207 \left( -0.167 - 1.167 \times 0.0405 + 0.833 \times 0.0405 + 1.167 \times 1.573 \right) = +4.07 \text{ m t}$$

Case 7  $p = 6.85 \text{ t/m}^2$  (7 - a, page 78)

$$\beta = 0.207 \quad \beta^2 = 0.0428 \quad \ln \beta = -1.573 \quad \text{so that}$$

$$k_1 = \beta^2 \frac{(1 - \nu) \beta^2 + (1 + \nu) (1 + 4\nu \beta^2 \ln \beta)}{1 - \nu + (1 + \nu) \beta^2}$$

$$= 0.0428 \times \frac{0.833 \times 0.0428 + 1.167 (1 - 4 \times 0.0428 \times 1.573)}{0.833 + 1.167 \times 0.0428} = 0.043$$

$$\rho = 1, \quad M_T = \frac{P a^2}{16} \left[ (1 + \nu) (1 - k_1) + 4 \beta^2 - (3 + \nu) - (1 - \nu) k_1 \right]$$

$$M_r = \frac{6.85 \times 3.5^2}{16} (1.167 \times 0.957 + 4 \times 0.0428 - 3.167 - 0.833 \times 0.043) \\ = -10.07 \text{ mt}$$

$$\rho = \beta, \quad M_r = 0 \quad \text{and} \quad \rho = \frac{1}{2} \quad M_r = +1.126 \text{ mt}$$

$$\rho = 1, \quad M_t = \frac{p a^2}{16} \left[ (1 + \nu) (1 - k_1) + 4\nu\beta^2 - (1 + 3\nu) + (1 - \nu) k_1 \right] \\ = \frac{6.85 \times 3.5^2}{16} (1.167 \times 0.957 + 4 \times 0.167 \times 0.0428 - 1.50 + 0.833 \\ \times 0.043) = -1.68 \text{ mt}$$

$$\rho = \beta, \quad M_t = \frac{p a^2}{16} \left[ (1 + \nu) (1 - k_1) + 4\nu\beta^2 - (1 + 3\nu) \beta^2 + (1 - \nu) \right. \\ \left. \frac{k_1}{\beta^2} + 4(1 + \nu) \beta^2 \ln \beta \right] \\ = \frac{6.85 \times 3.5^2}{16} (1.167 \times 0.957 + 4 \times 1.167 \times 0.0428 - 1.5 \times 0.0428 + \\ + 0.833 \times \frac{0.043}{0.0428} - 4 \times 1.167 \times 0.0428 \times 1.573) \\ = +8.42 \text{ mt}$$

### Fixed end moments

The fixed end moments of the two elements at the support are therefore :

For the cantilever ring

$$\bar{M} = -1.84 - 4.04 - 1.36 - 2.35 - 1.68 = -11.27 \text{ mt}$$

For the hollow circular slab

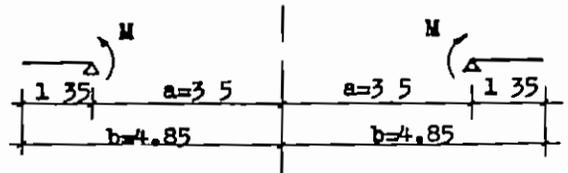
$$\bar{M} = -2.22 - 10.07 = -12.29 \text{ mt}$$

The unbalanced fixed end moment shall be distributed by the moment distribution method according to the relative stiffness of the cantilever ring and the hollow circular slab. Knowing that the stiffness is the moment causing an angle of rotation at the support equal to unity, it can be determined for each of the two elements in the following manner : (Refer to case 10 page 83 )

1) Stiffness of cantilever ring ( Fig VI-34 )

$$\beta = \frac{4.85}{3.50} = 1.386 \quad \beta^2 = 1.92$$

$$k_7 = \frac{1}{1-\beta^2} = \frac{1}{1-1.92} = -1.087$$



The angle of rotation  $\varphi$  is

given by

Fig VI-34

$$\varphi = \frac{M a}{D} \frac{k_7}{1+v} \left( \rho + \frac{1+v}{1-v} \beta^2 \frac{1}{\rho} \right)$$

Substituting  $\varphi = 1$  at  $\rho = 1$ , the resulting moment is a measure for the stiffness of the cantilever. Hence

$$\varphi = \frac{M a}{D} \frac{k_7}{1+v} \left( 1 + \frac{1+v}{1-v} \beta^2 \right) = 1$$

$$\text{or } \frac{M \times 3.5}{D} \times \frac{-1.087}{1.167} \left( 1 + \frac{1.167}{0.833} \times 1.92 \right) = 1$$

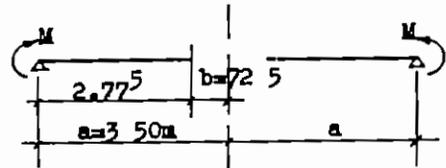
$$\text{i.e.} \quad M = 0.0831 D$$

in which  $D$  is the flexural rigidity of the cantilever

ii) Stiffness of hollow circular slab ( Fig VI-35 )

$$\beta = \frac{0.725}{3.50} = 0.207 \quad \beta^2 = 0.0428$$

$$k_7 = \frac{1}{1-\beta^2} = \frac{1}{1-0.0428} = 1.0447$$



$$\text{at } \rho = 1, \varphi = \frac{M a}{D} \frac{k_7}{1+v} \left( 1 + \frac{1+v}{1-v} \beta^2 \right) = 1 \quad \text{Fig VI-35}$$

$$\text{Hence } \frac{M \times 3.5}{D} \frac{1.0447}{1.167} \left( 1 + \frac{1.167}{0.833} \times 0.0428 \right) = 1$$

$$\text{i.e.} \quad M = 0.301 D$$

The connecting moment

The difference in the fixed end moments of the cantilever ring and the hollow circular slab being small, then the effect of the variation of the moment of inertia of the cantilever ring may be neglected without causing an appreciable error. Hence

$$\text{Relative stiffness of cantilever ring} \quad \frac{0.0831}{0.0831 + 0.301} = 0.216$$

$$\text{" " " hollow circular slab} \quad \frac{0.301}{0.0831 + 0.301} = 0.784$$

The connecting moment can accordingly be determined by the moment distribution method as follows :

	Cantilever ring	Hollow circular slab
Fixed end moment	+ 11.27	- 12.29
Balancing moment	+ 0.22	+ 0.80
Connecting moment	+ 11.49	- 11.49

In order to draw the final moment diagrams, one has to determine the radial and tangential moments due to the balancing moments as follows :

Case a : Cantilever ring subject to a balancing moment

$$M = -0.22 \text{ mt} \quad (\text{Refer to fig VI-34})$$

$$\beta = 1.386 \quad \beta^2 = 1.92 \quad k_7 = -1.087$$

$$\rho = 1, \quad M_r = -0.22 \quad \text{and} \quad \rho = \beta \quad M_r = 0$$

$$\rho = 1, \quad M_t = -M \beta^2 k_7 \left( \frac{1}{\beta^2} + 1 \right) = +0.22 \times 1.92 \times 1.087 \left( \frac{1}{1.92} + 1 \right) \\ = +0.69 \text{ mt}$$

$$\rho = \beta, \quad M_t = -M \beta^2 k_7 \frac{2}{\beta^2} = +2 \times 0.22 \times 1.087 = +0.479 \text{ mt}$$

Case b : Hollow circular slab subject to a balancing moment

$$M = +0.80 \text{ mt} \quad (\text{Refer to fig VI-35})$$

$$\beta = 0.207 \qquad \beta^2 = 0.0428 \qquad k_7 = 1.0447$$

$$\rho = 1, \quad M_r = 0.8 \text{ mt/m} \qquad \text{and} \quad \rho = \beta, \quad M_r = 0$$

$$\rho = \frac{1}{2}, \quad M_r = 0.694 \text{ mt}$$

$$\rho = 1, \quad M_t = M \beta^2 k_7 \left( \frac{1}{\rho^2} + 1 \right) \\ = 0.8 \times 0.0428 \times 1.0447 \left( \frac{1}{0.0428} + 1 \right) = +0.88 \text{ mt}$$

$$\rho = \beta \quad M_t = 2 M \cdot k_7 = 2 \times 0.8 \times 1.0447 = +1.67 \text{ mt}$$

The final values of the radial and tangential bending moments can now be determined by superposition as shown in the following tables

i) Moments in Cantilever Ring

Case	Radial moment $M_r$ in mt/m		Tangential moment $M_t$ in mt/m	
	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
1	-1.84	0	-0.30	-0.199
2	-4.04	0	-0.67	-0.398
3	-1.36	0	-0.22	-0.106
4	-2.35	0	-0.39	-0.156
5	-1.68	-1.40	-0.28	-0.560
a	-0.22	0	+0.69	+0.479
Total	-11.49	-1.40	-1.17	-0.94

ii) Moments in Hollow Circular Slab

Case	Radial moment $M_r$ in mt/m			Tangential moment $M_t$ in mt/m	
	$\rho = 1$	$\rho = \frac{1}{2}$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
6	-2.22	-0.77	0	-0.37	+4.07
7	-10.07	+1.13	0	-1.68	+8.42
b	+00.80	+0.69	0	+0.88	+1.67
Total	-11.49	+1.05	0	-1.17	+14.16

iii) Bending Moment Diagrams

(Fig VI-36)

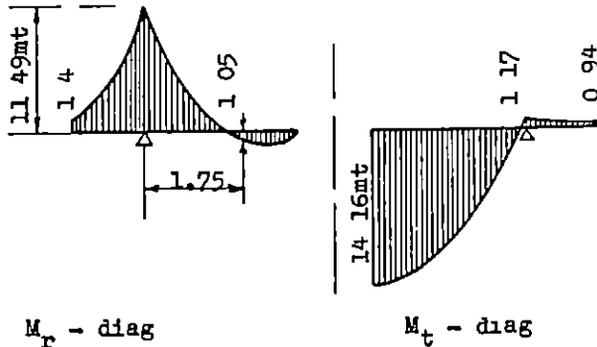
c) Design

Fig VI-36

The floor of the tank is to be designed for the following internal forces

Max radial tension in floor = 3.45 t/m

Max radial bending moment = -11.49 mt/m at supporting shaft

Max tangential ' ' = +14.16 mt/m at interm hole

For section at the supporting shaft

$M_r = 11.49 \text{ mt/m}$  (with tension on water side) and  $n = 3.45 \text{ t/m}$  tension

$$t = \sqrt{M_r/3} + 2 = \sqrt{11.49/3} + 2 = 64 \text{ cms} \quad \text{taken } 70 \text{ cms}$$

$$e = M_r/n = 11.49/3.45 = 3.33 \text{ ms} \quad e_B = 333 - \frac{70}{2} + 4 = 304 \text{ cms}$$

$$M_B = n \cdot e_B = 3.45 \times 304 = 1045 \text{ mt/m}$$

$$d = k_1 \sqrt{M_B} \quad i.e. 66 = k_1 \sqrt{10450} \quad \text{giving } k_1 = 0.645$$

$$A_B = \frac{M_B}{k_2 d} + \frac{n}{\sigma_B} = \frac{10450}{1300 \times 66} + \frac{3.45}{1400} = 12.13 + 2.47 = 14.6 \text{ cm}^2$$

choose  $\varphi 22 @ 20 \text{ cms}$  for the top and  $\varphi 13 @ 20 \text{ cms}$  for the bottom radial reinforcement

For the section at the intermediate shaft :

$M_t = 14.16 \text{ mt/m}$  (with tension on air side) , we have

$$64 = k_1 \sqrt{14.160} \quad \text{giving } k_1 = 0.54 \quad \text{and} \quad \sigma_o = 30.5 \text{ kg/cm}^2$$

$$A_B = \frac{14160}{1285 \times 64} = 17.2 \text{ cm}^2 \quad \text{chosen } \varphi 19 @ 12.5 \text{ cms}$$

The circular bottom reinforcement decreases as the distance from the center increases. The circular top reinforcement is chosen  $\varphi 13 @ 20$  cms as shown in figure VI 44

## 2) The Foundation Slab

The foundation slab shall be assumed as a circular plate with an overhanging cantilever ring simply supported on the shaft and subject to the soil pressure acting upwards

### a) Loads

Total weight of tank	= 260 ton
" " " water	= 300 "
" " " shaft	= 333 "
Weight of interm floors	= 103 "

Wind pressure  $w$  on cylindrical surfaces can be assumed = 0.45

the normally specified values acting on vertical surfaces i.e.  $w = 45 \text{ kg/m}^2$

Hence

$$W_1 = 9.7 \times 6.3 \times 45/1000 = 2.75 \text{ tons}$$

$$W_2 = 7.2 \times 29.3 \times 45/1000 = 9.5 \text{ tons}$$

$$\text{Total vertical load} = 9.96 \text{ ton}$$

Bending moment due to wind at Sec II - II

$$k_w = 2.75 \times \left( \frac{6.3}{2} + 29.3 \right) + 9.5 \times \frac{29.3}{2} = 228 \text{ mt}$$

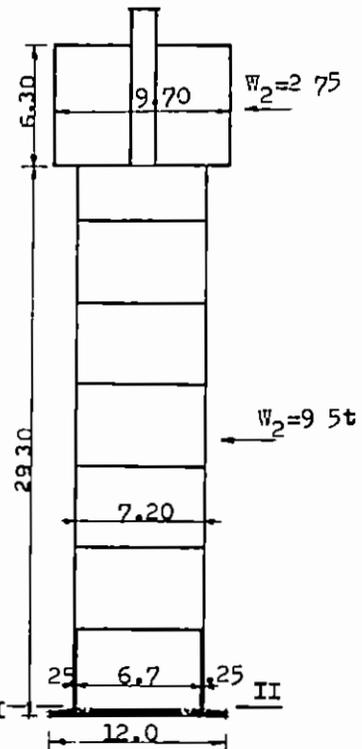


Fig VI-37

### b) Check of Stresses in Shaft at Foundation Slab

The stresses in the shaft shall be checked for the following two cases

1) Tank full + Wind pressure                      P = 996 t,                      M = 228 mt

Net area of cross-section of shaft

$$A_0 = \pi \times 6.95 \times 0.25 - (0.8 + 1.35) \times 0.25 = 5.44 - 0.54 = 4.9 \text{ m}^2$$

$$\text{Gross section modulus} = \frac{\pi}{32} \frac{(7.2^4 - 6.7^4)}{7.2} = 9.16 \text{ m}^3$$

Section modulus of openings (window and door)

$$(0.80 + 1.35) \times 0.25 \times 3.475^2 / 3.6 = 1.80 \text{ m}^3$$

$$\text{Net section modulus } Z_0 = 9.16 - 1.80 = 7.36 \text{ m}^3$$

Therefore, the stresses in the shaft are given by :

$$\sigma_{1/2} = -\frac{P}{A_0} \pm \frac{M}{Z_0} = -\frac{996}{4.9} \pm \frac{228}{7.36} = -203 \pm 31 \text{ t/m}^2 \quad \text{i.e.}$$

$$\underline{\sigma_1 = -234} \quad \text{and} \quad \underline{\sigma_2 = -172} \quad \text{kg/cm}^2$$

2) Tank empty + Wind pressure                      P = 696 t,                      M = 228 mt

$$\sigma_{1/2} = -\frac{696}{4.9} \pm \frac{228}{7.36} = -142 \pm 31 \text{ t/m}^2 \quad \text{i.e.}$$

$$\underline{\sigma_1 = -173} \quad \text{and} \quad \underline{\sigma_2 = -111} \quad \text{kg/cm}^2$$

c) Pressure Under Foundation Slab (Fig VI-38)

Own weight of foundation slab :

$$2.5 \left[ \frac{\pi}{4} \times 12.0^2 \times 0.8 - \frac{\pi}{4} (12.0^2 - 7.6^2) \times \frac{0.5}{2} \right] = 184 \text{ t}$$

$$\text{Total load} = 996 + 184 \quad P = 1180 \text{ t}$$

$$\text{Wind moment} \quad M = 228 \text{ mt}$$

$$\text{Area of foundation base} = \pi \times 12.0^2 / 4 = 113 \text{ m}^2$$

$$\text{Section modulus} \quad z = \pi \times 12.0^3 / 32 = 169.5 \text{ m}^3$$

$$\sigma_{1/2} = -\frac{1180}{113} \pm \frac{228}{169.5} = -10.44 \pm 1.34 \text{ ton/m}^2 \quad \text{i.e.}$$

$$\sigma_1 = -1.18 \quad \text{and} \quad \sigma_2 = -0.92 \text{ kg/cm}^2$$

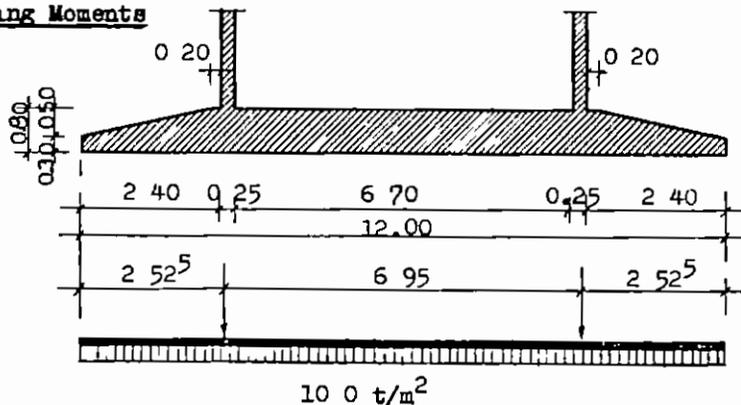
d) Bending Moments

Fig VI-38

In order to simplify the calculations, the internal forces in the foundation slab shall be determined for a net upward uniform pressure  $p = 10 \text{ t/m}^2$

Again here, we determine the radial and tangential bending moments for the cases shown in fig VI - 39 from the equations given in IV - 2

Proceeding in the same way as before we get the following results

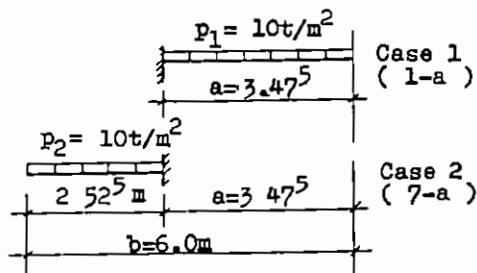


Fig VI-39

Case	$M_r$ in mt/m		$M_t$ in mt/m	
	$\rho = 1$	$\rho = 0$	$\rho_1$	$\rho_0$
1	- 15 14	+ 7 71	- 2 50	+ 7 71
2	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
	- 42 27	0	- 7 12	- 4 54

To redistribute the fixed end moments ( - 42 27 mt for the circular

plate and - 15 14 wt for the cantilever ring) one has to determine the stiffness of each of the two elements

1 ) Stiffness of circular slab Refer to page 77

$$\text{For } \rho = 1 \quad \varphi = \frac{M a}{D (1 + \nu)} = 1 \quad \text{thus}$$

$$M = \frac{1.167 D}{3.475} = 0.3357 D$$

ii) Stiffness of cantilever ring Refer to page 83

$$\text{For } \rho = 1 \quad \varphi = \frac{M a}{D} \times \frac{k_7}{1 + \nu} \left( 1 + \frac{1 + \nu}{1 - \nu} \beta^2 \right) = 1 \quad \text{in which}$$

$$\beta = 1.727, \quad \beta^2 = 2.98 \quad \text{and} \quad k_7 = \frac{1}{1 - \beta^2} = \frac{1}{1 - 2.98} = -0.521$$

$$\text{Therefore} \quad \frac{M \times 3.475}{D} \times -\frac{0.521}{1.167} \left( 1 + \frac{1.167}{0.833} \times 2.98 \right) = 1 \quad \text{or}$$

$$M = -0.1246 D$$

Neglecting the effect of the variation of the depth of the cantilever ring on its flexural rigidity, then

$$\text{Relative stiffness of the cantilever ring} = \frac{0.1246}{0.1246 + 0.3357} = 0.27$$

$$\text{" " " " circular slab} = \frac{0.3357}{0.1246 + 0.3357} = 0.73$$

The connecting moment

	Cantilever ring	Circular slab
Fixed end moment	- 42.27	+ 15.14
Balancing moment	+ 7.33	+ 19.80
	- 34.94	34.94

Effect of balancing moment (Fig VI-40)

For  $\rho = 1$  and  $\rho = 0$

$$M_r = M_t = M = -19.80 \text{ mt}$$

Case b 10 - page 83

$$\beta = 1.727 \quad \beta^2 = 2.98$$

$$k_7 = \frac{1}{1 - \beta^2} = \frac{1}{1 - 2.98} = -0.521$$

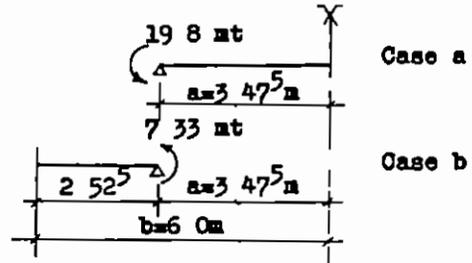


Fig VI-40

For  $\rho = 1, M_r = M = +7.33 \text{ mt}$  and for  $\rho = \beta, M_r = 0$

For  $\rho = 1, M_t = M k_7 (1 + \beta^2) = 7.33 \times (-0.521)(1 + 2.98) = -15.20 \text{ mt}$

For  $\rho = \beta, M_t = 2 M k_7 = -2 \times 7.33 \times 0.521 = -7.64 \text{ mt}$

Final values of the radial and tangential moments

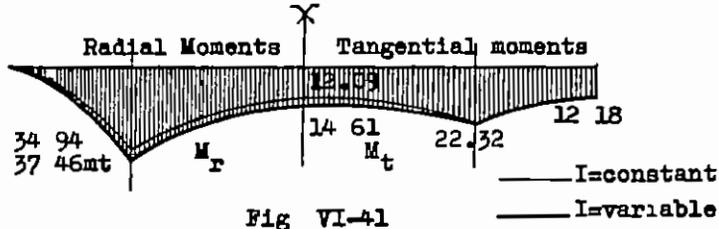
1) Circular Slab

Case	Radial moment $M_r$ in mt/m		Tangential moment $M_t$ in mt/m	
	$\rho = 1$	$\rho = 0$	$\rho = 1$	$\rho = 0$
1	-15.14	+7.71	-2.50	+7.71
a	-19.80	-19.80	-19.80	-19.80
Total	-34.94	-12.09	-22.30	-12.09

1i) Cantilever ring

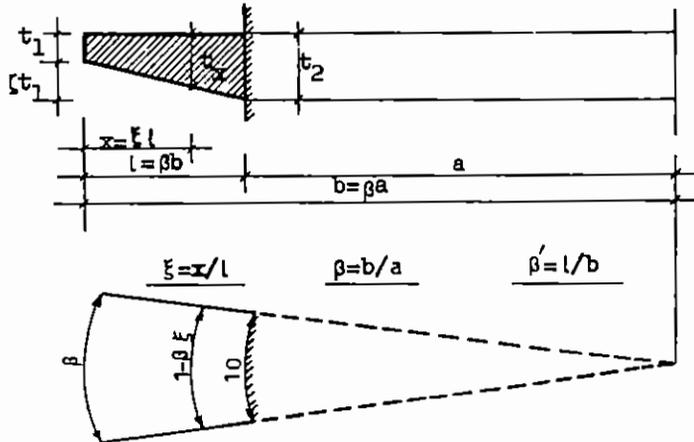
Case	Radial moment $M_r$ in mt/m		Tangential moment $M_t$ in mt/m	
	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
2	-42.27	0	-7.12	-4.54
b	+7.33	0	-15.20	-7.64
Total	-34.94	0	-22.32	-12.18

iii) Bending moment diagrams : (Fig VI-41)



Effect of variation of moment of inertia on radial moments

The effect of the variation of the moment of inertia of the cantilever ring on the flexural rigidity  $D$  and the corresponding effect on the radial moment can be determined according to Markus<sup>2</sup> in the following manner : (Fig VI-42)



Referring to data given in figure VI-42, we get

$$\beta' = \frac{l}{b} = \frac{b-a}{b} = 1 - \frac{1}{\beta}$$

$$t_1 + \xi t_1 = t_2$$

or

$$1 + \xi = t_2 / t_1 \quad \text{i.e.}$$

$$\xi = \frac{t_2}{t_1} - 1$$

<sup>2</sup> Markus : 'Theorie und Berechnung rotationssymmetrischer Bauwerke'

According to Markus, one can prove that :

$$D_x = D_1 (1 - \beta' \xi) (1 + \xi \xi)^3$$

Hence, at free edge 1, we have :

$$D_1 = \frac{E t_1^3}{12 (1 - \nu^2)}$$

and at fixed edge 2, where  $x = l$  and  $\xi = 1$ , we have :

$$D_2 = \frac{E t_1^3}{12 (1 - \nu^2)} (1 - \beta') \left[ 1 + \left( \frac{t_2}{t_1} - 1 \right) \right]^3 \quad \text{or}$$

$$D_2 = \frac{E t_2^3}{12 (1 - \nu^2)} \frac{1}{\beta}$$

Therefore

$$D_2 = \frac{E \times 0.8^3}{12 (1 - 0.167^2)} \times \frac{3.475}{6.00} = 2.541 \times 10^{-2} E$$

and the stiffness of the cantilever ring is

$$M = 0.1246 \times 2.541 \times 10^{-2} E = 0.3166 \times 10^{-2} E$$

Further, the stiffness of the circular slab is given by

$$M = 0.3357 D, \text{ in which } D = \frac{E t^3}{12 (1 - \nu^2)} = \frac{E \times 0.8^3}{12 (1 - 0.167^2)} = 4.39 \times 10^{-2} E$$

$$\text{thus } M = 0.3357 \times 4.39 \times 10^{-2} E = 1.4737 \times 10^{-2} E$$

Hence

$$\text{Relative stiffness of cantilever ring} = \frac{0.3166}{0.3166 + 1.4737} = 0.177$$

$$\text{" " " circular slab} = \frac{1.4737}{0.3166 + 1.4737} = 0.823$$

The connecting moment

	Cantilever ring	Circular slab	
Fixed end moment	- 42 27	+ 15 14	
Balancing moment	+ 4 81	+ 22 32	
Connecting moment	- 37 46	+ 37 46	mt

Effect of balancing moment Fig VI-43

The variation of the flexural rigidity affects mainly the radial moments, thus :

Case a 6 page 77

For  $\rho = 1$  and  $\rho = 0$   $M_r = M = -22.32$  mt

Case b 10 page 83

for  $\rho = 1$   $M_r = M = +4.81$  mt

For  $\rho = \beta$   $M_r = 0$

Therefore, the radial and tangential moments at the center of the plate is given by  $M_r = M_t = +7.71 - 22.32 = -14.61$  mt. The final moments may be assumed as shown by the heavy line in figure VI-41

e) Design

Section at Support  $M_r = 37.46$  mt,  $M_t = 22.32$  mt

Assuming total thickness of slab = 80 cms then

Theoretical depth  $d = 80 - 4 = 76$  cms and

for the radial direction, we have

$$d = k_1 \sqrt{M_r} \quad \text{or} \quad 76 = k_1 \sqrt{37460} \quad \text{giving} \quad k_1 = 0.304$$

assuming  $\sigma_s = 1400$  kg/cm<sup>2</sup>,  $n = 15$  &  $x/\alpha = 0.15$

then  $\sigma_c = 43$  kg/cm<sup>2</sup>, and  $k_2 = 1250$

$$\text{so that} \quad A_s = \frac{M}{k_2 d} = \frac{37460}{1250 \times 76} = 39.4 \text{ cm}^2 \quad \text{chosen } \varnothing 22 @ 10 \text{ cm}$$

for the tangential direction, we get

$$A_s = \frac{22320}{1300 \times 74} = 23.2 \text{ cm}^2 \quad \text{chosen } \varnothing 22 @ 15 \text{ cm}$$

Both radial and tangential bottom reinforcements may be reduced at the center of the circular slab to :

$$A_s = \frac{14610}{1300 \times 75} = 14.9 \text{ cm}^2 \quad \text{chosen } \varnothing 22 @ 25 \text{ cm}$$

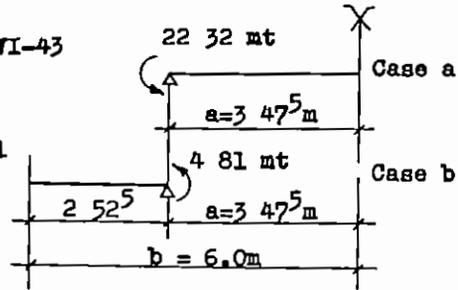


Fig VI-43

However, the foundation slab is reinforced by a rectangular mesh  $\phi$  22 mm @ 10 cms in its lower surface and by a mesh  $\phi$  13 mm @ 20 cms in its upper surface

The details of reinforcement of the water tower shown in figure IV-3a designed by the Misr Concrete Development Company is given in Fig VI-44

#### VI-4 A Treated Water Tank and a Flocculator

We give in Figs VI-45 and VI-46 two examples of circular tanks that have been recently designed by the author for Abu Quir Urea Plant for Fertilizers and Chemical Industries

The soil at the site of the plant is very weak it contains dark clays and peat for big depths that most of the units of the plant, including the tanks, are supported on mechanically driven cast-in-situ piles about 25 ms long and having a maximum capacity of 40 tons per pile

Fig VI-45 shows a covered treated-water-tank 23 ms diameter and about 8 ms deep. The roof slab is supported on nine interior columns and on the wall along its exterior edge. The joint between the roof and the walls is sliding (refer to Fig III-1), so that no stresses are created in the wall due to temperature changes of the roof

Fig VI-46 shows a flocculator 25 ms diameter and 7 ms deep

The main point of interest in these two tanks is the distribution of the piles which is chosen such that each pile is fully utilized and their arrangement gives a convenient thickness for the floors satisfying the requirements of the internal forces, water-tightness and utility of the tanks

VII DESIGN OF RECTANGULAR TANKS

VII 1 DEEP TANKS RESISTING HYDROSTATIC PRESSURE HORIZONTALLY

In closed rectangular tanks with sliding base the full water pressure is resisted horizontally. Also in deep tanks where  $H/L_1$

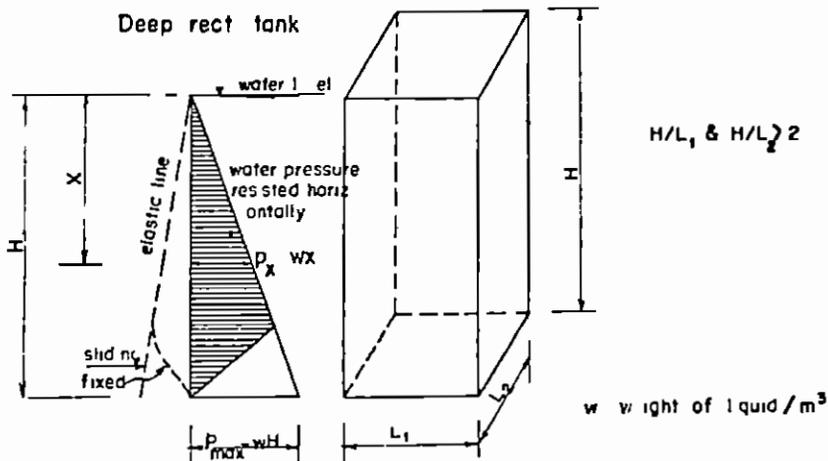


Fig VII-1

and  $H/L_2 > 2$  the effect of the fixation of the wall to the floor will be limited to a small part at the base of the wall the rest of the wall will resist the water pressure in the horizontal direction by closed frame action (Fig VII I)

a) Square section (Fig VII 2)

Due to symmetry of square sections subject to internal pressure the angles of rotation  $\theta$  of each side at its two ends must be equal to zero i.e. each side behaves as if it were totally fixed at both ends, and for a uniformly distributed horizontal pressure  $p$ , we get (Fig VII 2)

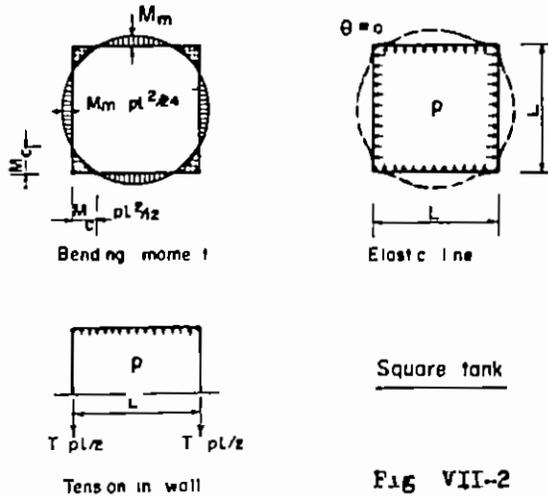


Fig VII-2

bending moment at middle of any side

$$l_m = p L^2 / 24 \quad (1-a)$$

bending moment at any of the corners

$$M_c = p L^2 / 12 \quad (1-b)$$

Each wall will further be subject to a tensile force given by

$$T = p L / 2 \quad (1-c)$$

giving a case of eccentric tension

For any tank of regular section (Fig VII 3) subject to uniform internal pressure  $p$ , we get similarly

$$\theta = 0$$

and

$$\begin{aligned}
 M_m &= p L^2 / 24 \\
 M_o &= p L^2 / 12 \\
 T &= p D / 2
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} M_m \\ M_o \\ T \end{aligned}} \right\} \quad (2)$$

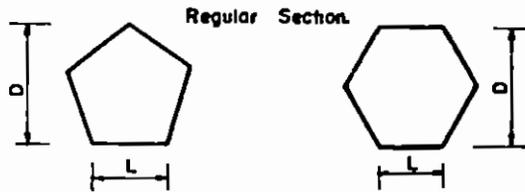


Fig VII-3

b) Rectangular Section (Fig VII 4)

Due to symmetry of loading, the corners of the cross-section will not move, and in order to determine the connecting moment  $M_c$  in the corner the equation of three moments can be applied thus :

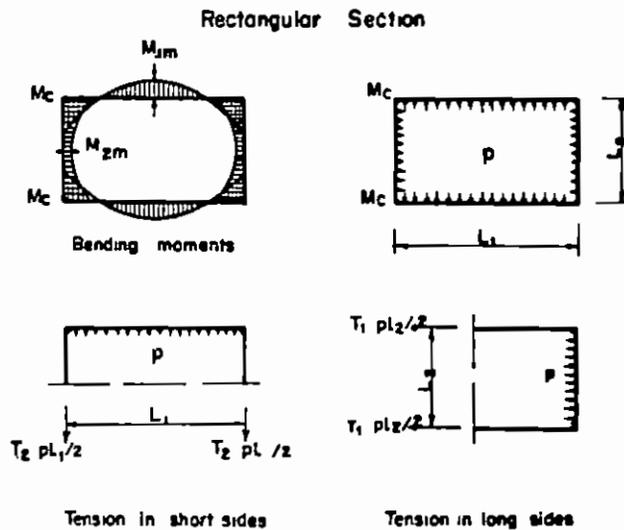


Fig VII-4

if the moment of inertia of the two sides  $L_1$  and  $L_2$  is the same then

$$M_c L_1 + 2 M_c (L_1 + L_2) + M_c L_2 = -6 \left( p \frac{L_1^3}{24} + p \frac{L_2^3}{24} \right) \quad \text{or}$$

$$3 M_c (L_1 + L_2) = -\frac{p}{4} (L_1^3 + L_2^3) \quad \text{i e}$$

$$M_c = -\frac{p}{12} \frac{L_1^3 + L_2^3}{L_1 + L_2} \quad \left. \vphantom{M_c} \right\} \quad (3a)$$

or

$$M_c = -\frac{p}{12} (L_1^2 - L_1 L_2 + L_2^2) \quad \left. \vphantom{M_c} \right\}$$

and accordingly we get

$$M_{1m} = p \frac{L_1^2}{6} + M_c = \frac{p}{24} (L_1^2 + 2 L_1 L_2 - 2 L_2^2) \quad (3b)$$

$$M_{2m} = p \frac{L_2^2}{6} + M_c = \frac{p}{24} (L_2^2 + 2 L_1 L_2 - 2 L_1^2) \quad (3c)$$

For  $L_1 = L_2 = L$ , then

$$M_c = -p L^2 / 12 \quad \text{and} \quad M_{1m} = M_{2m} = p L^2 / 24$$

Assuming further

$$M_{1m} = \alpha p L_2^2 / 12, \quad M_{2m} = \beta p L_2^2 / 12 \quad \text{and} \quad M_c = \gamma p L_2^2 / 12$$

we get

$L_1 / L_2$	1 00	1 10	1 20	1 30	1 40	1 50	1 60	1 70	1 80	1 90	2 00
$\alpha$	0 50	0 71	0 92	1 15	1 38	1 63	1 88	2 15	2 42	2 71	3 00
$\beta$	0 50	0 39	0 27	0 11	0 06	0 27	0 46	0 69	0 94	1 21	1 50
$\gamma$	1 00	1 11	1 24	1 40	1 56	1 77	1 96	2 10	2 44	2 71	3 00

If  $L_1$  varies much from  $L_2$  it is not economic to calculate with constant  $I$ , and assuming

$$\alpha = \frac{I_1}{L_1} \quad \frac{L_2}{I_2}$$

the equation of three moments can be given in the form :

$$\frac{I_1}{L_1} M_c \frac{L_1}{I_1} + 2 M_c \left( \frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_c \frac{L_2}{I_2} = -6 \left( p \frac{L_1^3}{24 I_1} + p \frac{L_2^3}{24 I_2} \right) \frac{I_1}{L_1}$$

or

$$M_c + 2 M_c (1 + x) + M_c = -\frac{p}{4} (L_1^2 + x L_2^2)$$

$$M_c = -\frac{p}{12} \frac{L_1^2 + x L_2^2}{1 + x} \quad (4)$$

Each wall is further subject to a tensile force given by :

$$T_1 = p L_2 / 2 \quad \text{for wall } L_1$$

and

$$T_2 = p L_1 / 2 \quad \text{for wall } L_2$$

### c) Determination of Internal Forces by the Tension Line Method

As the walls of any closed section are subject to bending moments and axial tension, it is possible to represent the internal forces by a tension line

For a closed frame of constant moment of inertia, subject to uniformly distributed internal pressure, the tension line is composed of a series of parabolas and can be assumed as a circle without any appreciable error

The radius of the tension line can be determined if we study the case of a square section as that shown in Fig VII 5 Thus,

$$T = p 2 R \quad \text{or} \quad T = p R$$

$$I_m = T \quad x = p R \quad x = p L^2 / 24$$

$$R x = L^2 / 24$$

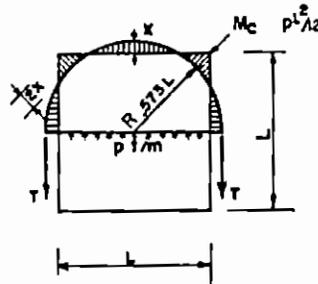


Fig VII-5

But  $x = R - L/2$  then

$$R(R - L/2) = L^2 / 24 \quad \text{or} \quad R^2 - RL/2 - L^2 / 24 = 0$$

$$\text{i.e.} \quad R = \frac{L/2 \pm \sqrt{L^2 / 4 + 4 L^2 / 24}}{2} = 0.573 L$$

which means that the area  $A_t$  enclosed by the tension line is given by

$$A_t = \pi (0.573 L)^2 = 1.03 L^2 \quad (5) \quad \text{i.e.}$$

area enclosed by tension line = 1.03 area of section of tank

This method can however be used for rectangular sections, Figure VII 6, as well as sections of irregular shape Fig VII 7, as can be proved from the following example

Assume a rectangular tank with  $L_1 = 2 L_2$  subject to internal pressure  $p$ , Fig VII 6

$$\text{Area of cross-section } A = 2 L_2^2$$

Area enclosed by tension line

$$A_t = 1.03 A = 1.03 \times 2 L_2^2 = 2.06 L_2^2$$

$$R = \sqrt{2.06 L_2^2 / \pi} = 0.81 L_2$$

So that

$$T = p R = 0.81 p L_2$$

We have further

$$\text{Length of diagonal } D = \sqrt{L_2^2 + (2 L_2)^2} = \sqrt{5} L_2 = 2.232 L_2$$

Therefore, the corner connecting moment  $M_c$  can be given by

$$M_c = T (R - D/2) = 0.81 p L_2 (0.81 L_2 - 2.232 L_2/2) = 0.248 p L_2^2$$

against

$$M_c = \gamma p L_2^2 / 12 = 3 p L_2^2 / 12 = 0.250 p L_2^2$$

and

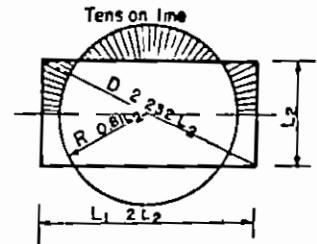


Fig VII-6

$$M_{1m} = p L_1^2 / 8 - M_c = p \cdot 4 L_2^2 / 8 - 0.248 p L_2^2 = 0.250 p L_2^2$$

against

$$M_{1m} = \alpha p L_2^2 / 12 = 3 p L_2^2 / 12 = 0.250 p L_2^2$$

$$M_{2m} = p L_2^2 / 8 - M_c = 0.125 p L_2^2 - 0.248 p L_2^2 = -0.123 p L_2^2$$

against

$$M_{2m} = \beta p L_2^2 / 12 = -0.15 p L_2^2 / 12 = -0.125 p L_2^2$$

This example shows that the method gives a very good approximate solution for the determination of the internal forces in isolated tanks of one cell. Its use simplifies the determination of the internal forces in tanks of special cross-section as that shown in Fig. VII-7 in which the area enclosed by the tension line is 1.03 the area of the cell & its center coincides on the center of gravity of the cross-section of the cell.

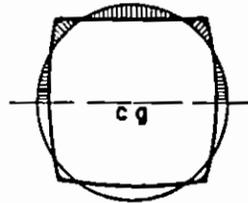


Fig. VII-7

As the liquid pressure increases with the depth, a trapezoidal vertical section for the wall gives a convenient form.

In deep tanks, the main steel reinforcement is horizontal; however, it is essential to arrange vertical secondary reinforcements having a minimum cross-section area of 20% of the main horizontal steel to fix the main reinforcements in position and to resist the shrinkage and temperature stresses.

In case of walls fixed to the floor slab, the fixing moment at the foot of the wall may be estimated from the relation

$$M_f \approx - p_{\max} L^2 / 24 \quad (6a)$$

the corresponding reaction is given by

$$R \approx 0.3 p_{\max} L \quad (6b)$$

Therefore, the maximum internal forces in an open square water tank 4 x 4 ms and 10 ms deep with fixed base can be calculated as follows :

Assuming that the maximum horizontal internal forces take place at a depth  $x = 0.75 H$  then :

The max horizontal fixing moment is

$$M_f = - p_x L^2/12 = - 0.75 \times 10 \times 4^2/12 = 10 \text{ mt}$$

The corresponding tension in the wall is

$$T = p_x L/2 = 0.75 \times 10 \times 4/2 = 15 \text{ t}$$

The fixing moment at the foot of the wall in the vertical direction is according to equation 6a given by

$$M_f = - p_{\max} L^2/24 = - 10 \times 4^2/24 = 6.66 \text{ mt}$$

The corresponding reaction = tension in floor, is according to equation 6b given by

$$R = 0.3 p_{\max} L = 0.3 \times 10 \times 4 = 12 \text{ t}$$

It is however generally sufficient to arrange vertical corner reinforcements of the same order as that used horizontally (Fig VII 8)

It is further recommended to add a mesh of min 5  $\phi$  8 mm/m on the compression side of the walls and to avoid the use of bent bars in vertical walls

The details of reinforcements of a horizontal section of a deep square tank may be done in the manner shown in figure VII 9

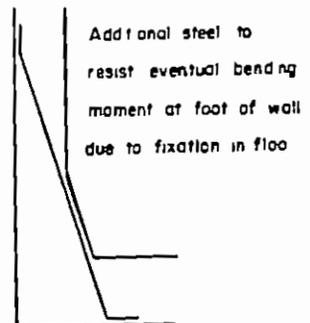
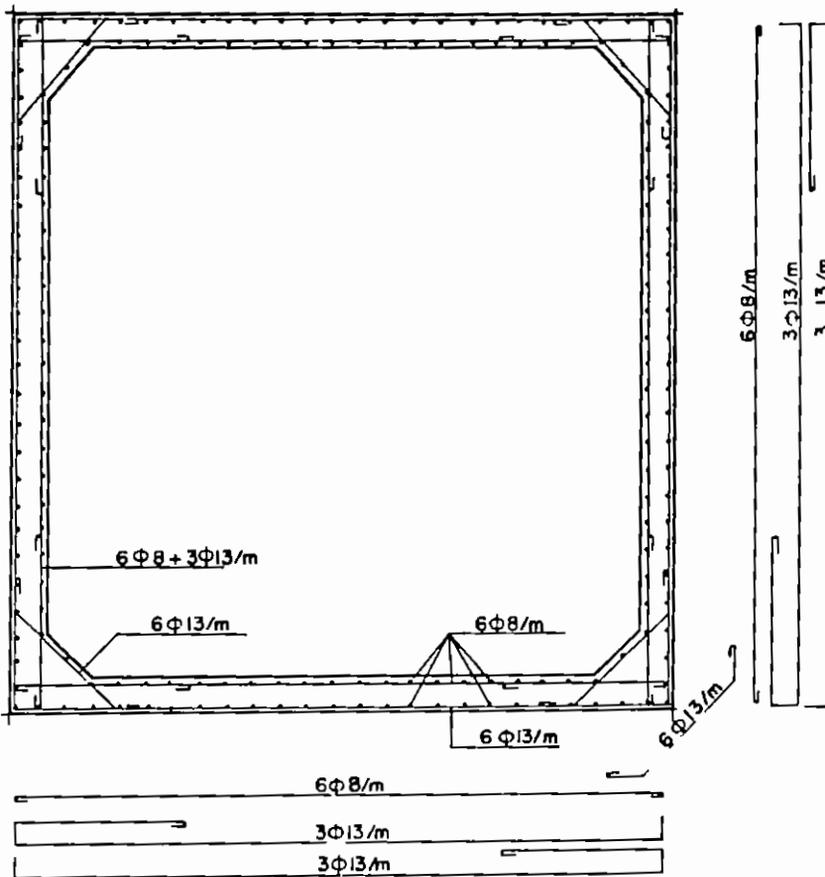
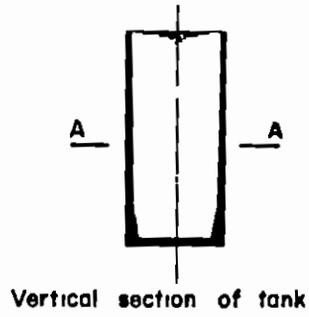


Fig VII-6



Details of horizontal section A A

Fig VII-9

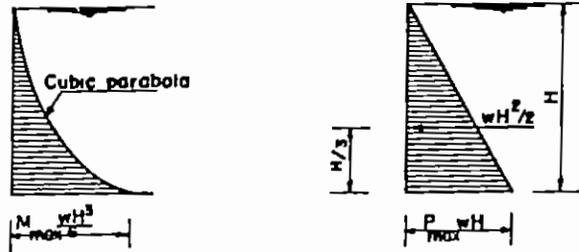
VII 2 WALLS RESISTING HYDROSTATIC PRESSURE IN VERTICAL DIRECTIONa) Cantilever Walls (Fig VII 10)

Fig VII-10

Walls fixed to the floor may act as simple cantilevers, such wall give a convenient thickness (max  $\approx 40$  cms) for a depth of water smaller than 30 ms

The max bending moment at the foot of the wall is given by

$$M_{\max} = w H^3 / 6 \quad (7a)$$

Reaction at base = Tension in floor, is given by

$$R_{\max} = w H^2 / 2 \quad (7b)$$

The convenient cross-section of the wall is trapezoidal with minimum thickness at top (15 - 20 cms) and maximum at bottom, or rectangular - of constant thickness - for small depths and eventually provided with a convenient haunch at its foot

For a cantilever wall of a water tank 3 ms deep, we get

$$M_{\max} = w H^3 / 6 = 1 \times 3^3 / 6 = 45 \text{ m t}$$

$$t_{\max} = \sqrt{M / 3} = \sqrt{4500 / 3} = 39 \text{ cms}$$

chosen 40 cms

The wall can be chosen trapezoidal with a minimum thickness of 15 cms

at the top, and a max thickness of 30 cms at the bottom, and provided with a haunch 10 x 45 cms at its base as shown in figure VII 11

Checking the section at beginning of haunch we get

$$M = w x^3 / 6 = 1 \times 2.55^3 / 6 = 2.76 \text{ m t}$$

$$\text{Required } t = \sqrt{M/3} = \sqrt{2760 / 3} \approx 30 \text{ cms}$$

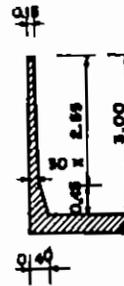


Fig VII-11

It has to be noted that free cantilevers of height H and supported on the two sides of their length L must be treated as slabs supported on three sides if their length L is smaller than four times their height H

Exact values of internal forces in walls free at top and fixed on the other three sides subject to hydrostatic pressure will be given later for ratios of H/L ( =  $L_y / L_x$  ) varying between 0.25 and 1.5

The fixing moments at the vertical edges of cantilever walls fixed at these edges and subject to hydrostatic pressure are to be considered in the design of water tanks because their values are relatively big

To give an idea about the values of the exact bending moments we consider a wall of depth H = 3 ms and length L = 12 ms free at top and fixed on the other three sides subject to hydrostatic pressure  
Max fixing moment at foot of wall

$$M_x = - w H^3 / 6.9 = - 1 \times 3^3 / 6.9 = - 3.9 \text{ m t}$$

Max fixing moment at top of vertical edges (in horizontal direction)

$$M_f = - w H^3 / 8.8 = - 1 \times 3^3 / 8.8 = - 3.06 \text{ m t}$$

The fixing moment at mid-height of vertical edges (in horiz direction)

$$M_f = - w H^3 / 18.2 = - 1 \times 3^3 / 18.2 = - 1.48 \text{ m t}$$



at the foot of the wall. If the wall is chosen trapezoidal with a bigger thickness at the bottom, the fixing moment is increased and the field moment is decreased<sup>■</sup> as shown in the following table :

$\beta = t_2 / t_1$	1 0	1 5	2 0	2 5	3 0
$M_f = w H^3 :$	15	13 2	12	11 2	10 7
$\lambda$	1 0	1 14	1 25	1 34	1 40

in which :  $\beta = \frac{t_2}{t_1} = \frac{\text{thickness of wall at bottom}}{\text{" " " " top}}$

$$\lambda = \frac{\text{real } M_f \text{ considering variation of } t}{M_f \text{ for wall of constant } t}$$

The table shows that if the variation of the thickness is neglected serious errors in  $M_f$  might take place. Such errors have a bad effect on the design especially because  $M_f$  causes tension on the water side.

The sedimentation tank shown in figure VII 13 gives a typical example for such walls.

In this tank, the walls, 4 0 ms deep, are fixed to the floor at their bottom edge and supported on a horizontal beam at their top edge. In order to have adequate supports for this beam, ties are arranged in both directions at distances of 3 0 ms at the upper surface of the tank. Columns at the points of intersection of the ties prevent their excessive sagging.

The top beam behaves as a continuous beam spanning between the ties and loaded by the top reaction of the wall, ( $p = R_1$ ). For equal spans  $L$ , we get

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■ Refer to "Theory of Elastically Restrained Beams" by M Hilal, Published by Fouad I University, 1944

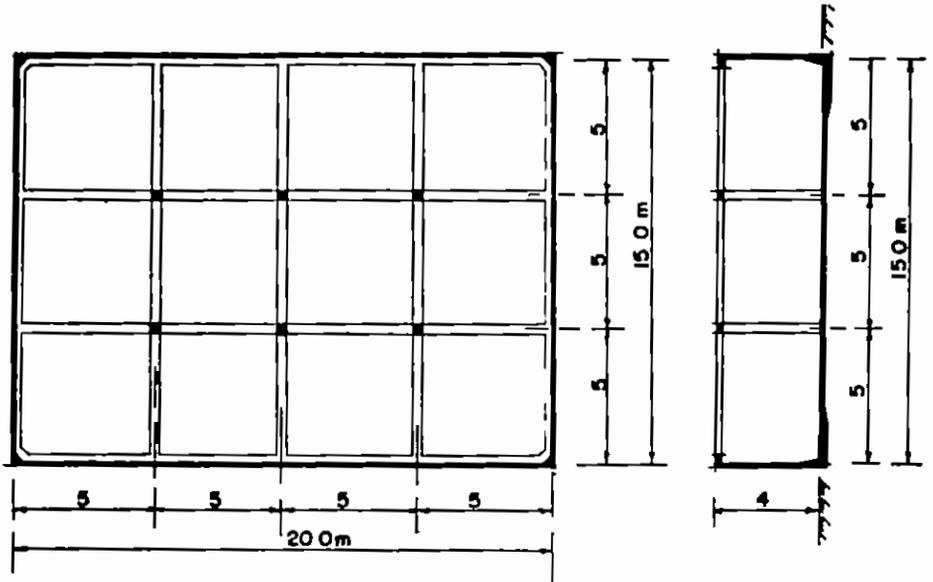


Fig VII-13

$$\begin{array}{l}
 \text{Field moment} \quad M_m = + p L^2 / 24 \\
 \text{Connecting} \quad M_c = - p L^2 / 12 \\
 \text{Tension in each side} \quad T = p L / 2
 \end{array}
 \quad \left. \vphantom{\begin{array}{l} M_m \\ M_c \\ T \end{array}} \right\} \quad (9)$$

The fixing moment at the vertical edges of one way slabs simply supported at the top and fixed at the other three sides, subject to hydrostatic pressure may be estimated by

$$M_f = w H^3 / 27 \quad (10a)$$

and lies at the middle of the height of the wall

The corresponding reaction is given by

$$R = 0.27 w H^2 \quad (10b)$$

The necessary provisions are to be taken to resist these values safely

c) Walls Fixed at Top and Bottom

If the wall in the previous example were fixed at top & bottom the bending moments and reactions due to a hydrostatic pressure  $p = w H$ , for the case of constant thickness, are given by : (Fig VII 14)

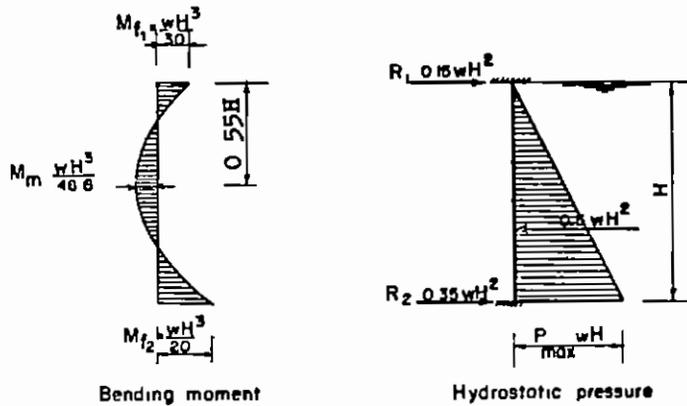


Fig VII-14

Fixing moment at top

$$M_{f1} = - w H^3 / 30 \quad (11a)$$

Fixing moment at bottom

$$M_{f2} = - w H^3 / 20 \quad (11b)$$

Max field moment at 0.55 H from top

$$M_m = + w H^3 / 46.6 \quad (11c)$$

Reaction at top edge

$$R_1 = 0.15 w H^2 \quad (11d)$$

Reaction at bottom edge

$$R_2 = 0.35 w H^2 \quad (11e)$$

For a trapezoidal wall, we get :

$\delta = t_2 / t_1$	1 0	1 5	2 0	2 5	3 0
$M_{F1} = w H^3$	30	38 7	52 6	63 6	76 5
$M_{F2} = w H^3$	20	16 7	14 7	13 7	12 8
$\lambda_{base}$	1 0	1 20	1 36	1 46	1 56

Neglecting the effect of the variation of the wall thickness, the possible error in the magnitude of the fixing moment at the base may be very serious

The fixing moment at the vertical edges of one way slabs fixed at all four sides and subject to hydrostatic pressure may be estimated from the relation

$$M_{F1} = - w H^3 / 33.8 \quad (12a)$$

and lies at 0.4 H from the base of the wall. The corresponding reaction is given by

$$R = 0.25 w H^2 \quad (12b)$$

In walls of tanks resisting the water pressure in the vertical direction, it is essential to arrange horizontal secondary reinforcements having a minimum cross-sectional area 20% of the main vertical steel (and not less than 5  $\phi$  8 mm/m) to fix the vertical reinforcements in position and to resist the temperature and shrinkage stresses as well as the eventual tensile stresses due to the reactions of the cross walls. The horizontal corner reinforcements may be calculated from the given relations. It is however generally

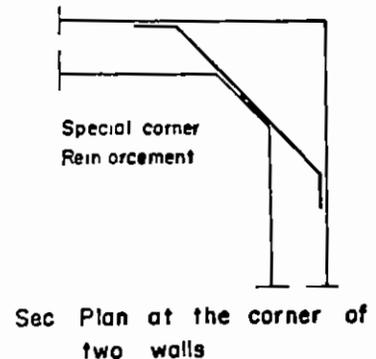


Fig VII-15

sufficient to arrange corner reinforcements of the same order as the max used vertically (Figure VII 15)

d) Walls Continuous with Roof & Floor

The walls of a tank may however be continuous with the roof and the floor or with one of them as shown in the tank given in Fig VII 16

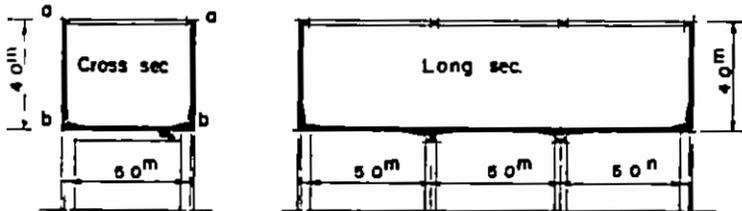


Fig VII-16

In this tank the longitudinal walls are simply supported on the top horizontal beam and continuous with the floor slab. In order to be able to assume that the top beam is a rigid support, ties are arranged every 5 ms

The effect of continuity can be calculated by any of the known methods of the theory of structures e.g. the method of virtual work, the moment distribution method, the equation of three moments etc

VII 3 WALLS AND FLOORS RESISTING HYDROSTATIC PRESSURE IN

TWO DIRECTIONS

a) The Strip Method :

This method gives an approximate solution for rectangular flat plates of constant thickness supported on four sides and subject to uniform or hydrostatic pressure. The solution is based on simplified assumptions generally adopted in structural problems.

Walls and floors supported on the four sides and having a ratio of length to breadth smaller than two are treated in this method as two way slabs.

In rectangular slabs with side lengths  $L_x$  and  $L_y$  subject to uniform loads (e.g. floor slabs), the load may be distributed in the two directions  $L_x$  and  $L_y$  according to the known coefficients of Grashoff. Due to the torsional resistance of the slab, the field (positive) moment may be reduced according to coefficients of Marcus as follows

$$\text{reduced } M_x = r_x M_x = \left[ 1 - \frac{5}{6} \left( \frac{L_x}{L_y} \right)^2 \frac{M_x}{M_{x0}} \right] M_x \quad \text{and}$$

$$\text{reduced } M_y = r_y M_y = \left[ 1 - \frac{5}{6} \left( \frac{L_y}{L_x} \right)^2 \frac{M_y}{M_{y0}} \right] M_y$$

in which  $M_x$  and  $M_y$  are the field moments if the effect of the torsional resistance is neglected, and

$$M_{x0} = p L_x^2 / 8 \quad \& \quad M_{y0} = p L_y^2 / 8$$

where

$$p = \text{total load on floor slab per m}^2$$

For a slab totally fixed on all four sides the reduction factors  $r$  can be extracted from the following table

$L_x / L_y$	1 0	1 1	1 2	1 3	1 4	1 5	1 6	1 7	1 8	1 9	2 0
$r_x = r_y$	86	86	87	88	89	90	91	91	92	93	94

It has been found that the load distribution on two way slabs subject to triangular load may approximately be assumed the same as in case of uniform loads using the coefficients of Grashoff i.e.

$$p = p_v + p_h$$

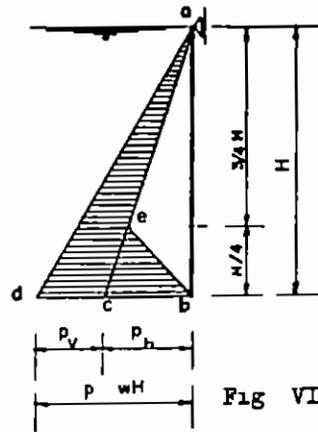
in which

$p$  = the hydrostatic pressure at any depth

$p_v$  = part of the pressure resisted in the vertical direction

$p_h$  = " " " horizontal "

As the wall is rigidly connected to the floor, no pressure can be resisted horizontally in the lowest strip and accordingly one can assume that the pressure resisted horizontally is the triangle a e b only and the small triangle e c b is resisted vertically (Figure VII 17) The position of point e varies according to ratio of sides of slab it lies at about  $\frac{3}{4} H$  from top in deep tanks where  $\frac{H}{L} \approx 2$  and at about  $\frac{H}{2}$  in shallow tanks where  $\frac{H}{L} \approx \frac{1}{2}$  It is however recommended to design the middle part of the wall (say the middle half) for a load equal to  $\frac{3}{4} p_h$



The bending moments and reactions in the vertical direction due to  $p_v$  (triangle a c d) can be determined as shown in VII 2-b whereas the bending moments and

reactions due to the small triangle e c b, for a wall of constant thickness simply supported at top and fixed at bottom can be determined as follows Fig VII 18

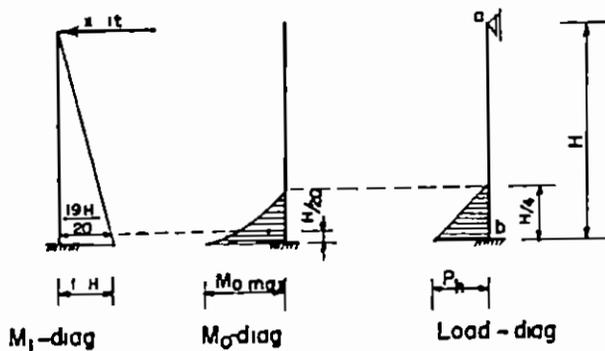


Fig VII-18

Using the method of virtual work and choosing the simple cantilever as a main system, we get

Max bending moment of main system is :

$$M_o \text{ max} = p_h (H/4)^2 / 6 = p_h H^2 / 96$$

$$\text{Area of } M_o \text{ - diagram} = \frac{1}{4} M_o \text{ max} \frac{H}{4} = p_h \frac{H^3}{1536}$$

Its center of gravity lies at  $1/5 (H/4)$  i.e.

$$E I \delta_o = \int M_o M_1 dH = - p_h \frac{H^3}{1536} \frac{19 H}{20} = - p_h \frac{19 H^4}{30720}$$

and

$$E I \delta_1 = \int M_1^2 dH = \frac{H}{2} \frac{H}{3} \frac{2 H}{3} = \frac{H^3}{3}$$

i.e. the statically indeterminate reaction  $X$  at  $a$  is given by

$$X = - \frac{\delta_o}{\delta_1} = p_h \frac{19 H^4}{30720} \frac{3}{H^3} = p_h \frac{H}{540}$$

The fixing moment at the fixed end  $b$  is therefore

$$M_f = p_h \frac{H}{540} H - p_h \frac{H^2}{96} = - p_h \frac{H^2}{117}$$

i.e. the total fixing moment at  $b$  for a wall of constant thickness due to the pressures resisted vertically (triangles  $a c d + e b c$ ) is given by

$$M_f = - (p_v \frac{H^2}{15} + p_h \frac{H^2}{117}) \quad (13a)$$

and the corresponding reaction at  $a$  is

$$R_a = p_v \frac{H}{10} + p_h \frac{H}{540} \quad (13b)$$

If the wall were rigidly connected to the floor, the effect of continuity can be considered by any of the known methods of the theory of elasticity. The max bending moments in the horizontal direction for the middle half of the wall can be determined for the maximum

pressure on the horizontal strip at  $3/4 H$  i e for max  $P_h = 3/4 P_h$

Example :

It is required to design the water tank shown in figure VII 16

1) Internal Forces in Cross-Sections of Tank (Fig VII 19)

The longitudinal wall of the tank (4 x 15 m) is a one way slab resisting the water pressure in the vertical direction only

The floor is a series of two way slabs (5 x 5 m) resisting half weight of water and floor in each direction Assuming the wall simply supported at a and totally fixed at b and subject to hydrostatic pressure with :

$$P_{\max} = w H = 4 \text{ t/m}^2 \text{ at b ,}$$

the fixing moment at b will be

$$M_f = - w H^3 / 15 = - 1 \times 4^3 / 15 = - 4.27 \text{ m t}$$

The load on the floor = weight of water + own weight of floor ( $\sim 0.7 \text{ t/m}^2$ )

or

$$= 4 + 0.7 = 4.7 \text{ t/m}^2 \quad \text{i e}$$

Load in each direction  $p = 4.7 / 2 = 2.35 \text{ t/m}^2$

Assuming the floor slab totally fixed at both ends, the fixing moment at b will be

$$M_f = - p L^2 / 12 = - 2.35 \times 5^2 / 12 = - 4.9 \text{ m t}$$

The unbalanced moment  $= 4.9 - 4.27 = 0.63 \text{ m t}$  is to be distributed on wall and floor slab according to their relative stiffness, but its value being small, the connecting moment  $M_b$  may be assumed as the average of the two values, i e

$$M_b = - \frac{4.27 + 4.9}{2} \approx - 4.6 \text{ m t}$$

Due to symmetry, the connecting moment at b can be determined from one equation of three moments; thus

$$2 M_b (H + L) + M_b L = -6 (r_1 + r_2)$$

in which

$r_1$  = the elastic reaction of the triangular load on wall

$$= p_{\max} H^3 / 45, \quad \text{and}$$

$r_2$  = the elastic reaction of the load on the floor

$$= p L^3 / 24, \quad \text{therefore}$$

$$2 M_b (4 + 5) + 5 M_b = -6 \left( \frac{4 \times 4^3}{45} + \frac{2.35 \times 5^3}{24} \right) \quad \text{or}$$

$$25 M_b = -6 (5.8 + 12.3) = -108.6 \quad \text{i e}$$

$$\underline{M_b = -4.7 \text{ m t}}$$

Accordingly, the field moment in the floor, neglecting its torsional resistance is given by

$$M_m = p \frac{L^2}{8} - M_b = \frac{2.35 \times 5^2}{8} - 4.7 = 2.65 \text{ m t}$$

Due to the torsional resistance of the slab, the final reduced field moment will be

$$M_{mr} = r M_m = 0.86 \times 2.65 \quad \text{or}$$

$$\underline{M_{mr} = 2.28 \text{ m t}}$$

The max field moment in the wall takes place at point of zero shear, thus :

Reaction at top edge of wall may either be calculated by super-position and is given by

$$R_a = 0.1 w H^2 - \frac{M_b - M_r}{H} = 0.1 \times 1 \times 4^2 - \frac{4.7 - 4.27}{4} = 1.5 \text{ t/m}$$

or from the equation of moments about corner b :

$$R_a H - p_{\max} H^2 / 6 = M_b \quad \text{or}$$

$$R_a \times 4 - 4 \times 4^2 / 6 = -4.7 \quad \text{or}$$

$$\underline{R_a = 1.5 \text{ t/m}}$$

If the point of zero shear lies at  $x$  meters from top end, then

$$R_a = 1.5 = \frac{x^2}{2} \quad \text{or} \quad \underline{x = 1.73 \text{ ms}}$$

The max field moment in the wall is therefore given by :

$$M_m = R_a x - p_x \frac{x^2}{6} = \frac{x^2}{2} x - \frac{x^3}{6} = \frac{x^3}{3} \quad \text{i.e.}$$

$$M_m = 1.73^3 / 3 \quad \text{or} \quad \underline{M_m = 1.73 \text{ mt}}$$

The reaction at the bottom edge of wall = tension in floor = water pressure - top reaction

$$R_b = p_{\max} H/2 - R_a = 4^2/2 - 1.5 = 6.5 \text{ t/m}$$

$$\text{Therefore Tension in floor} = \underline{6.5 \text{ t/m}}$$

We have further

$$\text{reaction of floor} = \text{Tension in all}$$

$$\text{average reaction on floor} = 4.7 \times 2.5/2 = 5.9 \text{ t/m}$$

This reaction is parabolically distributed in each span and the max ordinate at the middle is 1.5 times the average value

$$\text{Therefore, the max tension in the wall} = 1.5 \times 5.9 = 8.85 \text{ t/m}$$

The loads, bending moments and reactions are shown in figure VII 19

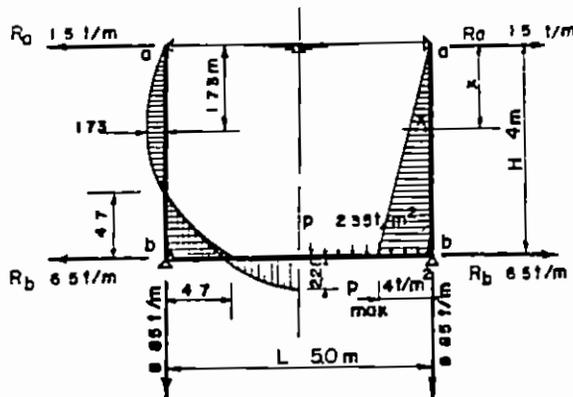


Fig VII-19

2) Internal Forces in Longitudinal Section of Tank

The side wall with  $H = 4$  ms and  $L = 5$  m is a two way slab The load will be distributed according to Grashoff, Fig VII 20 , thus

For  $L/H = 5/4 = 1.25$  , we get

$$\alpha = 0.7 \text{ and } \beta = 0.3 \text{ i.e.}$$

Load in vertical direction

$$p_v = \alpha p_{\max} = 0.7 \times 4 = 2.8 \text{ t/m}^2$$

Load in horizontal direction

$$p_h = \beta p_{\max} = 0.3 \times 4 = 1.2 \text{ t/m}^2$$

Assuming that the thickness of wall and floor is the same and applying

the equation of three moments at b and c, fig VII 21, we get

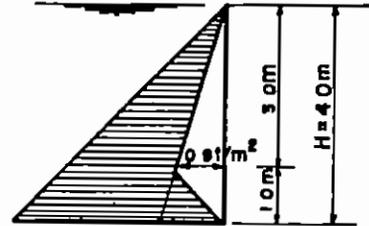


Fig VII-20

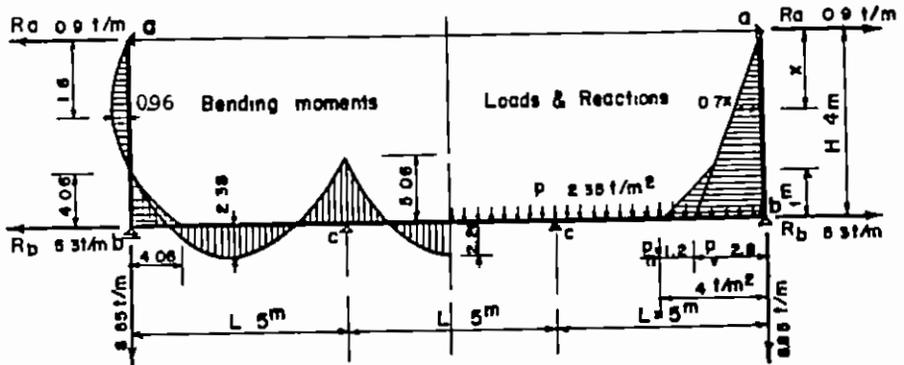


Fig VII-21

$$\text{sec b : } 2 M_b (H + L) + M_c L = -6 \left( p_v \frac{H^3}{45} + p_h \frac{H^3}{350} + p \frac{L^3}{24} \right)$$

$$\text{or } 2 M_b (4 + 5) + 5 M_c = -6 \left( \frac{2.8 \times 4^3}{45} + \frac{1.2 \times 4^3}{350} + \frac{2.35 \times 5^3}{24} \right)$$

$$\text{or } 18 M_b + 5 M_c = -98.6 \quad (\text{a})$$

$$\text{sec c } M_b L + 2 M_c \times 2 L + M_c L = -6 \left( 2 p \frac{L^3}{24} \right) = -\frac{2.35 \times 5^3}{2}$$

$$\text{or } 5 M_b + 25 M_c = -147 \quad (\text{b})$$

Equations (a) and (b) give :

$$\underline{M_b = -4.06 \text{ m t}} \quad \text{and} \quad \underline{M_c = -5.06 \text{ m t}}$$

Max moment in b c taking its torsional resistance in consideration is given by

$$M_{m1} = 0.86 \left[ \frac{2.35 \times 5^2}{8} - \frac{4.06 + 5.06}{2} \right] = 0.86 (7.32 - 4.56)$$

or 
$$\underline{M_{m1} = 2.38 \text{ m t}}$$

Field moment in c - c is

$$M_{m2} = \frac{2.35 \times 5^2}{8} - 5.06 = 7.32 - 5.06$$

or 
$$\underline{M_{m2} = 2.26 \text{ m t}}$$

N B The torsional resistance of span c - c has been neglected in order to have regular reinforcements for the floor slab

Average reaction at outside support of floor slab is

$$R = 0.5 \times 2.5 \times 4.7 = 5.9 \text{ t/m}$$

Max reaction = max tension in wall = 1.5 R = 1.5 \times 5.9

Therefore 
$$\underline{\text{max } R = 8.85 \text{ t/m}}$$

Reaction at a is determined from the equation of moments about b , thus :

$$R_a \times 4 - 2.8 \times \frac{4^2}{6} - 1.2 \times \frac{1^2}{6} = -4.06$$

or

$$\underline{R_a = 0.9 \text{ t/m}}$$

The reaction at the base of the wall = the tension in the floor and is given by :

or 
$$R_b = 2.8 \times \frac{4}{2} + 1.2 \times \frac{1}{2} - 0.9$$

$$\underline{R_b = 5.3 \text{ t/m}}$$

The max field moment  $M_{ma}$  in the wall takes place at the point of zero shear which lies at  $x$  ms from top, thus :

$$0.7 \frac{x^2}{2} = R_a = 0.9 \quad \text{or} \quad x = 1.6 \text{ ms} \quad \text{and}$$

$$M_m = 0.9 \times 1.6 - 0.7 \times \frac{1.6^3}{6} = 1.44 - 0.48$$

$$\text{or} \quad \underline{M_m = 0.96 \text{ m t}}$$

The loads, bending moments and reactions are shown in figure VII 21

### 3) Internal Forces in Horizontal Section of Tank

The section shown in figure VII 22 at mid-height of tank will be calculated for a horizontal load  $p = 0.9 \text{ t/m}^2$  acting on the cross wall (Refer to fig VII 20)

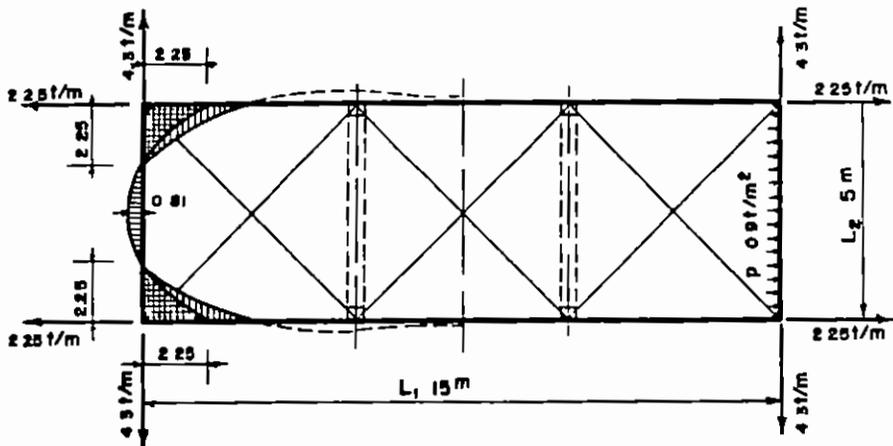


Fig VII-22

Assuming that the wall  $L_2$  is totally fixed at both ends, then the fixing moment  $M_f$  is given by :

$$M_f = - p L_2^2 / 12 = - 0.9 \times 5^2 / 12 = - 1.88 \text{ m t}$$

The longitudinal wall  $L_1$  is a one way slab resisting the full water pressure in the vertical direction, but due to its rigid connection to the cross-walls, bending moments will be created at the edges

in the horizontal direction Their magnitude may be estimated according to (10a) from the relation :

$$M_p = - w H^3 / 27 = - 1 \times 4^3 / 27 = - 2.37 \text{ m t}$$

for a wall totally fixed at its vertical edges

The difference of  $\Delta i = 2.37 - 1.88 = 0.49 \text{ m t}$  will be distributed between the two walls according to their relative stiffness thus

Stiffness of long wall  $\propto 1/15$

' " cross wall  $\propto 1/5 = 3/15$

The distribution factors are therefore :

Long wall  $1/4$  and cross wall  $3/4$

Accordingly, the connecting moment at the corner c will be

$$\underline{M_c = 2.25 \text{ m t}}$$

and the horizontal field moment is

$$M_m = p L^2/8 - M_c = \frac{0.9 \times 5^2}{8} - 2.25 = 2.82 - 2.25 = 0.57 \text{ m t}$$

The field moment should however be not smaller than the reduced moment of a totally fixed slab i e

$$\min M_m = 0.87 p L^2/24 = 0.87 \times 0.9 \times 5^2/24$$

$$\text{or} \quad \underline{M_m = 0.81 \text{ m t}}$$

Reaction of cross wall = Tension in long wall and is given by :

$$R = 0.9 \times 5/2 = 2.25 \text{ t/m}$$

Reaction of long wall = Tension in cross wall and according to equation (10b) is given by :

$$R = 0.27 w H^2 = 0.27 \times 1 \times 4^2 \quad \text{at mid-height ,}$$

or

$$\underline{R = 4.3 \text{ t/m}}$$

The loads, bending moments and reactions are shown in figure VII 22

4) Internal Forces in Top Horizontal Beam and Ties

The load on the top horizontal beam is equal to the top reaction  $R_a$  of the walls (Fig VII 23)

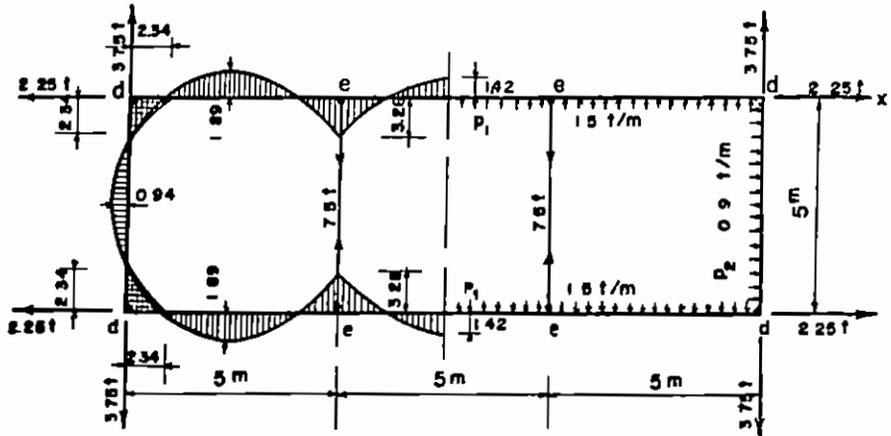


Fig VII-23

Applying the equation of three moments at d and e, we get

$$5 M_d + 20 M_e + 5 M_e = -6 \left( \frac{0.9 \times 5^3}{24} + \frac{1.5 \times 5^3}{24} \right)$$

$$\text{or } 5 M_d + M_e = -15 \quad \text{and}$$

$$5 M_d + 20 M_e + 5 M_e = -6 \left( 2 \times \frac{1.5 \times 5^3}{24} \right)$$

$$\text{or } M_d + 5 M_e = -18.75 \quad \text{giving}$$

$$\underline{M_d = -2.34 \text{ m t}} \quad \text{and} \quad \underline{M_e = -3.28 \text{ m t}}$$

Therefore, the field moment  $M_m$  in the different spans is given by

$$\text{Span d - d : } M_m = \frac{0.9 \times 5^2}{8} - 2.34 = 0.48 \text{ m t}$$

$$\text{but min } M_m = P_2 \frac{L^2}{24} \quad \text{or}$$

$$\text{min } M_m = \frac{0.9 \times 5^2}{24} = 0.94 \text{ m t}$$

$$\text{Span d - e : } M_m = \frac{1.5 \times 5^2}{8} - \frac{2.34 + 3.28}{2} = \underline{1.89} \text{ m t}$$

$$\text{Span e - e : } M_m = \frac{1.5 \times 5^2}{8} - 3.28 = \underline{1.42} \text{ m t}$$

Tension in span d-d

$$T_2 = 1.5 \times 5/2 = 3.75 \text{ t}$$

Tension in span d-e and e-e :

$$T_1 = 0.9 \times 5/2 = 2.25 \text{ t}$$

Tension in ties

$$T = 1.5 \times 5 = 7.5 \text{ t}$$

#### 5) Design of the Different Elements of the Tank

The thickness of the walls and floors is to be determined for the max field moments and the corresponding normal forces, haunches are provided to resist the bigger values of the connecting moments causing tension on the water side

In order to have sufficient water-tightness, the walls and floor are assumed 25 cms thick

#### A) Design of Longitudinal Walls

Section of max field moment  $M_m = \underline{1730 \text{ kgm}}$

The tensile stresses caused by the tensile force of 8850 kg/m acting at the foot of the wall reduce in the upper sections of the wall and vanish at the top. The tensile force at position of maximum moment is therefore given by

$$N = 8850 \times \frac{1.73}{4} \quad \text{or} \quad N = 3850 \text{ kg/m}$$

The tensile stresses in this section being on the air side, it is designed as ordinary reinforced concrete in stage II thus

$$\text{Eccentricity } e = M/N = 1730/3850 = 0.45 \text{ ms}$$

Eccentricity to tension steel

$$e_s = e - \frac{t}{2} + \text{cover} = 45 - \frac{25}{2} + 3 = 35.5 \text{ cms}$$

Moment about tension steel

$$M_s = N e_s = 3850 \times 0.355 = 1370 \text{ kgm}$$

$$d = k_1 \sqrt{M_s} \quad \text{or} \quad 22 = k_1 \sqrt{1370} \quad \text{giving } k_1 = 0.53$$

$$\text{for } \sigma_s = 1400 \text{ kg/cm}^2 \quad \text{and } \alpha = 0, \quad \text{we get}$$

$$\sigma_c = 31 \text{ kg/cm}^2 \quad \text{and } k_2 = 1285 \quad \text{so that}$$

$$A_s = \frac{M_s}{k_2 d} + \frac{N}{\sigma_s} = \frac{1370}{1285 \times 22} + \frac{3850}{1400}$$

$$\text{or} \quad A_s = 4.9 + 2.75 = 7.65 \text{ cm}^2 \quad 6 \text{ } \phi \text{ } 13 \text{ mm/m}$$

$$\text{Section at foot wall} \quad \underline{l = 4700} \text{ kgm} \quad \underline{N = 8850} \text{ kgs}$$

The tensile stresses in this section being on the water side, they should be smaller than the tensile strength of concrete in bending thus

$$t = \sqrt{\frac{l}{3}} + 2 \text{ cms} = \sqrt{\frac{4700}{3}} + 2 = 40 + 2 = 42 \text{ cms}$$

The thickness is chosen 45 cms and the wall will be provided with a convenient haunch

$$e = \frac{M}{N} = \frac{4700}{8850} = 0.53 \text{ ms}$$

$$e_s = 53 - \frac{45}{2} + 3 = 32.5 \text{ cms}$$

$$M_s = M e_s = 8850 \times 0.325 = 2880 \text{ kgm}$$

$$42 = k_1 \sqrt{2880} \quad \text{giving } k_1 = 0.78$$

$$\text{For } \sigma_s = 1400 \text{ kg/cm}^2 \quad \sigma_c = \text{low} \quad \text{and } k_2 = 1300, \quad \text{so that :}$$

$$A_B = \frac{2880}{1300 \times 42} + \frac{8850}{1400} = 5.3 + 6.3 = 11.6 \text{ cm}^2 \quad 6 \text{ } \varnothing 16 \text{ mm/m}$$

In the horizontal direction, the middle part of the longitudinal wall will be designed for a tensile force

$$N = 2250 \text{ kgs}$$

the bending moment is small and can be neglected, therefore the horizontal steel reinforcement is given by :

$$A_B = N/\sigma_B = 2250 / 1400 = 1.6 \text{ cm}^2$$

choose min steel of 5  $\varnothing$  8 mm/m ( $A_B = 2.5 \text{ cm}^2$ )

The corner, at the vertical edges, is to be designed for :

$$M = 2250 \text{ kgm} \quad \text{and} \quad N = 2250 \text{ kgs (tension)}$$

The tensile stresses being on the water side then,

$$t = \sqrt{M/3} + 1.5 \text{ cms} = \sqrt{2250/3} + 1.5 = 27.5 + 1.5 = 29 \text{ cms}$$

The vertical edges of the walls will be provided by a haunch 15 x 15 cms, the theoretical thickness at the corner will be

$$t = 25 + \frac{15}{3} = 30 \text{ cms}$$

Proceeding in the same way as before, we get the required area of steel reinforcement, which is given by  $A_B = 7.3 \text{ cm}^2$  6  $\varnothing$  13 mm/m

#### B) Design of Cross Walls

The internal forces, chosen thickness and reinforcements are shown in the following table :

sense	Section	M m t	N tons	Side of ten- sile stresses	t cm	$A_{B2}$ cm <sup>2</sup>	Rfmt
Vert	middle	0.96	3.54*	air	25	5.32	3 $\varnothing$ 13 + 3 $\varnothing$ 10
"	foot	4.06	8.85	water	45	10.60	6 $\varnothing$ 16
Horiz	middle	0.81	4.30	air	25	4.45	6 $\varnothing$ 10
"	vert edge	2.25	4.30	water	30	7.95	6 $\varnothing$ 13

\* This value gives the tension at position of max field moment and equals  $8.85 \times 1.6 / 4$

C) Design of Floor Slab

Sense	Section	M m t	N tons	Side of ten - sile stresses	t cm	$A_s$ cm <sup>2</sup>	Rfmt mm/m
Cross	middle	2 28	6 5	air	25	10 55	6 Ø 16
'	corner	4 70	6 5	water	45	10 95	6 Ø 16
Long	middle	2 38	5 3	air	25	10 50	6 Ø 16
'	corner	4 06	5 3	water	45	9 35	6 Ø 16
	intern sup	5 06	5 3	water	45	11 20	6 Ø 16

D) Design of Cross Beam

$$L = 4.9 \text{ ms}$$

The beam is assumed

$$30 \times 80 \text{ cms}$$

The load being triangular, the equivalent load for bending moment = water and floor load + own weight

$$p = 0.67 \times 5 \times 4.7 + 0.3 \times 0.55 \times 2.5 = 15.7 + 0.40 = 16.1 \text{ t/m}$$

$$M = p L^2 / 8 = 16.1 \times 4.9^2 / 8 = 48.2 \text{ m t}$$

$$\text{Effective breadth of flange } B = L/3 = 4.9/3 = 1.63 \text{ ms}$$

$$d = k_1 \sqrt{M/B} \quad \text{or} \quad 73 = k_1 \sqrt{\frac{48200}{1.63}} \quad \text{i.e.} \quad k_1 = 0.425$$

Using high grade steel with  $\sigma_s = 2000 \text{ kg/cm}^2$ , we get

$$\sigma_c = 49.5 \text{ kg/cm}^2 \quad \text{and} \quad k_2 = 1820 \text{ kg/cm}^2$$

Therefore ,

$$A_s = M/k_2 d = 48200/1820 \times 0.73 = 37 \text{ cm}^2 \quad \text{choose} \quad 10 \text{ } \phi 22$$

The average load on the beam is given by

$$p = 0.5 \times 5 \times 4.7 + 0.3 \times 0.55 \times 2.5 = 10.9 + 0.4 = 11.3 \text{ t/m}$$

The shearing force is therefore

$$Q = p L/2 = 11.3 \times 4.9/2 = 27.5 \text{ t} \quad \text{and}$$

the max shear stress is :

$$\tau = \frac{Q}{0.87 b d} = \frac{27500}{0.87 \times 30 \times 73}$$

$$= 14.5 \text{ kg/cm}^2$$

The corresponding diagonal tension will be resisted by four branch stirrups  $\phi 8 \text{ mm @ } 20 \text{ cms}$  and bent bars. The shear stress resisted by the stirrups

$$\tau_{st} = \frac{n A_s \sigma_s}{e b}$$

in which

$n$	= number of branches of stirrup	= 4	
$A_s$	= area of cross section of one branch	= $0.5 \text{ cm}^2$	
$e$	= spacing of stirrups	= 20 cm	
$b$	= breadth of web of beam	= 30 cm	i.e.

$$\tau_{st} = \frac{4 \times 0.5 \times 1400}{20 \times 30} = 4.67 \text{ kg/cm}^2$$

The bent bars resist the remaining area  $A$  of the diagonal tension diagram  $\times b$  Fig VII 24, thus

$$A_{sb} = \left( \frac{14.5 + 4.67}{2} \right) \frac{120 \times 30}{2000} = 10.0 \text{ cm}^2 \quad \text{chosen } 5 \phi 22$$

#### E) Walls as Deep Beams

The walls will act as deep beams (given later) supported on the columns

Load  $p$  = water and floor load + own weight of wall

$$= 0.67 \times 2.5 \times 4.7 + 0.25 \times 4 \times 2.5 = 7.9 + 2.5 = 10.4 \text{ t/m}$$

The max tension at the bottom of a simple deep beam is

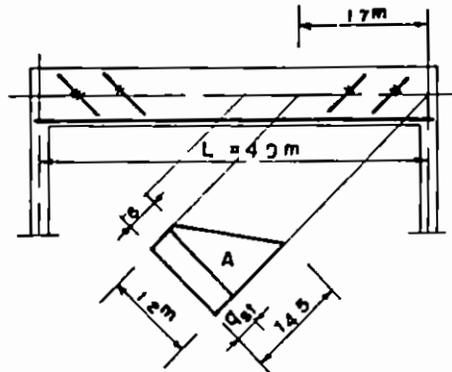


Fig VII-24

$$T = 0.2 p L = 0.2 \times 10.4 \times 5 = 10.4 \text{ ton}$$

$$A_s = T / \sigma_s = 10.4 / 1.4 = 7.4 \text{ cm}^2 \quad \text{chosen } 4 \text{ } \phi 16$$

The max tension at the bottom of the continuous wall at center line of spans is given by

$$T = 0.13 p L = 0.13 \times 10.4 \times 5 = 6.8 \text{ ton}$$

$$A_s = T / \sigma_s = 6.8 / 1.4 = 4.9 \text{ cm}^2 \quad \text{chosen } 4 \text{ } \phi 13$$

The tension over intermediate supports is

$$T = 0.2 p L \text{ giving } A_s = 7.4 \text{ cm}^2 \quad \text{or } 2 \text{ } \phi 16 + 6 \text{ } \phi 10$$

#### F) Design of Top Horizontal Beam and Ties

It is recommended to avoid the use of bent bars in this beam this means that the shear stresses must be low. Such a provision can be satisfied if the depth of the beam is chosen sufficiently big.

Assuming the beam to be 20 x 50 cms the reinforcements at the different sections will be as follows (see fig. VII-3)

Section at	l m t	V t	$A_s$ $\text{cm}^2$	Flat
c in long wall	3.28	2.25	6.5	) 2 $\phi$ 13 ) 2 $\phi$ 16
d in cross wall	2.34	3.75	5.2	
middle of d e	1.89	2.25	3.94	) 3 $\phi$ 13 ) )
' e e	1.42	2.25	3.1	
' ' d d	0.94	3.75	2.9	

$$\text{The tie is chosen } 20 \times 30 \text{ cms reinforced by } A_s = \frac{2.5}{1.4} = 1.8 \text{ cm}^2$$

or

$$4 \text{ } \phi 13$$

The details of the tank are shown in figure VII-25



b) General Theory of Flat Plates

The exact distribution of the internal forces in a slab subject to distributed - eventually concentrated - loads under different edge conditions is dealt with in text books of theory of elasticity e.g. "Theory of Plates and Shells" and "Theory of Elasticity" by Timoshenko

For a rectangular plate subject to a distributed load  $p$  and having any given boundary conditions the internal forces and stresses in any direction can be obtained mathematically according to the following assumptions and basic equations (Fig VII-26)

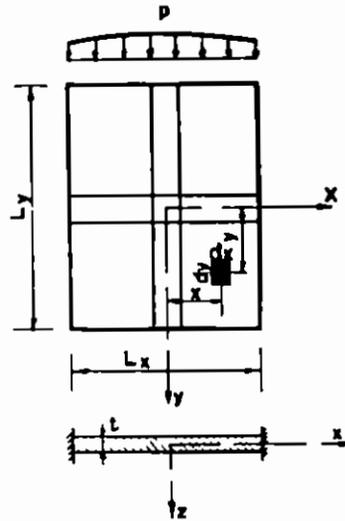


Fig VII-26

$\nu = 1/m = 1/5 - 1/6 =$  Poissons ratio

$D = \frac{E t^3}{12 (1-\nu^2)}$  = Flexural rigidity of a plate of thickness  $t$

$z$  = Lateral displacement of plate

$M_x, M_y$  = Bending moments per unit length of sections of a plate in directions  $x$  and  $y$  respectively

$M_{xy}, M_{yx}$  = Twisting moment per unit length of a section of a plate

$Q_x, Q_y$  = Shearing forces parallel to  $z$  - axis per unit length of sections of a plate perpendicular to  $x$  and  $y$  axes respectively

$R_x, R_y$  = Reactions per unit length of plate, acting on sections perpendicular to  $x$  and  $y$  axes respectively

Considering the equilibrium of the elementary prism  $dx, dy, t$ , we get



Adding these two equations we get

$$\frac{\partial^4 z}{\partial x^4} + 2 \frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = \frac{P}{D} \quad (14)$$

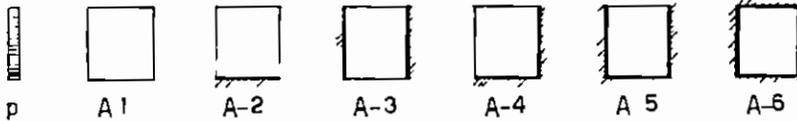
which gives the differential equation of the elastic surface of a plate loaded perpendicular to its plane

The previous investigation shows that the determination of the internal forces in a flat plate by the mathematical theory of elasticity requires the solution of a partial differential equation of the fourth degree

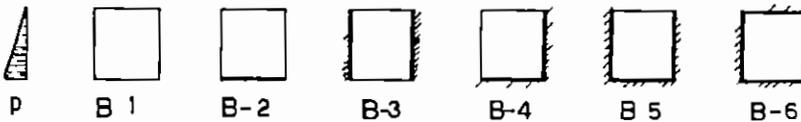
#### Internal Forces in Flat Plates by Czerny

The internal forces in the following individual typical cases of rectangular plates subject to uniform and triangular loads have been determined by Czerny

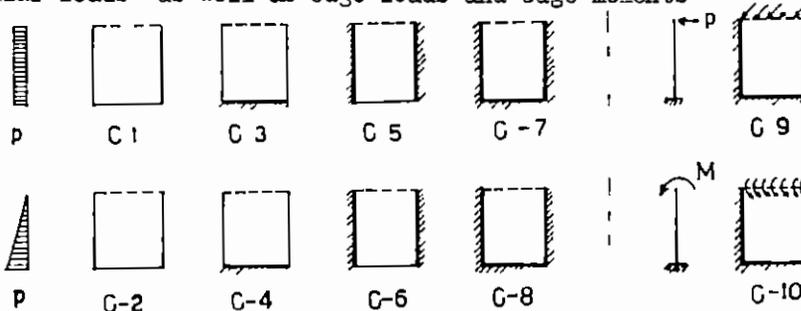
A) Slabs supported on four sides and subject to uniform load



B) Slabs supported on four sides and subject to triangular load



C) Slabs supported on three sides and subject to uniform and triangular loads as well as edge loads and edge moments



we give in the following tables the internal forces in the shown cases

Reactions of case C are given in table C 11

Notations

Loading Uniform or triangular

Spans  $L_x$  and  $L_y$

Supports ---- Free

— Hinged

~~~~~ Fixed

Bending Moments

In direction x  $M_x$

In direction y  $M_y$

At fixed end  $M_{min}$

Torsional Moment  $M_{xy}$

Reactions

In direction x  $R_x$

In direction y  $R_y$

At fixed end R

Shearing Forces

In direction x  $Q_x$

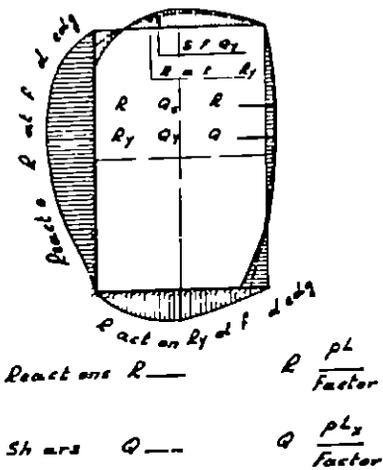
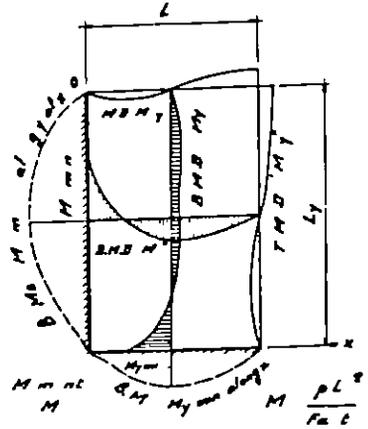
In direction y  $Q_y$

At fixed end  $Q = R$

E = Modulus of Elasticity

t = Thickness of Slab

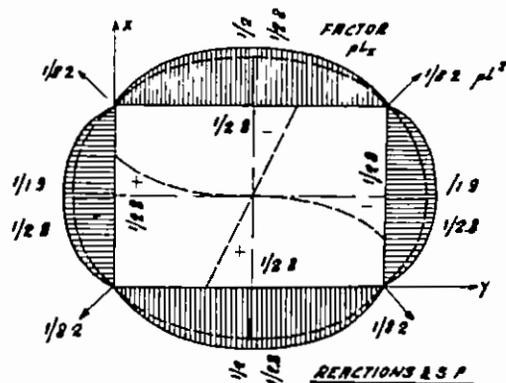
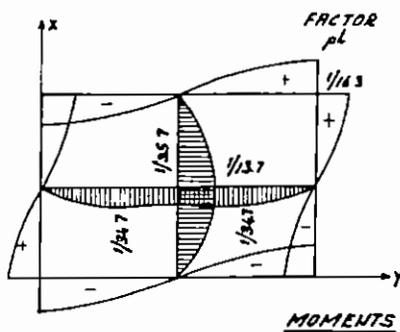
m = Deflection at mid-point  
of slab



R) SLABS SUPPORTED ON FOUR SIDES AND SUBJECT TO UNIFORM LOADS

A 1) SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES

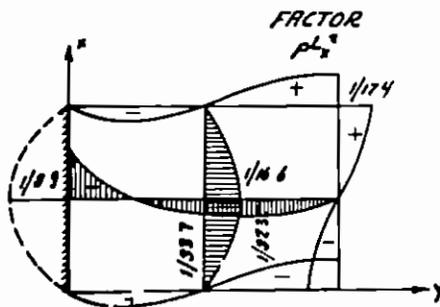
| $L_y/L_x$                              | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|----------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{x\text{m}} \quad \rho L^2$         | 272 | 224 | 191 | 168 | 150 | 137 | 127 | 119 | 113 | 108 | 104 |
| $M_{y\text{max}} = \rho L_x^2$         | 272 | 279 | 291 | 309 | 328 | 347 | 361 | 373 | 385 | 394 | 403 |
| $M_{y\text{ corner}} = +\rho L_x^2$    | 216 | 197 | 184 | 175 | 168 | 163 | 159 | 156 | 154 | 153 | 151 |
| $R_{\text{ corner}} \quad \rho L_x^2$  | 108 | 99  | 92  | 88  | 84  | 82  | 80  | 78  | 77  | 77  | 76  |
| $R_m = \rho L_x$                       | 22  | 21  | 20  | 20  | 20  | 20  | 19  | 19  | 19  | 19  | 19  |
| $R_{ym} \quad \rho L_x$                | 22  | 21  | 20  | 20  | 19  | 19  | 19  | 19  | 18  | 18  | 18  |
| $\delta_m \quad \frac{\rho L^4}{Et^3}$ | 049 | 058 | 068 | 073 | 085 | 093 | 100 | 106 | 112 | 117 | 122 |



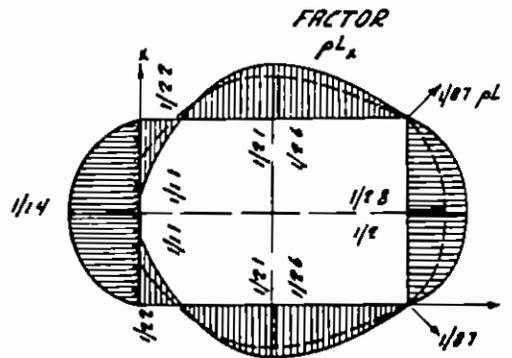
$$\underline{L_y/L_x = 15}$$

A 2) SLAB FIXED ON ONE OF THE SHORTER SIDES AND SIMPLY SUPPORTED ON THE OTHER THREE SIDES

| $L_y / L_x$                                 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|---------------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{x\text{m}} = \rho L^2$                  | 412 | 319 | 259 | 217 | 188 | 166 | 150 | 138 | 128 | 120 | 114 |
| $M_{y\text{m n}} = \rho L_x^2$              | 119 | 109 | 101 | 96  | 92  | 89  | 87  | 85  | 84  | 83  | 82  |
| $M_{y\text{max}} = \rho L_x^2$              | 294 | 288 | 283 | 277 | 268 | 263 | 256 | 249 | 242 | 235 | 228 |
| $M_{xy\text{ corner}} = \rho L_x^2$         | 262 | 232 | 210 | 194 | 183 | 174 | 168 | 163 | 159 | 156 | 154 |
| $R_{\text{corner}} = \rho L^2$              | 131 | 116 | 105 | 97  | 91  | 87  | 84  | 81  | 79  | 78  | 77  |
| $R_{x\text{m}} = \rho L_x$                  | 30  | 24  | 23  | 22  | 21  | 21  | 20  | 20  | 20  | 20  | 20  |
| $R_{ym} = \rho L$                           | 17  | 16  | 16  | 15  | 15  | 14  | 14  | 14  | 14  | 14  | 14  |
| $R_{ym} = \rho L$                           | 25  | 23  | 22  | 21  | 20  | 20  | 19  | 19  | 19  | 19  | 18  |
| $\delta_{\text{m}} = \frac{\rho L^4}{Et^3}$ | 033 | 042 | 051 | 060 | 069 | 077 | 085 | 093 | 099 | 106 | 111 |



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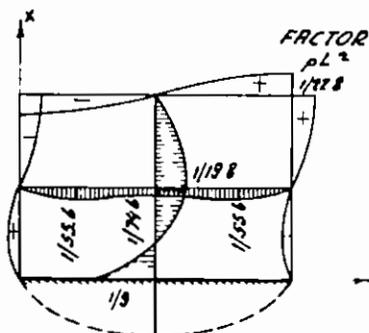


REACTIONS & SF

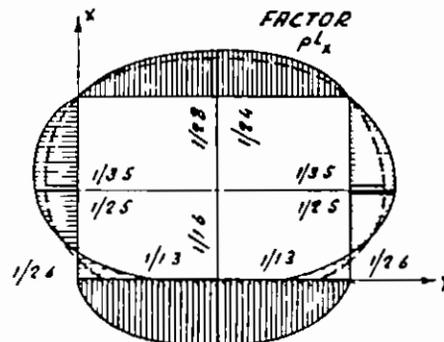
$L_y / L_x = 15$

A-2) SLAB FIXED ON ONE OF THE LONGER SIDES AND SIMPLY  
SUPPORTED ON THE OTHER THREE SIDES

| $L_y/L_x$                                        | 100 | 110 | 120 | 130 | 140 | SD  | 160 | 170 | 180 | 190 | 200 |
|--------------------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{x \text{ min}} = -\rho L_x^2$                | 119 | 109 | 102 | 97  | 93  | 90  | 88  | 86  | 84  | 83  | 83  |
| $M_{x \text{ m}} = \rho L_x^2$                   | 314 | 273 | 245 | 224 | 210 | 198 | 190 | 183 | 178 | 174 | 171 |
| $M_{y \text{ max}} = \rho L_x^2$                 | 412 | 451 | 488 | 518 | 543 | 556 | 568 | 578 | 586 | 590 | 592 |
| $M_{xy \text{ corner}} = \frac{1}{2} \rho L_x^2$ | 262 | 249 | 240 | 235 | 230 | 228 | 226 | 225 | 224 | 224 | 224 |
| $R_{\text{ corner}} = \rho L_x$                  | 131 | 124 | 120 | 117 | 115 | 114 | 113 | 112 | 112 | 112 | 112 |
| $R_{x \text{ m}} = \rho L_x$                     | 17  | 17  | 16  | 16  | 16  | 16  | 16  | 16  | 16  | 16  | 16  |
| $R_{x \text{ m}} = \rho L_x$                     | 25  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 25  | 25  | 25  |
| $R_{y \text{ m}} = \rho L_x$                     | 26  | 25  | 25  | 25  | 25  | 25  | 25  | 25  | 25  | 25  | 25  |
| $\delta_m = \frac{\rho L_x^4}{E t^3}$            | 033 | 038 | 042 | 046 | 049 | 051 | 053 | 055 | 056 | 058 | 059 |



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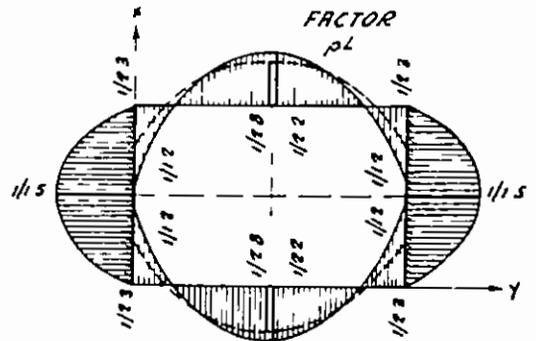
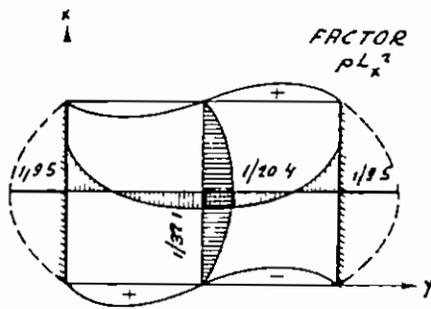


REACTIONS & S F

$L_y/L_x = 15$

R 3) SLAB FIXED ON THE TWO SHORTER SIDES AND SIMPLY SUPPORTED  
ON THE OTHER TWO SIDES

| $L_y/L_x$  |                         | 100  | 110  | 120  | 130  | 140  | 150  | 160  | 170  | 180  | 190  | 200  |
|------------|-------------------------|------|------|------|------|------|------|------|------|------|------|------|
| $M_{xm}$   | $\rho L^2$              | 63.3 | 46.1 | 35.5 | 28.5 | 23.7 | 20.4 | 17.9 | 16.0 | 14.6 | 13.4 | 12.5 |
| $M_{ymn}$  | $\rho L_x^2$            | 14.3 | 12.7 | 11.5 | 10.7 | 10.0 | 9.5  | 9.2  | 8.9  | 8.7  | 8.5  | 8.4  |
| $M_{ymax}$ | $= \rho L_x^2$          | 35.1 | 32.9 | 31.7 | 31.2 | 31.4 | 32.1 | 33.3 | 34.9 | 37.1 | 39.7 | 42.4 |
| $R_m$      | $\rho L_x$              | 30   | 27   | 25   | 24   | 23   | 22   | 21   | 21   | 20   | 20   | 20   |
| $R_{ym}$   | $= \rho L_x$            | 19   | 18   | 17   | 16   | 16   | 15   | 15   | 14   | 14   | 14   | 14   |
| $\sum m$   | $\frac{\rho L^4}{EI^3}$ | 0.23 | 0.30 | 0.38 | 0.47 | 0.55 | 0.64 | 0.72 | 0.80 | 0.88 | 0.95 | 1.01 |



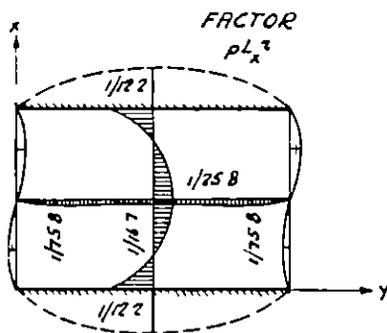
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REACTIONS & S F

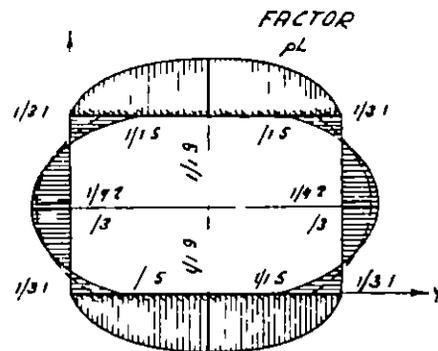
$L_y/L_x = 1.5$

R 3) SLAB FIXED ON THE TWO LONGER SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES

| $L_y / L_x$                          |  | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|--------------------------------------|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{xmn} = \rho L_x^2$               |  | 143 | 135 | 130 | 126 | 123 | 122 | 120 | 120 | 120 | 120 | 120 |
| $M_{xm} = \rho L_x^2$                |  | 351 | 317 | 294 | 278 | 266 | 258 | 252 | 247 | 244 | 243 | 241 |
| $M_{ymax} = \rho L_x^2$              |  | 617 | 672 | 715 | 735 | 746 | 758 | 770 | 770 | 770 | 770 | 770 |
| $R_{xm} = \rho L_x$                  |  | 19  | 19  | 19  | 19  | 19  | 19  | 19  | 19  | 19  | 20  | 20  |
| $R_{ym} = \rho L_x$                  |  | 30  | 30  | 30  | 30  | 30  | 30  | 30  | 30  | 30  | 30  | 30  |
| $\delta_m = \frac{\rho L_x^4}{Et^3}$ |  | 023 | 025 | 027 | 028 | 029 | 030 | 030 | 031 | 031 | 031 | 031 |



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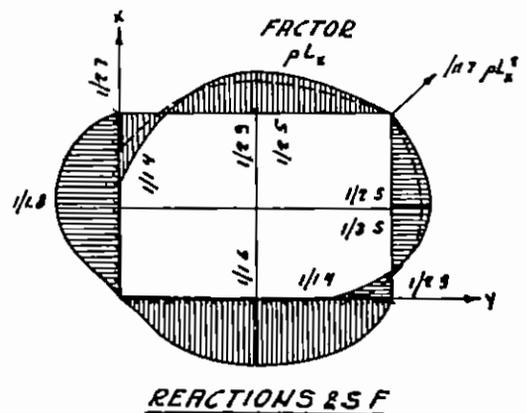
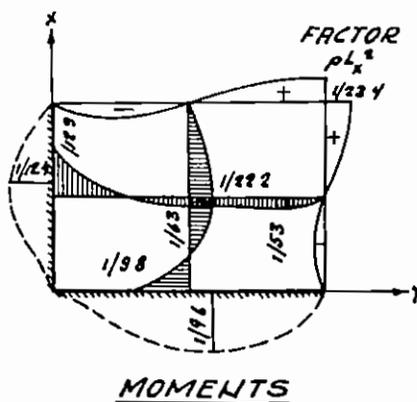


REACTIONS & F

$L_y / L_x = 1.5$

A 4) SLAB FIXED ON TWO ADJACENT SIDES AND SIMPLY SUPPORTED  
ON THE OTHER TWO SIDES

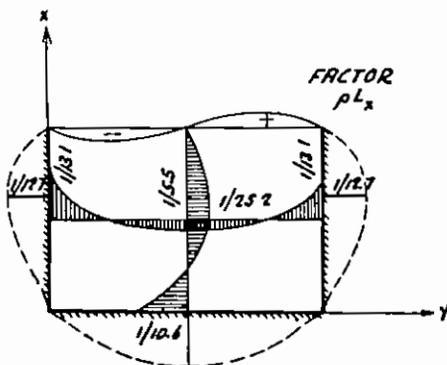
| $L_y / L_x$                        | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{x\min} = -\rho L_x^2$          | 143 | 127 | 115 | 107 | 100 | 96  | 92  | 89  | 87  | 85  | 84  |
| $M_{xm} = \rho L_x^2$              | 427 | 351 | 300 | 265 | 241 | 222 | 210 | 199 | 191 | 184 | 179 |
| $M_{y\min} = \rho L_x^2$           | 143 | 136 | 131 | 128 | 126 | 124 | 123 | 122 | 122 | 122 | 122 |
| $M_{ymax} = \rho L_x^2$            | 402 | 420 | 440 | 476 | 510 | 530 | 548 | 563 | 577 | 590 | 602 |
| $R_{xm} = \rho L_x$                | 20  | 18  | 18  | 17  | 17  | 16  | 16  | 16  | 16  | 16  | 16  |
| $R_{xm} = \rho L_x$                | 28  | 26  | 26  | 25  | 25  | 25  | 25  | 25  | 25  | 25  | 25  |
| $R_{ym} = \rho L_x$                | 20  | 19  | 19  | 19  | 18  | 18  | 18  | 18  | 18  | 18  | 18  |
| $R_{ym} = \rho L_x$                | 28  | 27  | 26  | 25  | 25  | 25  | 25  | 25  | 25  | 25  | 25  |
| $\sum m = \frac{\rho L_x^4}{Et^3}$ | 025 | 030 | 035 | 039 | 043 | 046 | 048 | 051 | 053 | 055 | 056 |



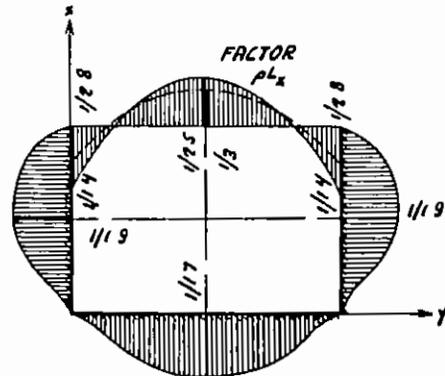
$$L_y / L_x = 15$$

**R-5) SLAB SIMPLY SUPPORTED ON ONE LONGER SIDE**  
**AND FIXED ON THE OTHER THREE SIDES**

| $L_y / L_x$      |                       | 1 00 | 1 10 | 1 20 | 1 30 | 1 40 | 1 50 | 1 60 | 1 70 | 1 80 | 1 90 | 2 00 |
|------------------|-----------------------|------|------|------|------|------|------|------|------|------|------|------|
| $M_x$ m n        | $-pL_x^2$             | 18 3 | 15 4 | 13 5 | 12 2 | 11 2 | 10 6 | 10 1 | 9 7  | 9 4  | 9 0  | 8 8  |
| $M$ m            | $= pL_x^2$            | 59 5 | 46 1 | 37 5 | 31 8 | 28 0 | 25 2 | 23 3 | 21 7 | 20 5 | 19 5 | 18 7 |
| $M_y$ m n        | $= pL_x^2$            | 16 2 | 14 8 | 13 9 | 13 3 | 13 0 | 12 7 | 12 6 | 12 5 | 12 4 | 12 3 | 12 3 |
| $M_y$ m          | $pL_x^2$              | 44 1 | 43 7 | 44 8 | 46 9 | 50 3 | 55 0 | 61 6 | 70 4 | 79 6 | 89 8 | 101  |
| $R_x$ m          | $-pL_x$               | 2 2  | 2 0  | 1 9  | 1 8  | 1 8  | 1 7  | 1 7  | 1 7  | 1 7  | 1 6  | 1 6  |
| $R_x$ m          | $pL_x$                | 3 0  | 2 8  | 2 7  | 2 6  | 2 5  | 2 5  | 2 5  | 2 5  | 2 5  | 2 5  | 2 5  |
| $R_y$ m          | $+pL_x$               | 2 1  | 2 0  | 1 9  | 1 9  | 1 9  | 1 9  | 1 8  | 1 8  | 1 8  | 1 8  | 1 8  |
| $\bar{\delta}_m$ | $\frac{pL_x^4}{Et^3}$ | 0 19 | 0 24 | 0 28 | 0 33 | 0 37 | 0 41 | 0 44 | 0 47 | 0 50 | 0 52 | 0 54 |



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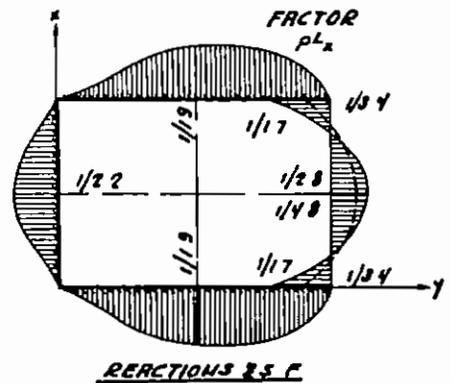
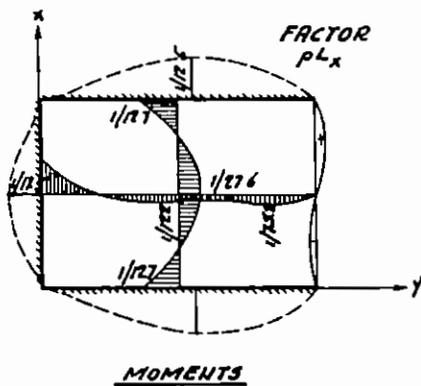


REACTIONS & S F

$L_y / L_x = 1.5$

**A-5) SLAB SIMPLY SUPPORTED ON ONE SHORTER**  
**SIDE AND FIXED ON THE OTHER THREE SIDES**

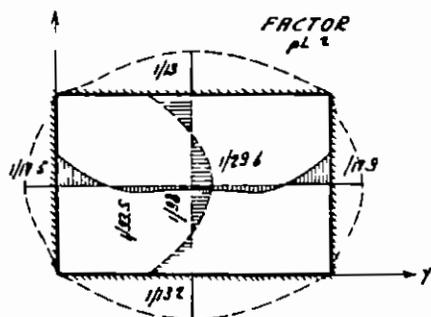
| $L_y / L_x$                           |  | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|---------------------------------------|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_x \text{ min} = \rho L_x^2$        |  | 162 | 148 | 139 | 132 | 127 | 125 | 123 | 122 | 121 | 120 | 120 |
| $M_x \text{ m} = \rho L_x^2$          |  | 441 | 379 | 338 | 310 | 290 | 276 | 265 | 257 | 251 | 247 | 246 |
| $M_y \text{ min} = -\rho L_x^2$       |  | 183 | 177 | 175 | 175 | 175 | 175 | 175 | 175 | 175 | 175 | 175 |
| $M_y \text{ ma} = \rho L_x^2$         |  | 559 | 603 | 662 | 690 | 720 | 752 | 787 | 825 | 868 | 917 | 970 |
| $R_x \text{ m} = \pm \rho L_x$        |  | 21  | 20  | 20  | 19  | 19  | 19  | 19  | 19  | 19  | 20  | 20  |
| $R_y \text{ m} = \rho L_x$            |  | 22  | 21  | 21  | 21  | 21  | 22  | 22  | 22  | 22  | 22  | 22  |
| $R_y \text{ m} = \rho L_x$            |  | 30  | 29  | 28  | 28  | 28  | 28  | 28  | 28  | 28  | 28  | 28  |
| $\delta_m = \frac{\rho L_x^4}{E t^3}$ |  | 019 | 021 | 024 | 025 | 027 | 028 | 029 | 030 | 030 | 031 | 031 |



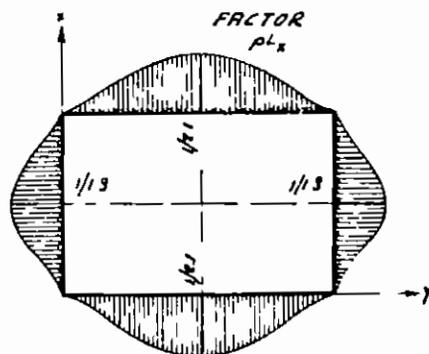
$$\underline{L_y / L_x = 15}$$

**A-6) SLAB FIXED ON ALL FOUR SIDES**

| $L_y / L_x$ |                         | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
|-------------|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $M_{xmn}$   | $\rho L_x^2$            | 194 | 171 | 155 | 145 | 137 | 132 | 128 | 125 | 123 | 121 | 120 |
| $M_{xm}$    | $-\rho L_x^2$           | 568 | 461 | 394 | 348 | 319 | 296 | 281 | 269 | 260 | 254 | 250 |
| $M_{ymn}$   | $\rho L_x^2$            | 194 | 184 | 179 | 176 | 175 | 175 | 175 | 175 | 175 | 175 | 175 |
| $M_{ymax}$  | $\rho L_x^2$            | 568 | 603 | 658 | 736 | 834 | 935 | 981 | 101 | 103 | 105 | 105 |
| $R_m$       | $\rho L$                | 22  | 21  | 20  | 20  | 19  | 19  | 19  | 19  | 19  | 19  | 19  |
| $R_{ym}$    | $\pm \rho L_x$          | 22  | 22  | 21  | 21  | 21  | 21  | 21  | 21  | 21  | 21  | 2   |
| $\delta_m$  | $\frac{\rho L^4}{EI^3}$ | 015 | 018 | 021 | 023 | 025 | 026 | 028 | 029 | 029 | 030 | 030 |



MOMENTS



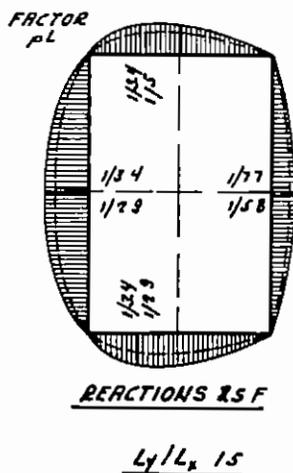
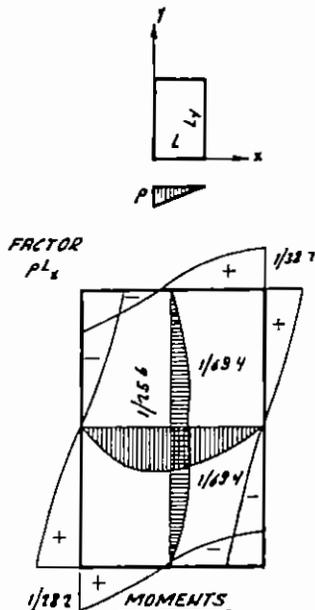
REACTIONS & F

$L_y / L_x = 15$

**B-SLABS SUPPORTED ON FOUR SIDES & SUBJECT TO TRIANGULAR LOADS**

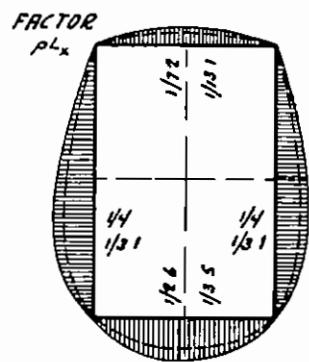
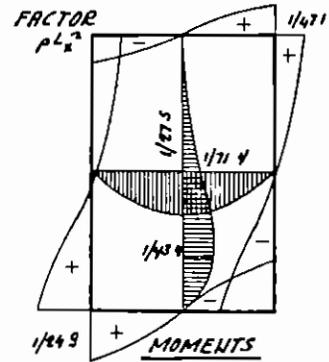
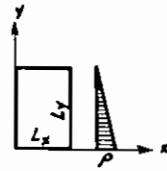
**B 1) SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES**

| POSITION            |                    | $L_y/L_x$                  | 10   | 11   | 12   | 13   | 14              | 15              | 20              |
|---------------------|--------------------|----------------------------|------|------|------|------|-----------------|-----------------|-----------------|
| y asly              | $M_x$ max          | $\rho L_x^2$               | 455  | 389  | 342  | 308  | 279             | 256             | 198             |
|                     | at                 | $\frac{x}{L}$              | 0.3  | 0.32 | 0.34 | 0.36 | 0.37            | 0.38            | 0.4             |
| y asly              | $M_x$ m            | $\rho L^2$                 | 54.3 | 44.6 | 38.2 | 33.5 | 30.1            | 27.5            | 20.8            |
| x=0.5L <sub>x</sub> | $M_y$ max          | $\rho L_x^2$               | 54.3 | 55.9 | 58.1 | 61.7 | 65.8            | 69.4            | 80.6            |
|                     | at                 | $\frac{y}{L_y}$            | 0.5  | 0.5  | 0.5  | 0.5  | $\frac{2}{6.6}$ | $\frac{2}{0.7}$ | $\frac{2}{0.8}$ |
| x=0.5L              | $M_y$ m            | $\rho L_x^2$               | 54.3 | 55.9 | 58.1 | 61.7 | 66.2            | 71.4            | 115             |
| x=0                 | $M_{xy}$ corner    | $+\rho L_x^2$              | 35.8 | 33.3 | 31.4 | 30.0 | 29.1            | 28.2            | 26.6            |
| x=L <sub>x</sub>    |                    |                            | 54.0 | 48.5 | 44.8 | 42.2 | 40.2            | 38.7            | 35.5            |
| x=0                 | Q m                | $\rho L$                   | 4.0  | 3.8  | 3.7  | 3.6  | 3.5             | 3.4             | 3.2             |
|                     | R <sub>x</sub> m   |                            | 3.2  | 3.1  | 3.0  | 3.0  | 3.0             | 2.9             | 2.9             |
| x=L <sub>x</sub>    | Q m                | $-\rho L_x$                | 11.1 | 10.0 | 9.2  | 8.6  | 8.1             | 7.7             | 6.6             |
|                     | R m                | $\rho L_x$                 | 7.0  | 6.6  | 6.3  | 6.1  | 5.9             | 5.8             | 5.7             |
| y=0                 | Q <sub>y</sub> max | $\rho L_x$                 | 5.2  | 5.1  | 5.1  | 5.0  | 5.0             | 5.0             | 4.9             |
| y=L <sub>y</sub>    | at                 | $\frac{x}{L_x}$            | 0.3  | 0.3  | 0.3  | 0.3  | 0.3             | 0.3             | 0.3             |
| y=0                 | Q <sub>y</sub> max | $\rho L_x$                 | 3.8  | 3.6  | 3.6  | 3.5  | 3.4             | 3.4             | 3.3             |
|                     | at                 | $\frac{x}{L_x}$            | 0.3  | 0.3  | 0.3  | 0.3  | 0.3             | 0.3             | 0.3             |
| x=0.5L <sub>x</sub> | δ m                | $\frac{\rho L_x^4}{E I^3}$ | 0.24 | 0.29 | 0.34 | 0.38 | 0.43            | 0.46            | 0.61            |
| y=0.5L <sub>y</sub> |                    |                            |      |      |      |      |                 |                 |                 |



**B 1) SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES**

| POSITION                     |                                | $L_y/L_x$              | 10   | 11   | 12   | 13   | 14   | 15   | 20   |
|------------------------------|--------------------------------|------------------------|------|------|------|------|------|------|------|
| $y = 0.5L_y$                 | $M_x \text{ max} \sim M_m$     | $pL^2$                 | 543  | 446  | 382  | 335  | 301  | 275  | 207  |
| $x = 0.5L_x$                 | $M_y \text{ max} = pL_x^2$     |                        | 455  | 442  | 435  | 429  | 427  | 431  | 431  |
|                              | at $\frac{y}{L_y}$             |                        | 0.30 | 0.28 | 0.26 | 0.24 | 0.22 | 0.20 | 0.16 |
| $x = 0.5L_x$                 | $M_y \text{ m}$                | $pL_x^2$               | 543  | 558  | 581  | 617  | 662  | 714  | 115  |
| $y = 0$                      | $M_y \text{ c v n r} + pL_x^2$ |                        | 358  | 324  | 298  | 277  | 262  | 249  | 213  |
| $y = L_y$                    |                                |                        | 540  | 507  | 488  | 476  | 471  | 471  | 524  |
| $x = 0$                      | $Q \text{ max}$                | $\pm pL_x$             | 52   | 49   | 46   | 43   | 42   | 40   | 35   |
| $x = L_x$                    | at $\frac{y}{L_y}$             |                        | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  |
| $x = 0$                      | $R \text{ max}$                | $pL_x$                 | 38   | 36   | 34   | 33   | 32   | 31   | 28   |
| $x = L_x$                    | at $\frac{y}{L_y}$             |                        | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.2  |
| $y = 0$                      | $Q_y \text{ max}$              | $pL_x$                 | 40   | 39   | 37   | 36   | 35   | 35   | 32   |
|                              | $R_y \text{ max}$              | $pL_x$                 | 32   | 30   | 29   | 27   | 26   | 26   | 23   |
| $y = L_y$                    | $Q_y \text{ max}$              | $pL_x$                 | 111  | 114  | 117  | 121  | 126  | 131  | 164  |
|                              | $R_y \text{ max}$              | $pL_x$                 | 70   | 69   | 69   | 70   | 71   | 72   | 85   |
| $x = 0.5L_x$<br>$y = 0.5L_y$ | $\delta \text{ m}$             | $\frac{pL_x^4}{E I^3}$ | 024  | 029  | 034  | 038  | 043  | 046  | 061  |

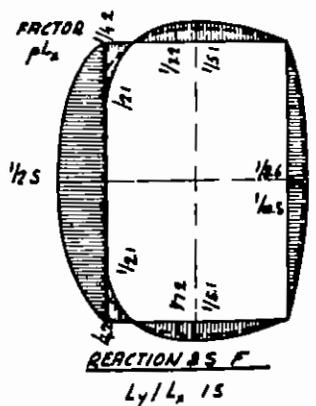
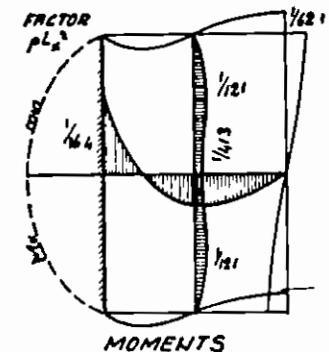
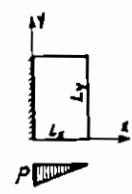


$L_y/L_x = 15$

**B 2) SLAB FIXED ON A LONGER SIDE AND SIMPLY**

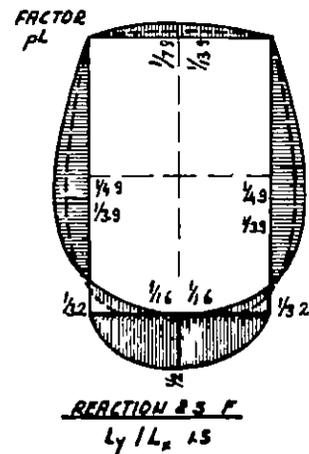
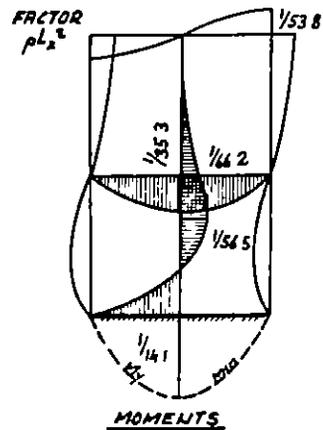
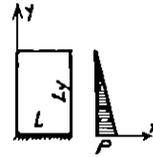
**SUPPORTED ON THE OTHER THREE SIDES**

| POSITION |                      | $L_y/L_x$               | 10   | 11   | 12  | 13   | 14   | 15   | 20   |
|----------|----------------------|-------------------------|------|------|-----|------|------|------|------|
| y asLy   | $M_x$ min            | $-pL_x^2$               | 207  | 131  | 102 | 74   | 68   | 64   | 53   |
| y asLy   | $M_x$ max            | $pL_x^2$                | 636  | 562  | 510 | 467  | 438  | 412  | 327  |
|          | at $\frac{x}{L_x} =$ |                         | 0.46 | 0.48 | 0.5 | 0.52 | 0.53 | 0.54 | 0.55 |
| y asLy   | $M_y$                | $pL_x^2$                | 645  | 565  | 510 | 469  | 441  | 418  | 325  |
| x asLx   | $M_y$ max            | $\pm pL_x^2$            | 321  | 290  | 269 | 252  | 238  | 228  | 178  |
|          | at $\frac{y}{L_y} =$ |                         | 0.5  | 0.4  | 0.3 | 0.2  | 0.1  | 0.2  | 0.1  |
|          |                      |                         | 0.6  | 0.8  | 0.7 | 0.8  | 0.8  | 0.8  | 0.8  |
| x asLx   | $M_y$ min            | $pL_x^2$                | 521  | 400  | 311 | 227  | 145  | 167  | 400  |
| x Lx     | $M_y$ c r m          | $\pm pL_x^2$            | 730  | 690  | 658 | 641  | 629  | 621  | 610  |
| x 0      | $Q_x$ max            | $\bar{R}_m$             | 2.6  | 2.5  | 2.5 | 2.5  | 2.5  | 2.5  | 2.5  |
| x Lx     | $Q_x$ min            | $-pL$                   | 132  | 123  | 116 | 111  | 108  | 105  | 100  |
|          | $R_x$ min            | $pL_x$                  | 89   | 86   | 85  | 85   | 85   | 86   | 91   |
| y 0      | $Q_y$ max            | $\pm pL_x$              | 71   | 72   | 72  | 72   | 72   | 72   | 73   |
|          | at $\frac{x}{L_x} =$ |                         | 0.4  | 0.4  | 0.4 | 0.4  | 0.4  | 0.4  | 0.4  |
| y 0      | $R_y$ max            | $pL_x$                  | 52   | 52   | 52  | 52   | 52   | 51   | 51   |
|          | at $\frac{x}{L_x} =$ |                         | 0.4  | 0.4  | 0.4 | 0.4  | 0.5  | 0.5  | 0.5  |
| x asLx   | $\delta_m$           | $= \frac{pL_x^4}{8L^3}$ | 015  | 017  | 019 | 021  | 022  | 023  | 026  |



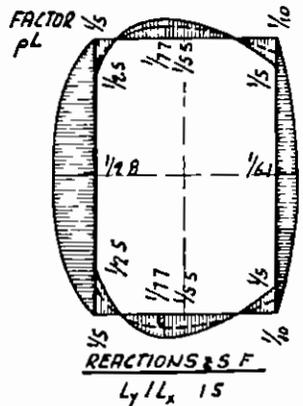
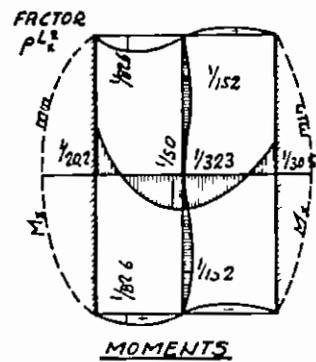
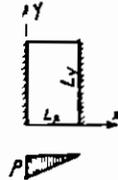
B 2) SLAB FIXED ON A SHORTER SIDE AND SIMPLY SUPPORTED ON THE OTHER THREE SIDE

| POSITION                     |                         | $L_y/L_x$                | 10   | 11   | 12   | 13  | 14   | 15   | 20   |
|------------------------------|-------------------------|--------------------------|------|------|------|-----|------|------|------|
| $x = 0.5L_x$                 | $M_y$ m                 | $pl^2$                   | 207  | 186  | 170  | 158 | 148  | 141  | 118  |
| $y = 0.5L_y$                 | $M_{max} \sim M_m$      | $pl_x^2$                 | 901  | 834  | 758  | 667 | 562  | 453  | 236  |
| $x = 0.5L_x$                 | $M_y$ max               | $pl^2$                   | 636  | 606  | 592  | 578 | 568  | 565  | 565  |
|                              | at $\frac{y}{L_y}$      |                          | 0.46 | 0.44 | 0.42 | 0.4 | 0.38 | 0.36 | 0.28 |
| $x = 0.5L_y$                 | $M_y$ m                 | $pl^2$                   | 645  | 621  | 617  | 621 | 637  | 662  | 926  |
| $y = L_y$                    | $M_y$ corner            | $\pm pl^2$               | 730  | 653  | 606  | 571 | 549  | 538  | 549  |
| $x = 0$<br>&<br>$x = L$      | $Q$ max                 | $\pm pl$                 | 71   | 65   | 60   | 56  | 52   | 49   | 41   |
|                              | $R_y$ ma                | $pl$                     | 52   | 48   | 45   | 42  | 40   | 39   | 33   |
|                              | at $\frac{y}{L_y}$      |                          | 0.4  | 0.4  | 0.4  | 0.4 | 0.4  | 0.4  | 0.3  |
| $y = 0$                      | $\bar{Q}_{ym}$ $R_{ym}$ | $pl_x$                   | 26   | 24   | 23   | 22  | 21   | 20   | 18   |
| $y = L_y$                    | $Q_{ym}$                | $pl$                     | 132  | 130  | 121  | 122 | 135  | 139  | 167  |
|                              | $R_{ym}$                | $pl_x$                   | 89   | 84   | 81   | 79  | 79   | 79   | 88   |
| $x = 0.5L_x$<br>$y = 0.5L_y$ | $\delta_m$              | $\frac{pl^4}{E\delta^3}$ | 015  | 020  | 024  | 028 | 032  | 037  | 054  |



**B 3) SLAB FIXED ON THE TWO LONGER SIDES AND**  
**SIMPLY SUPPORTED ON THE OTHER TWO SIDES**

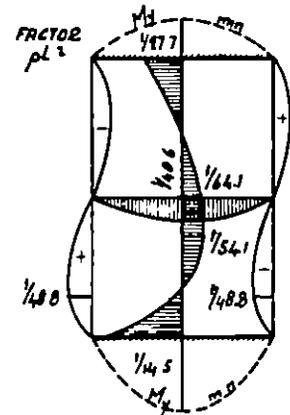
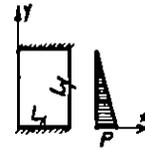
| POSITION                                       |                              | $L_y/L_x$                | 10           | 11           | 12           | 13           | 14           | 15           | 20           |
|------------------------------------------------|------------------------------|--------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $y \text{ o.s. } L_y$<br>$x \text{ o.s. } L_x$ | $M_x \text{ max}$            | $pL^2$                   | 232          | 222          | 215          | 209          | 206          | 202          | 200          |
| $y \text{ o.s. } L_y$<br>$x \text{ o.s. } L_x$ | $M_x \text{ min}$            | $pL_x^2$                 | 376          | 348          | 331          | 319          | 312          | 305          | 300          |
| $y \text{ o.s. } L_y$                          | $M \text{ max}$              | $pL_x^2$                 | 658          | 606          | 568          | 541          | 513          | 500          | 487          |
|                                                | at $\frac{x}{L}$             |                          | 0.42         | 0.43         | 0.44         | 0.45         | 0.45         | 0.45         | 0.46         |
| $y \text{ o.s. } L_y$                          | $M \text{ min}$              | $pL_x^2$                 | 704          | 683          | 688          | 656          | 632          | 615          | 611          |
| $x \text{ o.s. } L_x$                          | $M_y \text{ max}$            | $pL^2$                   | 124          | 133          | 143          | 147          | 149          | 152          | 154          |
|                                                | at $\frac{y}{L_y}$           |                          | 0.20<br>0.70 | 0.26<br>0.74 | 0.20<br>0.80 | 0.20<br>0.80 | 0.17<br>0.83 | 0.15<br>0.85 | 0.12<br>0.88 |
| $x \text{ o.s. } L_x$                          | $M_y \text{ min}$            | $pL^2$                   | 127          | 149          | 179          | 213          | 256          | 323          | 411          |
| $y \text{ o.s. } L_y$<br>$x \text{ o.s. } L_x$ | $M_y$                        | $pL^2$                   | 855          | 833          | 826          | 826          | 826          | 826          | 833          |
|                                                | at $\frac{x}{L_x}$           |                          | 0.16         | 0.16         | 0.16         | 0.16         | 0.16         | 0.16         | 0.16         |
| $x \text{ o.s. } L_x$<br>$L$                   | $Q_{xm} \hat{E}_{xm}$        | $pL$                     | 28           | 28           | 28           | 28           | 28           | 28           | 28           |
|                                                | $Q_{xm} \hat{E}_{xm} = pL_x$ |                          | 65           | 69           | 61           | 61           | 61           | 61           | 63           |
| $y \text{ o.s. } L_y$<br>$L$                   | $Q_y \text{ max}$            | $pL_x$                   | 75           | 76           | 76           | 77           | 77           | 77           | 77           |
| $y \text{ o.s. } L_y$                          | $R_y \text{ max}$            | $pL_x$                   | 55           | 54           | 54           | 55           | 55           | 55           | 55           |
|                                                | at $\frac{x}{L}$             |                          | 0.4          | 0.4          | 0.4          | 0.4          | 0.4          | 0.4          | 0.4          |
| $x \text{ o.s. } L_x$<br>$y \text{ o.s. } L_y$ | $\delta_m$                   | $\frac{pL^4}{E\delta^3}$ | 0.12         | 0.13         | 0.13         | 0.14         | 0.15         | 0.15         | 0.16         |



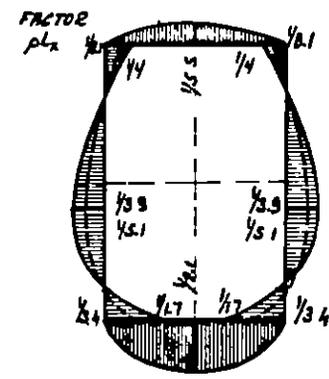
**B 3) SLAB FIXED ON THE TWO SHORTER SIDES AND**

**SIMPLY SUPPORTED ON THE OTHER TWO SIDES**

| POSITION                  |                               | $L_y/L_x$              | 10   | 11   | 12   | 13   | 14   | 15   | 20   |
|---------------------------|-------------------------------|------------------------|------|------|------|------|------|------|------|
| $x = 0.5L_x$              |                               |                        |      |      |      |      |      |      |      |
| $y = 0$                   | $M_y$ m                       | $pl_x^2$               | 22.2 | 20.4 | 18.3 | 16.7 | 15.5 | 14.5 | 11.9 |
| $y = L_y$                 | $M_y$ mm                      | $-pl_x^2$              | 37.6 | 33.7 | 31.2 | 29.5 | 28.3 | 27.7 | 23.5 |
| $y = 0.5L_y$              | $M_x$ max                     | $pl_x^2$               | 124  | 91.7 | 70.9 | 56.8 | 47.6 | 40.6 | 25.0 |
|                           | at                            | $\frac{x}{L_x} =$      | 0.8  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.8  |
| $x = 0.5L_x$              | $M_y$ max                     | $pl_x^2$               | 65.8 | 61.3 | 57.5 | 53.5 | 50.3 | 47.1 | 35.2 |
|                           | at                            | $\frac{y}{L_y}$        | 0.42 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 | 0.30 |
| $x = 0.5L_x$              | $M_y$ m                       | $pl_x^2$               | 70.4 | 65.8 | 62.3 | 58.5 | 55.5 | 52.1 | 35.5 |
| $x = 0$<br>$x = L$        | $M_{xy}$ max = $\pm pl_x^2$   |                        | 85.5 | 71.9 | 62.9 | 56.8 | 51.5 | 48.8 | 36.3 |
|                           | at                            | $\frac{y}{L_y}$        | 0.16 | 0.15 | 0.15 | 0.15 | 0.15 | 0.13 | 0.10 |
| $x = 0$<br>$x = L_x$<br>- | $Q$ max                       | $\pm pl_x$             | 7.6  | 6.8  | 6.2  | 5.8  | 5.4  | 5.1  | 4.1  |
|                           | $R_s$ max                     | $pl$                   | 5.5  | 5.0  | 4.6  | 4.3  | 4.1  | 3.9  | 3.3  |
|                           | at                            | $\frac{y}{L_y}$        | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.3  |
| $y = 0$                   | $\bar{Q}_{ym}$ $\bar{R}_{ym}$ | $pl_x$                 | 2.8  | 2.6  | 2.4  | 2.3  | 2.2  | 2.1  | 1.8  |
| $y = L_y$                 | $\bar{Q}_{ym}$ $\bar{R}_{ym}$ | $-pl_x$                | 6.5  | 6.0  | 5.7  | 5.5  | 5.5  | 5.5  | 5.9  |
| $x = 0.5L_x$              | $\delta_m$                    | $\frac{pl_x^2}{E t^3}$ | 0.12 | 0.15 | 0.19 | 0.23 | 0.28 | 0.32 | 0.51 |



**MOMENTS**



**REACTIONS & S.F.**

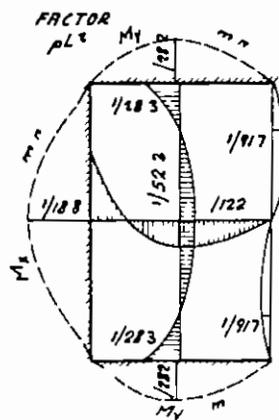
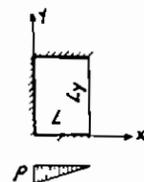
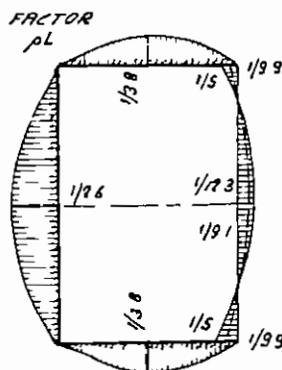
$L_y/L_x = 1.5$





**B 5) SLAB SIMPLY SUPPORTED ON A LONGER SIDE****AND FIXED ON THE OTHER THREE SIDES**

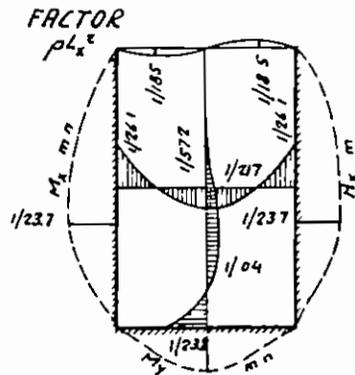
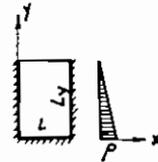
| POSITION     |                    | $L_y/L$                  | 10   | 11   | 12   | 13   | 14   | 15   | 20              |
|--------------|--------------------|--------------------------|------|------|------|------|------|------|-----------------|
| $y \ 0.5L_y$ | $M_{mn}$           | $\rho L_x^2$             | 290  | 255  | 229  | 211  | 196  | 188  | 161             |
| $y \ 0$      | $M_{ymn}$          | $\rho L^2$               | 345  | 321  | 303  | 292  | 285  | 282  | 273             |
| $y \ L_y$    | at $\frac{x}{L}$   |                          | 0.45 | 0.47 | 0.48 | 0.49 | 0.49 | 0.49 | 0.49            |
| $y \ 0.5L_y$ | $M_x \max$         | $\rho L^2$               | 104  | 869  | 752  | 649  | 578  | 523  | 394             |
|              | at $\frac{x}{L}$   |                          | 0.38 | 0.42 | 0.45 | 0.47 | 0.50 | 0.50 | 0.52            |
| $y \ 0.5L_y$ | $M_x m$            | $\rho L^2$               | 118  | 917  | 763  | 653  | 578  | 523  | 397             |
| $x \ 0.5L$   | $M_y \ ma$         | $\rho L_x^2$             | 952  | 952  | 980  | 103  | 112  | 122  | 182             |
|              | at $\frac{y}{L_y}$ |                          | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | $\frac{2}{0.7}$ |
| $0.5L$       | $M_y m$            | $\rho L^2$               | 952  | 952  | 980  | 103  | 112  | 122  | 233             |
| $x \ L$      | $M_y \ ma$         | $+\rho L^2$              | 149  | 128  | 114  | 103  | 971  | 917  | 855             |
|              | at $\frac{y}{L_y}$ |                          | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.17 | 0.14            |
|              |                    |                          | 0.82 | 0.82 | 0.82 | 0.82 | 0.82 | 0.83 | 0.86            |
| $x \ 0$      | $Q_{xm}$           | $\rho L_x$               | 31   | 29   | 28   | 27   | 27   | 26   | 25              |
|              | $R_{xm}$           |                          |      |      |      |      |      |      |                 |
| $L$          | $Q_x \ ma$         | $\rho L$                 | 213  | 181  | 158  | 142  | 131  | 123  | 104             |
|              | $R \ ma$           | $\rho L$                 | 127  | 112  | 103  | 97   | 93   | 91   | 89              |
| $y \ 0$      | $^+ Q_{yma}$       | $\rho L$                 | 41   | 41   | 39   | 38   | 38   | 38   | 37              |
| $y \ L_y$    | $R_{yma}$          |                          |      |      |      |      |      |      |                 |
|              | at $\frac{x}{L}$   |                          | 0.3  | 0.4  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5             |
| $x \ 0.5L$   | $\delta_m$         | $\frac{\rho L^4}{E t^3}$ | 009  | 011  | 013  | 015  | 017  | 019  | 024             |
| $y \ 0.5L_y$ |                    |                          |      |      |      |      |      |      |                 |

MOMENTSREACTIONS & S F  
 $L_y/L_x \ 1.5$

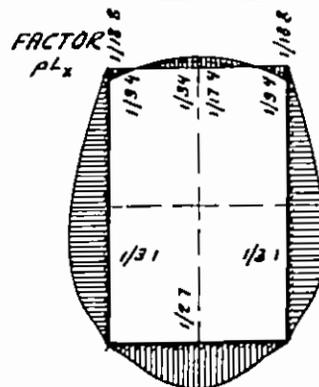
B 5) SLAB SIMPLY SUPPORTED ON A SHORTER

SIDE AND FIXED ON THE OTHER THREE SIDES

| POSITION    |                    | $L_y/L_x$              | 10   | 11   | 12   | 13   | 14   | 15   | 20   |
|-------------|--------------------|------------------------|------|------|------|------|------|------|------|
| x 0         | $M_{mn}$           | $pl_x^2$               | 345  | 311  | 287  | 267  | 253  | 237  | 202  |
| x $L_x$     | at                 | $\frac{y}{L_y}$        | 0.45 | 0.44 | 0.43 | 0.42 | 0.40 | 0.38 | 0.31 |
| x 0.5 $L_x$ | $M_{ymn}$          | $pl_x^2$               | 290  | 269  | 258  | 249  | 241  | 238  | 219  |
| y 0.5 $L_y$ | $M_{xmn} \sim M_m$ | $pl_x^2$               | 352  | 313  | 287  | 269  | 253  | 237  | 202  |
| 0.5 $L_x$   | $M_{y,max}$        | $pl_x^2$               | 104  | 104  | 104  | 104  | 104  | 104  | 104  |
|             | at                 | $\frac{y}{L_y}$        | 0.38 | 0.35 | 0.34 | 0.33 | 0.32 | 0.27 | 0.22 |
| x 0.5 $L_x$ | $M_{ym}$           | $pl_x^2$               | 118  | 128  | 143  | 161  | 185  | 217  | 288  |
| y $L_y$     | $M_{xy,max}$       | $pl_x^2$               | 149  | 154  | 159  | 167  | 175  | 185  | 244  |
|             | at                 | $\frac{x}{L_x}$        | 0.18 | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  |
|             |                    |                        | 0.82 | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  |
| x 0         | +R ma              | $pl$                   | 41   | 38   | 36   | 34   | 32   | 31   | 28   |
| x $L_x$     | R ma               |                        |      |      |      |      |      |      |      |
|             | at $\frac{y}{L_y}$ |                        | 0.3  |      |      |      |      |      |      |
| y 0         | $Q_{ym}$           | $pl_x$                 | 31   | 30   | 29   | 28   | 28   | 27   | 26   |
|             | $R_{ym}$           |                        |      |      |      |      |      |      |      |
| y $L_y$     | $Q_{ym}$           | $pl_x$                 | 213  | 234  | 258  | 285  | 313  | 340  | 474  |
|             | $R_{ym}$           | $= pl_x$               | 127  | 133  | 141  | 151  | 163  | 174  | 238  |
| x 0.5 $L_x$ | $\delta_m$         | $\frac{pl_y^4}{E t^3}$ | 009  | 010  | 011  | 012  | 013  | 014  | 015  |



MOMENTS

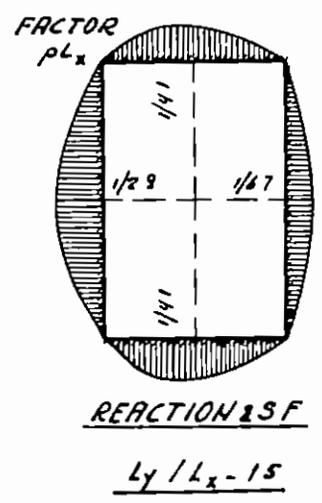
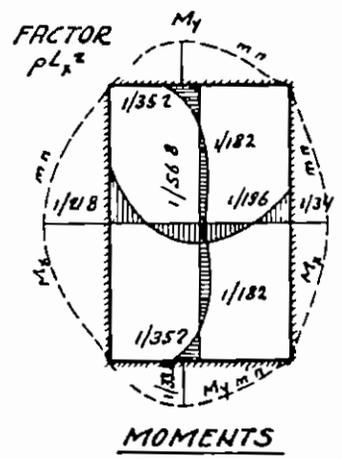
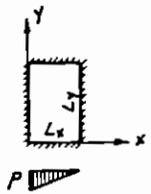


REACTIONS & S.F

$L_y/L_x = 15$

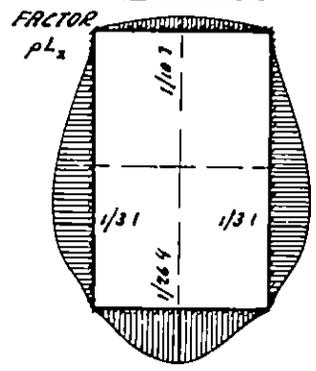
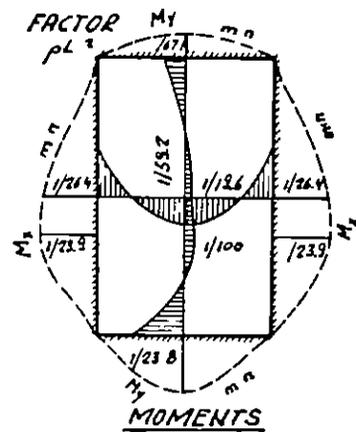
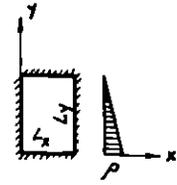
**B 6) SLAB FIXED ON ALL FOUR SIDES**

| POSITION   |                                                                      | $L_y/L_x$                 | 10   | 11   | 12   | 13   | 14   | 15   | 20   |
|------------|----------------------------------------------------------------------|---------------------------|------|------|------|------|------|------|------|
| $x=0$      | $M_{mn}$                                                             | $\rho L^2$                | 300  | 267  | 247  | 233  | 222  | 218  | 202  |
| $y=0.5L_y$ |                                                                      |                           |      |      |      |      |      |      |      |
| $x=L$      | $M_{xmn}$                                                            | $\rho L_x^2$              | 562  | 472  | 417  | 380  | 355  | 340  | 304  |
| $y=0$      | $M_{ymn}$                                                            | $\rho L^2$                | 369  | 360  | 351  | 346  | 344  | 338  | 338  |
| $y=L_y$    | at $x/L$                                                             | 0.4                       |      |      |      |      |      |      |      |
| $y=0.5L_y$ | $M_{mx}$                                                             | $\rho L^2$                | 98.0 | 82.6 | 73.0 | 65.8 | 60.6 | 56.8 | 48.5 |
|            | at $\frac{x}{L_x}$                                                   |                           | 0.34 | 0.38 | 0.40 | 0.41 | 0.42 | 0.44 | 0.45 |
| $y=0.5L_y$ | $M_{xm}$                                                             | $\rho L_x^2$              | 114  | 91.7 | 78.7 | 69.9 | 63.7 | 59.2 | 50.0 |
| $x=0.5L_x$ | $M_{y \max}$                                                         | $\rho L_x^2$              | 114  | 120  | 132  | 147  | 167  | 182  | 213  |
|            | at $\frac{y}{L_y}$                                                   |                           | 0.5  | 0.5  | 0.5  | 0.5  | 0.40 | 0.34 | 0.28 |
|            |                                                                      |                           |      |      |      |      | 0.40 | 0.66 | 0.77 |
| $x=0.5L_x$ | $M_{ym}$                                                             | $\rho L^2$                | 114  | 120  | 132  | 147  | 170  | 196  | 226  |
| $y=0.5L_y$ | $\left. \begin{matrix} a_m \\ R_m \end{matrix} \right\}$             | $\rho L$                  | 31   | 29   | 29   | 29   | 29   | 29   | 29   |
|            | at $x=0$                                                             | 0                         |      |      |      |      |      |      |      |
| $y=0.5L_y$ | $\left. \begin{matrix} a_m \\ R_{xm} \end{matrix} \right\}$          | $\rho L_x$                | 83   | 74   | 67   | 67   | 67   | 67   | 67   |
|            | at $x=-L_x$                                                          |                           |      |      |      |      |      |      |      |
| $y=0$      | $\left. \begin{matrix} \pm Q_{yma} \\ R_{ymx} \end{matrix} \right\}$ | $\rho L_x$                | 41   | 41   | 41   | 41   | 41   | 41   | 41   |
| $y=L_y$    | at $\frac{x}{L_x}$                                                   |                           | 0.3  | 0.3  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  |
| $y=0.5L_y$ | $\delta_m$                                                           | $\frac{\rho L_y^4}{Et^3}$ | 008  | 009  | 010  | 012  | 012  | 013  | 015  |



B 6) SLAB FIXED ON ALL FOUR SIDES

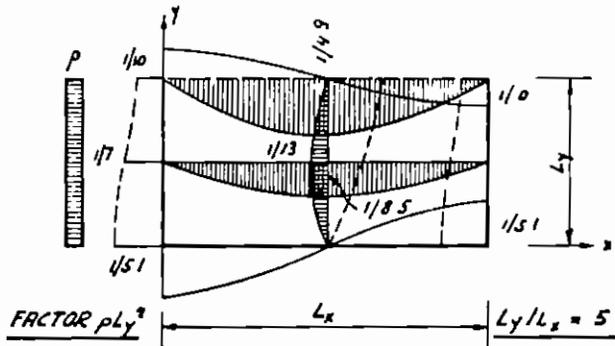
| POSITION      |                                                                | $L_y/L_x$             | 10   | 11   | 12   | 13   | 14   | 15   | 20   |
|---------------|----------------------------------------------------------------|-----------------------|------|------|------|------|------|------|------|
| x 0           | M m                                                            | $pL^2$                | 369  | 331  | 288  | 275  | 256  | 239  | 202  |
| x L           | at                                                             | $\frac{y}{L_y}$       | 0.42 | 0.40 | 0.40 | 0.40 | 0.38 | 0.37 | 0.33 |
| y 0           | $M_y m n$                                                      | $pL^2$                | 300  | 275  | 261  | 250  | 241  | 238  | 219  |
| y $L_y$       | $M_y m n$                                                      | $pL^2$                | 562  | 555  | 571  | 535  | 429  | 671  | 926  |
| y 0 & y $L_y$ | $M m a \sim M n$                                               | $pL^2$                | 114  | 917  | 787  | 699  | 637  | 592  | 500  |
| x 0 & x L     | $M_y m a$                                                      | $pL^2$                | 980  | 980  | 980  | 990  | 990  | 100  | 100  |
| x 0 & x L     | at                                                             | $\frac{y}{L_y}$       | 0.37 | 0.35 | 0.33 | 0.33 | 0.33 | 0.32 | 0.24 |
| x 0 & x L     | $M_y m$                                                        | $pL^2$                | 114  | 121  | 132  | 147  | 170  | 196  | 526  |
| x 0 & x L     | $\left. \begin{matrix} R_x m \\ R_x n s \end{matrix} \right\}$ | $pL$                  | 41   | 39   | 36   | 34   | 33   | 31   | 28   |
| x L           | at $y/L_y$                                                     | 0.3                   |      |      |      |      |      |      |      |
| y 0           | $\Phi_y R_y$                                                   | $pL_x$                | 31   | 29   | 28   | 28   | 27   | 26   | 25   |
| y $L_y$       | $\Phi_y R_y$                                                   | $pL$                  | 83   | 84   | 88   | 94   | 100  | 107  | 143  |
| 0 & x L       | $\delta m$                                                     | $\frac{pL_x^4}{EI^3}$ | 008  | 009  | 010  | 012  | 012  | 013  | 015  |



$\frac{L_y}{L_x} = 1.5$

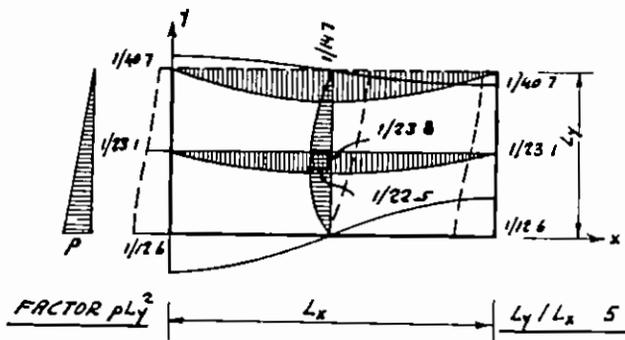
C1) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD

| POSITION           |             | $L_y/L_x$             | 0.25    | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |     |
|--------------------|-------------|-----------------------|---------|------|------|------|------|------|------|------|------|------|------|------|------|------|-----|
| $x=0SL_x$          | $M_{ym}$    | $ply^2$               | 84      | 89   | 105  | 130  | 165  | 213  | 276  | 357  | 469  | 594  | 759  | 965  | 122  | 154  |     |
| $x=0SL_x$          | $M_{yma}$   | $ply^2$               | 84      | 89   | 105  | 130  | 164  | 209  | 266  | 335  | 417  | 504  | 600  | 704  | 817  | 938  |     |
|                    | at          | $\frac{y}{L_y}$       | 0.5     | 0.5  | 0.5  | 0.48 | 0.45 | 0.42 | 0.4  | 0.38 | 0.38 | 0.28 | 0.26 | 0.25 | 0.24 | 0.22 |     |
| $y=0SL_y$          | $M_m$       | $ply^3$               | 79      | 79   | 80   | 85   | 91   | 100  | 110  | 123  | 137  | 152  | 170  | 188  | 209  | 230  |     |
| $yLy$              | $M_{ma}$    | $M_m$                 | $ply^2$ | 40   | 41   | 44   | 49   | 55   | 63   | 73   | 85   | 98   | 113  | 129  | 147  | 167  | 189 |
| $y=0$              | $M_{ycorr}$ | $+ply^2$              | 22      | 27   | 38   | 51   | 66   | 83   | 103  | 127  | 153  | 183  | 216  | 252  | 291  | 333  |     |
| $y=0SL_y$          | $M_{ym}$    | $+ply^2$              | 26      | 32   | 43   | 70   | 98   | 134  | 181  | 241  | 318  | 415  | 538  | 692  | 884  | 112  |     |
| $yLy$              | $M_{ycorr}$ | $+ply^2$              | 29      | 38   | 63   | 101  | 158  | 245  | 376  | 572  | 865  | 129  | 193  | 286  | 422  | 618  |     |
| $x=0SL_x$<br>$yLy$ | $\bar{J}_m$ | $= \frac{pL^4}{2t^3}$ | 0.22    | 0.30 | 0.48 | 0.67 | 0.85 | 1.00 | 1.13 | 1.20 | 1.26 | 1.32 | 1.37 | 1.48 | 1.53 | 1.64 |     |



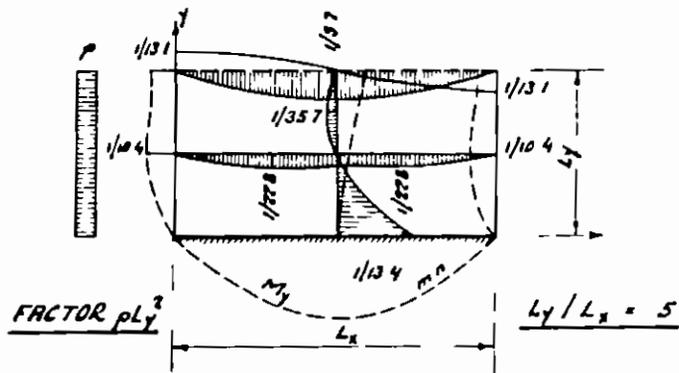
**C 2) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO TRIANGULAR LOAD**

| POSITION            |                        | $L_y/L$                 | 0.25 | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9  | 1.0  | 1.1  | 1.2  | 1.3   | 1.4   | 1.5   |
|---------------------|------------------------|-------------------------|------|-------|-------|-------|-------|-------|-------|------|------|------|------|-------|-------|-------|
| x 0.5L              | $M_{y \text{ min}}$    | $pl_y^2$                | 167  | 175   | 202   | 240   | 295   | 366   | 458   | 574  | 719  | 897  | 112  | 138   | 171   | 210   |
| x 0.5L <sub>x</sub> | $M_{y \text{ max}}$    | $pl_y^2$                | 162  | 169   | 191   | 225   | 267   | 316   | 376   | 438  | 513  | 605  | 702  | 79.8  | 89.9  | 101   |
|                     | at                     | $\frac{y}{L_y}$         | 0.4  | 0.4   | 0.4   | 0.4   | 0.35  | 0.32  | 0.3   | 0.28 | 0.25 | 0.23 | 0.22 | 0.2   | 0.19  | 0.18  |
| x 0.5L              | $M_{y \text{ min}}$    | $pl_y^2$                |      |       |       |       |       |       |       |      |      | 4566 | 1642 | 1175  | 1016  | 960   |
|                     | at                     | $\frac{y}{L_y}$         |      |       |       |       |       |       |       |      |      | 0.9  | 0.9  | 0.9   | 0.9   | 0.9   |
| y 0.5L <sub>y</sub> | $M_{x \text{ min}}$    | $pl_y^2$                | 235  | 232   | 232   | 23.8  | 25.0  | 26.8  | 28.9  | 31.4 | 34.3 | 37.4 | 40.9 | 44.7  | 48.6  | 52.9  |
| y L <sub>y</sub>    | $M_{x \text{ min}}$    | $pl_y^2$                | 121  | 123   | 132   | 147   | 16.8  | 19.6  | 23.0  | 27.3 | 32.4 | 38.5 | 45.6 | 54.0  | 63.6  | 74.7  |
| x 0.5L              | $M_{x \text{ max}}$    | $pl_y^2$                | 121  | 123   | 132   | 147   | 16.8  | 19.6  | 23.0  | 27.3 | 31.7 | 35.9 | 40.1 | 44.5  | 48.6  | 52.9  |
|                     | at                     | $\frac{y}{L_y}$         | 1.0  | 1.0   | 1.0   | 1.0   | 1.0   | 1.0   | 1.0   | 1.0  | 0.8  | 0.65 | 0.6  | 0.55  | 0.5   | 0.5   |
| y 0                 | $M_{y \text{ center}}$ | $pl_y^2$                | 6.4  | 7.5   | 10.0  | 12.6  | 15.5  | 18.7  | 22.2  | 26.0 | 30.1 | 34.4 | 39.1 | 44.0  | 49.0  | 54.5  |
| y 0.5L <sub>y</sub> | $M_{x \text{ y m}}$    | $pl_y^2$                | ±7.9 | ±10.1 | ±15.6 | ±23.1 | ±33.8 | ±49.4 | ±72.7 | ±109 | ±171 | ±285 | ±544 | ±1358 | ±2448 | ±4274 |
| y L <sub>y</sub>    | $M_{y \text{ n}}$      | $pl_y^2$                | 9.4  | 12.8  | 22.7  | ±40.7 | ±77.3 | 168   | 359   | 678  | 127  | +394 | +360 | 7353  | -374  | -402  |
| x 0.5L <sub>x</sub> | $\bar{\Delta} m$       | $\frac{pl_y^2}{2L_x^3}$ | 0.07 | 0.10  | 0.16  | 0.21  | 0.27  | 0.31  | 0.34  | 0.37 | 0.38 | 0.39 | 0.39 | 0.39  | 0.38  | 0.38  |
| y L <sub>y</sub>    |                        |                         |      |       |       |       |       |       |       |      |      |      |      |       |       |       |



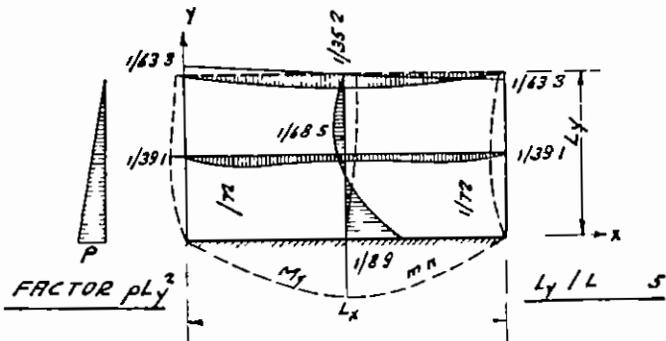
C 3) SLAB SIMPLY SUPPORTED ON TWO SIDES FIXED ON  
ONE SIDE AND SUBJECT TO UNIFORM LOAD

| POSITION              |                          | $L_y/L_x$             | 0.25         | 0.3          | 0.4        | 0.5        | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
|-----------------------|--------------------------|-----------------------|--------------|--------------|------------|------------|------|------|------|------|------|------|------|------|------|------|
| x o.s.l. <sub>x</sub> | $M_{y \min}$             | $pl_y^2$              | 23           | 24           | 29         | 34         | 41   | 49   | 60   | 71   | 85   | 100  | 118  | 137  | 158  | 181  |
|                       | $M_{y \min}$             | $pl_y^2$              | 177          | -172         | 436        | 00         | 820  | 570  | 522  | 540  | 592  | 675  | 787  | 929  | 110  | 132  |
|                       | $M_{y \max}$             | $pl_y^2$              | 170          | 90           | 468        | 357        | 333  | 354  | 397  | 454  | 545  | 664  | 797  | 937  | 110  | 127  |
|                       | at                       | $\frac{y}{L_y}$       | 0.88         | 0.85         | 0.79       | 0.75       | 0.71 | 0.68 | 0.65 | 0.62 | 0.6  | 0.57 | 0.53 | 0.5  | 0.46 | 0.43 |
| y Ly                  | $M_{x \min}$             | $pl_y^2$              | 26.4         | 18.0         | 11.8       | 9.7        | 9.0  | 9.1  | 9.6  | 10.4 | 11.4 | 12.6 | 14.0 | 15.7 | 17.6 | 19.6 |
|                       | $M_{x \max}$             | $= pl_y$              | 14.3         | 13.3         | 11.4       | 9.7        | 9.0  | 9.1  | 9.6  | 10.4 | 11.4 | 12.6 | 14.0 | 15.7 | 17.6 | 19.6 |
|                       | at                       | $\frac{x}{L_x}$       | 0.15<br>0.85 | 0.7<br>0.8   | 0.3<br>0.7 | 0.5        | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  |
| y o.s.l. <sub>y</sub> | $M_{x \min}$             | $pl_y^2$              | 732          | 523          | 317        | 240        | 206  | 189  | 186  | 189  | 197  | 207  | 220  | 235  | 262  | 272  |
|                       | $M_{x \max}$             | $= pl_y^2$            | 275          | 273          | 256        | 228        | 206  | 189  | 186  | 189  | 197  | 207  | 220  | 235  | 262  | 272  |
|                       | at                       | $\frac{x}{L_x}$       | 0.5<br>0.85  | 0.15<br>0.85 | 0.2<br>0.8 | 0.3<br>0.7 | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  |
| y Ly                  | $M_{x y \text{ corner}}$ | $pl_y^2$              | 87           | 90           | 105        | 131        | 171  | 232  | 322  | 454  | 645  | 923  | 133  | 191  | 274  | 394  |
| x o.s.l.<br>y Ly      | $\delta_m$               | $\frac{pL_x^4}{Et^3}$ | 005          | 009          | 020        | 035        | 051  | 070  | 087  | 098  | 109  | 120  | 131  | 142  | 153  | 164  |



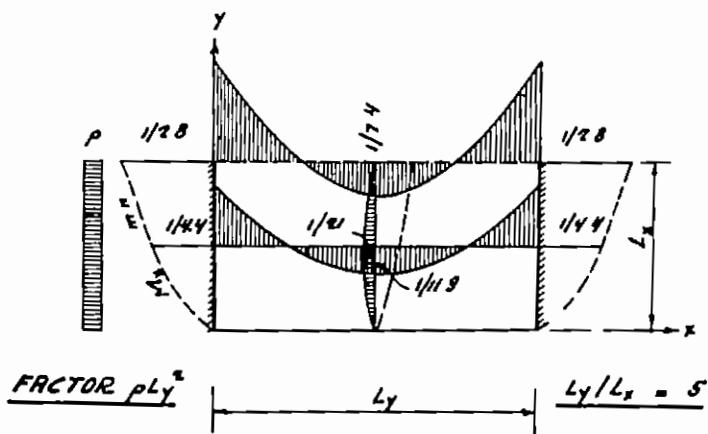
C-4) SLAB SIMPLY SUPPORTED ON TWO SIDES FIXED ON ONE  
SIDE AND SUBJECT TO TRIANGULAR LOAD

| POSITION                  | $L_y/L_x$                        | 0.25                          | 0.3                           | 0.4                           | 0.5                         | 0.6                         | 0.7  | 0.8  | 0.9  | 1.0   | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
|---------------------------|----------------------------------|-------------------------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|------|------|------|-------|------|------|------|------|------|
| $x = 0.5L_x$              | $M_{y\ min} = pLy^2$             | 66                            | 69                            | 78                            | 89                          | 103                         | 117  | 135  | 154  | 175   | 198  | 222  | 248  | 275  | 305  |
|                           | $M_{y\ m} = pLy^2$               | 114                           | 299                           | 175                           | 878                         | 714                         | 683  | 709  | 773  | 867   | 992  | 115  | 139  | 158  | 186  |
|                           | $M_{y\ max} = pLy^2$             | 220                           | 135                           | 84.0                          | 68.5                        | 64.2                        | 65.1 | 68.6 | 76.5 | 85.5  | 95.7 | 107  | 119  | 132  | 145  |
|                           | at $\frac{y}{L_y}$               | 0.75                          | 0.75                          | 0.63                          | 0.62                        | 0.58                        | 0.53 | 0.49 | 0.46 | 0.45  | 0.43 | 0.4  | 0.38 | 0.36 | 0.35 |
| $y = L_y$                 | $M_{x\ m} = pLy^2$               | 947                           | 46.0                          | 42.8                          | 35.2                        | 32.7                        | 32.9 | 34.7 | 37.9 | 42.6  | 47.6 | 54.2 | 62.0 | 71.1 | 81.9 |
|                           | $M_{x\ ma} = pLy^2$              | 495                           | 484                           | 41.2                          | 35.2                        | 32.7                        | 32.9 | 34.7 | 37.9 | 42.6  | 47.6 | 54.2 | 62.0 | 71.1 | 81.9 |
|                           | at $\frac{x}{L}$                 | 0.15<br>$\frac{2}{8}$<br>0.85 | 0.2<br>$\frac{2}{8}$<br>0.8   | 0.3<br>$\frac{2}{8}$<br>0.7   | 0.5                         | 0.5                         | 0.5  | 0.5  | 0.5  | 0.5   | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  |
| $y = 0.5L_y$              | $M_{x\ m} = pLy^2$               | 252                           | 184                           | 110                           | 80.5                        | 66.0                        | 592  | 56.0 | 54.9 | 55.0  | 56.3 | 582  | 607  | 636  | 67.0 |
|                           | $M_{x\ max} = pLy^2$             | 650                           | 75.0                          | 76.8                          | 72.0                        | 64.8                        | 592  | 56.0 | 54.9 | 55.0  | 56.3 | 58.1 | 60.6 | 63.6 | 66.9 |
|                           | at $\frac{x}{L_x}$               | 0.1<br>$\frac{2}{8}$<br>0.3   | 0.15<br>$\frac{2}{8}$<br>0.85 | 0.15<br>$\frac{2}{8}$<br>0.75 | 0.2<br>$\frac{2}{8}$<br>0.8 | 0.3<br>$\frac{2}{8}$<br>0.7 | 0.5  | 0.5  | 0.5  | 0.5   | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  |
| $y = L_y$                 | $M_{y\ corner} + pLy^2$          | 370                           | 391                           | 473                           | 633                         | 901                         | 143  | 258  | 618  | 15152 | 1122 | 678  | 560  | 524  | 521  |
| $x = 0.5L_x$<br>$y = L_y$ | $\delta_m = \frac{pL_x^4}{Et^3}$ | 001                           | 003                           | 006                           | 010                         | 015                         | 019  | 024  | 026  | 031   | 033  | 034  | 035  | 035  | 035  |



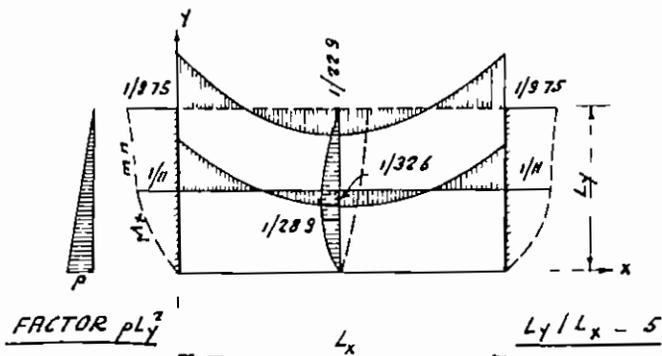
C 5) SLAB FIXED ON TWO SIDES SIMPLY SUPPORTED ON ONE  
SIDE AND SUBJECT TO UNIFORM LOAD

| POSITION              | $L_y/L_x$                            | 0.25 | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
|-----------------------|--------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $y=L_y$               | $M_{xmn} = pLy^2$                    | 10   | 13   | 19   | 28   | 38   | 52   | 68   | 87   | 108  | 132  | 157  | 186  | 216  | 249  |
|                       | $M_{xm} = pLy^2$                     | 44   | 46   | 58   | 74   | 96   | 122  | 154  | 191  | 234  | 282  | 335  | 394  | 458  | 526  |
| $y \text{ as } Ly$    | $M_{xmn} = -pLy^2$                   | 20   | 25   | 34   | 44   | 56   | 70   | 86   | 103  | 122  | 144  | 168  | 194  | 223  | 254  |
|                       | $M_{xm} = pLy^2$                     | 81   | 84   | 99   | 119  | 144  | 173  | 205  | 240  | 282  | 328  | 375  | 420  | 488  | 554  |
| $x \text{ as } L_x$   | $M_{ym} = pLy^2$                     | 96   | 114  | 153  | 216  | 305  | 432  | 611  | 850  | 122  | 171  | 240  | 331  | 462  | 652  |
|                       | $M_{ymax} = pLy^2$                   | 96   | 119  | 152  | 210  | 283  | 366  | 482  | 612  | 758  | 909  | 104  | 118  | 133  | 150  |
|                       | at $\frac{y}{L_y} =$                 | 0.5  | 0.48 | 0.47 | 0.43 | 0.38 | 0.33 | 0.28 | 0.25 | 0.22 | 0.19 | 0.16 | 0.13 | 0.1  | 0.1  |
| $x=0.5L_x$<br>$y=L_y$ | $\sum \sigma_m = \frac{pLy^4}{Et^3}$ | 0.15 | 0.20 | 0.25 | 0.30 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 |



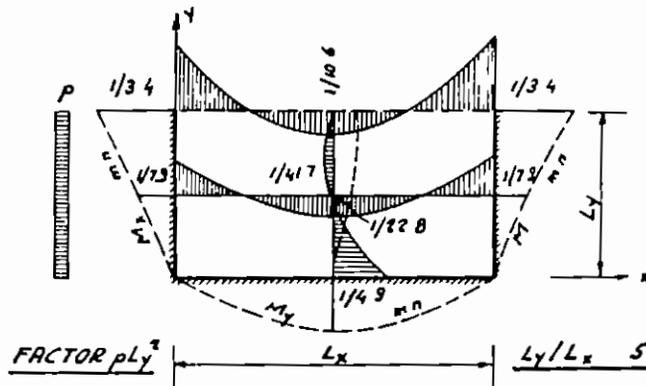
C 6) SLAB FIXED ON TWO SIDES, SIMPLY SUPPORTED ON ONE SIDE AND SUBJECT TO TRIANGULAR LOAD

| POSITION              |                                       | $L_y/L_x$                  | 0.25 | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
|-----------------------|---------------------------------------|----------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| y $L_y$               | $M_{xmn}$                             | $\rho L_y^2$               | 32   | 41   | 63   | 98   | 148  | 218  | 323  | 461  | 646  | 880  | 117  | 150  | 187  | 230  |
|                       | $M_{xm}$                              | $\rho L_y^2$               | 130  | 138  | 176  | 229  | 305  | 407  | 552  | 742  | 980  | 129  | 166  | 210  | 262  | 324  |
| y $0.5L_y$            | $M_{xmn}$                             | $\rho L_y^2$               | 57   | 67   | 88   | 110  | 133  | 160  | 189  | 223  | 260  | 299  | 346  | 395  | 451  | 513  |
|                       | $M_{xm}$                              | $\rho L_y^2$               | 243  | 253  | 283  | 326  | 377  | 434  | 496  | 563  | 636  | 722  | 814  | 915  | 102  | 115  |
| x $0$<br>x $L_x$      | $M_{ymn}$                             | $-\rho L_y^2$              | 57   | 67   | 88   | 110  | 132  | 160  | 186  | 216  | 246  | 279  | 314  | 347  | 392  | 435  |
|                       | at $\frac{y}{L_y}$                    |                            | 10   | 10   | 10   | 10   | 0.57 | 0.5  | 0.46 | 0.42 | 0.39 | 0.36 | 0.34 | 0.33 | 0.32 | 0.31 |
| x $0.5L_x$            | $M_{ym}$                              | $\rho L_y^2$               | 183  | 202  | 257  | 336  | 446  | 601  | 816  | 111  | 152  | 209  | 288  | 390  | 532  | 750  |
|                       | $M_{ymax}$                            | $\rho L_y^2$               | 177  | 191  | 235  | 289  | 359  | 444  | 542  | 655  | 784  | 935  | 113  | 132  | 153  | 175  |
|                       | at $\frac{x}{L_x}$                    |                            | 0.4  | 0.38 | 0.35 | 0.33 | 0.29 | 0.28 | 0.25 | 0.23 | 0.21 | 0.2  | 0.19 | 0.18 | 0.16 | 0.15 |
| x $0.5L_x$            | $M_{ymn}$ (at $\frac{x}{L_x} = 0.8$ ) | $\rho L_y^2$               | —    | —    | —    | —    | —    | —    | —    | 1386 | 060  | 957  | 936  | 1001 | 1148 | 1350 |
| x $0.5L_x$<br>y $L_y$ | $\omega$                              | $\frac{\rho L_y^2}{E I_s}$ | 005  | 005  | 008  | 009  | 009  | 009  | 009  | 008  | 007  | 007  | 006  | 006  | 005  | 005  |



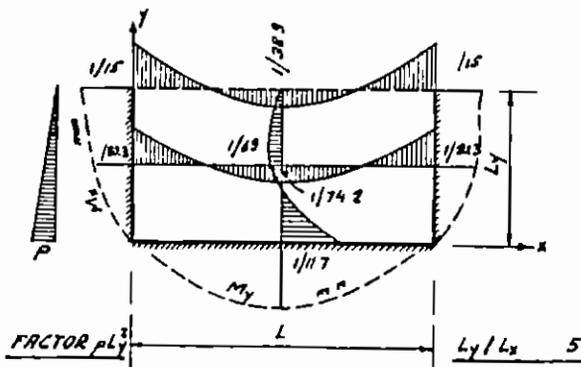
C 7) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD

| POSITION            |                 | $L_y/L$               | 0.25 | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
|---------------------|-----------------|-----------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| x 0.5L              | $M_y$ min       | $pL_y^2$              | 2.4  | 2.7  | 3.6  | 4.9  | 6.7  | 8.8  | 11.3 | 14.3 | 17.6 | 21.3 | 25.3 | 29.7 | 34.4 | 39.5 |
| y 0.5L <sub>y</sub> | $M$ min         | $= pL_y^2$            | 6.8  | 6.9  | 7.3  | 7.9  | 8.8  | 10.0 | 11.4 | 13.1 | 15.0 | 17.1 | 19.5 | 22.1 | 25.0 | 28.1 |
| y L <sub>y</sub>    | $M_x$ min (max) | $pL_y^2$              | 2.2  | 2.3  | 2.7  | 3.4  | 4.5  | 5.8  | 7.4  | 9.4  | 11.6 | 14.1 | 16.9 | 19.9 | 23.2 | 26.7 |
| x 0.5L              | $M_y$ max       | $pL_y^2$              | 16.1 | 26.7 | 26.8 | 88.1 | 62.9 | 63.8 | 73.4 | 89.6 | 113  | 145  | 190  | 250  | 333  | 444  |
| 0.5L                | $M_y$ max       | $pL_y^2$              | 104  | 62.5 | 43.2 | 41.7 | 48.0 | 58.5 | 72.7 | 89.6 | 109  | 132  | 157  | 184  | 213  | 245  |
|                     | at              | $\frac{y}{L_y}$       | 0.88 | 0.8  | 0.75 | 0.7  | 0.65 | 0.6  | 0.55 | 0.5  | 0.45 | 0.4  | 0.35 | 0.32 | 0.3  | 0.28 |
| y 0.5L <sub>y</sub> | $M$ min         | $pL_y^2$              | 57.1 | 37.9 | 25.4 | 22.8 | 23.2 | 24.9 | 27.5 | 30.7 | 34.3 | 38.4 | 43.0 | 48.1 | 53.6 | 59.7 |
| y L <sub>y</sub>    | $M$ min         | $pL_y^2$              | 13.2 | 13.9 | 10.7 | 10.6 | 11.9 | 13.9 | 16.7 | 20.2 | 24.3 | 29.0 | 34.4 | 40.3 | 46.7 | 53.7 |
| x 0.5L <sub>x</sub> | $\delta_m$      | $\frac{pL_x^4}{Et^3}$ | 0.04 | 0.07 | 0.15 | 0.21 | 0.27 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.34 |
| y L <sub>y</sub>    |                 |                       |      |      |      |      |      |      |      |      |      |      |      |      |      |      |



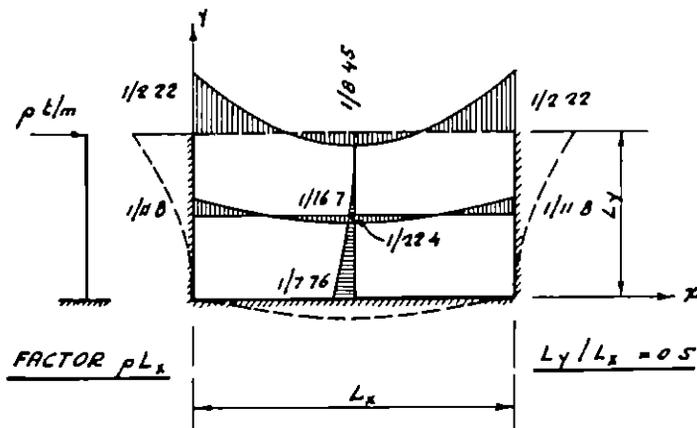
C 8) SLAB FIXED ON THREE SIDES AND SUBJECT TO TRIANGULAR LOAD

| POSITION             |                    | $L_y/L_x$  | 0.25                 | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9   | 1.0   | 1.1  | 1.2   | 1.3   | 1.4   | 1.5   |
|----------------------|--------------------|------------|----------------------|------|------|------|------|------|------|-------|-------|------|-------|-------|-------|-------|
| $x \text{ o.s.l.}$   | $M_y \text{ min}$  | $pl_y^3$   | 6.9                  | 7.6  | 9.4  | 11.7 | 14.4 | 17.5 | 20.9 | 24.3  | 28.5  | 32.8 | 37.4  | 42.4  | 47.7  | 53.3  |
| $y \text{ o.s.l.}$   | $M_x \text{ min}$  | $pl_y^3$   | 18.2                 | 18.4 | 19.1 | 20.3 | 22.0 | 24.1 | 26.7 | 29.8  | 33.2  | 37.1 | 41.5  | 46.3  | 51.7  | 57.6  |
| $y \text{ L}_y$      | $M \text{ min}$    | $pl_y^3$   | 8.8                  | 9.4  | 11.4 | 15.0 | 20.4 | 28.4 | 39.4 | 54.4  | 74.2  | 99.2 | 130   | 167   | 209   | 257   |
| $\text{o.s.l.}_x$    | $M_x \text{ min}$  | $pl_y^3$   | 8.8                  | 9.4  | 11.4 | 15.0 | 20.4 | 28.4 | 39.4 | 54.4  | 74.2  | 99.2 | 130   | 167   | 209   | 257   |
|                      | at $\frac{y}{L_y}$ |            | 1.0                  | 1.0  | 1.0  | 1.0  | 1.0  | 0.6  | 0.55 | 0.5   | 0.47  | 0.45 | 0.43  | 0.4   | 0.38  | 0.34  |
| $\text{o.s.l.}$      | $M_y \text{ max}$  | $pl_y^3$   | 24.3                 | 41.9 | 91.3 | 70.8 | 70.9 | 79.2 | 93.7 | 115   | 144   | 184  | 238   | 310   | 408   | 541   |
|                      | $M_y \text{ max}$  | $pl_y^3$   | 156                  | 103  | 72.7 | 69.0 | 70.9 | 79.0 | 86.5 | 103   | 118   | 137  | 160   | 186   | 213   | 242   |
|                      | at $\frac{y}{L_y}$ |            | 0.75                 | 0.7  | 0.65 | 0.6  | 0.5  | 0.47 | 0.45 | 0.4   | 0.35  | 0.32 | 0.31  | 0.3   | 0.3   | 0.28  |
|                      | $M_y \text{ min}$  | $pl_y^3$   |                      |      |      |      |      |      |      | 4.566 | 17.51 | 34.8 | 12.59 | 12.85 | 13.85 | 15.43 |
|                      | at $\frac{y}{L_y}$ |            |                      |      |      |      |      |      | 0.3  | 0.9   | 0.9   | 0.9  | 0.9   | 0.9   | 0.9   | 0.9   |
| $y \text{ o.s.l.}_y$ | $M_x \text{ min}$  | $pl_y^3$   | 9.16                 | 13.8 | 27.7 | 74.2 | 71.0 | 71.9 | 74.8 | 79.1  | 84.4  | 90.8 | 98.3  | 107   | 116   | 127   |
| $y \text{ L}_y$      | $M \text{ min}$    | $pl_y^3$   | 71.5                 | 51.3 | 39.0 | 39.9 | 43.9 | 53.1 | 64.1 | 84.5  | 108   | 139  | 177   | 224   | 281   | 347   |
| $\text{o.s.l.}$      | $M_x \text{ max}$  | $pl_y^3$   | 71.5                 | 51.3 | 39.0 | 39.9 | 43.9 | 53.1 | 64.1 | 73.7  | 87.1  | 90.3 | 98.3  | 106   | 115   | 125   |
|                      | at $\frac{y}{L_y}$ |            | 1.0                  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 0.8  | 0.7   | 0.6   | 0.55 | 0.5   | 0.47  | 0.45  | 0.42  |
| $\text{o.s.l.}$      | $y \text{ L}_y$    | $\delta_m$ | $\frac{pl_y^3}{24L}$ | 0.01 | 0.02 | 0.04 | 0.06 | 0.07 | 0.08 | 0.08  | 0.07  | 0.07 | 0.07  | 0.06  | 0.05  | 0.05  |



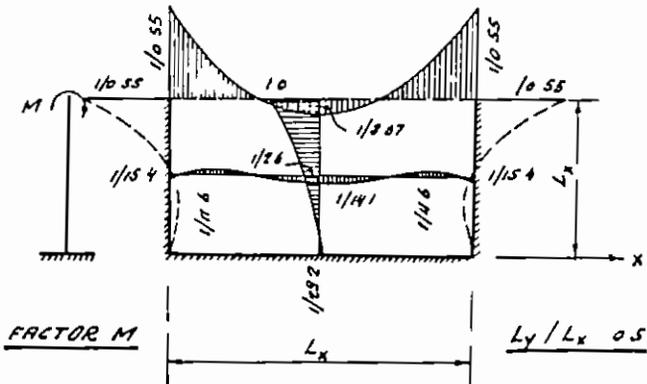
C 9) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM  
EDGE LOAD  $p_t/m$

| POSITION |          | $L_y/L_x$                         | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 |
|----------|----------|-----------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $x/L_x$  | $y/L_y$  |                                   |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 0.1      | 1        | $M_{mn} = p_t x$                  | 3.96 | 3.17 | 2.47 | 2.22 | 2.14 | 2.12 | 2.14 | 2.16 | 2.19 | 2.21 | 2.24 | 2.27 | 2.31 | 2.34 |
| 0.1      | 0.5      | $M_{xmn} = p_t x$                 | 15.0 | 13.0 | 11.6 | 11.8 | 13.0 | 15.3 | 19.2 | 25.0 | 33.5 | 46.3 | 66.3 | 98.0 | 155  | 250  |
| 0.5      | 1        | $M_m = p_t L$                     | 3.12 | 18.8 | 10.8 | 8.45 | 7.62 | 7.35 | 7.21 | 7.19 | 7.18 | 7.19 | 7.20 | 7.21 | 7.23 | 7.25 |
| 0.5      | 0.5      | $M_{xm} = -p_t x$                 | 93.6 | 53.2 | 28.5 | 22.4 | 21.4 | 22.5 | 26.1 | 31.2 | 38.6 | 48.3 | 63.0 | 83.4 | 111  | 150  |
| 0.5      | 0        | $M_{ymn} = -p_t x$                | 5.03 | 4.95 | 5.72 | 7.76 | 11.2 | 18.7 | 32.9 | 64.1 | 130  | 270  | —    | —    | —    | —    |
| 0.5      | 0.5      | $M_{ym} = -p_t x$                 | 11.8 | 12.2 | 14.3 | 16.7 | 19.0 | 21.0 | 23.6 | 26.8 | 30.7 | 36.0 | 42.6 | 51.0 | 62.1 | 76.5 |
| 0.5      | VARIABLE | $M_{ymn} = p_t x$<br>at           | —    | —    | —    | —    | —    | 70.0 | 19.7 | 19.5 | 19.7 | 20.0 | 20.3 | 20.6 | 20.8 | 20.9 |
|          |          | $y/L_y$                           | —    | —    | —    | —    | —    | 0.63 | 0.68 | 0.73 | 0.76 | 0.78 | 0.80 | 0.82 | 0.84 | 0.85 |
| 0.5      | 1        | $\delta_m = \frac{p_t L^3}{Et^3}$ | 0.46 | 0.67 | 0.92 | 1.03 | 1.20 | 1.26 | 1.26 | 1.33 | 1.33 | 1.26 | 1.26 | 1.26 | 1.26 | 1.26 |



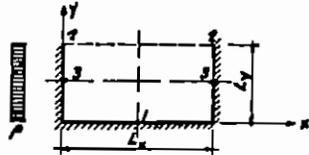
C 10) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM  
EDGE MOMENT  $M$  ml/m

| POSITION |          |            | $L_y/L_x$             |       |      |      |      |      |      |      |      |      |      |      |      |      |      |  |
|----------|----------|------------|-----------------------|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|
| $x/L$    | $y/L_y$  |            |                       | 0.25  | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 |  |
| 0.1      | 1        | $M_{mn}$   | $M$                   | 0.61  | 0.57 | 0.54 | 0.55 | 0.56 | 0.57 | 0.60 | 0.62 | 0.64 | 0.67 | 0.69 | 0.72 | 0.74 | 0.77 |  |
| 0.1      | 0.5      | $M_x$      | $M$                   | 5.98  | 8.29 | 4.76 | 15.4 | 7.91 | 6.17 | 5.59 | 5.56 | 5.94 | 6.67 | 7.55 | 8.66 | 10.2 | 12.4 |  |
| 0.1      | VARIABLE | $M_{xma}$  | $M$                   | 3.00  | 6.90 | 2.32 | 12.2 | 7.82 | 6.03 | 5.10 | 4.72 | 4.52 | 4.42 | 4.37 | 4.33 | 4.31 | 4.31 |  |
|          |          | at $y/L_y$ |                       | 0.14  | 0.19 | 0.29 | 0.39 | 0.47 | 0.54 | 0.59 | 0.63 | 0.66 | 0.69 | 0.72 | 0.74 | 0.76 | 0.77 |  |
| 0.5      | 1        | $M_{xm}$   | $M$                   | 5.4   | 4.0  | 3.14 | 3.07 | 3.18 | 3.29 | 3.40 | 3.44 | 3.48 | 3.49 | 3.50 | 3.52 | 3.53 | 3.54 |  |
| 0.5      | 0.5      | $M_{xm}$   | $M$                   | 16.90 | 12.3 | 10.6 | 14.1 | 29.4 | ∞    | 36.1 | 21.3 | 18.5 | 17.5 | 17.9 | 18.7 | 20.4 | 23.4 |  |
| VARIABLE | 0.5      | $M_{mn}$   | $M$                   |       | 7.00 | 8.2  | 11.6 | 18.5 | 29.5 | 27.8 | 21.3 | 18.5 | 17.5 | 17.9 | 18.7 | 20.4 | 23.4 |  |
|          |          | at $/L$    |                       | 0.5   | 0.8  | 1.3  | 1.5  | 2.1  | 2.7  |      | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  | 0.5  |  |
| 0.5      | 0        | $M_y$      | $M$                   | 1.52  | 2.08 | 6.05 | 2.92 | 8.03 | 6.90 | 7.55 | 9.00 | 12.7 | 18.5 | 27.8 | 41.7 | 66.7 | 115  |  |
| 0.5      | 0.5      | $M_{ym}$   | $M$                   | 1.32  | 1.52 | 2.01 | 2.60 | 3.36 | 4.39 | 6.07 | 8.12 | 13.1 | 21.7 | 40.0 | 1.0  | ∞    | 130  |  |
| 0.5      | 1        | $\delta$   | $\frac{ML_x^2}{Et^3}$ | 273   | 325  | 363  | 363  | 353  | 343  | 333  | 333  | 325  | 325  | 316  | 308  | 308  | 300  |  |



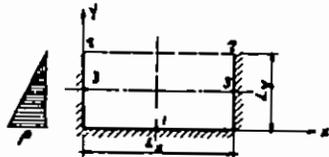
**c ii) REACTIONS OF SLABS FIXED ON THREE SIDES AND  
SUBJECT TO UNIFORM AND TRIANGULAR LOADS \***

**a. UNIFORM LOAD**



| POSITION |         | $L_y/L_x$   | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.25 | 1.50 |
|----------|---------|-------------|------|------|------|------|------|------|------|
| $x/L_x$  | $y/L_y$ |             |      |      |      |      |      |      |      |
| 0.5      | 0       | $R_1 = pLy$ | 1.44 | 1.70 | 1.95 | 2.20 | 2.50 | 3.22 | 4.02 |
| 0.5      | 1       | $R_2 = pLy$ | 0.80 | 0.98 | 1.17 | 1.37 | 1.59 | 2.20 | 2.85 |
| 0.5      | 0.5     | $R_3 = pLy$ | 2.00 | 2.00 | 2.07 | 2.17 | 2.30 | 2.53 | 3.05 |

**b. TRIANGULAR LOAD**



| POSITION |         | $L_y/L_x$   | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.25 | 1.50 |
|----------|---------|-------------|------|------|------|------|------|------|------|
| $x/L_x$  | $y/L_y$ |             |      |      |      |      |      |      |      |
| 0.5      | 0       | $R_1 = pLy$ | 2.40 | 2.70 | 2.90 | 3.14 | 3.33 | 4.05 | 4.82 |
| 0.5      | 1       | $R_2 = pLy$ | 6.50 | 8.70 | 11.6 | 15.8 | 22.2 | 33.5 | 45.0 |
| 0.5      | 0.5     | $R_3 = pLy$ | 4.40 | 4.40 | 4.50 | 4.60 | 4.80 | 5.35 | 6.13 |

\* TIMOSHENKO : PLATES AND SHELLS FOR  $\nu = 1/m = 1/6$

d) Direct Application of Czerny's-Tables in Tanks

The tables of Czerny can be directly used in some typical problems of rectangular tanks as can be seen from the following example

It is required to design a  $160 \text{ m}^3$  capacity open square water tank supported on four columns

The clear height of the tank will be assumed  $4.3 \text{ ms}$  then

$$\text{Clear area} = 160 / 4.3 = 37.2 \text{ m}^2$$

$$\text{Clear side length} = \sqrt{37.2} = 6.1 \text{ m}$$

To reduce the internal forces in the floor, four beams are arranged so that its span is  $4.5 \text{ ms}$  only. The chosen dimensions of the tank will be as shown in figure VII-27

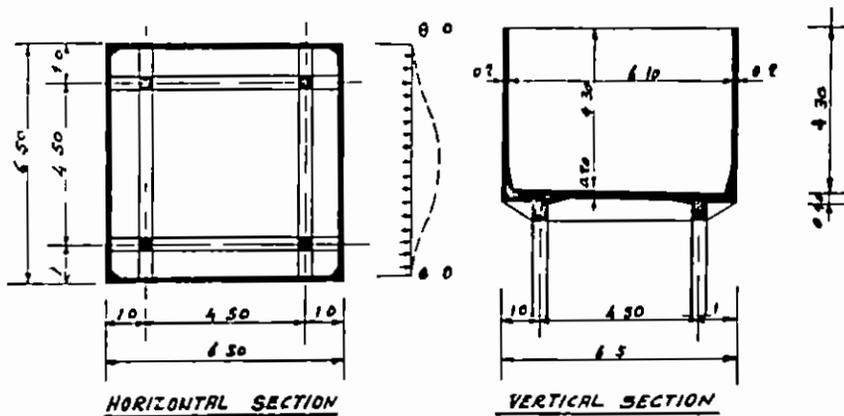


Fig VII-27

Each of the walls of the tank has

a theoretical height of  $H = 4.50 \text{ ms}$  and ,

a theoretical length of  $L = 6.30 \text{ ms}$

The walls can be assumed fixed at their vertical edges due to symmetry in shape and loading ( $\theta = 0$ ) and fixed to the floor due to the continuity with a short span of relatively big thickness

The floor slab 4.5 x 4.5 ms can be assumed as fixed on all four sides for the same reason

Max water pressure on walls

$$P_{\max} = 4.3 \text{ t/m}^2$$

Load on intermediate part of floor slab  $p_1$

= weight of water + own weight of floor slab (assumed 20 cms thick)  
or  $p_1 = 4.3 + 0.2 \times 2.5 = 4.3 + 0.5 = 4.8 \text{ t/m}^2$

Load on outside part of floor slab (assumed 40 cms thick)

$$p_2 = 4.3 + 0.4 \times 2.5 = 4.5 + 1.0 = 5.3 \text{ t/m}^2$$

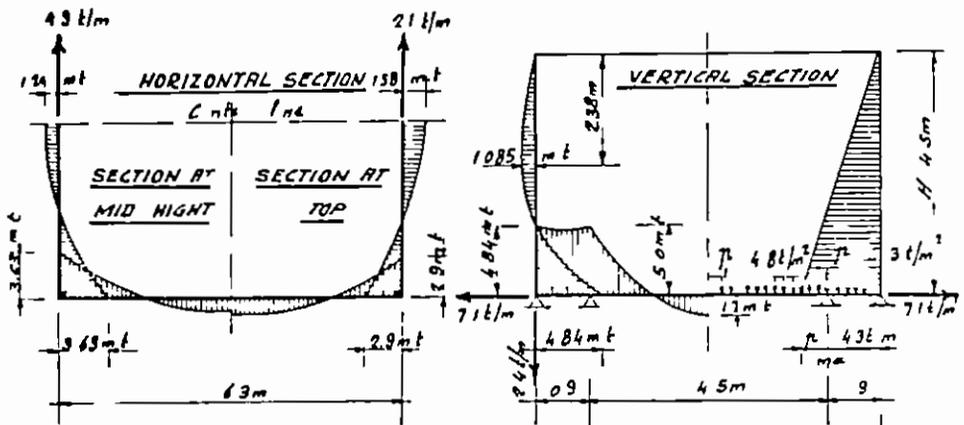


Fig VII-28

The internal forces in the walls free at top and fixed on the other three sides can be calculated according to table C-8 as follows (Fig VII 28)

$$H/L = 4.5 / 6.3 = 0.71 >$$

$$p_{\max} H^2 = 4.3 \times 4.5^2 = 87 \text{ mt}$$

Bending moments along middle vertical axis are therefore :

Max fixing moment at base

$$M_{y \min} = - p_{\max} H^2 / 18 = - 87 / 18 = - 4.84 \text{ mt}$$

Max field moment at 0.47 H from base :

$$M_{y \max} = p H^2 / 80 = 87 / 80 = 1.085 \text{ mt}$$

Bending moments in horizontal direction are

Section at top surface

at fixing moment

$$M_{x \min} = - p_{\max} H^2/30 = - 87/30 = - 2.90 \text{ m t}$$

Max field moment

$$M_{m \max} = p_{\max} H^2/55 = 87/55 = \underline{1.58} \text{ m t}$$

Section at 0.6 H from bottom

Max fixing and field moments

$$M_{x \min} = - p_{\max} H^2/24 = - 87/24 = - \underline{3.63} \text{ m t}$$

$$M_{m \max} = p_{\max} H^2/72.4 = 87/72.4 = \underline{1.20} \text{ m t}$$

The reactions can be calculated from the table of Timoshenko C-11b ,

thus for  $H/L = 0.715$  we get

Reaction at middle of base = max tension in floor

$$R_1 = pH/2.73 = 4.3 \times 5/2.73 = 7.1 \text{ t/m}$$

Reaction at top edge of wall = horizontal tension in side walls

$$R_2 = pH/9.14 = 4.3 \times 5/9.14 = 2.1 \text{ t/m}$$

Reaction at mid height of wall = horizontal tension in side walls

$$R_3 = pH/4.42 = 4.3 \times 5/4.42 = 4.9 \text{ t/m}$$

Tension in wall at its base

$$R_4 = p_2 \times 0.9/2 = 5.3 \times 0.45 = 2.40 \text{ t/m}$$

The internal forces in the floor slab, fixed on all four sides and subject to uniform load  $p_1 = 4.8 \text{ t/m}^2$  can be calculated according to table A-6 as follows

$$L_y = L_x = L = 4.5 \text{ m} \quad p_1 L^2 = 4.8 \times 4.5^2 = 97 \text{ m t}$$

Max fixing moment

$$M_{f \max} = - p L^2/19.4 = - 97/19.4 = - \underline{5.00} \text{ m t}$$

Max field moment

$$M_{m \max} = p L^2/56.8 = 97/56.8 = \underline{1.70} \text{ m t}$$

The vertical and horizontal sections of the tank are to be calculated for the internal forces shown in figure VII 28

The bending moments and shearing forces in the floor beams can be calculated as follows ( Fig VII 29 )

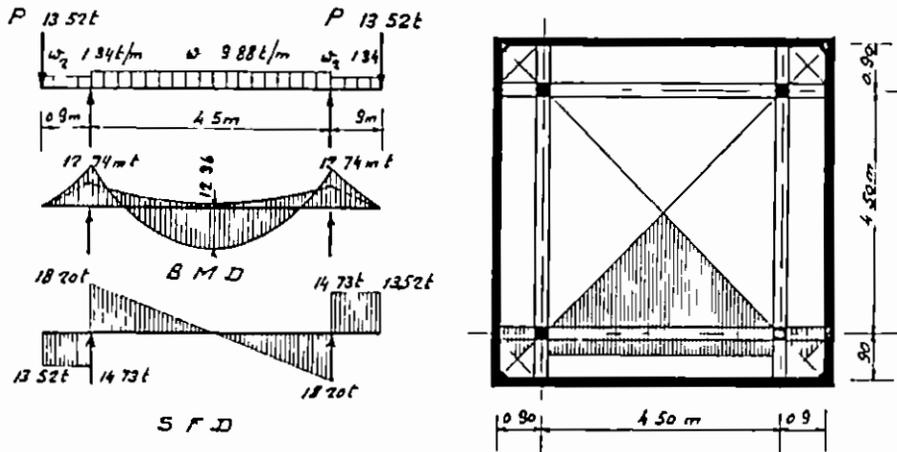


Fig VII-29

Each of the four supporting beams can be considered as a simple beam + 5 ms span with double cantilevers 0.90 ms each. The beam is assumed 30 x 60 cms

Load on intermediate span

$$w_1 = 0.3 \times 0.4 \times 2500 + 0.67 \times \frac{4.5}{2} \times 4800 + 0.45 \times 5300 \quad \text{or}$$

$$w_1 = 300 + 7200 + 2380 = 9880 \text{ kg/m}$$

Load on outside cantilevers

$$w_2 = 0.3 \times 0.4 \times 2500 \times \frac{1}{2} + \frac{0.45}{2} \times 5300 = 150 + 1190 = 1340 \text{ kg/m}$$

Concentrated load

$$\begin{aligned} P &= \text{own weight of wall} = 0.2 \times 4.5 \times 2500 \times \frac{6.3}{2} &= 7100 \text{ kg} \\ &+ \text{uniform floor load} = 0.45 \times 5300 \times \frac{4.5}{2} &= 5350 \text{ kg} \\ &+ \text{triangular floor load} = \frac{0.45}{2} \times 5300 \times 0.9 &= 1070 \text{ kg} \\ &\text{i e total P} &= \underline{13520 \text{ kgs}} \end{aligned}$$

Therefore, the max cantilever moment is given by

$$M_{\max} = 13.52 \times 0.9 + 1.34 \times \frac{0.9^2}{2} = 12.2 + 0.54 = 12.74 \text{ m t}$$

and the max field moment is

$$M_{\text{field}} = 9.88 \times \frac{4.5^2}{8} - 12.74 = 25.0 - 12.74 = 12.36 \text{ m t}$$

The loads, bending moments and shearing forces are shown in figure VII 29

One can see that the choice of the dimensions of the tank was convenient for the following reasons

- 1) The max fixing moment in the wall ( - 4.84 m t ) is approx equal to the max fixing moment in the floor ( - 5.00 m t )
- 2) The max moments are accumulated at the corner between walls and floor, so that only a small part of the slabs needs a relatively big thickness ( ~ 40 cms )
- 3) The max field moment in the wall ( 1.58 m t ) is approx equal to the max field moment in the floor ( 1.70 m t )
- 4) The thickness of slabs is generally governed by their field moments and as their values are small only the minimum thickness necessary for water-tightness ( ~ 20 cms ) is sufficient
- 5) The cantilever part has been chosen such that it gives approximately equal cantilever and field moments in the supporting beams

The same idea can be adopted for bigger capacities as shown in figure VII 30 in which the capacity of the tank is

$$4.00 \times 10.60 \times 10.60 \approx 450 \text{ m}^3$$

The floor slabs are again here fixed at the intermediate panelled frames, because they are arranged on the axes of symmetry

Each of the floor panelled frames has a span of 9 ms and carries, in addition to its own weight a floor load of  $p = \frac{4.5}{2} \times 4.5 = 10 \text{ t/m}$  on the intermediate span

The walls may however be supported at their top edge on cover slabs or horizontal beams as those shown in the example. Such beams can be assumed as supports for the walls if they are supported at convenient distances (smaller than ca 6 ms). In the shown example diagonal ties may give a convenient solution.

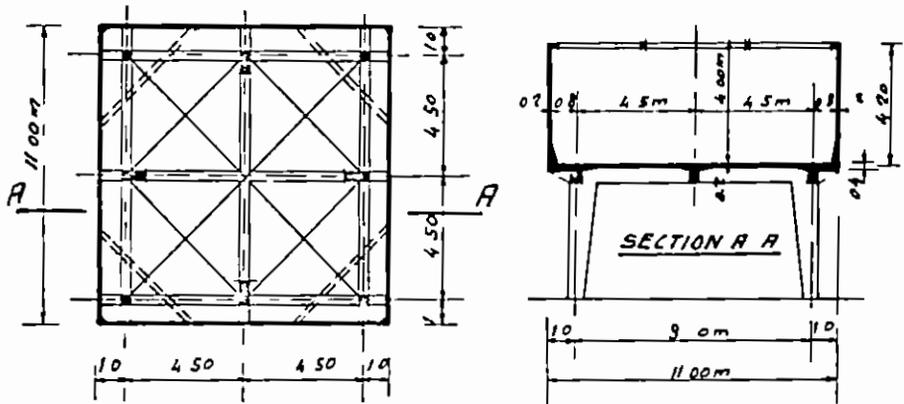


Fig VII-30

For the case of walls simply supported at top and fixed to the floor slab at bottom and resisting the full water pressure in the vertical direction the height of wall  $H$  and the span of square floor slabs  $L$  giving the same fixing moment in wall and floor can be determined as follows:

$$\text{Max hydrostatic pressure on wall } p_{\text{max}} \cong 0.95 H$$

$$\text{Load on floor slab including own weight } p \cong 1.10 H$$

$$\text{Fixing moment of wall} = \text{connecting moment of floor} \quad \text{or}$$

$$0.95 H^3 / 15 = 1.10 H L^2 / 19.4 \quad \text{or}$$

$$\underline{L = 1.06 H}$$

Figure VII 31 shows the concrete dimensions and details of reinforcements of a covered reinforced concrete square tank = 400 m<sup>3</sup>

capacity supported on four columns. The general arrangement of the supporting elements of the tank follows the given recommendations. The supporting elements of the floor are composed of four simple girders supported on the columns, each of them is provided with two cantilever arms supporting the walls as deep beams. The square floor enclosed between the main girders is subdivided by two parallel simple beams. Each of the four panels of the floor can be assumed as totally fixed at its edges. The walls being  $3.6 \times 10.80$  m they behave as one way slabs simply supported on the top horizontal beam and totally fixed to the floor. They resist the water pressure in the vertical direction only. In this manner, the bigger values of the bending moments in a vertical section of the tank are accumulated at the corner between the walls and the floor. However, the local horizontal bending moments along the vertical corners of the tank - equations 10 - are to be taken in consideration. In order to avoid vertical supports inside the tank, the top horizontal beam of the wall is supported on diagonal ties shown dotted. The roof is a reinforced concrete square pyramid supported on twelve posts arranged on the top of the walls at the corners and the third points of each side. The windows between the posts allow for good aeration of the water in the tank.

#### VII 4 COUNTERFORTED WALLS

In order to reduce the thickness of walls of deep rectangular tanks, it may be of advantage to support them on counterforts as can be seen in the following example of a diving pool  $5 \times 20 \times 20$  ms ( Fig. VII 32 )

It is obvious that, in this case, it is not possible to arrange any supports at the upper surface of the water, and, if the walls are constructed without any supports, i.e. free at top and totally fixed at the other three sides, we get for water pressure, according to Czerny's table C-8, the following

If  $L_y/L_x = H / L = 5/20 = 0.25$

and  $p H^2 = 5 \times 5^2 = 125 \text{ m t}$

the max fixing moments at foot of wall will be

$M_{y_{min}} = - p H^2 / 6.9 = - 125 / 6.9 = - 18.1 \text{ m t}$

and

the max fixing moment between the walls at their top surface

$M_{x_{min}} = - p H^2 / 8.8 = - 125 / 8.8 = - 14.2 \text{ m t}$

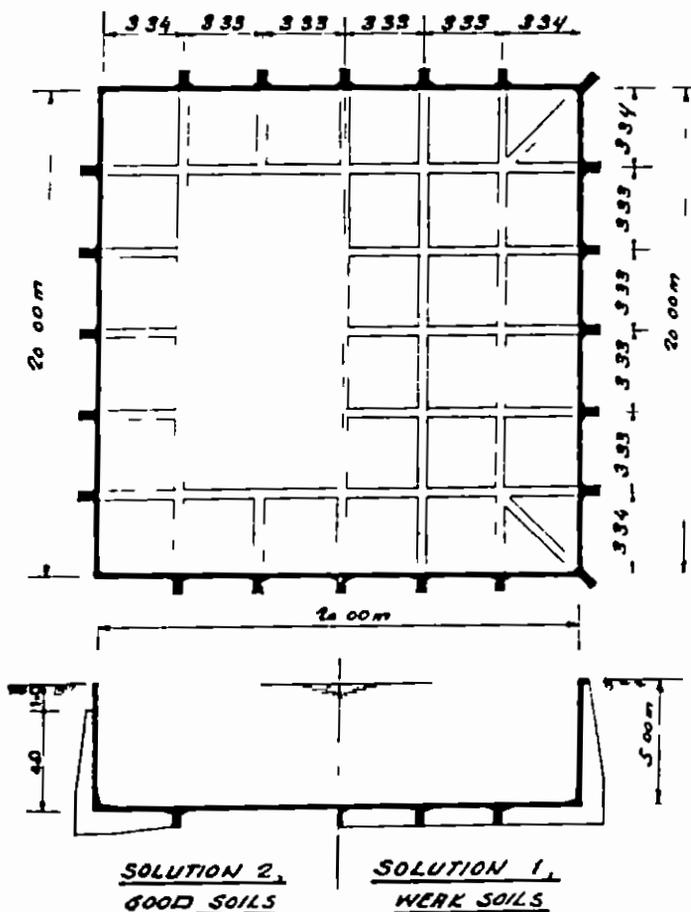


Fig VII-32

Both moments cause tensile stresses on the water side, so that, the thickness required for the wall at its base is given by

$$t_{\max} = \sqrt{H/3} = \sqrt{18100/3} = 77.5 \text{ cms}$$

and the corresponding tension steel will be

$$= 0.26 t_{\max} = 0.26 \times 77.5 = 20 \text{ cm}^2 \quad 7 \phi 19 \text{ mm/m}$$

Both values  $t$  and  $A_s$  are relatively big

We give in the following some trials for convenient solutions of the wall

### 1st Trial Wall Supported on Four Sides

As a first trial, we arrange horizontal beams at the top edge of the wall and vertical counterforts at say 3.33 m as shown in solution 1 of figure VII 2. In this manner, the walls behave as two way slabs simply supported at top and fixed on the other three sides. The internal forces can be calculated from Czerny's table B-5' as follows

$$\text{For } L_y / L_x = H / L = 3.33 / 3.33 = 1$$

$$\& \quad p L_x^2 = 5 \times 3.33^2 = 57.2 \text{ t}$$

We get the bending moments

$$M_{y \min} = -p L_x^2 / 23.5 = -57.2 / 23.5 = -2.43 \text{ mt} \quad \text{at base}$$

$$M_{y \max} = p L_x^2 / 104 = 57.2 / 104 = 0.549 \text{ mt at } 0.27 H \text{ from base}$$

$$M_{x \min} = -p L_x^2 / 23.7 = -57.2 / 23.7 = -2.39 \text{ mt at } 0.38 H$$

$$M_{x \max} = p L_x^2 / 57.2 = 57.2 / 57.2 = 0.97 \text{ mt at mid height}$$

and the reactions

$$\begin{aligned} R_{ym \max} &= \text{max tension in floor slab} \\ &= p L_x / 2.7 = 5 \times 3.33 / 2.7 = 6.2 \text{ t/m} \end{aligned}$$

$$\begin{aligned} R_{ym} &= \text{load at middle of top beam} \\ &= p L_x / 17.4 = 5 \times 3.33 / 17.4 = 0.96 \text{ t/m} \end{aligned}$$

$$R_{yB} = \text{load at support of top beam}$$

$$= -p L_x / 94 = -5 \times 3.33 / 94 = -1.78 \text{ t/m'}$$

The reaction of the top beam and the horizontal reaction of the wall give the loads acting on the counterfort, thus

If we assume that the load distribution on the top beam is parabolic, its reaction on the top edge of the counterfort will be nil ( $\approx 0$ ) because the negative value of the load at the support of the beam (1.78 t/m) is approximately equal to double the maximum positive value at the middle (0.96 t/m)

The maximum reaction of the wall on the counterfort is given by

$$R_{x_{max}} = 2 p L_x / 31 = 2 \times 5 \times 3.33 / 31 = 11 \text{ t/m at } 0.3 H \text{ from base}$$

In order to calculate the bending moments and reaction at base of counterfort, one may replace the load diagram acting on the counterfort by a polygon as shown in Fig VII 33 which shows the internal forces in the wall and supporting beams

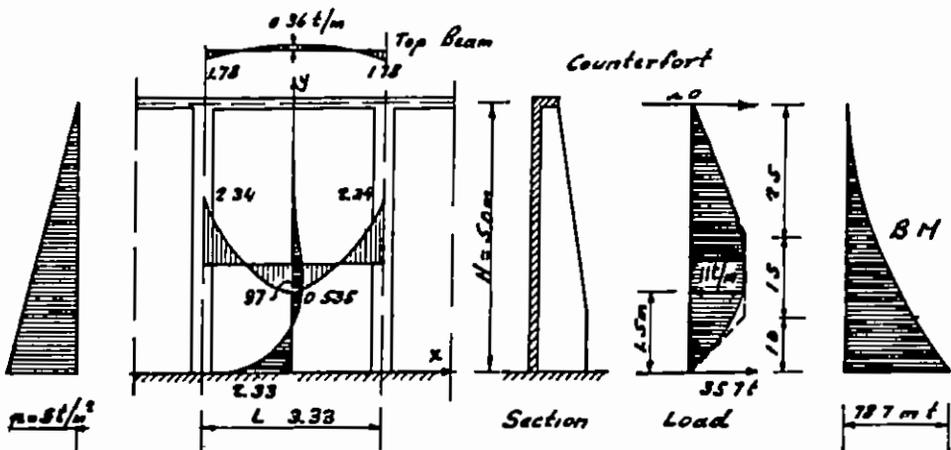


Fig VII-33

The figure shows further, the general arrangement of the counterforts in elevation and section, the bending moments in the wall and

in the vertical and horizontal directions, and the internal forces and reactions of the counterfort

Reaction of counterfort at its base

$$R = 11 \times \frac{2.5}{2} + 11 \times 1.5 + 11 \times \frac{1}{2} = 22.7 \text{ ton}$$

Maximum bending moment of counterfort

$$M_{\max} = \frac{11 \times 2.5}{2} \left( \frac{2.5}{3} + 2.5 \right) + 11 \times 1.5 \times 1.75 + \frac{11 \times 1}{2} \times \frac{2}{3} = 78.7 \text{ mt}$$

### 2nd trial Wall Supported on Three Sides

As we have seen, in the previous trial that the loads on the top beam are so small that it does not give an effective support for the wall. Further studies have shown that as long as the height of a wall subject to hydrostatic pressure is less than its span  $L$ , the internal forces are not affected if we discontinue the top beam, in which case the wall will be free at top and total, fixed on the other three sides. The bending moments can be determined from Cherny's table C-8 as follows

$$\text{For } L_y / L_x = H / L = 5 / 3.33 = 1.5$$

$$\text{and } p L_y^2 = 5 \times 5^2 = 125 \text{ mt}$$

We get the bending moments

$$M_{y \min} = -p L_y^2 / 53.5 = -125 / 53.5 = -2.34 \text{ mt} \quad \text{at base}$$

$$M_{y \max} = p L_y^2 / 242 = 125 / 242 = 0.516 \text{ mt} \quad \text{at } 0.2 H \text{ from base}$$

$$M_{x \min} = -p L_y^2 / 54.2 = -125 / 54.2 = -2.32 \text{ mt} \quad \text{at } 0.36 H "$$

$$M_{x \max} = p L_y^2 / 125 = 125 / 125 = 1.00 \text{ mt} \quad \text{at } 0.40 H "$$

The reactions of the slab can be calculated from table C-11, thus

$$R_1 = \text{max tension in floor slab}$$

$$= p L_y / 4.82 = 5 \times 5 / 4.82 = 5.2 \text{ t/m}$$

$$2R_2 = \text{load on an intermediate counterfort at its top edge} \\ = -2 p L_y / 250 = -2 \times 5 \times 5 / 250 = -0.2 \text{ t/m}$$

$$2R_3 = \text{load on an intermediate counterfort at mid-height} \\ = 2 p L_y / 6 l_3 = 2 \times 5 \times 5 / 6 \times 13 = 8.15 \text{ t/m}$$

The maximum load on the counterfort lies at  $\sim 0.3 H$  from base, so that the load diagram can be drawn as shown in figure VII 34, its ordinates being the same as those of the previous case, the bending moment of the counterfort and the reaction at its lower edge will be the same (Fig VII 34)

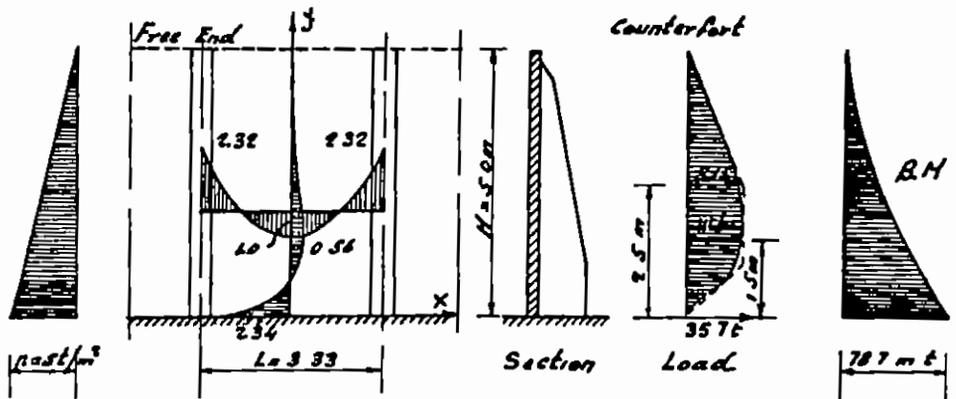


Fig VII-34

Comparing the values of the internal forces obtained for this trial, we find that they are nearly the same as those of the first trial

For such internal forces, it is sufficient to construct the walls with a thickness of 20 cms only at the middle of the spans and  $\approx 30$  cms at the supports

The maximum section of the counterfort may be chosen  $45 \times 110$  cms and the corresponding reinforcement  $\approx 9 \phi 25$

3rd Trial Cantilevering Wall Supported on Three Sides

In this new trial, the height of the counterfort will be reduced to 4.00 m. In this manner, the lower part of the wall will be fixed on three sides and free at its upper edge where we have 1 meter free cantilever. It is subject to a trapezoidal hydrostatic pressure with

$$p_1 = 1.00 \text{ t/m}^2 \text{ at its top edge} \quad \text{and}$$

$p = p_1 + p_2 = 1.00 + 4.00 = 5.00 \text{ t/m}^2$  at its lower edge in addition to a bending moment  $M_0$  given by :

$$M_0 = p_1 C^2/6 = 1.00^3 / 6 = 0.166 \text{ mt/m}$$

and a shearing force  $Q_0 = 1.00^2/2 = 0.50 \text{ t/m}$  acting at its free edge as shown in figure VII 35

The bending moments in the lower part of the slab can be calculated from the following tables

Table C-7 for the uniform load  $p_1 = 1.00 \text{ t/m}^2$

Table C-8 " triangular "  $p_2 = 4.00 \text{ t/m}^2$

Table C-9 for the shear  $Q_0 = 0.5 \text{ t/m}$  acting on the free edge

Table C-10 for the moment  $M_0 = 0.166 \text{ mt/m}$  ' ' "

The results are given in table 1.

The reactions due to  $M_0$  are so small that they can be neglected, while the reactions due to  $Q_0$  will be directly transmitted to the edges of the counterforts ( at points d ) and are given by

$$R_d = 0.5 \times 3.33 = 1.67 \text{ t}$$

The reactions of the lower part of the wall due to the uniform pressure  $p_1 = 1.0 \text{ t/m}^2$  and the triangular pressure  $p_2 = 4.0 \text{ t/m}^2$  can be calculated from table C-11 as shown in table 2.

$R_b$  given in the table indicates the maximum tension in the floor slab

**Table 1**

| Load     | Uniform Load<br>$P_1 = 1.0 \text{ t/m}^2$<br>Table C-7 |        | Triang Load<br>$P_2 = 4.0 \text{ t/m}^2$<br>Table C-8 |        | Edge Load<br>$Q_0 = 0.5 \text{ t/m}$<br>Table C-9     |        | Edge Moment<br>$M_0 = 0.166 \text{ mt/m}$<br>Table C-10 |        | Total  |
|----------|--------------------------------------------------------|--------|-------------------------------------------------------|--------|-------------------------------------------------------|--------|---------------------------------------------------------|--------|--------|
| B.M      | $M = \frac{P_1 L^2}{m} = \frac{1 \times 4^2}{m}$       |        | $M = \frac{P_2 L^2}{m} = \frac{4 \times 4^2}{m}$      |        | $M = \frac{Q_0 L^2}{m} = \frac{0.5 \times 3.32^2}{m}$ |        | $M = \frac{M_0}{m} = \frac{0.166}{m}$                   |        | B.M    |
|          | m                                                      | M      | m                                                     | M      | m                                                     | M      | m                                                       | M      | M      |
| $M_{xa}$ | 34.4                                                   | 0.465  | 177                                                   | 0.361  | 7.2                                                   | 0.232  | 3.5                                                     | 0.048  | 1.106  |
| $M_{xd}$ | -16.9                                                  | -0.950 | -130                                                  | -0.492 | -2.42                                                 | -0.746 | -0.69                                                   | -0.242 | -2.430 |
| $M_{xm}$ | 43.0                                                   | 0.372  | 90.3                                                  | 0.650  | 63.0                                                  | 0.027  | -17.9                                                   | -0.009 | 1.040  |
| $M_{xc}$ | -19.5                                                  | -0.820 | -41.5                                                 | -1.540 | -60.3                                                 | -0.025 | 7.55                                                    | 0.022  | -2.363 |
| $M_{ya}$ | $\infty$                                               | 0      | $\infty$                                              | 0      | $\infty$                                              | 0      | -1.00                                                   | -0.166 | -0.166 |
| $M_{ym}$ | 190                                                    | 0.084  | 238                                                   | 0.269  | -42.6                                                 | -0.039 | -0                                                      | -0.004 | 0.310  |
| $M_{yb}$ | -25.3                                                  | -0.632 | -37.4                                                 | -1.710 | -                                                     | -      | 115                                                     | 0.001  | -2.340 |

**Table 2**

| Loading  | Uniform Load<br>$P_1 = 1.0 \text{ t/m}^2$<br>Table C-11a |           | Triangular Load<br>$P_2 = 4.0 \text{ t/m}^2$<br>Table C-11b |            | Total    |
|----------|----------------------------------------------------------|-----------|-------------------------------------------------------------|------------|----------|
| Reaction | $R = \frac{P_1 L}{m} = \frac{1 \times 4}{m}$             |           | $R = \frac{P_2 L}{m} = \frac{4 \times 4}{m}$                |            | Reaction |
|          | m                                                        | $R = 4/m$ | m                                                           | $R = 16/m$ |          |
| $R_b$    | 3.06                                                     | 1.30      | 4.04                                                        | 3.96       | 5.26     |
| 2 $R_d$  | 2.08                                                     | 2 x 1.92  | 60                                                          | 2 x 0.27   | 4.38     |
| 2 $R_c$  | 2.48                                                     | 2 x 1.16  | 4.76                                                        | 2 x 3.35   | 9.02     |

+1.67 = 6.05t

The maximum load on the counterfort lies at  $0.5 H$  from base and can be estimated diagrammatically as shown in figure VII-35 in which the load is replaced by a polygon of the given ordinates

Accordingly, the reaction at the foot of the counterfort is given by:

$$R = \frac{6.05 + 9.02}{2} \times 2.5 + 10 \times 1.0 \times 0.5 = 33.9 \text{ t}$$

and its maximum moment is given by  $M = 65.25 \text{ mt}$

The internal forces in the wall and in an intermediate counterfort are shown in figure VII-35

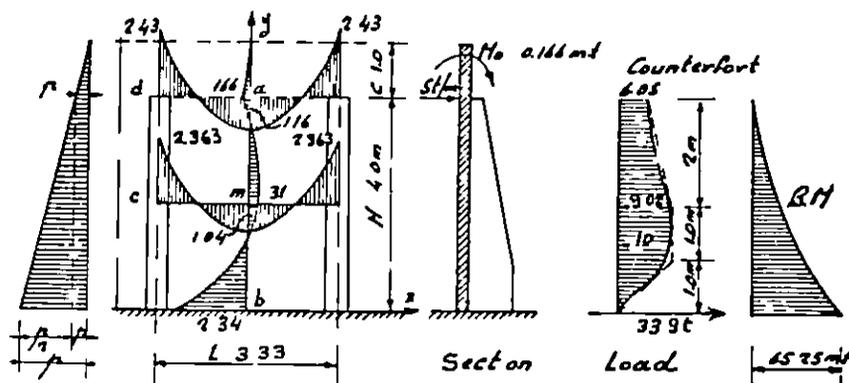


Fig VII-35

It is clear from the previous investigations that the choice of a convenient and simple solution as that shown in the third trial, requires a thorough knowledge of the distribution of the internal forces in flat plates under different loads for the various edge conditions

In all the shown cases, the counterforts behave as simple cantilevers subject to the reactions of the wall and the top beam, if any, giving maximum bending moments at their base. The tensile stresses in the counterfort due to the hydrostatic pressure are on the water side, and it is therefore essential to prevent tension cracks from being developed. In order to satisfy this requirement, the sections

of the counterfort will be designed as reinforced concrete rectangular sections because their flange lies in the tension zone and, in spite of that, the depth of the different sections must be chosen such that the maximum tensile stresses in the concrete are smaller than its tensile strength, i.e. no cracks are allowed in the flange of the beam. In this last case, we are allowed to assume the sections as plain concrete (or eventually reinforced concrete) T-sections with breadth of flange

$$B = 6 t_s + 2 b_s + b_o^{\frac{x}{h}}$$

and smaller than  $H/3$ . This means, the sections are to be treated as reinforced concrete rectangular sections in stage II and at the same time as plain concrete T-sections in stage I.

The walls must be designed for the full hydrostatic pressure assuming earth removed, and the full earth plus any ground water pressure + effect of surcharge, if any, with tank empty. In this last case, the wall sections behave as T both in stage I and stage II.

Corner counterforts may be arranged as shown in solution 1, in which case, they resist the horizontal reactions of the two adjoining panels of the walls. If they are not arranged, as shown in solution 2, the walls will be subject to horizontal forces, equal to the horizontal reactions of the end panels of walls, over their whole length.

The fixing moments and bottom reactions of counterforts must be transmitted to floor beams, (or eventually thick floor slabs). It is recommended to extend the floor beams over the whole width of

---

\*-  $t_s$  = thickness of flange       $b_s$  = breadth of haunches if any  
 $b_o$  = breadth of counterfort

the floor if it rests on relatively weak soils - solution 1 of figure VII-32 - and, to extend them on a short distance of the floor slab if it rests on stiff soils as shown in solution 2 of the same figure

It is generally not recommended to use bent bars in vertical slabs and beams of counterforted walls even in cases where such walls are subject to earth or water pressure from one side only because bent bars are liable to move from their position during concreting operations

Figure VII-36 shows the concrete dimensions and details of reinforcements of the third trial shown in figure VII-35 for solution 2 of figure VII-32 assuming that the internal forces due to earth pressure are 0.65 the values due to water pressure

It has to be noted here that no bent bars are used in the different elements of the pool and that high grade steel stressed to 2000 kg/cm<sup>2</sup> in tension is used as reinforcement for the counterforts

The required concrete dimensions and reinforcements of the different elements are as follows

#### 1 ) Wall and floor slabs

Thickness 20 cms provided with haunches 10 x 30 cms at the supporting beams

Tension steel 7  $\phi$  13 mm/m for water pressure and 7  $\phi$  10 mm/m for earth pressure

#### 2 ) Counterforts

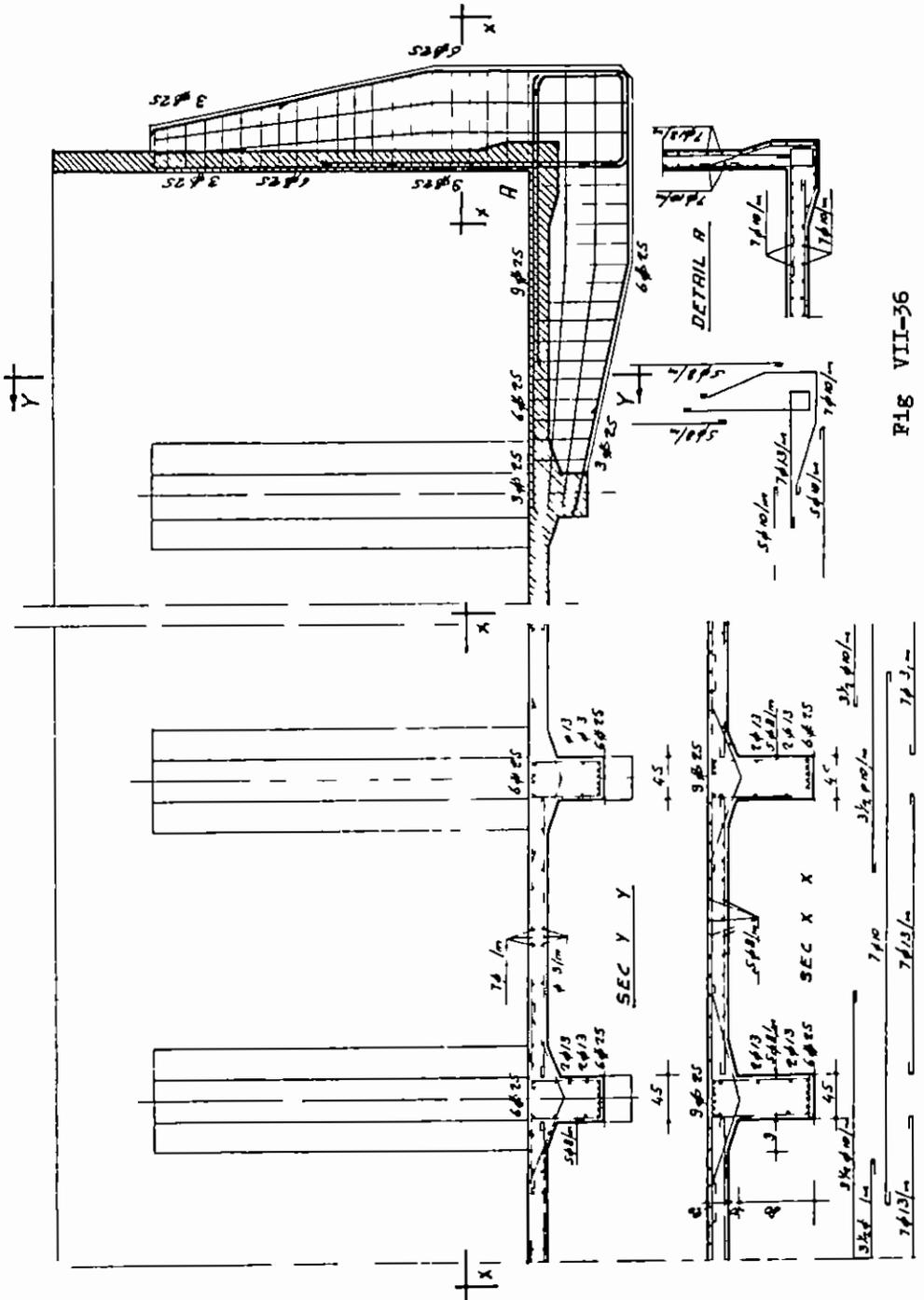
Cross-section max 45 x 110 cm and min 45 x 60 cm

Tension steel 9  $\phi$  25 mm on water side and 6  $\phi$  25 mm on earth side

#### 3 ) Intermediate floor beams:

Cross-section: 45 x 60 cm

Reinforcement: 3  $\phi$  25 mm top and bottom



F18 VII-36

## VIII TANKS DIRECTLY BUILT ON THE GROUND

In tanks directly built on the ground, we recognize three different cases, namely

- a) Tanks on fill or soft-weak soils,
- b) Tanks on rigid foundation and
- c) Tanks on compressible soils

### VIII 1 TANKS ON FILL OR SOFT WEAK SOILS

The stresses on the soil due to weight of tank and water is generally low ( $\sim 0.6 \text{ kg/cm}^2$  for a depth of water of 5 ms) but in spite of this fact, it is not recommended to construct a tank directly on unconsolidated fill as this may expose the tank to very serious differential settlements due to the unhomogenous texture of the fill

Soft weak clayey layers, peat and similar soils may consolidate to big values even under small stresses

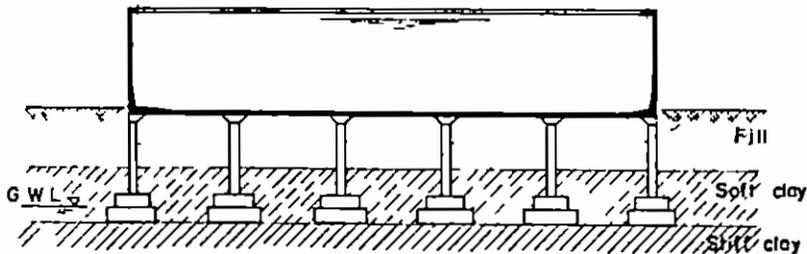
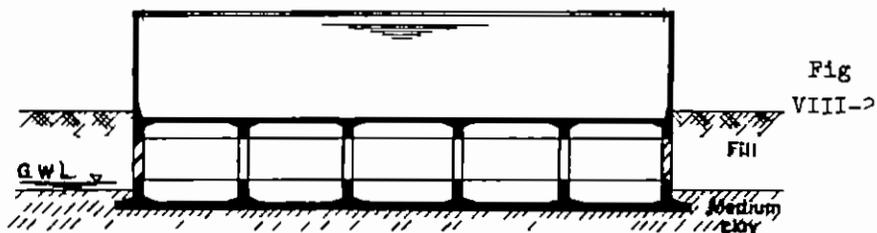


Fig VIII-1

In such cases, it is recommended to support the tank on columns and isolated or strip footings if the stiff soil layers are at a reasonable depth from the ground surface and no big difficulties

are liable to arise due to high ground water as shown in fig VIII 1 in which the floor of the tank, acting as a flat slab, is supported on columns and isolated footings resting on the stiff clay

In case of medium soils at foundation level, one may need a raft over the whole area as shown in figure VIII 2 The floor of the tank and the lower raft may be arranged as flat slabs If it is required to increase the stiffness of the tank, longitudinal and cross-girders may be arranged as shown



If the incompressible layers are deep or the ground water level is high one may support the tank on piles. It is recommended to drive the piles, where possible, to the incompressible layers and to arrange them such that we get just the floor thickness sufficient to give the necessary water-tightness ( 20 - 25 cms ) The pile caps may act as column heads for a flat slab system as shown in figure VIII 3

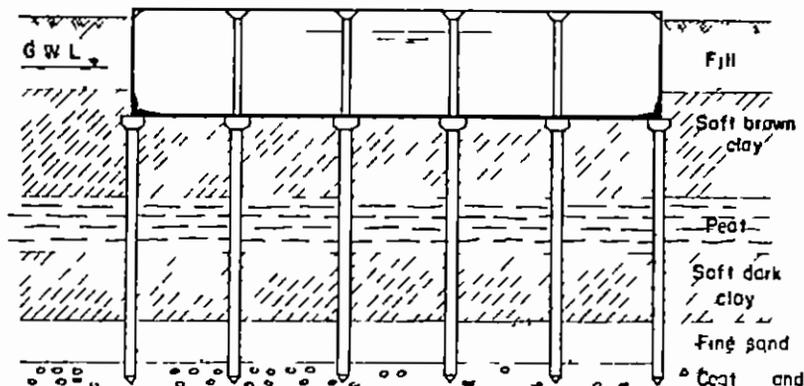
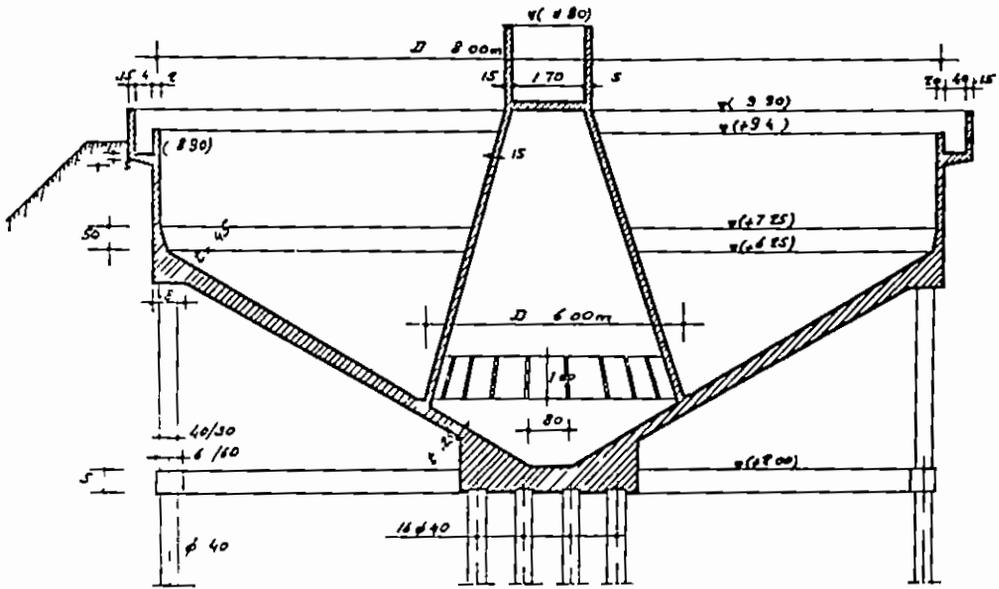
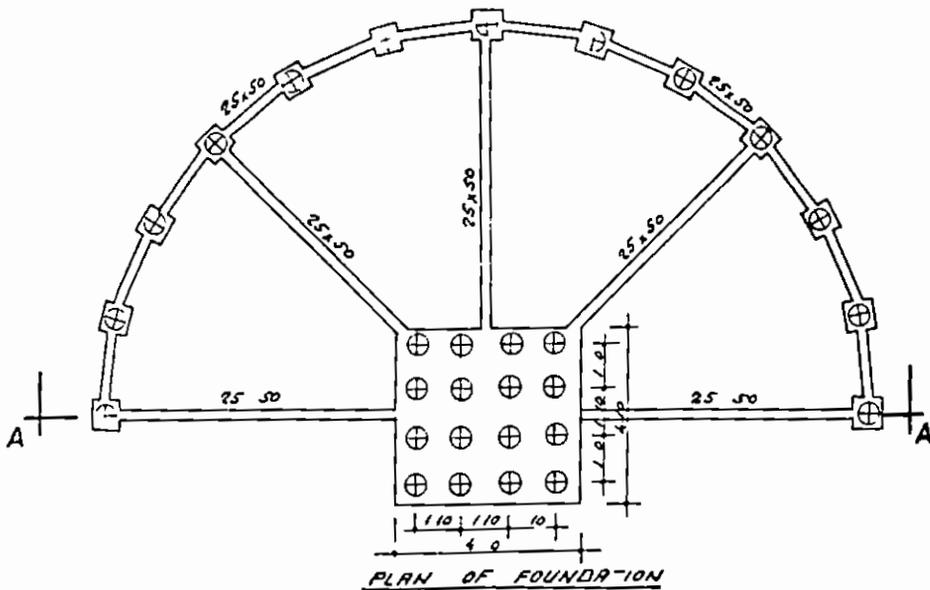


Fig VIII-3

In this example, the soil layers being soft and compressible, the weight of the tank and liquid will be transmitted to the incompressible



SECTION ELEVATION A A



PLAN OF FOUNDATION

WATER SUPPLY FLOCCULATOR

**Fig VIII-4**

sand layers through cast in place piles

The roof may be a flat slab or of the slab and beam type supported on columns. The columns and the floor, as a flat slab, are supported on piles, with the pile caps acting as column heads. Single piles are arranged such that each pile carries its full capacity and the required thickness of the floor slab is 20 to 25 cms. The steel tubes in which the concrete of the piles is cast are driven from the ground surface to the coarse sand layer, they are filled with concrete to the level of the floor and with sand fill to the ground surface.

We give in figure VIII 4 a circular flocculator which is to be constructed in very soft swampy soils on piles arranged in the outside perimeter below the outside vertical wall and in the center underneath the solid part shown.

VIII 2 TANKS ON RIGID FOUNDATION

If we assume that a tank is supported on a rigid foundation,

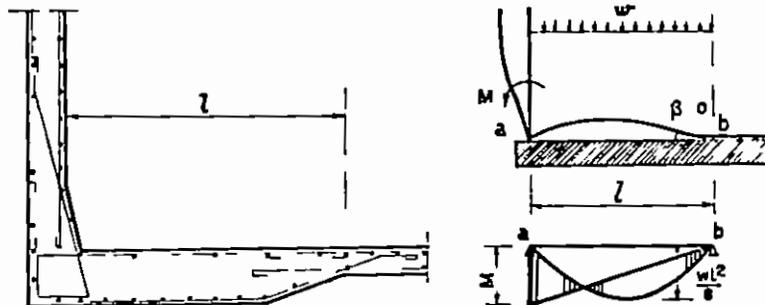


Fig VIII-5

then the vertical reaction of the wall will be directly resisted by the area beneath it, while the bending moment  $M$  will deflect the floor in a length  $l$  beyond which no deformation or bending moments are created. The deformation due to  $M$  will be counter balanced by the weight of the liquid and the floor  $w$  (Fig VIII 5) accordingly, we should have at point  $b$  :  $M_b = 0$  and  $\beta = 0$

But 
$$\beta = 0 = \frac{w l^3}{24 E I} - \frac{M l}{6 E I}$$

so that 
$$l = 2 \sqrt{\frac{M}{w}}$$

The part  $l$  of the floor slab is to be designed to resist the bending moments shown in figure VIII 5 plus an axial tensile force  $N$  equal to the reaction at the base of the wall. The middle part of the floor slab is to be designed to resist the tension  $N$  with a minimum thickness of 15 - 20 cms giving sufficient water-tightness. The minimum reinforcement is  $5 \phi 8$  mm/m in each direction on both surfaces of the slab.

### VIII 3 TANKS ON COMPRESSIBLE SOILS

Floors of tanks resting on medium clayey or sandy soils may be calculated in the following manner (Fig VIII 6)

We assume that the internal forces transmitted from the wall to the floor at  $b$  are distributed on the soil by the part  $bc = l$  of the floor where  $l = 0.4$  to  $0.6 H$ . The length  $l$  is chosen such that the max stress  $\sigma_1$  is smaller than the allowed soil bearing pressure, and  $\sigma_2 > \sigma_1 / 2$  on clayey soils and  $\sigma_2 > 0$  on sandy soils.

These limitations are recommended in order to prevent relatively big rotations of the floor at  $b$ .

The internal forces in the part  $bc$  of the floor are to be determined for the downward forces

- $G_1$  = weight of wall and roof (if any)
- $G_2$  = " floor  $cb$
- = ' ' water on  $cb$

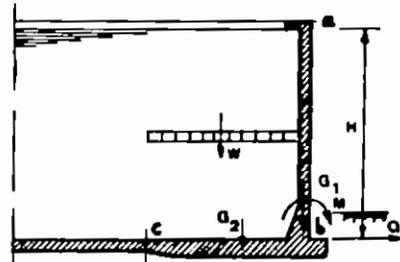


Fig VIII-6  $\sigma_2$   $\sigma_1$

plus the bending moment  $M$  and the shearing force  $Q$  at the foot of the wall and the corresponding upward stresses  $\sigma$

The part of the floor to the left of  $c$  is subject to tension equal to  $Q$ . Its thickness is chosen 15 to 20 cms to give the necessary watertightness, reinforced by two meshes of area  $A_B$  in any direction given by :

$$A_B = Q / \sigma_B$$

For the details of reinforcements refer to figure VIII 5. The floor of a tank directly built on stiff medium or weak compressible soils can however be treated as a floor on elastic foundation if the soil is homogenous and its elastic properties are known

#### General Theory of Beams on Elastic Foundation<sup>m</sup>

In order to determine the deformation and internal forces in a floor on elastic foundation, one has to know the general theory of beams on elastic foundation. We give in the following, the outlines of the theory as may be needed for our purpose

A beam  $A B$  supported along its entire length by an elastic medium and subjected to vertical forces  $p$  will deflect producing continuously distributed stresses  $\sigma$  in the supporting medium ( Fig VIII 7 )

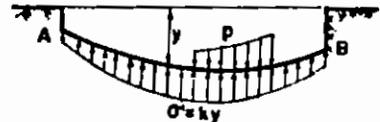


Fig VIII-7

The fundamental assumption of elastic supports is :

" the intensity of the stress  $\sigma$  at any point is proportional to the deflection of the beam  $y$  at that point " ; that is :

$$\sigma = k y$$

The elasticity of the supporting medium is characterized by the stress  $\sigma$  which causes a deflection equal to unity. This constant of the supporting medium  $k_0$   $\text{kg/cm}^3$  is called the modulus of the foundation

z Hetényi " Beams on Elastic Foundation "

Assume that the beam under consideration has a uniform cross section and that  $b$  is its constant width, which is supported on the foundation. A unit deflection of this beam will cause a stress  $b k_0$  in the foundation, consequently, at a point where the deflection is  $y$  the intensity of distributed reaction (per unit length of the beam) will be

$$\sigma \text{ kg/cm} = b k_0 y$$

For the sake of brevity we shall use the symbol  $k \text{ kg/cm}^2$  for  $b \text{ cms} \times k_0 \text{ kg/cm}^2$  in the following derivations, but it is to be remembered that this  $k$  includes the effect of the width of the beam and will be numerically equal to  $k_0$  only if we deal with a beam of unit width

Considering the equilibrium of an infinitely small element enclosed between two vertical cross-sections a distance  $dx$  apart on the beam we find (Fig VIII 8)

$$Q - (Q + dQ) + k y dx - p dx = 0$$

which gives

$$\frac{dQ}{dx} = k y - p \quad (a)$$

knowing that  $\frac{dM}{dx} = Q$ , we can write

$$\frac{dQ}{dx} = \frac{d^2 M}{dx^2} = k y - p \quad (b)$$

We know further that

$$\frac{d^2 y}{dx^2} = - \frac{M}{E I} \quad (c)$$

Differentiating this relation twice, we obtain

$$E I \frac{d^4 y}{dx^4} = - \frac{d^2 M}{dx^2} \quad (d)$$

Relations (b) and (d) give

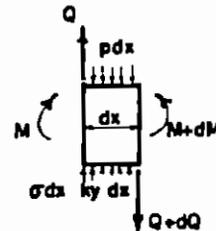


Fig VIII-8

$$E I \frac{d^4 y}{dx^4} = -k y + p \quad (e)$$

which gives the differential equation of the elastic line of a beam on an elastic foundation. Along the unloaded parts of the beam, where no distributed load is acting,  $p = 0$ , and equ (e) will take the form :

$$E I \frac{d^4 y}{dx^4} = -k y \quad (f)$$

It will be sufficient to consider below only the general solution of (f) from which solutions will be obtained also for cases implied in (e) by adding to it a particular integral corresponding to  $p$ . Assuming further that

$$n = \sqrt[4]{\frac{k}{E I}} \quad (g)$$

we can give the solution of the differential equation (f) in the form

$$y = e^{nx} (A_1 \cos nx + B_1 \sin nx) + e^{-nx} (B_1 \cos nx + B_2 \sin nx) \quad (15)$$

Here  $n$  includes the flexural rigidity of the beam as well as the elasticity of the supporting medium, and is an important factor influencing the shape of the elastic line. For this reason the factor  $n$  is called the characteristic of the system, and, since its dimension is  $\text{length}^{-1}$ , the term  $1/n$  is frequently referred to as the characteristic length. Consequently,  $nx$  will be an absolute number.

Differentiating equation (15), we get

$$\begin{aligned} \frac{1}{n} \frac{dy}{dx} &= e^{nx} \left[ A_1 (\cos nx - \sin nx) + A_2 (\cos nx + \sin nx) \right] \\ &\quad - e^{-nx} \left[ B_1 (\cos nx + \sin nx) - B_2 (\cos nx - \sin nx) \right] \\ \frac{1}{2n^2} \frac{d^2 y}{dx^2} &= -e^{nx} (A_1 \sin nx - A_2 \cos nx) + e^{-nx} (B_1 \sin nx - B_2 \cos nx) \\ \frac{1}{3n^3} \frac{d^3 y}{dx^3} &= -e^{nx} \left[ A_1 (\cos nx + \sin nx) - A_2 (\cos nx - \sin nx) \right] \\ &\quad + e^{-nx} \left[ B_1 (\cos nx - \sin nx) + B_2 (\cos nx + \sin nx) \right] \end{aligned} \quad (16)$$

Knowing that

$$\frac{dy}{dx} = \tan \theta, \quad -EI \frac{d^2y}{dx^2} = M \quad \text{and} \quad -EI \frac{d^3y}{dx^3} = Q$$

we can obtain the general expressions for the slope  $\theta$  ( $\theta = \tan \theta$ ) of the deflection line as well as for the bending moment  $M$  and the shearing force  $Q$ . The intensity of pressure in the foundation will be found from equation (15) to be  $\sigma = k y$

In applying these general equations, or corresponding ones including the term depending on  $p$ , to particular cases the next step is to determine the integration constants  $A_1, A_2, B_1$  and  $B_2$ . These integration constants depend on the manner in which the beam is subjected to the loading and have constant values along each portion of the beam within which the elastic line and all its derivatives are continuous. Their values can be obtained from the conditions existing at the two ends of such continuous portions. Out of the four quantities ( $y, \theta, M$  and  $Q$ ) characterizing the condition of an end two are usually known at each end from which sufficient data are available for the determination of the constants  $A$  and  $B$ .

#### Internal Forces for Particular Cases of Loading

In the following formulae, we have

|                          |                               |                     |
|--------------------------|-------------------------------|---------------------|
| The characteristic value | $n = \sqrt[4]{\frac{k}{4EI}}$ |                     |
| The foundation modulus   | $k_0$                         | in $\text{kg/cm}^3$ |
|                          | $k = b k_0$                   | in $\text{kg/cm}^2$ |

Constant width of beam in contact with the foundation =  $b$  cm

The flexural rigidity of the beam =  $E I$

The deflection  $y$ , slope angles  $\theta$ , bending moments  $M$ , and shearing forces  $Q$  are given in the following for beams with free ends under some particular cases of loading

The numerical values of the trigonometric and hyperbolic functions needed in the different equations are given in the appendix

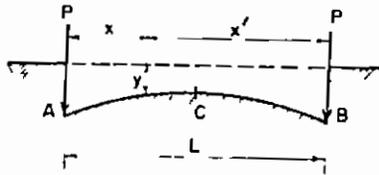
1) Equal Concentrated Forces at Both Ends

Fig VIII-9

$$y = \frac{2 P n}{k} \frac{\cosh nx \cos nx' + \cosh nx' \cos nx}{\sinh nL + \sin nL} \quad (17)$$

Deflection at the end points

$$y_A = y_B = \frac{2 P n}{k} \frac{\cosh nL + \cos nL}{\sinh nL + \sin nL} \quad (17a)$$

Deflection at the middle

$$y_C = \frac{4 P n}{k} \frac{\cosh \frac{nL}{2} \cos \frac{nL}{2}}{\sinh nL + \sin nL} \quad (17b)$$

The deflection  $y_C = 0$  when  $nL = \pi, 3\pi, 5\pi, \text{ etc}$ 

Slope angles at end points A and B

$$\theta_A = -\theta_B = -\frac{2 P n^2}{k} \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL} \quad (18)$$

The bending moment

$$= -\frac{P}{n} \frac{\sinh nx \sin nx' + \sinh nx' \sin nx}{\sinh nL + \sin nL} \quad (19)$$

The bending moment at the middle

$$M_C = -\frac{2 P}{n} \frac{\sinh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad (19a)$$

 $M_C = 0$  when  $nL = 2\pi, 4\pi, 6\pi, \text{ etc}$  $M_C$  is negative when  $nL < 2\pi$  $M_C$  is positive when  $2\pi < nL < 4\pi$ 

The shearing force

$$Q = P \frac{1}{\sinh nL + \sin nL} (\sinh nx \cos nx' - \cosh nx \sin nx' + \cosh nx' \sin nx - \sinh nx' \cos nx) \quad (20)$$

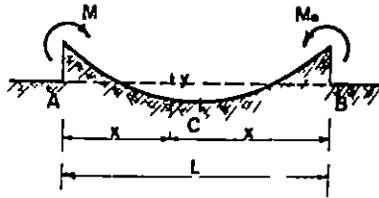
2) Equal Concentrated Moments at Both Ends :

Fig VIII-10

$$y = - \frac{2 M_0 n^2}{k} \frac{1}{\sinh nL + \sin nL} (\sinh nx \cos nx' - \cosh nx \sin nx' + \sinh nx' \cos nx - \cosh nx' \sin nx) \quad (21)$$

Deflection at end points

$$y_A = y_B = - \frac{2 M_0 n^2}{k} \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL} \quad (21a)$$

Slope at end points

$$\theta_A = - \theta_B = \frac{4 M_0 n^3}{k} \frac{\cosh nL - \cos nL}{\sinh nL + \sin nL} \quad (22)$$

The bending moment

$$M = M_0 \frac{1}{\sinh nL + \sin nL} (\sinh nx \cos nx' + \cosh nx \sin nx' + \sinh nx' \cos nx + \cosh nx' \sin nx) \quad (23)$$

The bending moment at the middle

$$M_C = 2 M_0 \frac{\sinh \frac{nL}{2} \cos \frac{nL}{2} + \cosh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad (23a)$$

$$M_0 = 0 \text{ when } \cos \frac{nL}{2} = - \sin \frac{nL}{2}, \text{ that is,}$$

$$\text{when } nL = \frac{3}{2} \pi, \frac{7}{2} \pi, \frac{11}{2} \pi, \text{ etc}$$

The shearing force :

$$Q = 2 M_0 n \frac{\sinh nx \sin nx' - \sinh nx' \sin nx}{\sinh nL + \sin nL} \quad (24)$$

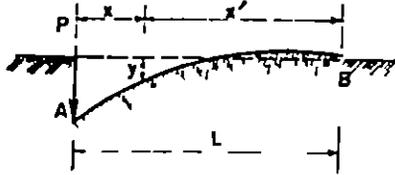
3) Concentrated Force at One End

Fig VIII-11

$$y = \frac{2 P n}{k} \frac{\sinh nL \cos nx \cosh nx' - \sin nL \cosh nx \cos nx'}{\sinh^2 nL - \sin^2 nL} \quad (25)$$

Deflection at the end points

$$\left. \begin{aligned} y_A &= \frac{2 P n}{k} \frac{\sinh nL \cosh nL - \sin nL \cos nL}{\sinh^2 nL - \sin^2 nL} \\ y_B &= \frac{2 P n}{k} \frac{\sinh nL \cos nL - \sin nL \cosh nL}{\sinh^2 nL - \sin^2 nL} \end{aligned} \right\} \quad (25a)$$

The deflection  $y_B = 0$  when  $nL = \frac{5\pi}{4}, \frac{9}{4}\pi, \text{ etc}$

Slopes at the end points :

$$\left. \begin{aligned} \theta_A &= - \frac{2 P n^2}{k} \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL} \\ \theta_B &= - \frac{4 P n^2}{k} \frac{\sinh nL \sin nL}{\sinh^2 nL - \sin^2 nL} \end{aligned} \right\} \quad (26)$$

The slope  $\theta_B = 0$  when  $nL = \pi, 2\pi, 3\pi, \text{ etc}$

The bending moment

$$M = - \frac{P}{n} \frac{\sinh nL \sin nx \sinh nx' - \sin nL \sinh nx \sin nx'}{\sinh^2 nL - \sin^2 nL} \quad (27)$$

The shearing force

$$Q = - P \frac{1}{\sinh^2 nL - \sin^2 nL} \left[ \sinh nL (\cos nx \sin nx' - \sin nx \cosh nx') - \sin nL (\cosh nx \sin nx' - \sinh nx \cos nx') \right] \quad (28)$$

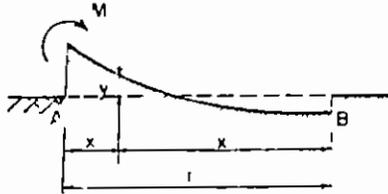
4) Concentrated Moment at One End

Fig VIII-12

$$y = \frac{2 M_0 n^2}{k} \frac{1}{\sinh^2 nL - \sin^2 nL} \left[ \sinh nL (\cos nx \sin nx - \sinh nx' \cos nx) + \sin nL (\sinh nx \cos nx' - \cosh nx \sin nx) \right] \quad (29)$$

Deflection at the end points

$$\left. \begin{aligned} y_A &= \frac{2 M_0 n^2}{k} \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL} \\ y_B &= \frac{4 M_0 n^2}{k} \frac{\sinh nL \sin nL}{\sinh^2 nL - \sin^2 nL} \end{aligned} \right\} \quad (29a)$$

The deflection  $y_B = 0$  when  $nL = \pi, 2\pi, 3\pi, \text{ etc}$ 

Slopes at the end points

$$\left. \begin{aligned} \theta_A &= \frac{4 M_0 n^3}{k} \frac{\sinh nL \cosh nL + \sin nL \cos nL}{\sinh^2 nL - \sin^2 nL} \\ \theta_B &= \frac{4 M_0 n^3}{k} \frac{\sinh nL \cos nL + \sin nL \cosh nL}{\sinh^2 nL - \sin^2 nL} \end{aligned} \right\} \quad (30)$$

The slope  $\theta_B = 0$  when  $nL = \frac{3}{4}\pi, \frac{7}{4}\pi, \frac{11}{4}\pi, \text{ etc}$ 

The bending moment

$$M = M_0 \frac{1}{\sinh^2 nL - \sin^2 nL} \left[ \sinh nL (\sinh nx' \cos nx + \cosh nx' \sin nx) - \sin nL (\sinh nx \cos nx' + \cosh nx \sin nx) \right] \quad (31)$$

The shearing force

$$Q = 2 M_0 n \frac{\sinh nL \sinh nx' \sin nx + \sin nL \sinh nx \sin nx}{\sinh^2 nL - \sin^2 nL} \quad (32)$$

Foundation Modulus  $k_0$  :The magnitude of the foundation or subgrade modulus  $k_0$  depends

mainly on the following factors

- 1) Kind of soil and its properties
- 2) Internal friction between soil particles and their water content
- 3) Size and form of loaded area
- 4) Depth of foundation base from ground level

We give in the following the approximate average values of  $k_0$

|                                      |           |                    |
|--------------------------------------|-----------|--------------------|
| Peat                                 | 0 5 - 1 0 | kg/cm <sup>3</sup> |
| Fill of sand and gravel              | 1 0 - 2 0 | "                  |
| Wet clayey soil                      | 2 0 - 3 0 | '                  |
| Moistured clay                       | 4 0 - 5 0 | '                  |
| Dry clay                             | 6 0 - 8 0 | '                  |
| Hard dry clay                        | 10        | kg/cm <sup>3</sup> |
| Coarse sand                          | }         | 8 - 10             |
| Coarse sand + small amount of gravel |           |                    |
| Fine gravel + small amount of sand   | }         | 10 - 12            |
| Middle size gravel + fine sand       |           |                    |
| Middle size gravel + coarse sand     | 12 - 15   | '                  |
| Large size gravel + coarse sand      | 15 - 20   |                    |

### Examples

- 1) In figure VIII 13 is shown the cross-section of an aqueduct It is required to find the pressure distribution in the sub-soil and the moment diagram for the bottom plate

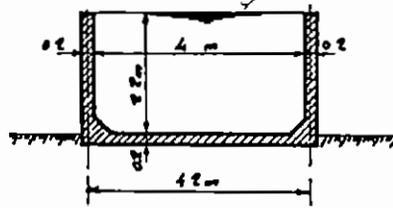
Assume :

$$E_c = 200\ 000\ \text{kg/cm}^2 \quad \text{and} \quad k_0 = 5\ \text{kg/cm}^2$$

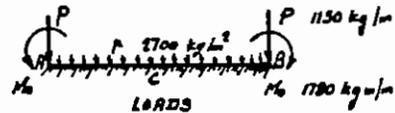
Solution :

The bottom plate can be regarded as a beam on elastic foundation subject to

- i) A uniformly distributed loading  $p$  equal to the weight of the bottom plate itself plus the weight of the water,
- ii) Two concentrated forces  $P$  due to the weight of the side walls
- iii) Two moments  $M_0$  due to the hydrostatic pressure



Computing the loads of the section per meter, we get



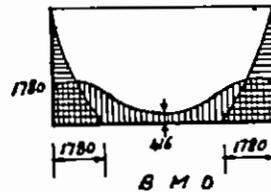
Own weight of bottom plate  
 $= 0.2 \times 2500 = 500 \text{ kg/m}^2$

Own weight of water  
 $= 2.2 \times 1000 = 2200 \text{ ''}$

Total  $p = 2700 \text{ ''}$



Corresponding stress on soil  
 $= 0.27 \text{ kg/cm}^2$



Own weight of wall  
 $P = 2.30 \times 500 = 1150 \text{ kg/m}$

Fig VIII-13

Moment due to water pressure  $M_0 = -2200 \times 2^2 / 6 = -1780 \text{ kg-m}$

The deflection and moments of the floor can be calculated with the help of equations 17, 19 and 21, 23 thus:

$$k = b k_0 = 100 \times 5 = 500 \text{ kg/cm}^2$$

$$I = b t^3 / 12 = 100 \times 20^3 / 12 = 6.67 \times 10^4 \text{ cm}^4$$

$$n = \sqrt[4]{\frac{k}{4 E I}} = \sqrt[4]{\frac{500}{4 \times 200000 \times 6.67 \times 10^4}} \approx 0.010$$

The deflection at the edges  $y_A$  and  $y_B$ , and at the middle  $y_G$  due to the forces  $P = 11.5 \text{ kg/cm}$  is, according to equation 17-a & b, given by:

$$y_A = y_B = \frac{2 P n}{k} \frac{\cosh nL + \cos nL}{\sinh nL + \sin nL} \quad \& \quad y_C = \frac{4 P n}{k} \frac{\cosh \frac{nL}{2} + \cos \frac{nL}{2}}{\sinh nL + \sin nL}$$

in which

$$nL = 0.01 \times 420 = 4.20, \quad nL/2 = 0.01 \times 420/2 = 2.1$$

$$\sin nL = -0.872, \quad \cos nL = -0.490, \quad \sin \frac{nL}{2} = 0.863, \quad \cos \frac{nL}{2} = -0.505$$

$$\sinh nL = 33.3357, \quad \cosh nL = 33.3507, \quad \sinh \frac{nL}{2} = 4.0219, \quad \cosh \frac{nL}{2} = 4.1443$$

So that

$$y_A = y_B = \frac{2 \times 11.5 \times 0.1}{500} \times \frac{33.3507 - 0.490}{33.3357 - 0.872} = 4.65 \times 10^{-4} \text{ cm}$$

$$\text{and } y_C = \frac{4 \times 11.5 \times 0.1}{500} \times \frac{-4.1443 \times 0.505}{33.3357 - 0.872} = -0.59 \times 10^{-4} \text{ cm}$$

The stresses due to the forces  $P$  are therefore

$$\sigma_A = \sigma_B = k y_A = 500 \times 4.65 \times 10^{-4} = 0.233 \text{ kg/cm}^2$$

$$\text{and } \sigma_C = k y_C = -500 \times 0.59 \times 10^{-4} = -0.030 \text{ "}$$

The bending moment  $M_C$  at the middle point of the bottom due to the forces  $P$  is, according to equation (19a) given by

$$M_C = - \frac{2 P}{n} \frac{\sinh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad \text{or}$$

$$M_C = - \frac{2 \times 11.5}{0.01} \times \frac{4.0219 \times 0.863}{33.3357 - 0.872} = -246 \text{ kg cm/cm}$$

The deflection at the edge and middle points of the bottom plate due to the moments  $M_0 = -1780 \text{ kgcm/cm}$  is according to equation (21), given by

$$y_A = y_B = - \frac{2 M_0 n^2}{k} \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL}$$

$$y_C = - \frac{4 M_0 n^2}{k} \frac{\sinh \frac{nL}{2} \cos \frac{nL}{2} - \cosh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL}$$

So that :

$$y_A = y_B = \frac{2 \times 1780 \times 0.1^2}{500} \frac{33\ 3357 + 0\ 872}{33\ 3357 - 0\ 872} = 7\ 50 \times 10^{-4} \text{ cm}$$

$$y_C = \frac{4 \times 1780 \times 0.1^2}{500} \frac{-4\ 0219 \times 0\ 505 - 4\ 1443 \times 0\ 863}{33\ 3357 - 0\ 872} = -2\ 45 \times 10^{-4} \text{ cm}$$

The stresses due to the moments  $M_0$  are therefore :

$$\sigma_A = \sigma_B = k y_A = 500 \times 7\ 5 \times 10^{-4} = 0\ 375 \text{ kg/cm}^2$$

$$\sigma_C = k y_C = 500 \times 2\ 45 \times 10^{-4} = -0\ 123 \text{ "}$$

The bending moment at the middle point of the bottom plate due to the edge moments  $M_0$  is, according to equation 23-a, given by :

$$M_C = 2 M_0 \frac{\sinh \frac{nL}{2} \cos \frac{nL}{2} + \cosh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad \text{or}$$

$$M_C = -2 \times 1780 \frac{-4\ 0219 \times 0\ 505 + 4\ 1443 \times 0\ 863}{33\ 3357 - 0\ 872} = -170 \text{ kgcm/cm}$$

The total stresses and bending moments are therefore : (Fig. VIII 13)

$$\sigma_A = \sigma_B = 0\ 270 + 0\ 233 + 0\ 375 = 0\ 878 \text{ kg/cm}^2$$

$$\sigma_C = 0\ 270 - 0\ 030 - 0\ 123 = 0\ 117 \text{ "}$$

$$M_C = -246 - 170 = -416 \text{ kgm/m}$$

2) It is required to determine the stress distribution and bending moments of the floor of the diving pool shown in figure VII 32 for the case of water pressure assuming  $E = 2000 \text{ kg/cm}^2$  and  $k_0 = 10 \text{ kg/cm}^3$ . The solution will be done for a strip  $B = 3\ 33 \text{ m}$  (distance between center lines of floor panels)

In order to simplify the computations, the floor girders will be assumed with a constant average depth of 90 cms

The floor strip is subject to : (Fig VIII 14)

i) The weight of water and floor  $p$  where

$$p = 3\ 33 (5\ 0 + 0\ 2 \times 2\ 5 + 0\ 45 \times 0\ 7 \times 2\ 5) = 19\ 1 \text{ t/m}$$

ii) The weight of the wall slab and counterfort  $P$  where

$$P = 3\ 33 \times 0\ 2 \times 5 \times 2\ 5 + 0\ 45 \times 0\ 7 \times 4\ 5 \times 2\ 5 = 11\ 9 \text{ t}$$

iii) A concentrated bending moment at the outer edge M due to the fixing moments of the counterfort and the wall given by :

$$M = 65.23 + \frac{2}{3} \times 2.34 \times 3.33 = 65.23 + 5.2 \approx 70 \text{ mt}$$

Area of the section : (Fig VIII 15)

$$A = 0.2 \times 3.33 + 0.7 \times 0.45 = 0.981 \text{ m}^2$$

Statical moment about mid-height of flange:

$$S = 0.45 \times 0.7 \times 0.45 = 0.142 \text{ m}^2$$

Distance between center of gravity of flange and center of gravity of section is :

$$\frac{0.142}{0.981} = 0.145 \text{ m}$$

So that the distance of the center of gravity is equal to :

0.245 ms from top surface of the flange

0.655 ms from bottom " " " web

The moment of inertia of the section about its center of gravity :

$$I = \frac{0.45 \times 0.7^3}{12} + 0.45 \times 0.7 \times 305^2 + 3.33 \times \frac{0.2^3}{12} + 2.33 \times 0.2 \times 145^2 = 1853 \times 10^4 \text{ cm}^4$$

The characteristic value n :

$$n = \sqrt[4]{\frac{B k_0}{4 E I}} = \sqrt[4]{\frac{333 \times 10}{4 \times 20 \times 10^4 \times 1853 \times 10^4}} = 0.00385 \text{ cm}^{-1}$$

Therefore :

$$nL = \frac{3.85}{1000} \times 333 = 1.28$$

$$\frac{nL}{2} = 0.64$$

$$\sin nL = 0.958$$

$$\sin \frac{nL}{2} = 0.597$$

$$\cos nL = 0.287$$

$$\cos \frac{nL}{2} = 0.802$$

$$\sinh nL = 1.6593$$

$$\sinh \frac{nL}{2} = 0.6846$$

$$\cosh nL = 1.9373$$

$$\cosh \frac{nL}{2} = 1.2119$$

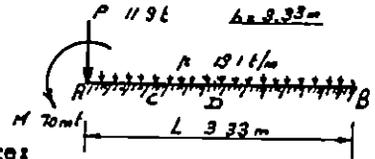


Fig VIII-14

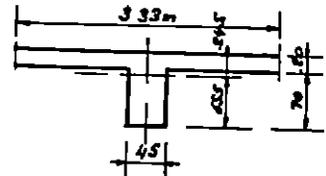


Fig VIII-15

The deflection at the edges  $y_A$  and  $y_B$  due to the force

$$P = \frac{11900}{333} = 35.7 \text{ kg/cm is given according to equations(25a) by}$$

$$y_A = \frac{2 P n}{k} \frac{\sinh nL \cosh nL - \sin nL \cos nL}{\sinh^2 nL - \sin^2 nL}$$

$$y_B = \frac{2 P n}{k} \frac{\sinh nL \cos nL - \sin nL \cosh nL}{\sinh^2 nL - \sin^2 nL}$$

So that

$$y_A = \frac{2 \times 35.7 \times 0.00385}{3330} \frac{1.6593 \times 1.9373 - 0.958 \times 0.287}{1.6593^2 - 0.958^2} = 1.32 \times 10^{-4} \text{ cm}$$

$$y_B = \frac{2 \times 35.7 \times 0.00385}{3330} \frac{1.6593 \times 0.287 - 0.958 \times 1.9373}{1.6593^2 - 0.958^2} = -6.25 \times 10^{-4} \text{ cm}$$

The deflection at the quarter point C and middle point D due to the force P is given according to equation 25, by :

$$y = \frac{2 P n}{k} \frac{\sinh nL \cos nx \cosh nx' - \sin nL \cosh nx \cos nx'}{\sinh^2 nL - \sin^2 nL}$$

$$\text{For point C } x = L/4 \quad x' = 3L/4 \quad nx = 0.34 \quad nx' = 1.02$$

$$\text{For point D } x = L/2 \quad x' = L/2 \quad nx = nx' = 0.68$$

$$\cos 0.34 = 0.943 \quad \cosh 0.34 = 1.0584$$

$$\cos 0.68 = 0.778 \quad \cosh 0.68 = 1.2402$$

$$\cos 1.02 = 0.523 \quad \cosh 1.02 = 1.5669$$

So that :

$$y_C = 0.865 \times 10^{-4} \text{ cm}$$

$$y_D = 0.306 \times 10^{-4} \text{ cm}$$

The bending moments at points C and D due to the concentrated load P can be calculated from equation 27, thus :

$$M = -\frac{P}{n} \frac{\sinh nL \sin nx \sinh nx' - \sin nL \sinh nx \sin nx'}{\sinh^2 nL - \sin^2 nL}$$

in which :

$$\text{For point C } nx = 0.34 \quad nx' = 1.02$$

$$\text{For point D } nx = nx' = 0.68$$

$$\sin 0.34 = 0.334 \quad \sinh 0.34 = 0.3466$$

$$\sin 0.68 = 0.629 \quad \sinh 0.68 = 0.7336$$

$$\sin 1.02 = 0.852$$

$$\sinh 1.02 = 1.2063$$

So that :

$$M_C = -1940 \text{ kgcm/cm}$$

$$M_D = -1630 \text{ kgcm/cm}$$

The deflection at the edges  $y_A$  and  $y_B$  due to the moment

$$M = -\frac{70 \times 10^5}{333} = -21000 \text{ kgcm/cm} \text{ is given according to equations}$$

29-a, by the relations :

$$y_A = -\frac{2 M n^2}{k} \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL}$$

$$y_B = \frac{4 M n^2}{k} \frac{\sinh nL \sin nL}{\sinh^2 nL - \sin^2 nL}$$

So that :

$$y_A = \frac{2 \times 21000}{3330} \left( \frac{3.85}{1000} \right)^2 \frac{1.6593^2 + 0.958^2}{1.6593^2 - 0.958^2} = 3.76 \times 10^{-4} \text{ cm}$$

$$y_B = -\frac{4 \times 21000}{3330} \left( \frac{3.85}{1000} \right)^2 \frac{1.6593 \times 0.958}{1.6593^2 - 0.958^2} = -3.23 \times 10^{-4} \text{ cm}$$

The deflection at the quarter point C and middle point D due to M can be calculated from equation 29 giving :

$$y_C = 1.73 \times 10^{-4} \text{ cm} \quad y_D = -0.143 \times 10^{-4} \text{ cm}$$

The bending moments at C and D due to M can be calculated from equation 31 giving :

$$M_C = -19500 \text{ kgcm/cm} \quad M_D = -10950 \text{ kgcm/cm}$$

The stress due to the uniform load p is given by :

$$\sigma = \frac{19100}{100 \times 333} = 0.573 \text{ kg/cm}^2$$

The corresponding deflection y is therefore :

$$y = \sigma / k = 0.573/3330 = 1.72 \times 10^{-4} \text{ cm}$$

The deflections, stresses and bending moments are as shown in the following table : Fig VIII 16a

| Case                               | Point | A                       | C                        | D                        | B                           |
|------------------------------------|-------|-------------------------|--------------------------|--------------------------|-----------------------------|
| deflection y<br>due to             | P     | $1\ 720 \times 10^{-4}$ | $1\ 720 \times 10^{-4}$  | $1\ 720 \times 10^{-4}$  | $1\ 720 \times 10^{-4}$ cm  |
|                                    | P     | $1\ 320 \times 10^{-4}$ | $0\ 865 \times 10^{-4}$  | $0\ 30 \times 10^{-4}$   | $-0\ 625 \times 10^{-4}$ cm |
|                                    | M     | $3\ 760 \times 10^{-4}$ | $1\ 730 \times 10^{-4}$  | $-0\ 143 \times 10^{-4}$ | $-3\ 230 \times 10^{-4}$ cm |
|                                    | P+P+M | $6\ 800 \times 10^{-4}$ | $4\ 315 \times 10^{-4}$  | $1\ 883 \times 10^{-4}$  | $-2\ 125 \times 10^{-4}$ cm |
| Stress $\sigma = ky$<br>$= 3330 y$ |       | 2 266                   | 1 438                    | 0 628                    | $-0\ 708 \text{ kg/cm}^2$   |
| B 1<br>due to                      | P     | 0                       | - 1940                   | - 1630                   | 0 kgcm/cm                   |
|                                    | M     | - 21000                 | - 19500                  | - 10950                  | 0 "                         |
|                                    | P + M | - 21000<br>$= M_0$      | - 21440<br>$= 1\ 02 M_0$ | - 12580<br>$= '0\ 6 M_0$ | 0 ' "                       |

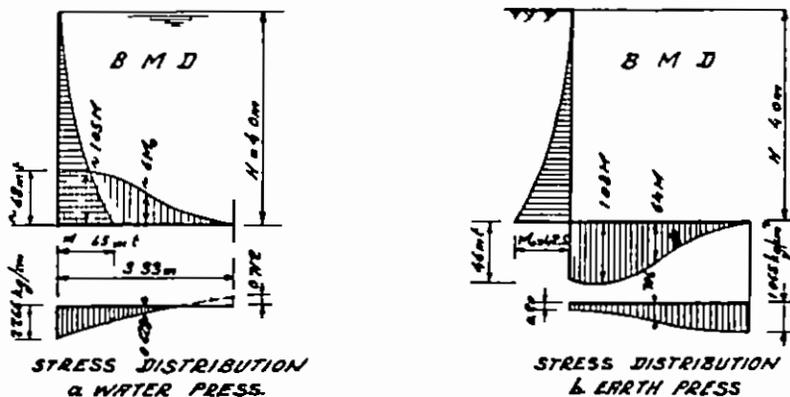


Fig VIII-16

The maximum bending moment can therefore be estimated by  $1\ 05 M_0$ , that is, for a maximum bending moment in the counterfort  $M_0 = 65 \text{ m t}$ , the maximum bending moment in the floor beam  $= 1\ 05 \times 65 = 68 \text{ m t}$ . Assuming the bending moment due to earth pressure is  $0\ 65$  that

of water pressure and of opposite sense, the deflections, stresses bending moments of the floor beam will be as shown in the following table Fig VIII 16b

| Case                               | Point | A                        | C                         | D                        | B                           |
|------------------------------------|-------|--------------------------|---------------------------|--------------------------|-----------------------------|
| Deflection $y$<br>due to           | p     | $1\ 720 \times 10^{-4}$  | $1\ 720 \times 10^{-4}$   | $1\ 720 \times 10^{-4}$  | $1\ 720 \times 10^{-4}$ cm  |
|                                    | P     | $1\ 320 \times 10^{-4}$  | $0\ 865 \times 10^{-4}$   | $0\ 306 \times 10^{-4}$  | $-0\ 625 \times 10^{-4}$ cm |
|                                    | d     | $-2\ 440 \times 10^{-4}$ | $-1\ 120 \times 10^{-4}$  | $0\ 093 \times 10^{-4}$  | $2\ 100 \times 10^{-4}$ cm  |
|                                    | p+P+d | $0\ 600 \times 10^{-4}$  | $1\ 465 \times 10^{-4}$   | $2\ 119 \times 10^{-4}$  | $3\ 195 \times 10^{-4}$ cm  |
| Stress $\sigma = ky$<br>$= 3330 y$ |       | 0 20                     | 0 488                     | 0 706                    | 1 065 kg/cm <sup>2</sup>    |
| B i<br>due to                      | P     | 0                        | - 1940                    | - 1630                   | 0                           |
|                                    | d     | - 13700                  | - 12700                   | - 7100                   | 0                           |
|                                    | P + d | - 13700<br>$= M_0$       | - 14640<br>$= 1\ 07\ M_0$ | - 8730<br>$= 0\ 64\ M_0$ | 0                           |

## IX BUNKERS AND SILOS

### IX 1 DEFINITIONS

Bunkers and silos are containers used for storing dry materials such as cement, corn, coal etc They are generally filled from the top and emptied from the bottom

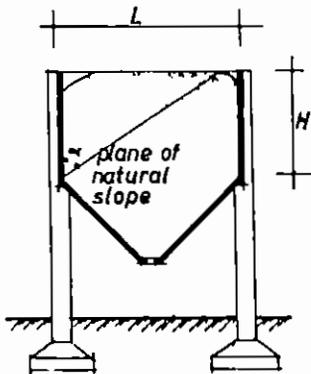


Fig IX-1 BUNKER

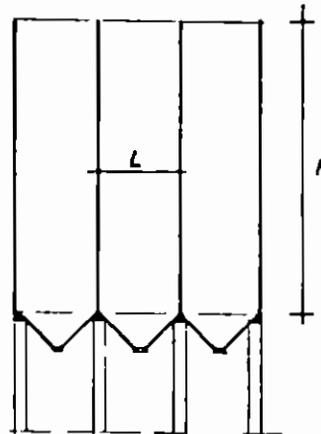


Fig IX-2 SILO

In bunkers, figure IX-1, the plane of natural repose of the filling material must not intersect the opposite wall i e

$$H \tan \alpha < L \quad \text{practically} \quad H < 1.5 L_{\max}$$

whereas in silos, figure IX-2, the depth is big in proportion to the section, and the plane of natural repose generally intersects the opposite wall i e

$$H \tan \alpha > L \quad \text{practically} \quad H > 1.5 L_{\max}$$

IX 2 LAYOUT OF SILO CELLS

Silos are generally composed of a series of square, hexagonal, octagonal or circular deep cells as shown in fig IX-3. The dead zones

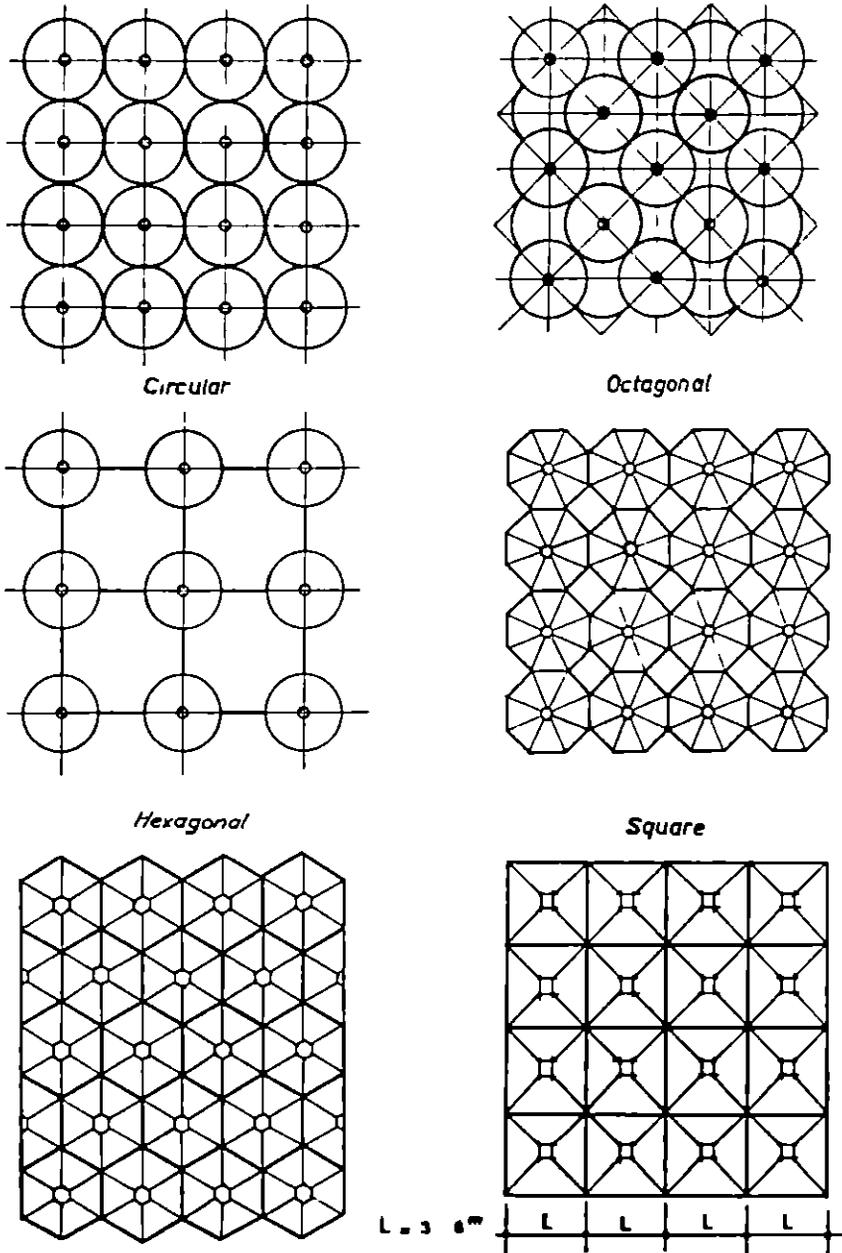


Fig IX-3 Arrangement of Silo-Cells in Plan

between octagonal or circular cells do not exist in case of square, rectangular or hexagonal cells ; these zones may however be used for ventilation, piping, lifts etc

The bending moments in walls of circular cells are nil and in walls of octagonal cells are smaller than in walls of hexagonal or rectangular cells

In order to reduce the bending moments in exterior walls, the cells may be constructed in the form shown in figure IX-4

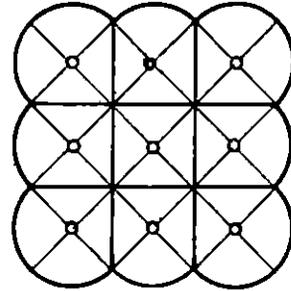


Fig IX-4 CELLS WITH CURVED OUTSIDE WALL

Silos are generally provided with hoppers, the slope of their sides is to be chosen 5% more than the natural slope of the filling material in order to empty the cells without difficulty. The sloping sides of the hoppers may be of reinforced concrete, but due to the difficulties encountered in determining the stress distribution and in the shuttering and reinforcements, it may be easier to make a flat floor for the cells and to make the sloping sides of the hopper by using lean plain concrete as shown in figure IX-5, in which the

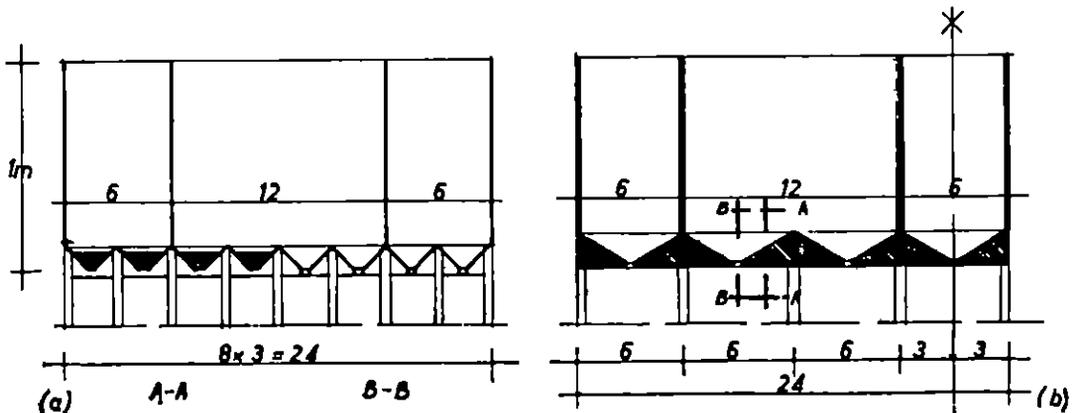


Fig IX-5

silos are 24 meters wide, 42 meters long and 11 meters deep and composed of two small cells 6 x 6 m and two inside big cells 12 x 12 m

The thickness of the walls is generally chosen to follow the values of the internal forces, which are small at the top and increase with the depth i e the walls are chosen of a trapezoidal shape with maximum thickness at the base Recently, big silos are constructed by using sliding forms which take a relatively short time in construction In order to be able to use such forms, the choice of a constant wall thickness is essential

If a bunker or a silo is used for storing a material which sticks to the concrete such as cement or flour, one may cover the inside surface of the hopper and part of the walls at the base with steel or zinc plates or with timber planks

### IX 3 DETERMINATION OF PRESSURE INTENSITIES ACCORDING TO CLASSIC THEORY

#### IX 3-1 Bunkers

When calculating the side pressure on the walls and floor of a bunker, it is allowed due to the small depth of the bunker, to neglect the friction between the filling material and the walls

The side pressure can be calculated according to the known theories of earth pressure Thus, assuming

$\gamma$  = weight of filling material per cubic meter

$p_1$  = vertical pressure intensity at any depth  $x$

$p_2$  = horizontal " " " "  $x$

$\rho$  = angle of internal friction of filling material

$\rho'$  = angle of friction between filling material and walls,

the vertical and horizontal pressure intensities can be calculated from the relations:

$$p_1 = \gamma H$$

$$p_2 = H \gamma \tan^2 (45 - \rho/2)$$

## IX 3-2 Silos Theory of Janssen

In deep silo - cells the earth pressure theory is not valid because the wedge of material causing the pressure on a wall intersects the opposite wall and hence not fully acting. The pressure intensities on the walls and floors of prismatic and circular cells are under continual study since 1896. The classic theory of Janssen was generally used. Recent tests have shown, in many cases big deviations that must be considered when determining the pressure intensities in silo-cell (Refer to figure IX 8 )

If we consider the equilibrium of the forces acting on an element  $dx$  of the material, as shown in figure IX 6, we get

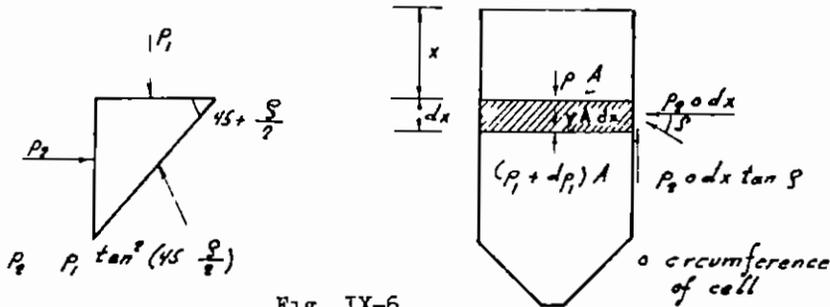


Fig IX-6

$$(p_1 + dp_1) A - p_1 A - \gamma A dx + p_2 O dx \tan \rho = 0 \quad \text{or}$$

$$dp_1 A = \gamma A dx - p_2 O dx \tan \rho \quad \text{i e}$$

dividing by  $A dx$ , we get

$$\frac{dp_1}{dx} = \gamma - p_2 \frac{O}{A} \tan \rho = \gamma - p_1 \frac{O}{A} \tan \rho \tan^2 (45 - \rho/2)$$

in which

$O$  = circumference of cell

$A$  = area of cell

Assuming further that

$$k = \frac{O}{A} \tan \rho \tan^2 (45 - \rho/2)$$

then

$$\frac{dp_1}{dx} = \gamma - k p_1 \quad \text{and} \quad \frac{dx}{dp_1} = \frac{1}{\gamma - k p_1}$$

giving

$$x = -\frac{1}{k} \log (\gamma - k p_1) + C$$

But for  $x = 0$  ,  $p_1 = 0$  , thus

$$0 = -\frac{1}{k} \log \gamma + C \quad \text{or} \quad C = \log \gamma / k$$

Substituting this value in the equation of  $x$  , we get

$$-k x = \log (\gamma - k p_1) - \log \gamma = \log \frac{\gamma - k p_1}{\gamma}$$

Therefore

$$\frac{\gamma - k p_1}{\gamma} = e^{-kx} = \frac{1}{e^{kx}}$$

in which  $e = 2.71828$  is the base of natural logarithm

Accordingly, we get

$$p_1 = \frac{\gamma}{k} \left( 1 - \frac{1}{e^{kx}} \right)$$

and

$$\max p_1 = \gamma / k$$

Substituting for  $k$  we get

$$\max p_1 = \frac{\gamma}{\frac{A}{\tan \rho} \tan^2 (45 - \rho/2)}$$

We have further

$$p_2 = p_1 \tan^2 (45 - \rho/2) = \frac{\gamma}{k} \left( 1 - \frac{1}{e^{kx}} \right) \tan^2 (45 - \rho/2)$$

substituting for  $k$ , we get

$$p_2 = \gamma \left( 1 - \frac{1}{e^{kx}} \right) \frac{A}{\tan \rho}$$

$$\max p_2 = \frac{\gamma}{\frac{A}{\tan \rho}} \quad \text{at } \underline{x = \infty}$$

Therefore

$$\max p_1 = \frac{\max p_2}{\tan^2 (45 - \rho/2)}$$

The theoretical horizontal pressure distribution can approximately be replaced by the broken line O A B C D shown in fig IX 7 According to

this approximation, max  $p_2$  is assumed to take place at a distance  $2h$ , where

$$h = \frac{\max p_2}{\gamma \tan^2 (45 - \rho/2)}$$

$h$  is, in this manner, the depth at which the horizontal pressure  $p_2$  calculated from the relation  $p_2 = \gamma x \tan^2 (45 - \rho/2)$  is equal to  $\max p_2$  calculated according to given theory of silos. Accordingly we get further :

$$p_x = \max p_2 \left( \frac{0.368}{h} x + 0.264 \right)$$

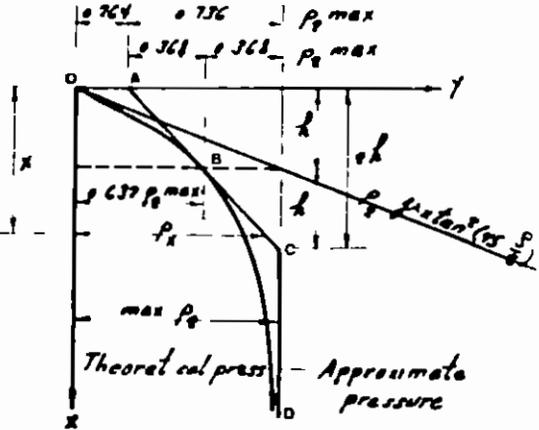


Fig IX-7

Assuming the intensity of the upward frictional force =  $p_3$  and  $p_3 / p_1 = x$ , the value of  $x$  may be estimated = 0.2 - 0.25

Results of recent researches are shown in figure IX-8

Horizontal pressures on wall of circular cells

Stored material corn

Curve 1 theoretical values of Janssen

Curve 2 measured horizontal pressure

after filling of cell

Curve 3 measured pressure by emptying of cell

Curve 4 : measured pressure after 8 days from filling of cell

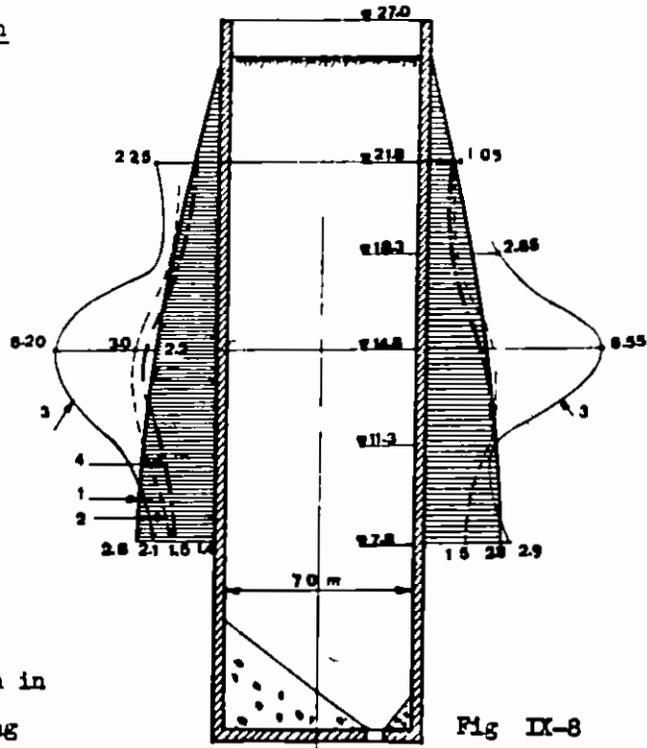


Fig IX-8

It is clear that even in case of granular filling



## 2 Determination of Internal Pressure Intensities

### 2.1 Data

The form of the cross-section will be included by the relation  $A/O$  meter, in which

$A$  = net area of cross-section of cell in  $m^2$

$O$  = internal perimeter of cell in  $m$

The depth  $x$  in meter is measured from the real assumed upper plane surface of the filling material (Fig IX 10)



Fig IX-10

The weight of the filling material  $\gamma$  in tons per cubic meter and the angle of internal friction  $\rho$  may be taken from the following table

|          | Coal    | Cement | Lime | Gypsum | Tabac | Sugar | Beans & Grains | Flour |
|----------|---------|--------|------|--------|-------|-------|----------------|-------|
| $\gamma$ | 0.8-1.2 | 1.7    | 1.7  | 1.5    | 0.5   | 1.0   | 0.8            | 0.6   |
| $\rho$   | 30°     | 20°    | 45°  | 25°    | -     | 35°   | 30°            | 25°   |

The angle of friction  $\rho'$  between silo-wall and filling material and the coefficient of friction  $\mu$  are given by :

$$\tan \rho' = P_3 / P_2 = \mu$$

$\rho'$  is a factor of  $\rho$ . It can be assumed according to the following table

| Filling material                              | angle of friction $\rho'$ in degrees |                       |
|-----------------------------------------------|--------------------------------------|-----------------------|
|                                               | by filling $\rho'_f$                 | by emptying $\rho'_e$ |
| coarse fill having grain-diameter $> 0.20$ mm | 0.75 $\rho$                          | 0.60 $\rho$           |
| dusty fill having grain-diameter $< 0.06$ mm  | 1.00 $\rho$                          | 1.00 $\rho$           |

Linear interpolation may be used for grain-diameters between 0.06 and 0.20

The relation between the horizontal and vertical pressure intensities

$$\lambda = p_2 / p_1$$

will be assumed constant over the whole depth of the silo

By filling, assume  $\lambda_f = 0.5$

By emptying, assume  $\lambda_e = 1.00$

## 2.2 Governing Cases of Loading

The biggest loads generally take place for the cases shown in the following table

| Load                           | Coarse Fill  |                    | Dusty Fill   |                    |
|--------------------------------|--------------|--------------------|--------------|--------------------|
|                                | finite depth | infinite depth     | finite depth | infinite depth     |
| Vert pressure intensity $p_1$  | filling      | filling            | filling      | filling            |
| Horiz pressure intensity $p_2$ | emptying     | emptying           | emptying     | Filling = emptying |
| Frictional force $p_3$         | emptying     | filling = emptying | emptying     | filling = emptying |

## 2.3 Pressure Intensities at an Infinite Depth

The pressure intensities reach their maximum values at a depth equal to infinity

By filling

$$\max p_1 = \gamma A / \lambda_f \quad \mu_f \quad 0$$

$$\max p_2 = \gamma A / \mu_f \quad 0$$

$$\max p_3 = \gamma A / 0$$

By emptying

$$\max p_1 = \gamma A / \lambda_e \quad \mu_e \quad 0$$

$$\max p_2 = \gamma A / \mu_e \quad 0$$

$$\max p_3 = \gamma A / 0$$

## 2 4 Pressure Intensities at a Finite Depth

The pressure intensities  $p$  increase with the depth  $x$  according to the relation

$$p_x = \max p \cdot \psi$$

in which

$$\psi = \left( 1 - e^{-\frac{x}{x_0}} \right)$$

where

$$x_0 = \Delta / \lambda_f \quad \mu_f = 0 \quad \text{by filling}$$

$$x_0 = \Delta / \lambda_e \quad \mu_e = 0 \quad \text{by emptying}$$

Values of  $\psi$  as a factor of  $x / x_0$

| $x/x_0$ | ,0   | ,1   | ,2   | ,3   | ,4   | ,5   | ,6   | ,7   | ,8   | ,9   |
|---------|------|------|------|------|------|------|------|------|------|------|
| 0,      | 0,00 | 0,10 | 0,18 | 0,26 | 0,33 | 0,39 | 0,45 | 0,50 | 0,55 | 0,59 |
| 1,      | 0,63 | 0,67 | 0,70 | 0,73 | 0,75 | 0,78 | 0,80 | 0,82 | 0,83 | 0,85 |
| 2,      | 0,86 | 0,88 | 0,89 | 0,90 | 0,91 | 0,92 | 0,93 | 0,93 | 0,94 | 0,94 |
| 3,      | 0,95 | 0,96 | 0,96 | 0,96 | 0,97 | 0,97 | 0,97 | 0,98 | 0,98 | 0,98 |

## 3 Factors Increasing Pressure Intensities

### 3 1 Arch Action of Filling Materials

The pressure intensities are much increased by the sudden failure of the material - arches-existing inside the cells. For this reason, the vertical pressure intensity on the hopper may be double but is not to be assumed in any case bigger than  $\gamma x$ .

This rule may not be applied, only if it is proved by practical experience that arching of the filling material is not liable to take place.

### 3 2 Eccentric Hopper-Opening

By emptying of silo-cells provided with eccentric hopper-opening, non-uniform horizontal forces acting on the perimeter are created over the whole depth of the cell.

These additional horizontal forces  $p_2'$  are to be considered

in positions giving bigger internal stresses

Assuming

$p_2$  = horizontal pressure intensity of original silo - cell according to article 2

$p_2^1 = P_{21} - p_2$  = an anti-symmetrical load acting on the original cell keeping the equilibrium of the frictional forces (Fig IX 11)

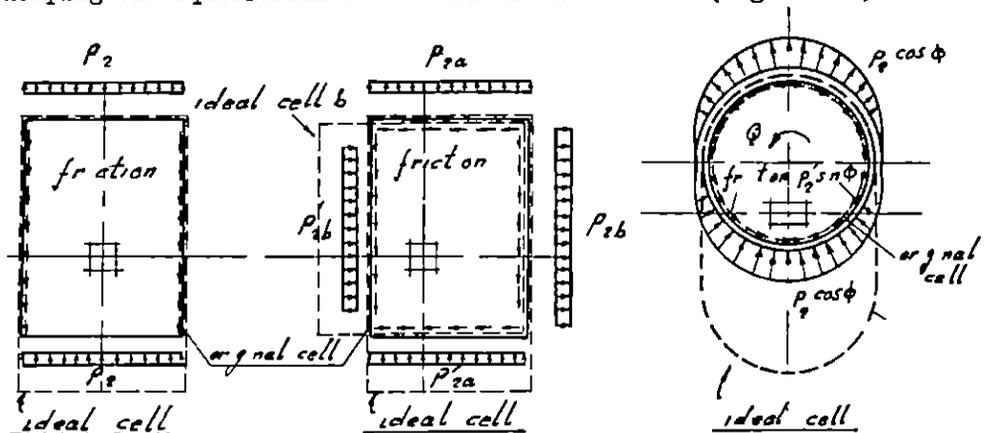


Fig IX-11

$P_{21}$  = horizontal pressure intensity created by emptying an ideal symmetrical silo-cell chosen according to figure IX 11

These additional horizontal pressure intensities may be neglected if the eccentricity of the hopper opening is smaller than  $d/6$  or if the height of the cell  $H$  is smaller than  $2d$  where  $d$  is the diameter of the circle that can be drawn inside the cross-section of the cell

### 3 3 Filling Material under Pneumatic Pressure

The horizontal pressure intensity in silo-cells provided with piping to pressurise the filling material depends on its grain size. In cells used for storing cohesionless granular filling materials, the horizontal pressure intensity is to be increased by an amount equal to the air-pressure and is to be gradually decreased to zero from the highest pressure-opening in the cell to the upper surface of the filling material. No increase in the magnitude of the horizontal pressure intensity need to be considered for dusty filling materials.

#### 4 Factors Decreasing Pressure Intensities

##### 4 1 Floor of Cells

Due to the fixation of the walls to the floor, the horizontal pressure intensity may be reduced in the manner shown in figure IX 12

##### 4 2 Special Emptying Plant

If a silo is provided with a special emptying plant which enables the emptying of the upper layers of the filling material without moving the lower layers, the emptying pressure intensities need not be considered in the design. In this case, it is essential to take the neces-

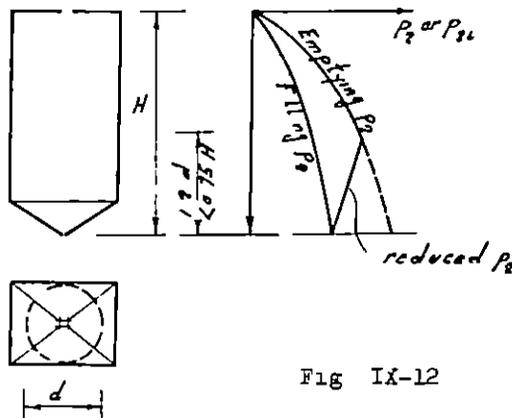


Fig IX-12

sary provisions to prevent the emptying of the silo from the hopper-opening

#### 5 Special Cases

##### 5 1 Special Mixing Plant

If a silo-building is provided with a special pneumatic mixing plant giving a homogenous regularly distributed dusty mix the pressure intensities are to be calculated according to article 2 provided that their magnitude is bigger than the values given by the relation

$$p_1 = p_2 = 0.6 \gamma x$$

##### 5 2 Silos for Fermented Seeds

Fermented seeds do not follow the rules of granular or dusty materials. The forces due to such materials depend on the included amount of water and on the fermentation process. They may be assumed according to the following table

|                                                   | Category I<br>material<br>highly<br>fermented | Category II<br>fermented<br>material | Category III<br>wet stored<br>material |
|---------------------------------------------------|-----------------------------------------------|--------------------------------------|----------------------------------------|
| Dry weight in % of<br>fresh weight                | > 35                                          | 23 - 35                              | < 23                                   |
| Weight to be assumed<br>in $t/m^3$                | 0.5                                           | 0.75                                 | 1.00                                   |
| Vertical pressure in-<br>tensity $p_1$ in $t/m^2$ | $\gamma x$                                    | $\gamma x$                           | $\gamma x$                             |
| Horizontal pressure<br>intensity $p_2$ in $t/m^2$ | $0.75 \gamma x$                               | $0.70 \gamma x$                      | $1.00 \gamma x$                        |
| Friction force $p_3$<br>in $t/m^2$                | $0.16 p_2$                                    | $0.14 p_2$                           | $0.10 p_2$                             |

If the stored materials given under categories I & II are wet, the silo should not be filled to more than half its depth and to provide it with a draining outlet, so that the material-juice is not more than one meter deep

#### IX 5 ILLUSTRATIVE EXAMPLE

A silo-cell 5 x 5 ms and 40 ms deep is used for storing corn determine the maximum pressure intensities  $p_1$ ,  $p_2$  and  $p_3$  during filling and emptying according to both the classic theory and the german specifications Assume  $\gamma = 0.8 t/m^3$  &  $\rho = 30^\circ$

##### a) According to classic theory

area of cell  $A = 5 \times 5 = 25 \text{ m}^2$

perimeter of cell  $O = 4 \times 5 = 20 \text{ m}$

angle of friction between corn and wall  $\rho$  according to the classic theory is assumed the same ( $= 25^\circ$ ) both during filling and emptying

So that

$$k = \frac{O}{A} \tan \rho \tan^2 (45 - \rho/2) = \frac{20}{25} \tan 25^\circ \tan^2 (45 - 15^\circ) \quad \text{or}$$

$$k = 0.8 \times 0.466 \times 0.577^2 = 0.124$$

$$\max p_1 = \gamma/k = 0.80/0.124 = 6.45 \text{ t/m}^2$$

$$\max p_2 = \frac{\gamma}{\frac{O}{A} \tan \rho} = \frac{0.8}{0.8 \times 0.466} = 2.14 \text{ ms}$$

max  $p_2$  acts at a depth  $2h$  where

$$h = \frac{\max p_2}{\gamma \tan^2 (45 - \rho/2)} = \frac{2.14}{0.8 \times 0.577^2} = 8.03 \text{ ms}$$

i.e. the max horizontal pressure acts at a depth of  $\sim 16$  ms

$$\max p_3 = \max p_2 \tan \rho = 2.14 \times 0.466 = 1.00 \text{ t/m}^2$$

b) According to DIN 1055

$$\gamma A / O = 0.8 \times 25 / 20 = 1 \text{ t/m}^2$$

$$\lambda = p_2 / p_1, \quad \lambda_f = 0.5, \quad \lambda_e = 1.0$$

$$\mu = \tan \rho = p_3 / p_2$$

$$\text{By filling } \rho_f = 0.75 \rho = 0.75 \times 30 = 22.5^\circ \text{ and } \mu_f = 0.414$$

$$\text{By emptying } \rho_e = 0.60 \rho = 0.60 \times 30^\circ = 18^\circ \text{ and } \mu_e = 0.325$$

So that the maximum pressure intensities at an infinite depth are given by

$$\left. \begin{aligned} \max p_1 &= \gamma A / \lambda_f \mu_f O = 1/0.5 \times 0.414 = 4.85 \text{ t/m}^2 \\ \max p_2 &= \gamma A / \mu_f O = 1/0.412 = 2.42 \text{ t/m}^2 \\ \max p_3 &= \gamma A / O = 1 \text{ t/m}^2 \end{aligned} \right\} \text{filling}$$

$$\left. \begin{aligned} \max p_1 &= \gamma A / \lambda_e \mu_e O = 1/1.0 \times 0.325 = 3.08 \text{ t/m}^2 \\ \max p_2 &= \gamma A / \mu_e O = 1/0.325 = 3.08 \text{ t/m}^2 \\ \max p_3 &= \gamma A / O = 1 \text{ t/m}^2 \end{aligned} \right\} \text{emptying}$$

The results are shown in the following table

| Pressure Intensity |                    | $\text{t/m}^2$ | $p_1$ | $p_2$ | $p_3$ |
|--------------------|--------------------|----------------|-------|-------|-------|
| Classic Theory     | Filling & Emptying |                | 6.45  | 2.14  | 1.00  |
| DIN 1055           | Filling            |                | 4.85  | 2.42  | 1.00  |
|                    | Emptying           |                | 3.08  | 3.08  | 1.00  |

The table shows that the difference in this case reaches  $\pm 50\%$

IX 6 DESIGN OF WALLS AND FLOORSIX 6 1 Design of Walls of Circular Cells

Walls of circular silo-cells are to be calculated for

a) ring tension, b) compression due to frictional forces, c) temperature stresses (if any)

a) Due to the horizontal pressure  $p_2$ , the walls of a circular cell, with diameter  $D$  and radius  $r$ , will be subject to ring tension  $T$ /meter height given by

$$T = p_2 r$$

The corresponding ring reinforcement must be capable to resist the full tension  $T$

$$A_s = T/\sigma_s$$

The upper part of the walls (2-3 ms) may be reinforced by one mesh of reinforcements and the lower parts by double meshes. The reinforcements may be chosen as follows

$\phi$  of rings  $\leq 16$  mm and  $\geq 8$  mm at a distance  $\leq 20$  cm and  $\geq 10$  cm  
Minimum vertical reinforcement  $4 \phi 8$  mm/m

IX 6 2 Width and Spacing of Cracks

The thickness of the wall depends on the allowed crack width which can be calculated as follows

Assuming

Spacing between cracks =  $e$  cms

Width of cracks =  $\Delta e$  cms

Ratio of tension steel in section =  $A_s / A_c$  =  $\mu$

Diameter of tension steel =  $\phi$

Tensile stress in steel =  $\sigma_s$

Stress in steel at which cracking occurs =  $\sigma_{s0}$

Then

$$e = \frac{0.4 \phi}{\mu} \quad \text{and} \quad \frac{\Delta e}{e} = \frac{\sigma_s - \sigma_{s0}}{E_s}$$

$\sigma_{s0}$  depends on the concrete quality and the percentage of the steel in the section. It can be calculated from the relation

$$\sigma_{s0} = \left( \frac{1}{16\mu} + 1.75 \right) \sigma_{c28}$$

Assuming  $\sigma_{c28} = 200 \text{ kg/cm}^2$ , then

$$\sigma_{s0} = \frac{12.5}{\mu} + 350$$

For normal cases, a width of crack  $< 0.2 \text{ mm}$  may be accepted

If the walls are to be free from cracks then the thickness of the wall may be chosen from the relation

$$t \text{ in cms} = 0.8 T \text{ in tons/m}$$

#### Example

A circular silo cell 15 meters diameter is subject to an internal pressure  $p = 5 \text{ t/m}^2$ . If cold twisted steel with an allowable stress  $\sigma_s = 2 \text{ t/cm}^2$  is used for the reinforcement, determine the spacing and width of cracks if the wall is chosen 20 cms thick

Ring tension  $T = p D/2 = 5 \times 15/2 = 37.5 \text{ t/m}$

Ring reinforcement  $A_s = T / \sigma_s = 37.5 / 2 = 18.75 \text{ cm}^2$

Choose 7  $\phi 13$  on each face giving an area of  $18.5 \text{ cm}^2$

Ratio of tension steel  $\mu = A_s/bt = 18.5 / 100 \times 20 = 0.0925$

Assuming a concrete quality C200, we get

$$\sigma_{s0} = \left( \frac{12.5}{0.0925} + 350 \right) = 1250 + 350 = 1600 \text{ kg/cm}^2$$

Therefore

The average spacing of the cracks is

$$e = 0.4 \phi / \mu = 0.4 \times 13 / 0.0925 = 56 \text{ cms}$$

The average width of cracks is

$$\Delta e = \frac{\sigma_s - \sigma_{s0}}{E_s} \times e = \frac{2000 - 1600}{2100000} \times 56 = 0.0107 \text{ cms} = 0.107 \text{ mm}$$

In order to have a wall free from cracks, its thickness  $t$  is to be chosen  $> 0.8 T$  i.e.  $t = 0.8 \times 37.5 = 30 \text{ cms}$  in which case

$$\mu = 18.5 / 100 \times 30 = 0.062 \quad \text{and}$$

$$\sigma_{s0} = \frac{12.5}{0.062} + 350 = 2350 \text{ kg/cm}^2 > \sigma_s \quad \text{i.e. no cracks!}$$

b) The horizontal sections of the wall are subject to compressive forces due to the own weight of the wall and the frictional forces from the filling materials

The frictional force  $N_x$  at any depth  $x$  is given by

$$N_x = \frac{(\gamma x - p_{1x}) \frac{\pi}{4} D^2}{\pi D}$$

c) Eventual effect of temperature differences (as will be shown later) may be included in the design

### IX 6 3 Design of Conical Hoppers

Circular silo-cells are generally provided with conical hoppers. These hoppers are subject to meridian and ring forces due to the vertical pressure  $p_1$  of the filling material and the own weight of the hopper\* (Fig IX-13)

The tensile meridian force  $N_s$  at any section a - a per unit length circumference is given by

$$N_s = \frac{\pi r_1^2 p_1 + G}{2 \pi r_1 \sin \varphi}$$

in which,  $G$  is the own weight of the hopper below a - a

The tensile ring force  $N_\theta$  per unit length meridian is given by

$$N_\theta = p_n r_1 / \sin \varphi$$

where  $p_n$  = component of the pressure of the filling material normal to the plane of the hopper. It is given by

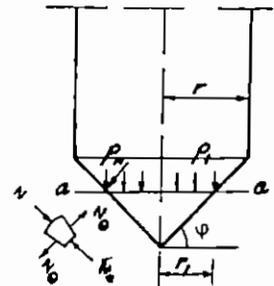


Fig IX-13

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\* Refer to "Fundamentals of Reinforced and Prestressed Concrete", by M Hilal

$$p_n = p_1 \cos^2 \varphi + p_2 \sin^2 \varphi$$

The component of the pressure of the filling material parallel to the plane of the hopper is given by

$$p_t = p_1 \sin^2 \varphi + p_2 \cos^2 \varphi$$

$N_s$  and  $N_\theta$  are zero at the bottom of the hopper and maximum at its top edge

The reinforcements in the hopper are

longitudinal bars  $A_{s1} = N_s / \sigma_s$

rings :  $A_{s2} = N_\theta / \sigma_s$

Due to the rigid connection between the cylindrical wall and the conical floor, bending moments  $M$  and shearing forces  $Q$  are created, they may be roughly estimated by the relations (Fig IX 14)

$$M = p_2 r^2 / 6 \quad Q = p_2 r / 2$$

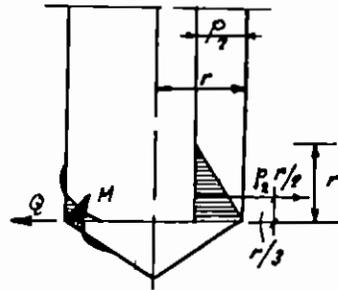


Fig IX-14

The connecting moment  $M$  can however be determined by the moment distribution method as explained in IV 3 6

#### IX 6 4 Design of Rectangular Cells

The bending moments and tension in the walls of rectangular silo-cells due to the internal horizontal pressure  $p_2$  can be determined according to the methods given in VII 1 Cells of approximately equal spans may be considered as isolated i e effect of continuity of different cells is neglected Effect of continuity must however be taken in consideration if the spans of the cells differ much as for example the silos shown in figure IX 5

Due to the rigid connection between the wall and the floor, the fixed end moment  $M$  and the shearing force  $Q$  can be roughly estimated from the relations

$$M = p_2 L^2 / 24 \quad \& \quad Q = 0.3 p_2 L$$

#### IX 6 5 Width & Spacing of Cracks

The spacing and width of cracks in elements subject to simple bending can be calculated from the relations

$$e = \frac{0.24 \phi}{\mu} \quad \text{and} \quad \frac{\Delta e}{e} = \frac{\sigma_s - \sigma_{s0}}{E_s}$$

$$\sigma_{s0} = \left( \frac{1}{25\mu} + 1.75 \right) \sigma_{c28}$$

Assuming  $\sigma_{c28} = 200 \text{ kg/cm}^2$ , then

$$\sigma_{s0} = \left( \frac{8}{\mu} + 350 \right)$$

#### IX 6 6 Design of Floor of Rectangular Cells

If the floor of a cell is flat it may be treated as a two way slab hung to the walls, and carries its own weight and the weight of the lean concrete forming the required shape of the hopper plus the maximum vertical pressure of the filling material

If the hopper is a reinforced concrete pyramid, it may approximately be calculated as follows

The sides of the hoppers are subjected to the pressure of the filling material plus their own weight. The pressure of the filling material is resolved to the two components  $p_n$  and  $p_t$  normal and parallel to the sides of the hopper

$$p_n = p_1 \cos^2 \phi + p_2 \sin^2 \phi$$

$$p_t = p_1 \sin^2 \phi + p_2 \cos^2 \phi$$

The triangular or trapezoidal sides of the hopper may be replaced by rectangular slabs as follows<sup>2</sup> (Fig IX 15)

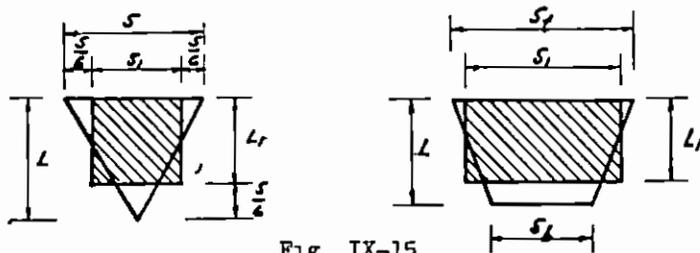


Fig IX-15

For a triangular side  $S_1 = \frac{2}{3} S$  and  $L_1 = L - \frac{S}{6}$

and for a trapezoidal side  $S_1 = \frac{2}{3} \frac{S_t^2 + 2 S_t S_b}{S_t + S_b}$

and  $L_1 = L - \frac{S_t (S_t - S_b)}{6 (S_t + S_b)}$

The load  $p_n$  is distributed on the two directions of the slab according to the ratio of its reduced sides  $L_1 / S_1$ . In horizontal direction, the different sectors of the hopper behave as closed frames subjected to bending moments and tension in the inclined direction, each side behaves as a slab continuous with the wall at the top and simply supported at the bottom. Due to the tangential component  $p_t$ , the sides are subjected to tensile stresses and are to be treated as folded plates.

The exact values of the bending moments in isotropic triangular and trapezoidal flat plates with different edge conditions and subjected to uniform and triangular pressure are given in the text book of Bares 'Tables for the Analysis of Plates, Slabs and Diaphragms' Published by Bauverlag Berlin. Dr. Shaker El-Beahry in his reinforced concrete Design Handbook has extracted the following tables from the book of Bares.

#### IX 6 7 Calculation of Wall as a Beam

The shear stress in the wall due to its own weight plus the force due to friction must be less than the allowable values. Hence

$$\begin{aligned} \text{Own weight of wall} &= L H t \times 2.5 \text{ tons} \\ \text{Frictional force on the two sides of the wall} &= 2 \left[ \frac{A}{4} (\gamma H - p_{lmax}) \right] \text{ tons} \\ \text{Total } Q &= \text{tons} \end{aligned}$$

$$\text{Max Shear Stress } \tau_{max} \approx \frac{Q}{2} \frac{1.5}{t H} = \frac{3 Q}{4 t H}$$

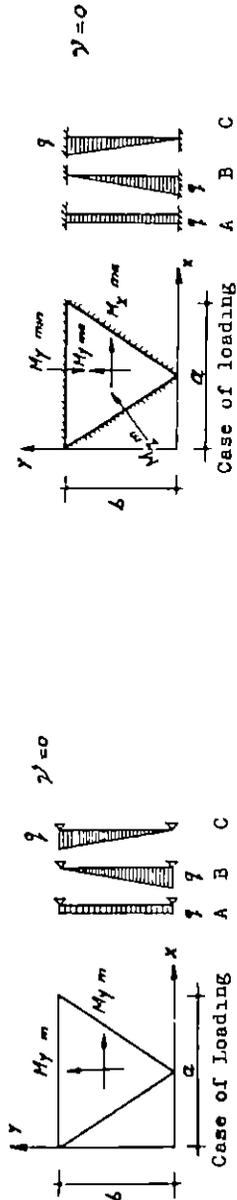
The wall is however to be calculated as a deep beam carrying its own weight + weight of floor + weight of stored material (Refer to Chapter XI)

#### IX 7 FOUNDATIONS

The column loads in silo-buildings are generally high and the foundations are to be carefully designed and executed.

Due to the big rigidity of the concrete silos, differential settlements are generally not possible. For good soils, one may use isolated footings either plain or reinforced. For medium bearing soils, strip continuous foundations may be used whereas for weak soils or

Bending Moments in Simply Supported and Totally Fixed Triangular Plates  
due to Rectangular and Triangular Loads



|     |        | a) simply supported triangular plate |                    |                    |                    |        |                    | b) totally fixed triangular plate |                    |                    |        |                    |                    |                    |                    |                    |                    |
|-----|--------|--------------------------------------|--------------------|--------------------|--------------------|--------|--------------------|-----------------------------------|--------------------|--------------------|--------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| a/b | Case A | Case B                               |                    | Case C             |                    | Case A | Case B             |                                   | Case C             |                    | Case A | Case B             |                    | Case C             |                    |                    |                    |
|     |        | M <sub>x</sub> max                   | M <sub>y</sub> max | M <sub>x</sub> max | M <sub>y</sub> max |        | M <sub>x</sub> max | M <sub>y</sub> max                | M <sub>x</sub> max | M <sub>y</sub> max |        | M <sub>x</sub> max | M <sub>y</sub> max | M <sub>x</sub> max | M <sub>y</sub> max | M <sub>x</sub> min | M <sub>y</sub> min |
| 0.5 | 0209   | 0125                                 | 0027               | 0271               | 0182               | 0187   | 0089               | 0210                              | 0378               | 0046               | 0007   | 0036               | 0085               | 0141               | 0022               | 0174               | 0293               |
| 0.6 | 0343   | 0197                                 | 0030               | 0226               | 0167               | 0164   | 0058               | 0219                              | 0335               | 0052               | 0022   | 0042               | 0087               | 0116               | 0076               | 0177               | 0248               |
| 0.7 | 0303   | 0186                                 | 0108               | 0195               | 0154               | 0150   | 0100               | 0220                              | 0297               | 0053               | 0028   | 0444               | 0087               | 0097               | 0072               | 0176               | 0210               |
| 0.8 | 0270   | 0178                                 | 0100               | 0035               | 0170               | 0131   | 0096               | 0215                              | 0264               | 0048               | 0028   | 0446               | 0086               | 0083               | 0068               | 0169               | 0178               |
| 0.9 | 0241   | 0172                                 | 0091               | 0038               | 0150               | 0114   | 0089               | 0206                              | 0234               | 0043               | 0024   | 0446               | 0081               | 0071               | 0065               | 0160               | 0152               |
| 1.0 | 0214   | 0166                                 | 0084               | 0041               | 0125               | 0098   | 0082               | 0196                              | 0207               | 0034               | 0020   | 0446               | 0071               | 0061               | 0062               | 0150               | 0136               |
| 1.1 | 0192   | 0161                                 | 0078               | 0044               | 0114               | 0087   | 0076               | 0184                              | 0183               | 0034               | 0018   | 0444               | 0062               | 0053               | 0058               | 0140               | 0120               |
| 1.2 | 0172   | 0154                                 | 0072               | 0044               | 0100               | 0077   | 0071               | 0173                              | 0162               | 0031               | 0017   | 0043               | 0056               | 0046               | 0054               | 0130               | 0106               |
| 1.3 | 0155   | 0145                                 | 0067               | 0041               | 0088               | 0069   | 0063               | 0163                              | 0145               | 0029               | 0017   | 0042               | 0051               | 0040               | 0050               | 0121               | 0094               |
| 1.4 | 0141   | 0135                                 | 0063               | 0038               | 0078               | 0061   | 0053               | 0153                              | 0131               | 0026               | 0017   | 0041               | 0043               | 0035               | 0046               | 0112               | 0083               |
| 1.5 | 0128   | 0126                                 | 0058               | 0035               | 0070               | 0055   | 0039               | 0144                              | 0120               | 0024               | 0017   | 0041               | 0046               | 0031               | 0042               | 0103               | 0074               |
| 1.6 | 0118   | 0118                                 | 0055               | 0033               | 0063               | 0049   | 0036               | 0136                              | 0111               | 0021               | 0017   | 0041               | 0044               | 0028               | 0038               | 0095               | 0067               |
| 1.7 | 0108   | 0111                                 | 0051               | 0031               | 0057               | 0044   | 0033               | 0128                              | 0102               | 0019               | 0017   | 0040               | 0042               | 0025               | 0035               | 0088               | 0061               |
| 1.8 | 0099   | 0105                                 | 0047               | 0030               | 0052               | 0039   | 0030               | 0120                              | 0096               | 0017               | 0017   | 0038               | 0040               | 0022               | 0033               | 0088               | 0056               |
| 1.9 | 0090   | 0099                                 | 0042               | 0029               | 0048               | 0034   | 0028               | 0112                              | 0091               | 0015               | 0017   | 0035               | 0039               | 0019               | 0031               | 0077               | 0052               |
| 2.0 | 0081   | 0094                                 | 0036               | 0028               | 0045               | 0029   | 0029               | 0103                              | 0087               | 0013               | 0016   | 0025               | 0038               | 0016               | 0029               | 0072               | 0049               |

Bending moments coefficients

**Bending Moments in Trapezoidal Plates with Different  
Edge Conditions for Rectangular & Triangular Loads**

Bending moments = coeff  $q a^2/c^4$      $\gamma = 0.16$      $b = (a-c) \frac{\sqrt{3}}{2}$

| 1          |                  | 2     |       | 3                |       |       |                  |       |       |
|------------|------------------|-------|-------|------------------|-------|-------|------------------|-------|-------|
|            |                  |       |       |                  |       |       |                  |       |       |
| M at point | Shape of loading |       |       | Shape of loading |       |       | Shape of loading |       |       |
|            | A                | B     | C     | A                | B     | C     | A                | B     | C     |
| $M_{x6}$   | 2446             | 1349  | 2097  | 1417             | 0547  | 087   | 1266             | 0546  | 0720  |
| $M_{x7}$   | 1823             | 0541  | 1292  | 0756             | 0147  | 0609  | 0895             | 0290  | 0605  |
| $M_{x8}$   | -0782            | -0.56 | -0222 | -0424            | -0393 | -0021 | -0168            | -0.02 | 0092  |
| $M_{x10}$  | 5909             | 2708  | 3201  | 4109             | 2003  | 2106  | 5100             | 1646  | 1454  |
| $M_{x11}$  | 4189             | 1804  | 2385  | 2952             | 1329  | 1623  | 2693             | 1400  | 1293  |
| $M_{x12}$  | 0382             | 0545  | 0163  | -0295            | -0411 | 0016  | 0242             | -0002 | 0245  |
| $M_{x14}$  | 5880             | 3062  | 2818  | 4762             | 2630  | 2122  | 2748             | 1629  | 1119  |
| $M_{x15}$  | 0516             | 0241  | 0.02  | 0586             | 0270  | 0316  | 1549             | 0995  | 05.4  |
| $M_{x17}$  | 4731             | 2747  | 1984  | 4066             | 2402  | 1574  |                  |       |       |
| $M_{x18}$  | 2373             | 1528  | 0845  | 2192             | 1458  | 0734  |                  |       |       |
| $M_{y6}$   | 6225             | 1742  | 4483  | 0981             | -0287 | 1268  | 1663             | 0191  | 1472  |
| $M_{y7}$   | 5223             | 1435  | 3788  | 1248             | -0026 | 1374  | 1728             | 0277  | 1451  |
| $M_{y8}$   | 2996             | 0771  | 2225  | 1253             | 013   | 1122  | 1656             | 0545  | 1111  |
| $M_{y10}$  | 7374             | 2940  | 4434  | 5424             | 2183  | 3221  | 595              | 3027  | 2923  |
| $M_{y11}$  | 6331             | 2529  | 3802  | 4742             | 1919  | 2823  | 5102             | 2583  | 2519  |
| $M_{y12}$  | 2092             | 1261  | 1831  | 2260             | 1053  | 1207  | 2692             | 1383  | 1309  |
| $M_{y14}$  | 5024             | 2956  | 2068  | 4936             | 2922  | 2014  | 4920             | 3297  | 1633  |
| $M_{y15}$  | 1816             | 1313  | 0503  | 2202             | 1457  | 0742  | 2493             | 1851  | 0642  |
| $M_{y17}$  | 2903             | 2520  | 0283  | 2294             | 2671  | 0223  |                  |       |       |
| $M_{y18}$  | 0382             | 1263  | -0381 | 1429             | 1476  | -0037 |                  |       |       |
| $M_{z9}$   | -2860            | -0922 | -1983 | -1582            | -0456 | -1126 | -1585            | -0563 | -1022 |
| $M_{z13}$  | -8689            | -3303 | -5336 | -5621            | -2160 | -3461 | -5295            | -2374 | -2921 |
| $M_{z16}$  | -1303            | -5935 | -7102 | -9742            | -4681 | -2061 | -7911            | -4238 | -3675 |
| $M_{z19}$  | -1317            | -6956 | -6217 | -1079            | -6042 | -4750 | 0                | 0     | 0     |
| $M_{x9}$   | -2265            | -0730 | -1535 | -1253            | -0361 | -0892 | -1250            | -0441 | -0809 |
| $M_{x13}$  | -8843            | -4579 | -4264 | -4449            | -1709 | -2740 | -4190            | -1878 | -2312 |
| $M_{x16}$  | -1032            | -4699 | -5623 | -7711            | -3705 | -4006 | -6261            | -3354 | -2907 |
| $M_{x19}$  | -1042            | -5507 | -4922 | -844             | -4783 | -3761 | 0                | 0     | 0     |
| $M_{y9}$   | -1073            | -0346 | -0727 | -0593            | -0171 | -0422 | -0592            | -0209 | -0383 |
| $M_{y13}$  | -4189            | -0216 | -2020 | -2108            | -0810 | -1238 | -1985            | -0890 | -1095 |
| $M_{y16}$  | -4889            | -2226 | -2663 | -3653            | -1755 | -1838 | -2963            | -1586 | -1377 |
| $M_{y19}$  | -4941            | -2609 | -2332 | -4047            | -2266 | -1781 | 0                | 0     | 0     |
| $M_{y1}$   |                  |       |       | -1120            | -4590 | -6310 | -1006            | -4600 | -5465 |
| $M_{y2}$   |                  |       |       | -9780            | -3774 | -6006 | -8890            | -3896 | -4994 |
| $M_{y3}$   |                  |       |       | -561             | -1935 | -3677 | -5346            | -2153 | -3193 |
| $M_{y4}$   |                  |       |       | -1582            | -0456 | -1126 | -1599            | -0577 | -1022 |

| Loment<br>at P nt | Shape of Loading |       |       | Shape of Loading |       |       | Shape of Loading |       |       |
|-------------------|------------------|-------|-------|------------------|-------|-------|------------------|-------|-------|
|                   | A                | B     | C     | A                | B     | C     | A                | B     | C     |
| M <sub>x6</sub>   | 1318             | 0488  | 0830  | 1033             | 0415  | 0023  | 3174             | 1195  | 1979  |
| M <sub>x7</sub>   | 0791             | 0161  | 0620  | 0415             | 0301  | 0614  | 1830             | 0539  | 1291  |
| M <sub>x8</sub>   | 0403             | -0318 | 0015  | 0039             | 0162  | 0201  | -0596            | -0455 | -0141 |
| M <sub>x10</sub>  | 3567             | 1679  | 1888  | 2145             | 1100  | 0145  | 5045             | 2272  | 2773  |
| M <sub>x11</sub>  | 2738             | 1201  | 1537  | 2077             | 1047  | 1030  | 3853             | 1618  | 2239  |
| M <sub>x12</sub>  | 0409             | -0604 | 0135  | 0617             | 0219  | 0398  | -0332            | -0332 | 0     |
| M <sub>x14</sub>  | 3777             | 2041  | 1736  | 1325             | 0815  | 0210  | 4668             | 2367  | 2291  |
| M <sub>x15</sub>  | 0921             | 0411  | 0510  | 1040             | 0699  | 0341  | 0796             | 0399  | 0397  |
| M <sub>x17</sub>  | 2272             | 1418  | 0854  |                  |       |       | 2590             | 1538  | 1058  |
| M <sub>x18</sub>  | 1321             | 0935  | 0386  |                  |       |       | 1347             | 0945  | 0402  |
| M <sub>y6</sub>   | 1285             | -0105 | 1390  | 2022             | 0397  | 1625  | 6200             | 1728  | 4472  |
| M <sub>y7</sub>   | 1466             | 0022  | 1444  | 1957             | 0408  | 1349  | 5169             | 1805  | 3764  |
| M <sub>y8</sub>   | 1313             | 0167  | 1146  | 1472             | 0321  | 1151  | 2998             | 0755  | 2243  |
| M <sub>y10</sub>  | 5381             | 2171  | 3210  | 4942             | 245   | 249   | 6688             | 2841  | 3847  |
| M <sub>y11</sub>  | 4697             | 1892  | 2805  | 4316             | 2133  | 2183  | 6168             | 2437  | 3731  |
| M <sub>y12</sub>  | 2679             | 1180  | 1499  | 2390             | 1211  | 1179  | 3028             | 1225  | 1803  |
| M <sub>y14</sub>  | 4266             | 2521  | 1742  | 1730             | 1465  | 0265  | 4276             | 2510  | 1757  |
| M <sub>y15</sub>  | 2523             | 1296  | 1237  | 0743             | 0848  | -0105 | 1525             | 1148  | 0377  |
| M <sub>y17</sub>  | 0588             | 0931  | -0543 |                  |       |       | -0937            | 0618  | -1055 |
| M <sub>y18</sub>  | 0397             | 0376  | -0773 |                  |       |       | -1218            | 0073  | -1291 |
| M <sub>z9</sub>   | -1561            | -0443 | -1118 | -1486            | -0504 | -0902 | -2762            | -0866 | -1896 |
| M <sub>z13</sub>  | -5359            | -2003 | -3356 | -4553            | -1949 | -1604 | -8190            | -3020 | -5170 |
| M <sub>z16</sub>  | -8687            | -4049 | -4638 | -5744            | -2996 | -2748 | -1583            | -5111 | -6472 |
| M <sub>z19</sub>  | -7973            | -4219 | -3754 | -2712            | -1356 | -1356 | -1170            | -5220 | -4890 |
| M <sub>z21</sub>  | -2989            | -1805 | -1184 |                  |       |       | -3460            | -1980 | -1480 |
| M <sub>x9</sub>   | -1236            | -0351 | -0885 | -1176            | -0399 | -777  | -2187            | -0686 | -1501 |
| M <sub>x13</sub>  | -4240            | -1584 | -2656 | -3605            | -1543 | -2062 | -7590            | -3497 | -4093 |
| M <sub>x16</sub>  | -6876            | -3205 | -3671 | -4546            | -2371 | -2175 | -9170            | -8046 | -5124 |
| M <sub>x19</sub>  | -6575            | -3604 | -2971 | -2712            | -1356 | -1356 | -8003            | -4132 | -3871 |
| M <sub>x21</sub>  | -2989            | -1805 | -1184 |                  |       |       | -3860            | -1980 | -1480 |
| M <sub>y9</sub>   | -0585            | -0166 | -0419 | -0557            | -0189 | -0368 | -1032            | -0325 | -0707 |
| M <sub>y13</sub>  | -2009            | -0751 | -1258 | -1708            | -0731 | -0977 | -3595            | -1656 | -1939 |
| M <sub>y16</sub>  | -3257            | -1518 | -1739 | -2153            | -1123 | -1030 | -4344            | -1917 | -2427 |
| M <sub>y19</sub>  | -3115            | -1707 | -1408 |                  |       |       | -3794            | -1960 | -1834 |
| M <sub>y20</sub>  | -8347            | -4993 | -3354 |                  |       |       | -9740            | -5510 | -4230 |
| M <sub>y21</sub>  | -2984            | -1805 | -1184 |                  |       |       | -3460            | -1980 | -1418 |
| M <sub>y1</sub>   | -1070            | -411  | -6588 | -8431            | -3562 | -4869 |                  |       |       |
| M <sub>y2</sub>   | -9184            | -3417 | -5767 | -7470            | -3083 | -4387 |                  |       |       |
| M <sub>y3</sub>   | -5397            | -1806 | -3591 | -4740            | -1806 | -2934 |                  |       |       |
| M <sub>y4</sub>   | -1561            | -0443 | -1118 | -1486            | -0504 | -0982 |                  |       |       |
| M <sub>y5</sub>   | 0                | 0     | 0     | 0                | 0     | 0     |                  |       |       |

heavy loads raft or pile foundations are generally chosen. The fundamentals and details of the design of any of these systems can be done according to the known rules in foundation - design.

IX-8 CONSTRUCTIONAL DETAILS

(Fig IX-16)

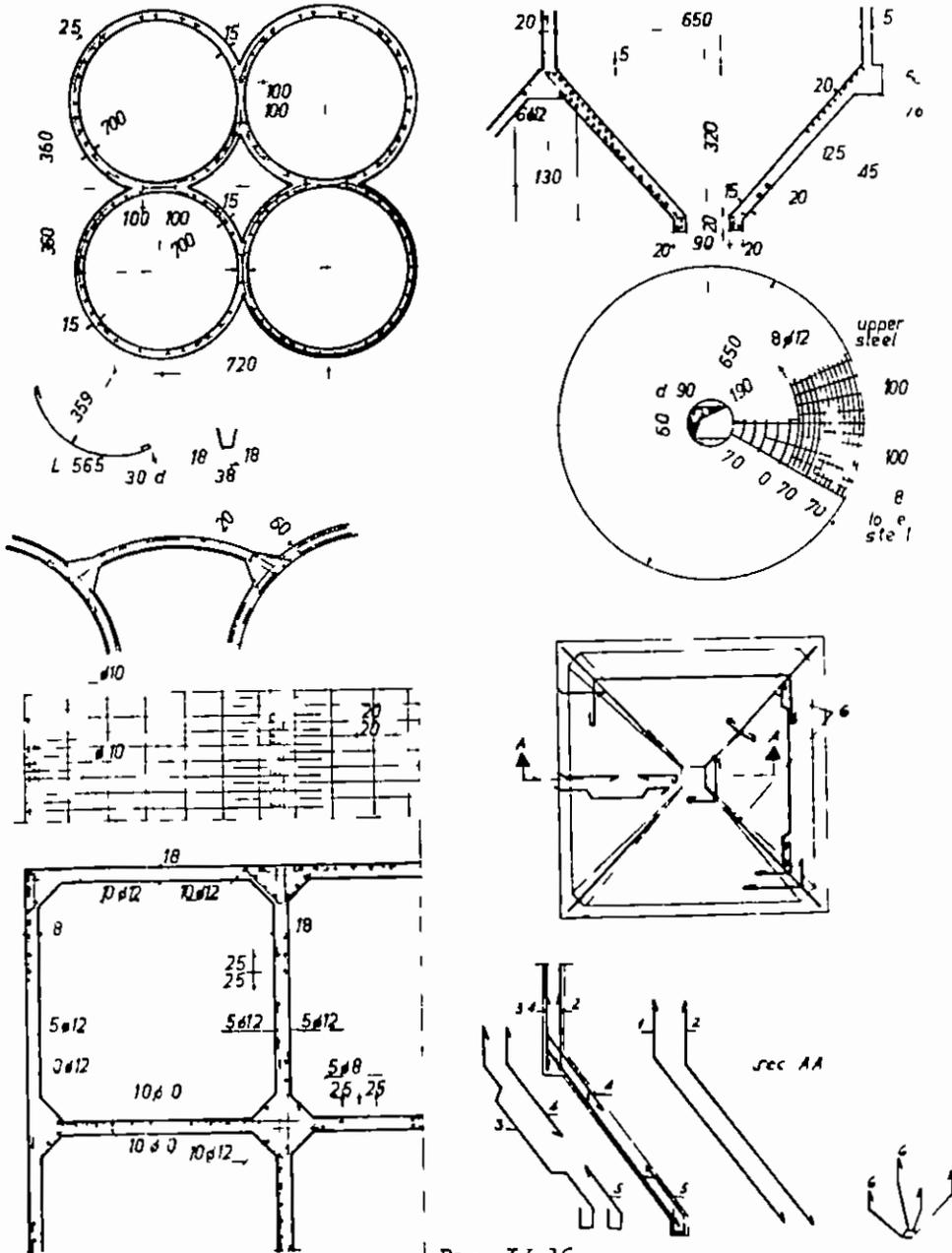


Fig IX-16

\* Refer to "Fundamentals of Reinforced and Prestressed Concrete", by M. Hilal

IX 9 APPENDIX CIRCULAR CEMENT SILOS

Silos used for storing cement (and flour) must be precisely treated due to the big variation that is liable to take place in its properties. Fischer\* proposes the direct use of test results which give a parabolic distribution of the horizontal pressure on the walls according to the relation

$$p_2 = \sqrt{7x} \quad \text{in t/m}^2 \quad \text{for } x \text{ in meters}$$

Leonhardt\* gives the following recommendations for the design of circular cement silos

1) Horizontal pressure on the walls

Up to 4 m silo depth calculation according to conventional theory of earth pressure is

$$p_2 = \gamma x \tan^2 (45 - \rho/2)$$

where  $\gamma = 2 \text{ t/m}^3$ ,  $\rho = 17.5$ ,  $\rho = 0$

Above 9 m silo depth calculation in accordance with Jansen's theory multiplying the results by a factor  $K = 2$

$$p_2 = \gamma \left( 1 - \frac{1}{e^{2x}} \right) \frac{A}{\sigma \tan \rho} K$$

with  $\gamma = 1.6 \text{ t/m}^3$ ,  $\sigma = \rho = 30$

Between 4 and 9 m silo depth linear interpolation between the pressure values at 4 and 9 m

2) Vertical loads on the walls

The vertical loads acting on the walls can be determined according to the following assumptions

a) For maximum wall load  $\max p_3$

$\gamma = 1.6 \text{ t/m}^3$ ,  $\rho = \rho = 35$  (determined from the equations of Jansen)

The high value for the angle of friction makes allowance for the arch (bridging) effect

\* 1- Fischer 'Silos and Bunker in Stahlbeton Der Verlag Für Bauwesen Berlin 1966

\* 2- Leonhardt The safe design of cement silos In german language Beton & Stahlbeton Vol 57 to 13 March 1960

## b) For minimum wall load

The own weight of the wall only (empty silo) is to be considered

3) Vertical and horizontal bending moments in circular silo walls

Horizontal bending moments may be ignored. Vertical moments due to restraint of wall at base of the silo depend on the type of construction employed, the thickness and reinforcement at the base must be sufficient to resist the induced bending moments and normal forces. Other vertical moments are to be resisted by nominal reinforcements - distributors - and are not determined by calculation.

4) Design load for floor slab

The weight of all the contents of the silo are to be assumed acting on the floor slab. The weight per cubic meter can be chosen according to Fig IX-17.

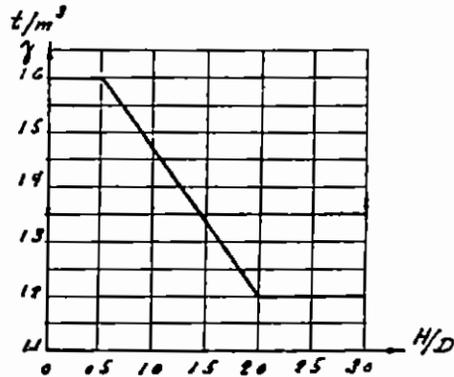


Fig IX-17

5) Constructional precautions

Figure IX-18 shows a carefully designed circular cement silo in which a high grade concrete quality C600 was used.

5.1 Concrete quality

The minimum allowed concrete quality is C300.

5.2 Circumferential reinforcement

The full ring tension is to be resisted by thin circumferential reinforcement arranged near the external face of the wall only, spacing 4-12 cms. This close spacing of the bars is essential in order to avoid wide cracks. Splices in bars should be staggered. Non-tensioned reinforcement is adequate for the purpose.

5.3 Design of base of wall

Hinged connection or, preferably, horizontally sliding bearing employing rubber rings or inserts of bituminous or synthetic substances of plastic consistence are recommended. Fig IX-19a shows a rubber bearing especially suitable where pneumatic discharge with air at high



pressure is employed, because the walls receive hardly any vertical loads from the material in the silo but do undergo considerable deformations. Fig IX 19b shows another arrangement using some convenient bituminous compound.

#### 5.4 Vertical reinforcement in wall

Not less than  $5 \phi 8$  mm/m, to be installed within one layer of circumferential reinforcement or between two layers (if any) in the regions where bending moments occur at the base of the wall, vertical reinforcements (duly calculated) should be provided both externally and internally.

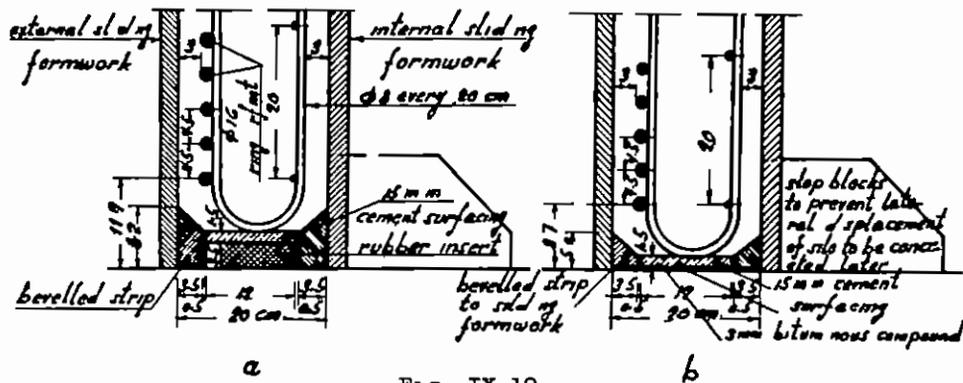


Fig IX-19

#### 6) Temperature stresses

The stresses in the walls of cement silos for a difference of temperature of  $30^{\circ}\text{C}$  are to be included in the design. The temperature stresses are however liable to become serious only if the wall is made too thick and is fixed at the base.

As soon as cracks occur, the temperature stresses are considerably reduced. The floor usually does not undergo the same amount of thermal-expansion as the wall, because the floor is protected from the high temperature by a layer of residual cement which acts as an effective insulator.

\* Refer to XI-3



## X P U M P S T A T I O N S

Under-ground pump rooms, tanks and channels are amongst the main elements of pump stations. The statical behavior of pump rooms is similar to that of under-ground tanks, all provisions, precautions and methods of design can be directly applied.

We give in the following examples some special pump stations and settling tanks constructed in the pipe factory at Helwan.

### X 1 P U M P S T A T I O N F O R R E S I D U E C H A N N E L

Figure X-1 shows the general layout giving the main elements and dimensions of the station, namely

- 1 ) The main hall of the station  $\sim 12$  ms wide,  $\sim 15$  ms long and  $\sim 7$  ms high. The roof of the hall is a reinforced concrete solid slab supported on longitudinal secondary beams, 40 ms apart, and on cross frames 6 ms apart. The outside columns of the supporting frames extend to the ground level and are supported by a horizontal beam. Their vertical axes are chosen such that they coincide on that of the 60 cms reinforced concrete wall of the pump room. It was however not possible to support the 75 cm columns of the other side on the 30 cm thick walls underneath, so it appeared necessary to extend them to the floor of the pump room. The hall is provided with a mono-rail crane supported on the main frames; it is used for mounting and maintenance of the pumps. The horizontal thrust of the frame has been safely resisted by the floor foot path slab of the hall existing in its direction.
- 2 ) The underground pump room is located underneath the station having the same width,  $\sim 12$  ms, but  $\sim 10$  ms long only. Its floor is  $\sim 6$  ms below

mean ground level. It includes the pumps which pump the clear water in the attached 2.5 ms wide tank (refer to section B-B) back to the main factory to be re-used. It is directly connected to the following two main elements:

3 ) The steel-filing residue container, ~ 4 ms wide, 10 ms long and ~ 10 ms deep (refer to fig X-1, plan, sec A-A and sec C-C) receives a mix of steel-filing residues and water transferred to it through the channel appearing in sec B-B left with its inlet appearing in sec C-C left. The steel-filing, being heavy, settles at the bottom of the container and the water at the top is transferred to the chamber appearing in sec C-C right.

4 ) The under-ground tank, < 5 ms wide and 12 ms long (refer to fig X-1, plan and sec B-B right) is used to collect the clear water of the process. From there, the water is pumped for re-use.

5 ) The steel-filing residues collected in the container given in (3) are transferred by the crane girder shown in sec A-A right to a higher container 4.65 ms wide and 3.35 ms deep. From there, they are removed from the site by cars.

The soil at the site is good incompressible cemented coarse sand at the level of the floor slab of the pump room, the attached container and clear water tank. For this reason, the three elements are directly supported on the soil, all other elements, elevated container, channels, etc., are supported on columns with isolated footings resting on the same layer.

In order to protect the containers including the steel-filing against the shocks of the bucket of the crane girder during the lifting operations, they are provided with steel rails fixed to their inner surface (refer to sections A-A and C-C). The vertical and horizontal rails are well anchored in the floor slab and wall so that they share in resisting the tensile stresses on their inner surfaces.

The details of reinforcements of the main two sections of the under-ground elements are shown in fig X-2.

## X 2 SETTLING TANK OF STEEL FILING

Figure X-3 shows the general layout and main dimensions of the tank. The soil at the site is composed of fill and loose materials to a depth of 5.5 ms from ground level underlaid by good incompressible cemented coarse sand. The weight of the filling material being relatively big,  $\sim 6 \text{ t/m}^3$ , it was decided to replace the fill and loose materials by stabilized sand ( 100 kg cement /  $\text{m}^3$  sand ) and to construct the tank directly on this layer. Its area is chosen such that the extra bearing stress at level -5.50 is within the allowed value of  $1.5 \text{ kg/cm}^2$  and with its center of gravity coinciding on that of the full tank. In order to confine the stabilized area, so that no horizontal displacement is liable to take place, it was surrounded to its full height by a masonry wall 40 cms thick.

Due to the heavy loads acting on the deep part of the tank, the concrete dimensions of its elements are big relative to those of the lightly loaded shallower part. In order to get a tank of approximately the same rigidity, the floor of the shallow part is provided with longitudinal and cross stiffening beams.

For the same reasons stated in the previous example, the deeper part of the tank is provided with steel rails fixed to the inner face of the walls and floor.

The details of reinforcements of three main sections are shown in figure X-4.

It is generally recommended to arrange longitudinal reinforcements at the top and bottom of walls to resist the eventual tensile stresses due to temperature changes or differential settlements. Such reinforcements can be seen in figures X-2, 4 and 6.

## X 3 MAIN PUMP STATION

The function of this station is to supply the different factory halls, workshops, etc. of the plant by the water necessary for the different processes of the industry. The general layout and dimensions

of the main supporting elements are shown in figures X-5 and 6

The sumps between axes A and B receive the clear water from the main sources of water - supply, while each of the tanks between axes B and C supplies certain sectors of the plant by the water required for the industry and receives the remaining part again, generally in a hot state. The water in these tanks is cooled by adding clear cold water from the sumps equal to the amount lost in the industry before being pumped again in a new cycle.

The pumps exist in the pump halls lying between axes D and E and the middle part between axes E and F.

The sumps, water tanks and pump halls are 8.5 ms below the floor level of the pump hall.

The side areas between axes E and F are used for storing. The cable ducts exist in a mezzanine floor at level 38 between the same axes.

The roof of the main hall between axes D and E is a reinforced concrete solid slab 10 cms thick and 9.40 ms above floor level. It is supported on simple girders 12 ms span and 3 ms apart, while the roof of the hall between axes E and F lies at 5.20 ms from floor level and is similarly supported on simple girders ~ 9.2 ms span.

The natural ground lies at level -7.9 and the under ground water contains relatively high percentage of sulfates ( $\text{HO}_3$ ) which attack the concrete. The soil in the site at the ground water level and below being of medium sand, it was decided to choose the bottom of the foundation few cms above the ground water and to design them for a bearing stress of  $1.5 \text{ kg/cm}^2$ . The roads and passages around the station are planned to be at level -2.0 so that the ramp and retaining walls shown in figures X-5 and 6 were essential.

The details of reinforcements of the sumps, tanks and retaining walls are shown in figure X-7.

## XI C O M P L E M E N T A R Y   D E S I G N S

### XI 1 W A L L S   A C T I N G   A S   D E E P   B E A M S

#### a) I n t r o d u c t i o n

In elevated tanks, silos, bunkers and pump houses, the walls may act as beams spanning between the supporting columns. If the depth of such walls is big, they may act as deep beams and the stress distribution can differ much from that of ordinary slender beams.

According to Navier, the stress - distribution in the homogeneous elastic, slender beams is assumed linear and, the stresses are proportional to their distance from the neutral axis which passes through the center of gravity of the section (Fig XI 1)

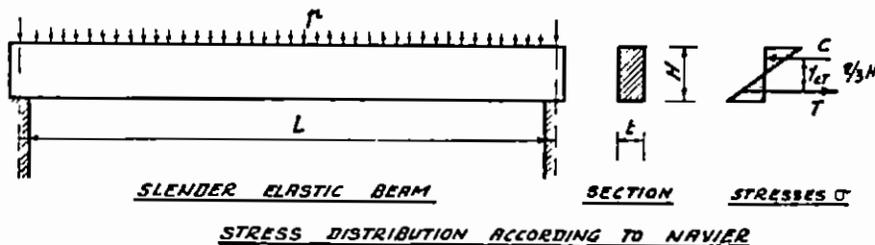


Fig XI-1

This assumption is valid only in slender beams where the ratio of depth  $H$  to span  $L$  is smaller than  $1/2$  in simple beams and  $2/5$  in continuous beams, beyond these limits the stress distribution is not linear and the arm of internal forces  $Y_{CT}$  is much smaller than according to Navier.

Fig XI 2 shows the stress distribution at the middle and

over the supports of a continuous beam subject to uniform load  $p$  at its top surface and having a depth  $H$  equal to span  $L$

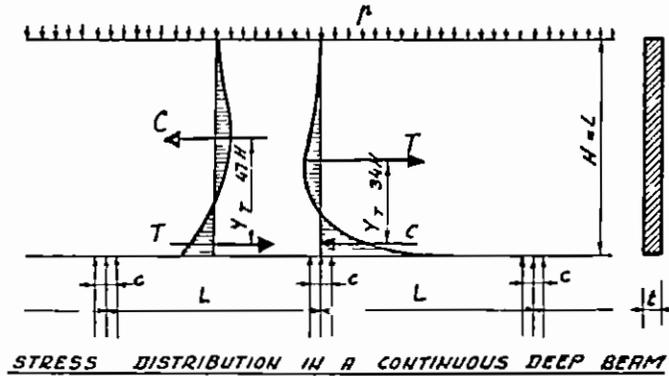


Fig XI-2

### b) Fundamental Equations of Stress Distribution

In figure XI 3 we give the stresses acting on an elemental area  $dx \ dy$  in the middle plane of a deep beam subject to uniform load  $p$

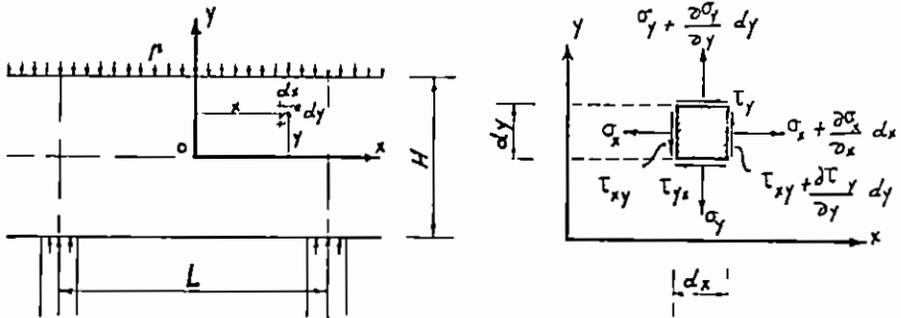


Fig XI-3

According to the mathematical theory of elasticity the stresses must satisfy the condition

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

in which  $F(x, y)$  is the Airy biharmonic stress function, whose derivatives give the stresses in the form :

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} \quad \tau_{xy} = \tau_{yx} = - \frac{\partial^2 F}{\partial x \partial y}$$

The given basic equation of the stress function includes no elasticity constants ( E , G and  $\nu = \frac{1}{m}$  ) It can be applied for every homogeneous isotropic elastic material that follows Hook's law It can also be applied to reinforced concrete beams before the formation of tension cracks ( Stage I ) After the formation of cracks, the tension reinforcements must be sufficient to resist all the tensile stresses in the section it is recommended to arrange these reinforcements according to the tension trajectories The calculation according to the theory of elastic beams gives sufficient safety as far as the amount of the tension steel is concerned, it is however essential to make good anchorage for the tension steel at the supports

The Airy function can be imagined as the elastic surface of an elastically restrained flat plate under certain edge conditions In this manner , the displacement of this surface is a measure for the normal stresses  $\sigma_x$  and  $\sigma_y$  and its twist is a measure for the shear stresses  $\tau_{xy}$

The basic equation used for solving deep beams is as shown a linear partial differential equation of the fourth degree , its integration constants must satisfy the edge conditions of the case under consideration

The solution of this differential equation was done by Dischinger<sup>(1)</sup> using the Fourier -Series, the results of his mathematical investigation were presented by the American Portland Cement Association<sup>(2)</sup> Bay<sup>(3)</sup> and recently El-Darwish<sup>(4)</sup> have used the method of differences Chow<sup>(5)</sup> and others solved some special problems of deep beams Theimer<sup>(6)</sup> has given a valuable series of tables

and curves for the design of reinforced concrete deep beams. A theoretical investigation has been recently published by Worch<sup>(7)</sup>. We give in the following some of the important results of these investigations as they may be required for tank problems<sup>(8)</sup>.

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- (1) F Dtschinger " Beitrag Zur Theorie der Halbscheibe und des Wandartiger Balkens " Publications of the International Association of Bridge and Structural Engineering Zurich, Switzerland, Vol 1, 1932 pp 69-93
- (2) ' Design of Deep Girders " Pamflet No St 66, Concrete Information, Structural Bureau, Portland Cement Association, Chicago, III
- (3) H Bay ' Wandartiger Trager und Bogenscheibe " Published by Konrad Wittwer Stuttgart, 1960
- (4) I A El Darwish ' Stresses in Simply Supported Deep Beams ' Bulletin of the Faculty of Engineering, Alexandria University , Vol V 1966
- (5) Chow, Li , Conway, H D , and Winter , G , " Stresses in Deep Beams " Transactions, A S C E , Vol 118 , 1953 , p 686
- (6) O F Theimer " Hilfstafeln zur Berechnung Wandartiger Stahlbetontrager " Published by Wilhelm Ernst & Sohn Berlin, 1963
- (7) G Worch " Elastische Scheiben " Beton Kalender 1967 Vol II pp 1 128
- (8) Leonhardt Wandartiger Trager  
Deutscher Ausschuss fur Beton und Stahlbeton

c) Simply Supported Deep BeamsNotations Fig XI 4

|                      |       |
|----------------------|-------|
| Span of beam         | $L$   |
| Height of beam       | $H$   |
| Wall thickness       | $t$   |
| Breadth of support   | $c$   |
| Support width        | $t_s$ |
| Total length of beam | $L_t$ |
| Load intensity       | $p/t$ |

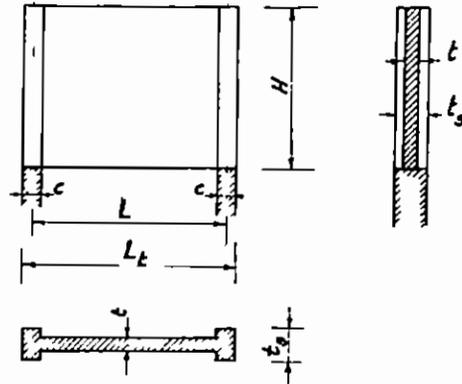


Fig XI-4

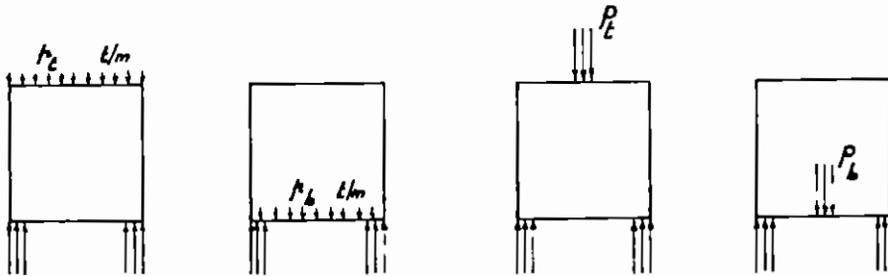
Types of Loading Fig XI 5

Fig XI-5

Stress Distribution

The stress distribution depends on  $\frac{H}{L}$ ,  $\frac{c}{L}$ , type ( uniform or concentrated ) and position ( top or bottom ) of loading , as well as  $\frac{t_s}{t}$

Figure XI 6 shows the stress distribution in a simply supported beam subject to uniformly distributed load  $p$  acting on its top surface for  $\frac{c}{L} = 1/10$ ,  $t_s = t$  and various values of  $\frac{H}{L}$  in stage I

Notes

- 1) Stress distribution is linear for  $H < \frac{L}{2}$  and curved for  $H > \frac{L}{2}$
- 2) For  $\frac{L}{2} < H < L$  the magnitude of the compressive stresses decreases with increasing depth
- 3) No stresses are resisted by the upper part of a deep beam above  $H = L$
- 4) The max tensile stress at the lower fiber is for  $H = \frac{L}{2}$  equal to 1.5 times and for  $H = L$  equal to 2.2 times the values according to Navier
- 5) The arm of the internal forces  $Y_{CT} = 0.62 - 0.78 H$  i.e. its magnitude as a factor of  $H$  does not vary much
- 6) If the load  $p$  acts at the bottom, the normal stresses  $\sigma_x$  and the shear stresses  $\tau$  are not much affected, while the compressive stresses  $\sigma_y$  are much reduced and high tensile stresses are created at the lower zones of the beam as shown in fig XI 8

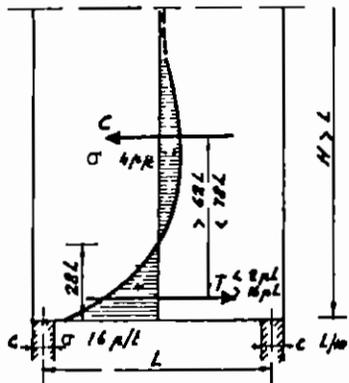
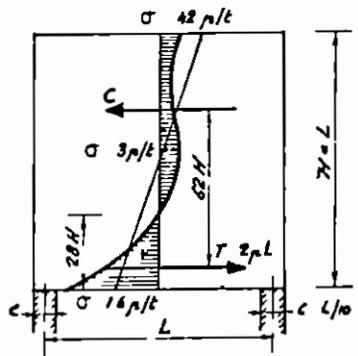
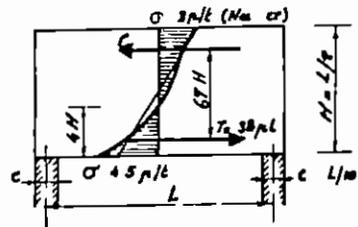
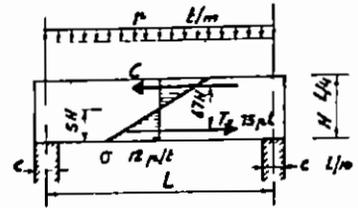


Fig XI-6

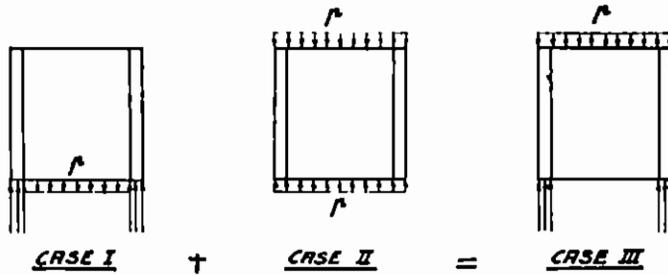


Fig XI-7

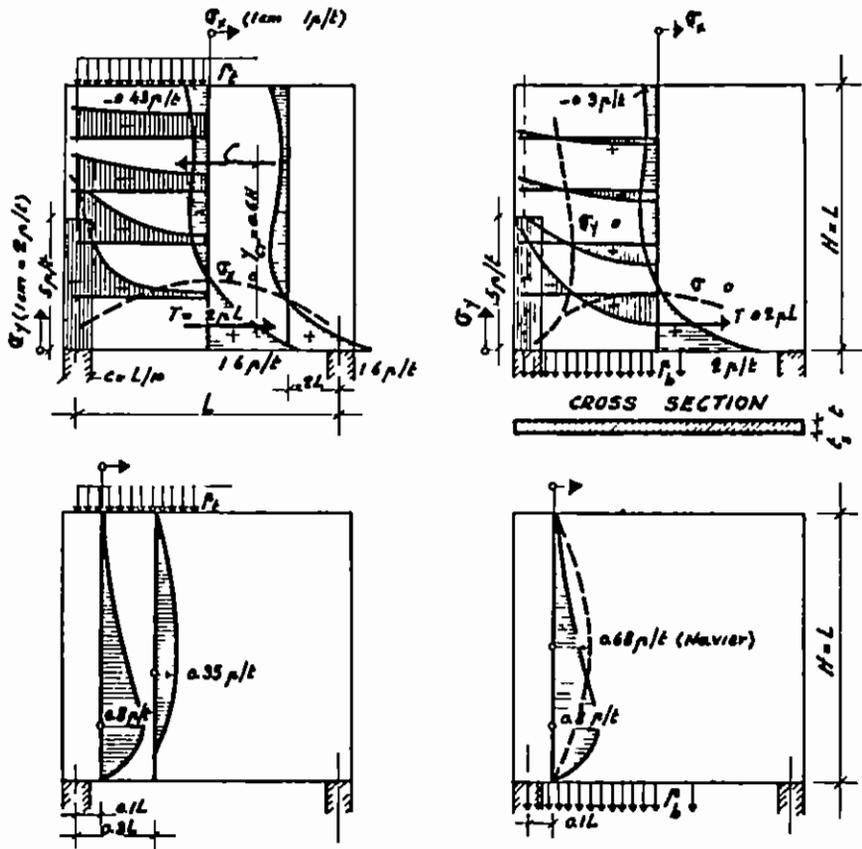


Fig XI-8

The distribution of normal stresses in simple deep beams subject to concentrated loads acting at the middle of top surface is given in figure XI 9

In deep beams subject to concentrated loads acting at the lower surface, the stresses are distributed in a depth equal to  $L$  only, bigger depths are not effective

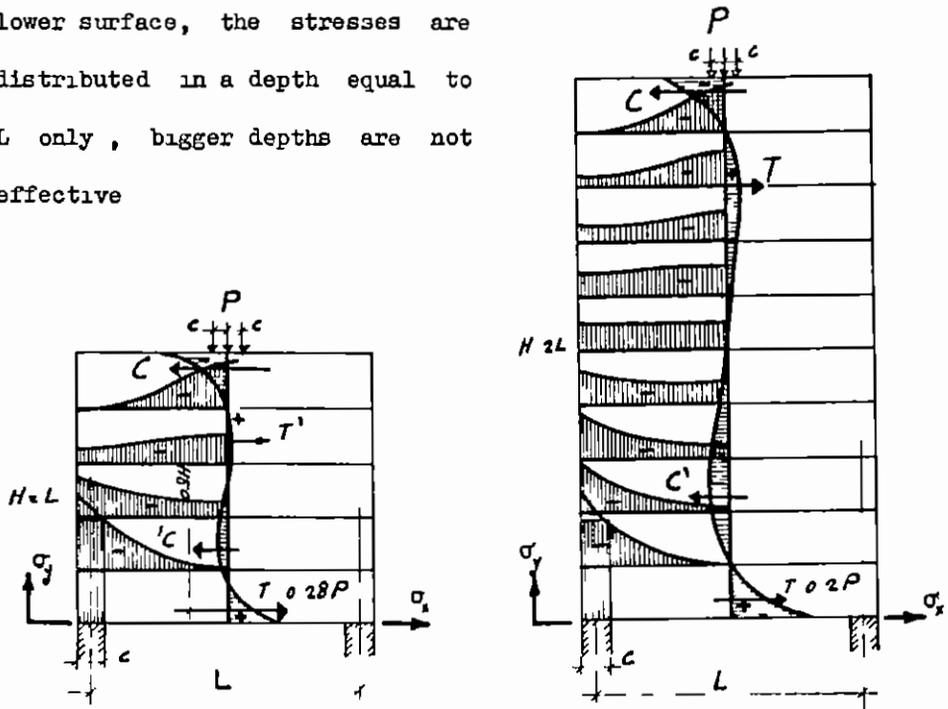


Fig XI-9

The total tension  $T$  is the same whether the load  $P$  acts at top or bottom. Big tensile stresses  $\sigma_y$ , especially in the lower part of the beam are created and must be resisted by vertical reinforcements

Stiffening the edges of the beam affects the stresses in a manner similar to that of uniform loads

#### d) Continuous Deep Beams

The stress distribution at the center lines of the spans and

over the supports of continuous deep beams subject to uniform loads acting on their top surface is shown in figure XI 10

It has to be noticed that the stress distribution at the middle of the spans is similar to that of simple beams but with smaller values due to continuity, whereas at the supports, the arm of internal forces is relatively small and the compressive stresses at the lower fiber are high and may govern the design

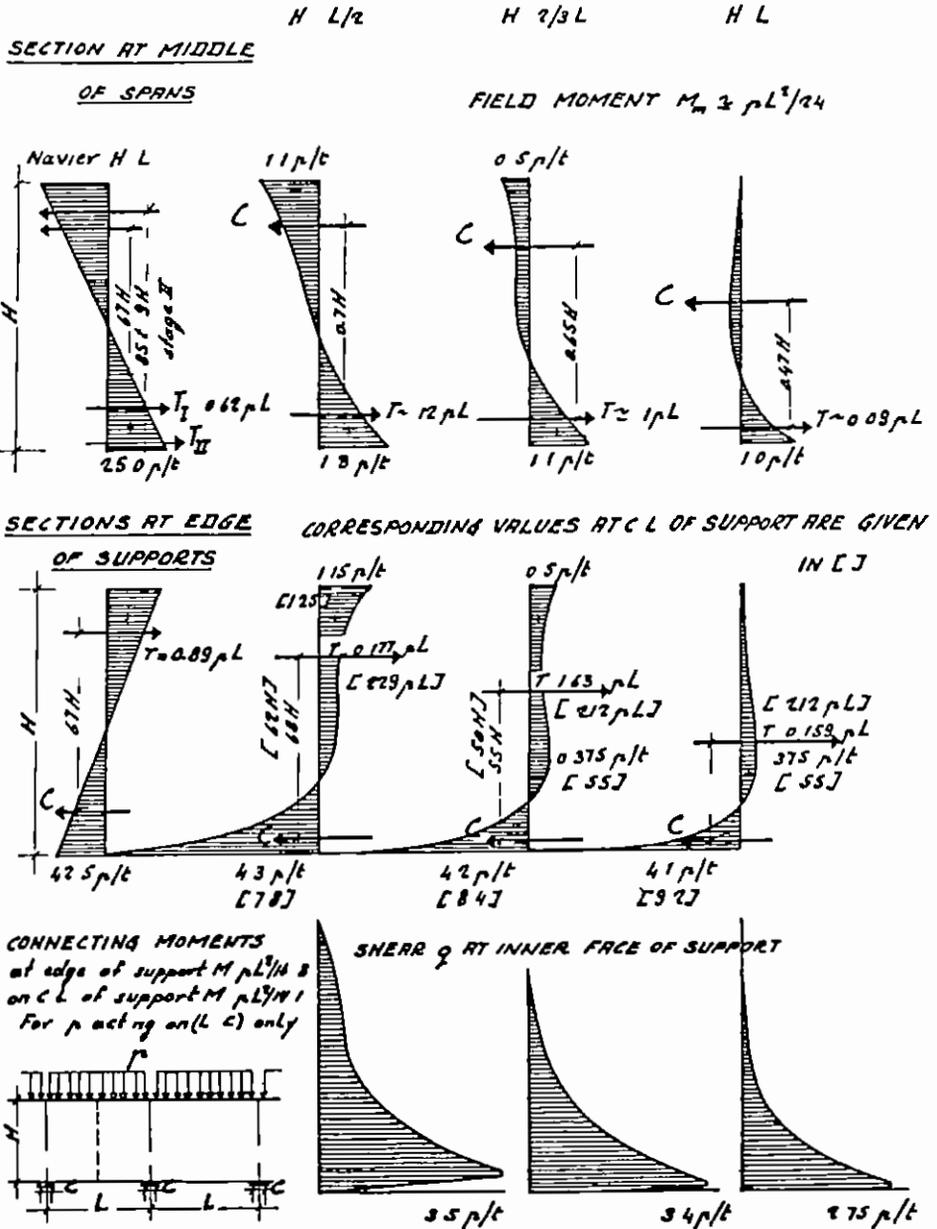
It has further been found that the connecting moments in continuous deep beams is smaller than that of slender beams of the same spans due to the high compressive strains that are liable to be developed at the supports and as a result of this fact, the field moments are bigger than those of slender beams

Figure XI 11 gives the stress distribution at the center lines of continuous deep beams subject to concentrated loads at the top or bottom surfaces. The stresses over the center lines of the supports have the same values but with opposite signs

#### e) Guide Lines for the Design of Reinforced Concrete Deep Beams

##### Introduction

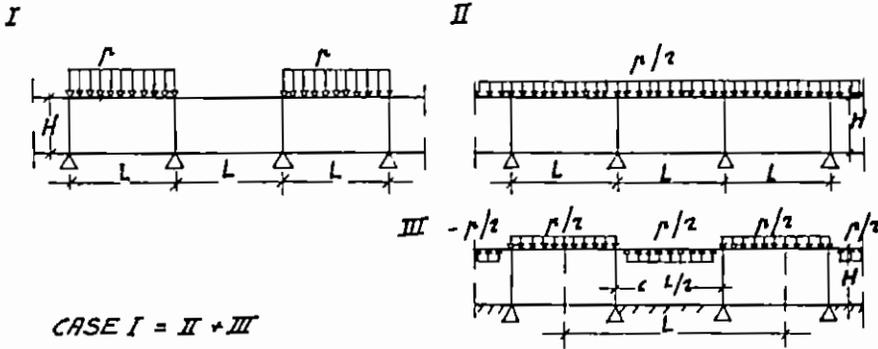
Experiments have shown that the theory of elastic deep beams can be applied to reinforced concrete before the formation of tension cracks (Stage I). After the formation of cracks, which generally take place under working loads, the real stresses differ



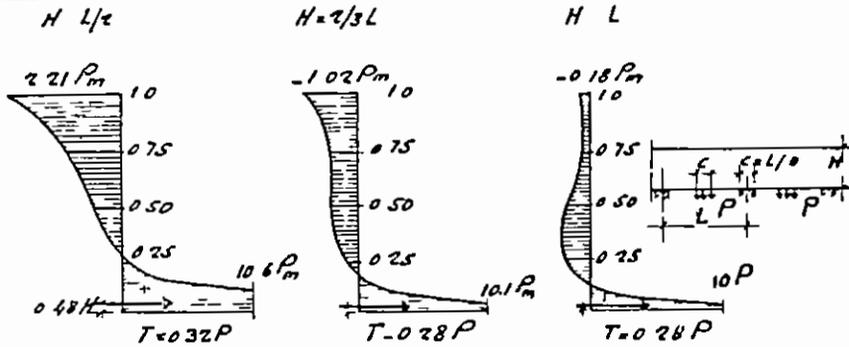
NORMAL AND SNERR STRESSES IN DEEP CONTINUOUS BEAM  
 SUBJECT TO UNIFORM LOADS

Fig XI-10

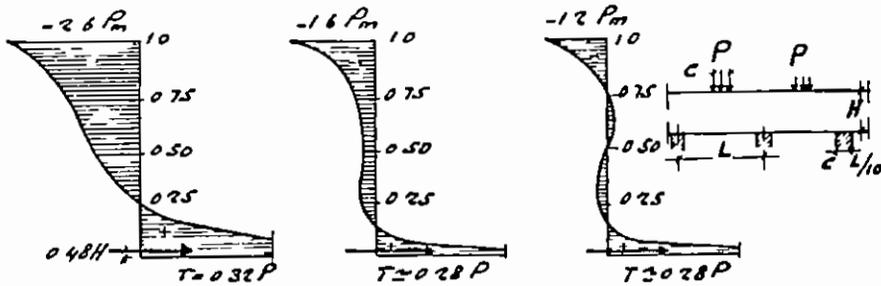
**SUPERPOSITION OF LOADING**



**P ACTING AT BOTTOM**



**P ACTING AT TOP**



STRESSES AT CENTER LINE OF SUPPORT HAVE THE SAME VALUES WITH OPPOSITE SIGN

STRESSES AT THE MIDDLE OF THE SPANS OF CONTINUOUS DEEP BEAMS SUBJECT TO CONCENTRATED LOADS

Fig XI-11

much from the theoretical values. The arm of the internal forces is increased and the stresses in the steel will be low if its magnitude is determined according to the elastic theory. On the other hand, the inclined principal compressive stresses at the supports are higher than the values determined by the elastic theory and they may be critical in thin heavily loaded deep beams. By the increase of the width of cracks under heavy loads, the compressive bending stresses increase by a ratio bigger than the increase of the load, but they generally do not govern the failure load.

Due to the big moment of inertia of the cross-sections of deep beams, any movement of the supports causes high internal stresses and is not to be neglected.

The compressive stresses at the middle of the spans is generally low and need not to be checked. The beam must however be sufficiently thick ( $> 12$  cms) that it does not buckle, otherwise stiffeners or compression flanges are to be arranged. In thin deep beams, stiffeners at the supports are essential, they must be introduced to a sufficient height from the bottom surface of the beam.

The tension steel is generally determined such that it resists the full tensile force calculated according to the elastic theory of deep beams in stage I.

The shear stresses vary much along the depth of the beam and are generally low so that no special bent bars or stirrups are needed for this purpose.

### Determination of Tension Reinforcements

#### Simply supported deep beam subject to uniform loads

The maximum tension at the bottom of the beam is given by

$$\max T = \frac{\max M_o}{Y_{OT}}$$

in which

$$Y_{CT} \approx 0.6 H \text{ for } H \leq L \quad \text{and} \quad Y_{CT} \approx 0.6 L \text{ for } H > L$$

so that

$$\max T = \frac{\max M_o}{0.6 H} \quad \text{and} \quad \max T = \frac{\max M_o}{0.6 L} \quad "$$

In case of uniformly distributed load  $p/m'$

$$\max M_o = p \frac{L^2}{8}$$

so that

$$\max T = 0.2 p L \frac{L}{H} \quad \text{for } H \leq L$$

and

$$\max T = 0.2 p L \quad \text{for } H > L$$

The max area of tension steel  $A_s$  is therefore

$$A_s = \max T / \sigma_s$$

It is recommended to extend the tension steel over the whole length of the span and to anchor it well at the supports Fig XI 12

For loads hung at the bottom vertical reinforcements carrying the full reaction are to be introduced for the full height  $H$  over a length of  $0.7 L$

### Concentrated Loads

Deep beams subject to concentrated loads may be calculated in the same way as those subject to uniform loads introducing the corresponding values of  $M_o$ . The tension can also be directly determined from the triangle of forces shown in figure XI 13

### Continuous Deep Beams

It has been stated before that connecting moments in deep continuous beams are smaller, while field moments are bigger than those

UNIFORM LOAD

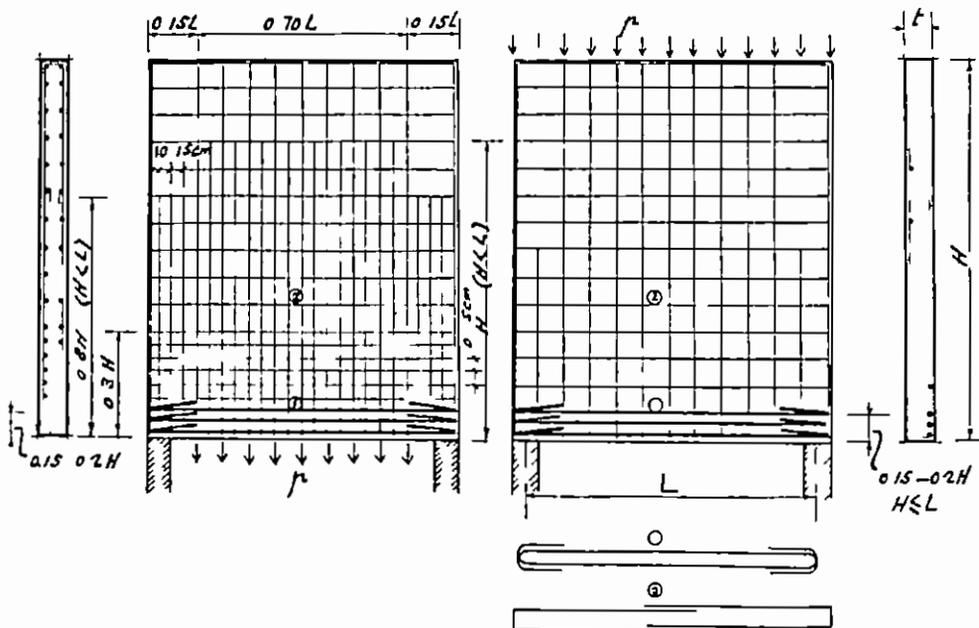


Fig XI-12

CONCENTRATED LOAD

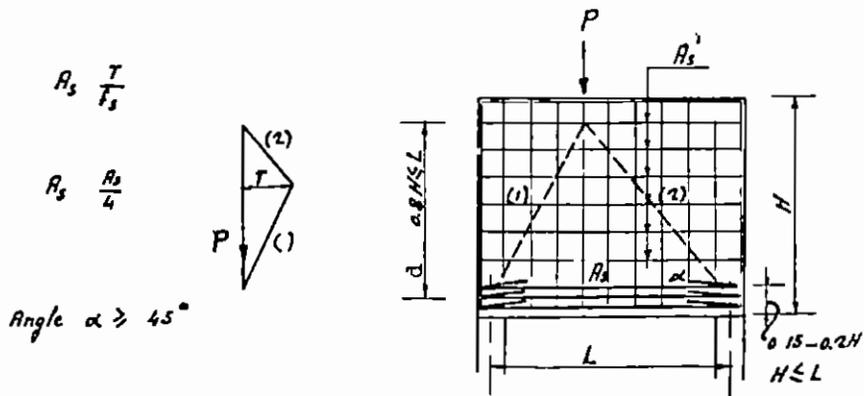


Fig XI-13

of ordinary slender continuous beams. In order to make a simple, sufficiently safe, design taking this fact in consideration one may calculate the bending moments in the usual way according to the theory of elasticity as applied to normal slender beams and, to increase  $Y_{CT}$  at the supports and decrease it at the center lines in such a way as to give results conforming with the data extracted from the experiments in the following manner

$$Y_{CT} = 0.5 H \text{ for } H \leq L \quad \text{and} \quad Y_{CT} = 0.5 L \text{ for } H > L$$

The tension  $T$  being equal to 
$$T = \frac{M}{Y_{CT}} \quad \text{then}$$

for uniform load and  $H \leq L$ , we get

At middle of outer spans 
$$T_m \approx 0.18 p L$$

At middle of inner spans 
$$T_m \approx 0.13 p L$$

At center line of inner support of  
outer span 
$$T_s \approx 0.25 p L$$

At center line of other inner  
supports 
$$T_s \approx 0.20 p L$$

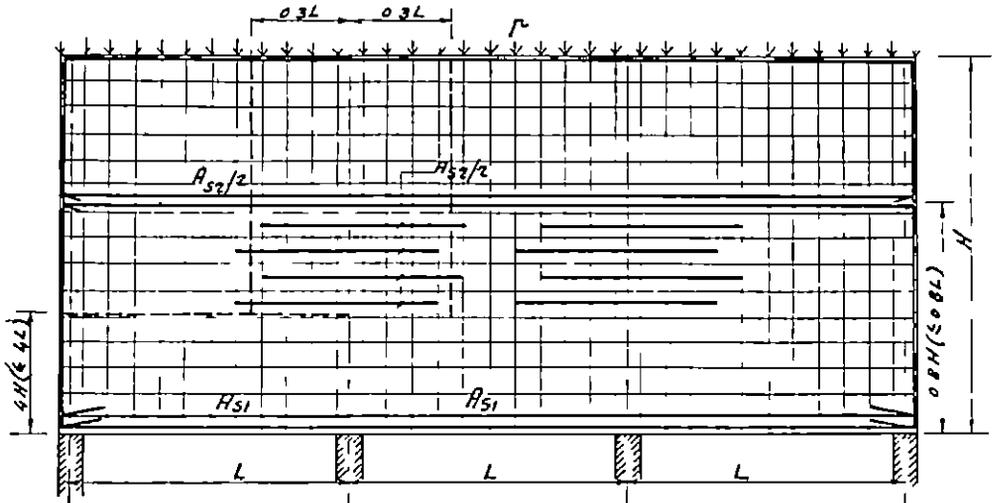
The steel reinforcement is given by 
$$A_s = T / \sigma_s$$

It is recommended to extend the full amount of the tension steel at the middle of the spans to the supports, half the tension steel required at the center lines of the supports may be arranged over the full length of the adjoining spans and the other half to be extended to a distance equal to  $0.3 L$  on each side of the center line of the supports as shown in figure XI 14

Loads hung at the lower surface of a deep continuous beam can be treated in the same way as given before for simple beams

It is however essential to note that the governing design

criterion for continuous deep beams is the stress conditions in the support regions ( support reaction & principal compressive stresses),



DETAILS OF REINFORCEMENTS OF A CONTINUOUS DEEP BEAM SUBJECT  
TO UNIFORM LOADS AT TOP

Fig XI-14

while for the reinforcements - particularly for the tension chord - constructive requirements ( crack limitation & anchorage) are governing rather than the statical ones. Therefore no great accuracy is necessary for the design of chord reinforcement because on the one hand the cross section of the required steel area is anyhow small & on the other hand the opening of wide cracks has to be prevented over a high tensile zone. Thus it is sufficient in most cases to calculate the required tension reinforcements from the approximate formulae given before.

A danger of shear failure, which had to be resisted by stirrups or bentup bars, does not exist in case of girders with loading at top edge. A light orthogonal reinforcement mesh of vertical and horizontal stirrups, therefore, suffices. In cases of indirect loading or of hanging loads, an additional web reinforcement may be essential.

## XI 2 DESIGN OF PYRAMID ROOFS

Pyramid roofs<sup>\*</sup> give in many cases a convenient solution for the cover of rectangular containers and pump houses

Under the effect of vertical loads, a pyramid roof as shown in figure XI 15 is subject to inclined meridian forces  $T_1$  acting along the triangular sides of the pyramid, and horizontal cross forces  $T_2$ . Both  $T_1$  and  $T_2$  are compression

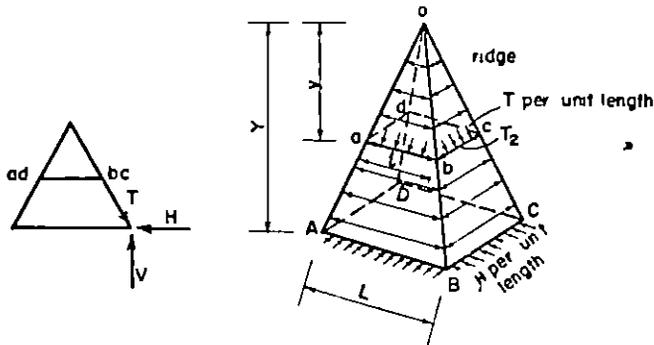


Fig XI-15

In addition the triangular plane surfaces of the pyramid are subject to bending moments both in the inclined and horizontal directions

### a) The Meridian Force

The value of the meridian force  $T_1$  at any intermediate depth  $y$  from the vertex  $O$  in a pyramid whose sides make an angle  $\alpha$  with the vertical is obtained by equating the vertical components of the meridian forces  $T_1$  to the vertical loads of the portion above

Hence, if the pyramid weighs  $g$  per unit of area, the weight of the triangular portion above  $a$   $b$  is given by ( Fig XI 16 )

---

\* Terington 'Pyramid Roofs'

$$\frac{1}{2} g l y'$$

in which

$$l = 2 y \tan \alpha$$

and

$$y' = y / \cos \alpha$$

$y$  is the height of any triangular inclined surface between  $o$  and  $a b$  or  $b c$

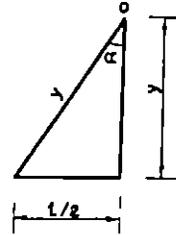


Fig XI-16

The vertical component of the meridian force  $T_1$  along one side is given by

$$T_1 l \cos \alpha$$

Therefore

$$\frac{1}{2} g l y = T_1 l \cos \alpha \quad \text{or}$$

$$\frac{1}{2} g y / \cos \alpha = T_1 \cos \alpha \quad \text{and}$$

$$\underline{T_1 = \frac{1}{2} g y / \cos^2 \alpha} \quad \text{or} \quad \underline{T_1 = g l / 4 \sin \alpha \cos \alpha}$$

These expressions give the magnitude of the meridian force per unit width at any intermediate level due to dead loads  $g$  per unit area

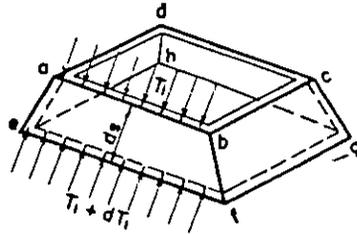
For a superimposed load per unit horizontal area we get

$$p l^2 / 4 l = T_1 \cos \alpha \quad \text{or}$$

$$\underline{T_1 = p l / 4 \cos \alpha}$$

#### b) The Horizontal Thrust

The horizontal thrust  $T_2$  exists at all intermediate levels. This may be demonstrated by considering an element strip of length  $ds$  cut from the structure by two horizontal planes, one  $a b c d$  at a depth  $y$  from the vertex and the other,  $e f g h$  at a depth  $y + dy$  from the vertex (Fig XI 17)



$ab = l$

$ef = l$

$2(y + dy) \tan \alpha$

Fig VI-17

The horizontal component of  $T_1$ , which is  $T_1 \sin \alpha$ , causes outward thrust on the elemental portion  $ds$ , and the horizontal component of the supporting thrust  $T_1 + dT_1$  causes inward thrust. The outward force per strip  $ds = T_1 \sin \alpha$

$$= T_1 \sin \alpha \cdot 2y \tan \alpha$$

but 
$$T_1 = \frac{\rho y}{2 \cos^2 \alpha}$$

then the outward force per strip = 
$$\frac{\rho y}{2} \frac{\sin \alpha}{\cos^2 \alpha} \cdot 2y \tan \alpha$$

The inward force due to  $T_1 + dT_1$  per strip  $ds$

$$= (T_1 + dT_1) \sin \alpha \cdot l$$

$$= (T_1 + dT_1) \sin \alpha \cdot 2(y + dy) \tan \alpha$$

The net result of these two forces is an inward thrust over the width  $ds$ . This is resisted by two equal horizontal reactions in the planes at right angles. Let the magnitude of this resultant reaction be  $T_2$  per unit width of strip. The horizontal compression induced in the adjacent planes is therefore

$$T_2 ds = (T_1 + dT_1) \sin \alpha (y + dy) \tan \alpha - T_1 \sin \alpha y \tan \alpha \quad \text{or}$$

$$\frac{T_2 ds}{\cos \alpha} = d(T_1 \sin \alpha y \tan \alpha)$$

For a dead load  $\rho/m^2$

$$T_1 = \frac{\rho y}{2 \cos^2 \alpha}$$

then

$$\frac{T_2}{\cos \alpha} = \frac{d}{dy} \left( \frac{g y}{2 \cos^2 \alpha} y \sin \alpha \tan \alpha \right) \quad \text{but} \quad \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

then

$$\frac{T_2}{\cos \alpha} = \frac{d}{dy} \left( \frac{g}{2} y^2 \frac{\tan^2 \alpha}{\cos \alpha} \right) \quad \text{and}$$

$$T_2 = \frac{g}{2} 2 y \tan^2 \alpha \quad \text{or}$$

$$\underline{T_2 = g y \tan^2 \alpha} \quad \text{or} \quad \underline{T_2 = \frac{1}{2} g l \tan \alpha}$$

These expressions give the magnitude of the cross horizontal compression  $T_2$  at any depth  $y$  from the vertex due to dead loads  $g$  per unit area

For a superimposed load  $p$  per unit horizontal area, we have

$$T_1 = \frac{p l}{4 \cos \alpha} \quad \text{but} \quad l = 2 y \tan \alpha \quad \text{then}$$

$$T_1 = \frac{p 2 y \tan \alpha}{4 \cos \alpha} = \frac{p y \tan \alpha}{2 \cos \alpha}$$

Therefore

$$\frac{T_2}{\cos \alpha} = \frac{d}{dy} \left( \frac{p y \tan \alpha}{2 \cos \alpha} y \sin \alpha \tan \alpha \right)$$

$$T_2 = \frac{d}{dy} \left( \frac{1}{2} p y^2 \tan^2 \alpha \sin \alpha \right) \quad \text{i e}$$

$$\underline{T_2 = p y \tan^2 \alpha \sin \alpha} \quad \text{or} \quad \underline{T_2 = \frac{1}{2} p l \tan \alpha \sin \alpha}$$

### c) The bending Moments :

The bending moments in the different surfaces of a pyramid depend on the actual proportions of each triangle forming a panel as they influence the manner in which the panels bend

In an equilateral triangle the bending effect is symmetrical and analogous to the conditions existing in a square panel

In an acute-angled triangle, as would occur in a sharply pointed pyramid, the most of the bending is taken horizontally from ridged to ridge i e analogous to one way slabs in the horizontal direction

At the opposite extreme is the case of a panel of an obtuse angled triangle in which the base is very long in comparison with the sides In this case, the most of the bending is taken in the sloping direction of the slab

It will be noted that the moments compared in these three cases are those that occur horizontally and in the sloping direction of the slab Actually in any triangular panel, the distribution of maximum moments is undoubtedly across the corners as indicated in figure XI 18 a particularly in triangular panels which approximate to equilateral proportions From the point of view of the moments, therefore, the reinforcements might more resonably be disposed along the diagonals shown in figure (a)



Fig XI-18

However, it is generally more convenient to reinforce the slab in two directions at right angles, that is, horizontally and up and down the slab as indicated in figure (b) Hence, it is necessary to estimate the moments and direct thrusts in these two directions at the point of maximum deflection, which is approximately situated at the center of pressure

The exact mathematical analysis of bending moments in triangular slabs is too complicated for practical design, but a fairly close estimate of the effective span may be made with sufficient accuracy

For the purpose of the calculation this effective span may be taken as the diameter of the inscribed circle which touches all the sides figure XI 19 and for convenience the center of this circle is referred to as the center of pressure

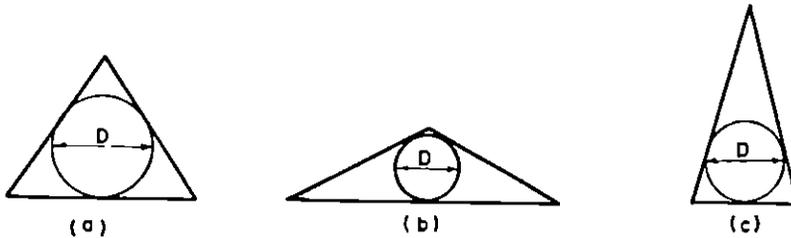


Fig XI-19

To find the bending moments to be allowed for it is convenient to consider three conditions

- 1) A panel approximating to an equilateral triangle
- 2) A panel with an obtuse angle at the vertex
- 3) A panel with an acute angle at the vertex

### C e I

The conditions at the supports are an indeterminate fixity of the slab at the lower edge along the supporting beam and continuity in the horizontal direction along the ridges forming the side supports. Comparing these conditions with a square slab spanning in two directions at right angles, it is a fundamental law that if two opposite edges are continuous supports and the other two edges are only partly fixed, the larger moment is taken in the direction of greater fixity. From this analogy, the moment in the horizontal direction will tend to be greater than the moment in the sloping direction. For a panel approximating to an equilateral triangle, the following moments may be taken to cover own weight and the effects of superimposed loading including wind, which actually causes a thrust on the wind ward side and a suction on the leeward side

In the horizontal direction in the span and over the ridges

$$M = \pm w \sin \alpha \frac{D^2}{18}$$

In the sloping direction

$$M = w \sin \alpha \frac{D^2}{18}$$

in which  $w = g + p$

### Case 2

Where a panel has an obtuse angle at the vertex, practically all the bending is taken in the sloping direction. The following bending moment may therefore be taken to cover effects of superimposed loading and own weight

$$M = v \sin \alpha \frac{D^2}{18}$$

in the span and over the ridges

In the horizontal direction distribution steel, equal to about 20% of the main reinforcement is required

### Case 3

In the other extreme case, where a panel has a particularly acute angle at the vertex, practically all the moment is taken in the horizontal direction and

$$M = \pm w \sin \alpha \frac{D^2}{12}$$

in the span and over the ridges

In the sloping direction, distribution steel, equal to about 20% of the main reinforcement is required

d) Forces at Bottom Edge

The lower edge of the pyramid is subject to a vertical reaction  $V$  per meter equal to

$$V = g \frac{Y}{2} + p \frac{L}{4}$$

and a horizontal thrust  $H$  giving tension in each of the lower edges equal to  $T$  where

$$T = \frac{H L}{2}$$

in which

$$H = V \tan \alpha$$

For a dead load  $g$ , we get

$$T = \frac{H L}{2} = V \tan \alpha \frac{L}{2} = g \frac{Y}{2} \tan \alpha \frac{L}{2}$$

But  $Y' = \frac{L}{2 \sin \alpha}$  then

$$T = \frac{g L}{4 \sin \alpha} \frac{\sin \alpha}{\cos \alpha} \frac{L}{2} \quad \text{i.e.}$$

$$T = \frac{g L^2}{8 \cos \alpha}$$

and for a superimposed load  $p$ , we get

$$T = \frac{H L}{2} = V \tan \alpha \frac{L}{2} = p \frac{L}{4} \tan \alpha \frac{L}{2} \quad \text{i.e.}$$

$$T = p \frac{L^2 \sin \alpha}{8 \cos \alpha}$$

Horizontal bending is also induced along the lower edges, and the magnitude of the moments is

In the span  $M = \frac{H L^2}{24}$

$$M = \frac{L^3}{96 \cos \alpha} (g + p \sin \alpha)$$

and at the corners

$$M = \frac{H L^2}{12}$$

or

$$M = \frac{L^3}{48 \cos \alpha} (g + p \sin \alpha)$$

It is however possible to consider each side of the pyramid as a deep beam subject to a load equal to the maximum meridian force at the base of the pyramid. The steel at the bottom edge should be sufficient to resist the tension of the deep beam  $T = \frac{L}{\sqrt{CT}}$  plus the tension due to the reaction of the load on the two sides normal to that under consideration and given by  $T = \frac{H L}{2}$

e) Example

It is required to cover a tank as that shown in figure VII 30 by a pyramid roof of the form shown in figure XI 20

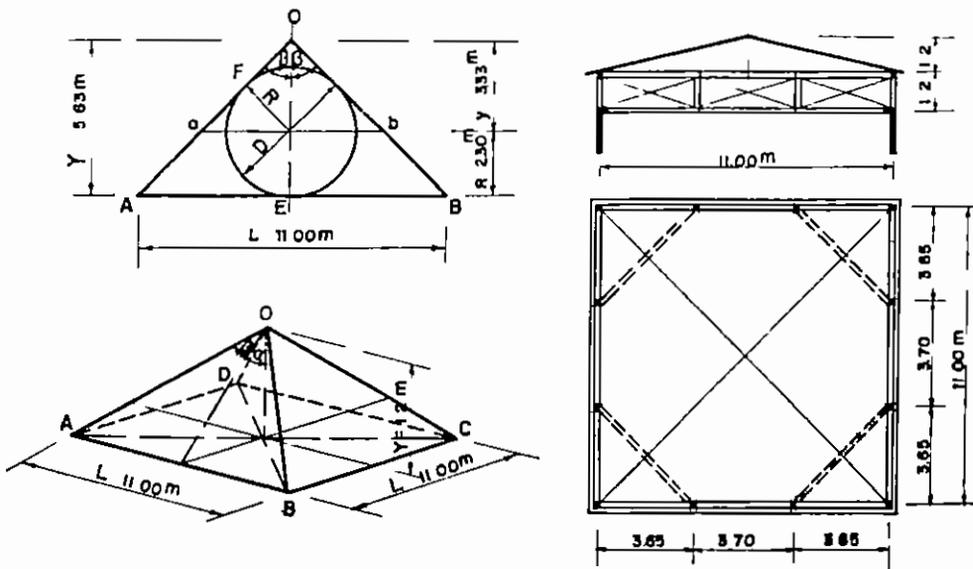


Fig XI-20

Data

Base L = 11 00 m

Height Y = 1 20 m

Thickness of pyramid slab

t = 10 cms

Total dead + Superimposed loads

w = 350 kg/m<sup>2</sup> surfaceSolution

$$\text{Length } \overline{OE} = Y' = \sqrt{1.2^2 + 5.5^2} = \sqrt{31.69} = 5.63 \text{ ms}$$

$$\sin \alpha = \frac{5.5}{5.63} = 0.978, \quad \cos \alpha = \frac{1.2}{5.63} = 0.213, \quad \tan \alpha = \frac{5.5}{1.2} = 4.58$$

$$\text{Length of ridge} = \overline{OA} = \sqrt{5.63^2 + 5.5^2} = \sqrt{61.94} = 7.85 \text{ ms}$$

$$\overline{AF} = \overline{AE} = 5.5 \text{ m} \quad \frac{R}{OF} = \tan \beta$$

$$\overline{OF} = \overline{OA} - \overline{AF} = 7.85 - 5.5 = 2.35 \text{ ms}$$

$$\tan \beta = \frac{\overline{AE}}{Y'} = \frac{5.5}{5.63} = 0.978$$

Therefore

$$R = D/2 = \overline{OF} \tan \beta = 2.35 \times 0.978 = 2.30 \text{ ms} \quad \text{ie}$$

$$D = 4.6 \text{ ms} \quad \text{and}$$

$$\text{the height } y' = 5.63 - 2.30 = 3.33 \text{ ms} \quad \text{ie}$$

the height of the vertex of the pyramid from the center of pressure

$$y = y' \cos \alpha = 3.33 \times 0.213 = 0.71 \text{ ms}$$

The average meridial force  $T_1$  across section a b

$$T_1 = \frac{1}{2} \frac{w y}{\cos^2 \alpha} = \frac{1}{2} \frac{350 \times 0.71}{0.213^2} = 2740 \text{ kg/m}$$

The bending moment in the inclined and horizontal directions at section a b

$$M = w \sin \alpha \frac{D^2}{16} = 350 \times 0.978 \times \frac{4.6^2}{16} = 450 \text{ kg/m}$$

The horizontal cross compression force  $T_2$  at section a b

$$T_2 = w y \tan^2 \alpha = 350 \times 0.71 \times 4.58^2 = 5220 \text{ kg/m}$$

 $T_2$  can further be calculated from the relation :

$$T_2 = \frac{1}{2} w l \tan \alpha$$

knowing that  $l/L = y'/Y'$  then  $l = 11 \times \frac{3.33}{5.63} = 6.5 \text{ m}$  and

we get

$$T_2 = \frac{1}{2} \times 350 \times 6.5 \times 4.85 = \underline{5220} \text{ kg/m}$$

The max meridian force at A B

$$\max T_1 = \frac{1}{2} \frac{w Y}{\cos^2 \alpha} = \frac{1}{2} \frac{350 \times 12}{0.213^2} = \underline{4520} \text{ kg/m}$$

The max horizontal cross compressive force  $T_2$  at section A B

$$\max T_2 = \frac{1}{2} w L \tan \alpha = \frac{1}{2} \times 350 \times 11 \times 4.85 = \underline{8900} \text{ kg/m}$$

The vertical load on the lower horizontal beam

$$V = \frac{1}{2} w Y' = \frac{1}{2} \times 350 \times 5.63 = \underline{985} \text{ kg/m}$$

The horizontal load / m

$$H = V \tan \alpha = 985 \times 4.58 = \underline{4510} \text{ kg/m}$$

The beam at the bottom of the pyramid is supported by vertical posts arranged at the four corners and at the third points of each side and horizontal diagonal ties arranged at the third points as shown in figure XI 20. The max bending moment in the vertical beam

$$M_{\max} \approx \frac{1}{12} V \left( \frac{L}{3} \right)^2 = \frac{1}{12} \times 985 \left( \frac{11}{3} \right)^2 = \underline{1100} \text{ kgm}$$

The load per post

$$P = V L/3 = 985 \times 11/3 = \underline{3600} \text{ kgs}$$

The max bending moment in the horizontal beam

$$M_{\max} \approx \frac{1}{12} H \left( \frac{L}{3} \right)^2 = \frac{1}{12} \times 4510 \left( \frac{11}{3} \right)^2 = \underline{5050} \text{ kgm}$$

The tension in the outside panels of the horizontal beam

$$T = \frac{1}{2} H \left( \frac{L}{3} \right) = \frac{1}{2} \times 4510 \times \frac{11}{3} = \underline{8270} \text{ kgs}$$

The tension in the diagonal tie

$$T = H \left( \frac{L}{3} \right) \sqrt{2} = 4510 \times \frac{11}{3} \times \sqrt{2} = \underline{23400} \text{ kgs}$$

The tension in the intermediate panel of the horizontal beam

$$T = 3 \times 8270 = 25000 \text{ kgs}$$

It can be seen from the previous investigation that the slab of the pyramid is subjected to max horizontal compressive stresses at its base while the bottom edge beam is subjected to tensile stresses. The corresponding difference of strain will cause shearing forces at the joint between the pyramid slab and the edge beam, and bending moments of the sense shown in figure IV 25 will be developed in the inclined direction of the slab. It is therefore recommended in flat pyramid roofs of relatively big dimensions to increase the thickness of the slab at its bottom edge and to reinforce it with top reinforcements of the order of the main steel used in the pyramid slab.

### XI 3 TEMPERATURE STRESSES IN SILO WALLS

If the temperature of the stored material inside a silo-cell is increased, the walls are subject to temperature stresses. Assume that the temperature is  $T_1$  in the inner face,  $T_2$  in the outer face and that the temperature decreases uniformly from inner to outer face,  $T_1 - T_2$  being denoted as  $\Delta T$ . Figure XI 21 shows a segment of a circular silo wall in two positions, one before and one after a uniform increase in temperature. The original length of the arc of the wall has been increased, but an increase that is uniform throughout will not create any stresses so long as the ring is supposed to be free and unrestrained at its edges. It is the temperature differential only  $\Delta T$ , which creates stresses.

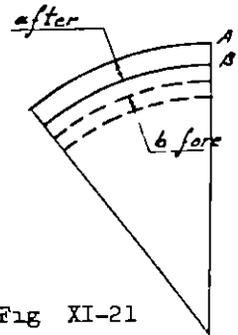


Fig XI-21

The inner fibers being hotter tend to expand more than the outer fibers, so if the segment is cut loose from the adjacent por-

tions of the wall, point A in figure XI 22 will move to A', B will move to B', and section A B, which represents the stressless condition due to a uniform temperature change throughout, will move to a new position A' B'. Actually the movements from A to A' and B to B' are prevented since the circle must remain a circle and stresses will be created that are proportional to the horizontal distances between A B and A' B'

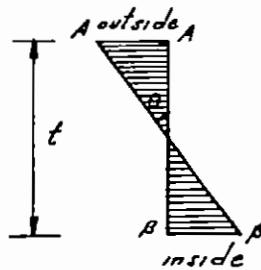


Fig XI-22

It is clear that  $AA' = BB' =$  movement due to a temperature change of  $\Delta T/2$  or when  $\alpha$  is the coefficient of expansion, that  $AA' = BB' = \frac{1}{2} \Delta T \alpha$  per unit length of arc and

$$\theta = \frac{AA'}{\frac{1}{2} t} = \frac{\alpha \Delta T}{t}$$

In a homogeneous section, the moment  $M$  required to produce an angle change  $\theta$  in an element of unit length may be written as

$$M = EI \theta$$

Eliminating  $\theta$  gives

$$M = \frac{EI \alpha \Delta T}{t}$$

The stresses in the extreme fibers created by  $M$  are

$$\sigma = \frac{M}{I} \frac{t}{2} = 1/2 E \alpha \Delta T$$

The stress distribution across the cross-section is as indicated in figure XI 22. The stresses are numerically equal at the two faces but have opposite signs. Note that the equation applies to uncracked sections only, and that this procedure of stress calculation is to be considered merely as a method by which the problem can be approached.

The variables  $E$  and  $I$  in the equations are uncertain quantities.  $E$  may vary from 100 000 to 300 000  $\text{kg/cm}^2$ , and  $I$  may also vary

considerably because of deviations from the assumption of linear relation between stress and strain. Finally, if the concrete cracks  $M$  can no longer be set equal to  $E I \theta$ , nor  $\sigma$  equal to  $\frac{1}{I} \frac{t}{2}$  as a result, the equation  $\sigma = 1/2 E \alpha \Delta T$  is to be regarded as merely indicative rather than formally correct.

The value of  $\alpha$  may be taken 0.00001 and for the purpose of this problem one may choose  $E = 100,000 \text{ kg/cm}^2$  so that  $E \alpha = 100,000 \times 0.00001 = 1$ . Knowing further that  $I = 100 \frac{t^3}{12}$  we get

$$= \frac{100 t^2}{12} \Delta T \quad \text{in kg cms when } t \text{ is in cms} \quad \text{and}$$

$$M = \frac{t^2}{12} \Delta T \quad \text{in kg ms when } t \text{ is in cms} \quad \text{and}$$

$$\sigma = \frac{\Delta T}{2} \quad \text{in kg/cm}^2$$

The value of  $\Delta T$  is the difference between the temperature in the two surfaces of the concrete which may be computed from the temperature of the stored material and the outside air. When the flow of heat is uniform from the inside to the outside of the wall section in figure XI-23, the temperature difference,  $\Delta T = T_1 - T_2$ , is smaller than the difference  $T_1 - T_0$ , between the inside liquid and the outside air.

Standard text books give

$$\Delta T = (T_1 - T_0) \frac{t/\lambda_0}{1/\lambda}$$

$$\frac{1}{\lambda} = \frac{t}{\lambda_0} + \frac{t_1}{\lambda_1} + \frac{t_2}{\lambda_2} + \frac{1}{\epsilon}$$

$\lambda_0$  = coefficient of conductivity of gravel concrete = 0.67

$\lambda_1, \lambda_2$  = " " insulating layers

= 0.05 for cork, 0.40 for brick walls, 0.6 for air etc

$\epsilon$  = outside surface coefficient = 10

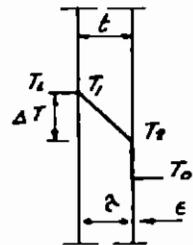


Fig XI-23

$t$  = thickness of reinforced concrete wall in meters

$t_1$   $t_2$  = insulating layers in meters

Thus for an uninsulated reinforced concrete wall

$$\Delta T = (T_1 - T_0) \frac{\frac{t}{0.67}}{\frac{t}{0.67} + \frac{1}{10}} = (T_1 - T_0) \frac{t}{t + 0.067}$$

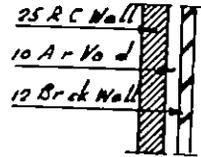
Assuming  $t = 0.25$  ms, then

$$\Delta T = 0.8 (T_1 - T_0)$$

If the wall were insulated by a 12 cms brick wall arranged as shown in figure XI-24, we get

$$\frac{1}{\lambda} = \frac{0.25}{0.67} + \frac{0.10}{0.60} + \frac{0.12}{0.40} + \frac{1}{10} = 0.670$$

$$\Delta T = \frac{T_1 - T_0}{0.67} \cdot \frac{0.25}{0.67} = 0.557 (T_1 - T_0)$$



If the air void were replaced by 5 cms cork, we get

$$\frac{1}{\lambda} = \frac{0.25}{0.67} + \frac{0.05}{0.05} + \frac{0.12}{0.40} + \frac{1}{10} = 1.503$$

$$\Delta T = \frac{T_1 - T_0}{1.503} \cdot \frac{0.25}{0.67} = 0.25 (T_1 - T_0)$$

Consider a circular silo wall 25 cms thick storing a material with a maximum temperature of 50 C while the temperature of the outside air is 20 C, then

$$\Delta T = 0.8 (50 - 20) = 24^\circ\text{C}$$

The corresponding bending moment due to this temperature change is

$$M = t^2 \Delta T / 12 = 25^2 \times 24 / 12 = 1250 \text{ kgm}$$

While the temperature stresses are

$$\sigma = \pm \Delta T / 2 = \pm 12 \text{ kg/cm}^2$$

Fig XI-24

These stresses are tension in the outside and compression in the inside face. If the uniformly distributed ring tension is  $15 \text{ kg/cm}^2$  the combined stress will be

$$\text{Outside fiber } 15 + 12 = 27 \text{ kg/cm}^2 \quad (\text{tension})$$

$$\text{Inside fiber } 15 - 12 = 3 \text{ kg/cm}^2 \quad (\text{tension})$$

In reality too much significance should not be attached to the temperature stress computed from the equation derived. The stress equation is developed from the strain equation  $\Delta A/A = 1/2 \alpha \Delta T$  based on the assumption that stress is proportional to strain. This assumption is rather inaccurate for the case under discussion. The inaccuracy may be rectified to some extent by using a relatively low value for  $E_c$ , such as  $E_c = 100,000 \text{ kg/cm}^2$  which has been used.

As computed in the example, a temperature differential of  $24^\circ \text{C}$  gives a stress of  $12 \text{ kg/cm}^2$  in the extreme fiber. This is probably more than the concrete can take in addition to the regular ring tension stress without cracking on the colder surface. The temperature stress may be reduced by means of insulation which serves to decrease the temperature differential, or additional horizontal reinforcement may be provided closer to the colder surface. A procedure will be illustrated for determination of temperature steel. It is not based upon a rigorous mathematical analysis but will be helpful as a guide and as an aid to engineering judgement.

In the given example, the area of horizontal steel at the colder face computed as for a cracked section is given by the relation

$$A_s = M/y_{CT} \sigma_s$$

For  $y_{CT} \approx \frac{7}{8} d$ , where  $d$  = the theoretical depth of the section and  $\sigma_s = 1400 \text{ kg/cm}^2$ , then

$$A_s = M/1200 d$$

For  $M$  in  $\text{kgm}$  and  $d$  in  $\text{ms}$ ,  $A_s$  will be in  $\text{cm}^2$ . Accordingly

$$A_s = 1250 / 1200 \times 0.22 = \underline{4.75} \text{ cm}^2$$

This area is in addition to the regular ring steel at the outer face of the wall.

**A P P E N D I X**

**Tables of Trigonometric and Hyperbolic  
Functions.**

**—————**

| x   | sin x | cos x | sinh x | cosh x | x    | sin x | cos x | sinh x | cosh x |
|-----|-------|-------|--------|--------|------|-------|-------|--------|--------|
| 0   | 0     | 1     | 0      | 1      | 0 20 | 0 199 | 0 980 | 0.2013 | 1 0201 |
| 001 | 0 001 | 1 000 | 0 0010 | 1 0000 | 0 21 | 0.208 | 0 978 | 0.2116 | 1 0221 |
| 002 | 0 002 | 1.000 | 0 0020 | 1 0000 | 0 22 | 0 218 | 0 976 | 0.2218 | 1 0243 |
| 003 | 0 003 | 1.000 | 0 0030 | 1 0000 | 0 23 | 0 228 | 0 974 | 0 2320 | 1 0265 |
| 004 | 0 004 | 1 000 | 0 0040 | 1 0000 | 0 24 | 0 238 | 0 971 | 0 2423 | 1 0289 |
| 005 | 0 005 | 1 000 | 0 0050 | 1 0000 | 0 25 | 0 247 | 0 969 | 0.2526 | 1 0314 |
| 006 | 0 006 | 1 000 | 0 0060 | 1 0000 | 0 26 | 0 257 | 0 966 | 0 2629 | 1 0340 |
| 007 | 0 007 | 1 000 | 0 0070 | 1 0000 | 0 27 | 0 267 | 0 964 | 0 2733 | 1 0367 |
| 008 | 0 008 | 1 000 | 0 0080 | 1 0000 | 0.28 | 0.276 | 0 961 | 0 2837 | 1 0395 |
| 009 | 0 009 | 1 000 | 0 0090 | 1 0000 | 0 29 | 0 286 | 0.958 | 0.2941 | 1 0424 |
| 010 | 0 010 | 1 000 | 0 0100 | 1 0000 | 0 30 | 0.296 | 0 955 | 0 3045 | 1 0453 |
| 011 | 0 011 | 1 000 | 0 0110 | 1 0001 | 0 31 | 0 305 | 0 952 | 0 3150 | 1 0484 |
| 012 | 0 012 | 1 000 | 0 0120 | 1 0001 | 0 32 | 0 315 | 0 949 | 0 3255 | 1 0516 |
| 013 | 0 013 | 1 000 | 0 0130 | 1 0001 | 0 33 | 0 324 | 0 946 | 0 3360 | 1.0550 |
| 014 | 0 014 | 1 000 | 0 0140 | 1 0001 | 0 34 | 0 334 | 0 943 | 0 3465 | 1 0584 |
| 015 | 0.015 | 1 000 | 0 0150 | 1 0001 | 0 35 | 0 343 | 0 939 | 0 3572 | 1 0619 |
| 016 | 0 016 | 1 000 | 0 0160 | 1 0001 | 0 36 | 0 352 | 0 936 | 0 3678 | 1 0655 |
| 017 | 0 017 | 1 000 | 0.0170 | 1 0001 | 0 37 | 0 362 | 0 932 | 0 3785 | 1 0692 |
| 018 | 0 018 | 1 000 | 0 0180 | 1 0002 | 0 38 | 0 371 | 0 929 | 0 3892 | 1 0731 |
| 019 | 0 019 | 1 000 | 0 0190 | 1 0002 | 0 39 | 0 380 | 0 925 | 0 4000 | 1 0770 |
| 020 | 0 020 | 1 000 | 0 0200 | 1 0002 | 0 40 | 0 389 | 0 921 | 0 4108 | 1 0811 |
| 030 | 0 030 | 1 000 | 0 0300 | 1 0004 | 0 41 | 0 399 | 0 917 | 0 4216 | 1 0852 |
| 040 | 0 040 | 0 999 | 0 0400 | 1 0008 | 0 42 | 0 408 | 0 913 | 0 4325 | 1 0895 |
| 050 | 0 050 | 0 999 | 0 0500 | 1 0012 | 0 43 | 0 417 | 0 909 | 0 4434 | 1 0939 |
|     |       |       |        |        | 0 44 | 0 426 | 0.905 | 0 4543 | 1 0984 |
| 060 | 0 060 | 0 998 | 0 0600 | 1 0018 | 0 45 | 0 435 | 0 900 | 0 4653 | 1 1030 |
| 070 | 0 070 | 0 998 | 0 0701 | 1 0024 | 0 46 | 0 444 | 0 896 | 0 4764 | 1 1077 |
| 080 | 0 080 | 0 997 | 0 0801 | 1 0032 | 0 47 | 0 453 | 0 892 | 0 4875 | 1 1125 |
| 090 | 0 090 | 0 996 | 0 0901 | 1 0040 | 0 48 | 0 462 | 0.887 | 0 4986 | 1 1174 |
|     |       |       |        |        | 0 49 | 0 471 | 0 882 | 0 5098 | 1 1225 |
| 100 | 0 100 | 0.995 | 0 1002 | 1 0050 | 0 50 | 0 479 | 0 878 | 0 5211 | 1 1276 |
| 110 | 0 110 | 0 994 | 0 1102 | 1.0061 | 0 51 | 0 488 | 0 873 | 0 5324 | 1 1329 |
| 120 | 0 120 | 0 993 | 0 1203 | 1 0072 | 0 52 | 0 497 | 0 868 | 0 5438 | 1 1383 |
| 130 | 0 130 | 0.992 | 0 1304 | 1 0085 | 0 53 | 0 506 | 0 863 | 0 5552 | 1 1438 |
| 140 | 0 140 | 0 990 | 0 1405 | 1 0098 | 0 54 | 0 514 | 0 858 | 0 5666 | 1 1494 |
| 150 | 0 149 | 0 989 | 0 1506 | 1 0113 | 0 55 | 0 523 | 0 853 | 0 5782 | 1.1551 |
| 160 | 0 159 | 0.987 | 0 1607 | 1 0128 | 0 56 | 0 531 | 0.847 | 0 5897 | 1 1609 |
| 170 | 0.169 | 0 986 | 0 1708 | 1 0145 | 0 57 | 0 540 | 0 842 | 0 6014 | 1.1669 |
| 180 | 0 179 | 0 984 | 0 1810 | 1 0162 | 0 58 | 0 549 | 0 837 | 0 6131 | 1 1730 |
| 190 | 0 189 | 0 982 | 0 1912 | 1 0181 | 0 59 | 0 556 | 0 831 | 0 6248 | 1 1792 |

| x    | sin x | cos x | sinh x | cosh x | x    | sin x | cos x | sinh x | cosh x |
|------|-------|-------|--------|--------|------|-------|-------|--------|--------|
| 0.60 | 0.565 | 0.825 | 0.6366 | 1.1853 | 1.00 | 0.842 | 0.540 | 1.1752 | 1.5431 |
| 0.61 | 0.573 | 0.820 | 0.6405 | 1.1919 | 1.01 | 0.847 | 0.532 | 1.1907 | 1.5549 |
| 0.62 | 0.581 | 0.814 | 0.6405 | 1.1984 | 1.02 | 0.852 | 0.523 | 1.2063 | 1.5669 |
| 0.63 | 0.589 | 0.808 | 0.6425 | 1.2051 | 1.03 | 0.857 | 0.515 | 1.2220 | 1.5790 |
| 0.64 | 0.597 | 0.802 | 0.6446 | 1.2119 | 1.04 | 0.862 | 0.506 | 1.2379 | 1.5913 |
| 0.65 | 0.605 | 0.796 | 0.6468 | 1.2188 | 1.05 | 0.867 | 0.498 | 1.2539 | 1.6038 |
| 0.66 | 0.613 | 0.790 | 0.6490 | 1.2258 | 1.06 | 0.872 | 0.489 | 1.2700 | 1.6164 |
| 0.67 | 0.621 | 0.784 | 0.6513 | 1.2330 | 1.07 | 0.877 | 0.480 | 1.2862 | 1.6292 |
| 0.68 | 0.629 | 0.778 | 0.6536 | 1.2402 | 1.08 | 0.882 | 0.471 | 1.3025 | 1.6421 |
| 0.69 | 0.637 | 0.771 | 0.6561 | 1.2476 | 1.09 | 0.887 | 0.463 | 1.3190 | 1.6552 |
| 0.70 | 0.644 | 0.765 | 0.6586 | 1.2552 | 1.10 | 0.891 | 0.454 | 1.3356 | 1.6685 |
| 0.71 | 0.652 | 0.758 | 0.6612 | 1.2628 | 1.11 | 0.896 | 0.445 | 1.3524 | 1.6820 |
| 0.72 | 0.659 | 0.752 | 0.6638 | 1.2706 | 1.12 | 0.900 | 0.436 | 1.3693 | 1.6956 |
| 0.73 | 0.667 | 0.745 | 0.6666 | 1.2785 | 1.13 | 0.904 | 0.427 | 1.3863 | 1.7092 |
| 0.74 | 0.674 | 0.739 | 0.6694 | 1.2865 | 1.14 | 0.909 | 0.418 | 1.4035 | 1.7233 |
| 0.75 | 0.682 | 0.732 | 0.6723 | 1.2947 | 1.15 | 0.913 | 0.409 | 1.4208 | 1.7374 |
| 0.76 | 0.689 | 0.725 | 0.6753 | 1.303  | 1.16 | 0.917 | 0.399 | 1.4382 | 1.7517 |
| 0.77 | 0.696 | 0.718 | 0.6784 | 1.3114 | 1.17 | 0.921 | 0.390 | 1.4558 | 1.7662 |
| 0.78 | 0.703 | 0.711 | 0.6815 | 1.3199 | 1.18 | 0.925 | 0.381 | 1.4736 | 1.7808 |
| 0.79 | 0.710 | 0.704 | 0.6848 | 1.3286 | 1.19 | 0.928 | 0.372 | 1.4914 | 1.7956 |
| 0.80 | 0.717 | 0.697 | 0.6881 | 1.3374 | 1.20 | 0.932 | 0.362 | 1.5095 | 1.8107 |
| 0.81 | 0.724 | 0.690 | 0.6915 | 1.3464 | 1.21 | 0.936 | 0.353 | 1.5276 | 1.8258 |
| 0.82 | 0.731 | 0.682 | 0.6950 | 1.3555 | 1.22 | 0.939 | 0.344 | 1.5460 | 1.8412 |
| 0.83 | 0.738 | 0.675 | 0.6986 | 1.3647 | 1.23 | 0.943 | 0.334 | 1.5645 | 1.8568 |
| 0.84 | 0.745 | 0.668 | 0.7023 | 1.3740 | 1.24 | 0.946 | 0.325 | 1.5831 | 1.8725 |
| 0.85 | 0.751 | 0.660 | 0.7061 | 1.3835 | 1.25 | 0.949 | 0.315 | 1.6019 | 1.8884 |
| 0.86 | 0.758 | 0.652 | 0.7100 | 1.3932 | 1.26 | 0.952 | 0.306 | 1.6209 | 1.9045 |
| 0.87 | 0.764 | 0.645 | 0.7140 | 1.4029 | 1.27 | 0.955 | 0.296 | 1.6400 | 1.9208 |
| 0.88 | 0.771 | 0.637 | 0.7181 | 1.4128 | 1.28 | 0.958 | 0.287 | 1.6593 | 1.9373 |
| 0.89 | 0.777 | 0.630 | 0.7222 | 1.4229 | 1.29 | 0.961 | 0.277 | 1.6788 | 1.9540 |
| 0.90 | 0.783 | 0.621 | 0.7265 | 1.4331 | 1.30 | 0.964 | 0.268 | 1.6984 | 1.9709 |
| 0.91 | 0.790 | 0.614 | 0.7309 | 1.4434 | 1.31 | 0.966 | 0.258 | 1.7182 | 1.9880 |
| 0.92 | 0.796 | 0.606 | 0.7354 | 1.4539 | 1.32 | 0.969 | 0.248 | 1.7381 | 2.0053 |
| 0.93 | 0.802 | 0.598 | 0.7400 | 1.4645 | 1.33 | 0.971 | 0.239 | 1.7583 | 2.0228 |
| 0.94 | 0.808 | 0.590 | 0.7447 | 1.4753 | 1.34 | 0.974 | 0.229 | 1.7786 | 2.0404 |
| 0.95 | 0.813 | 0.582 | 0.7495 | 1.4862 | 1.35 | 0.976 | 0.219 | 1.7991 | 2.0583 |
| 0.96 | 0.819 | 0.574 | 0.7544 | 1.4973 | 1.36 | 0.978 | 0.209 | 1.8198 | 2.0764 |
| 0.97 | 0.825 | 0.565 | 0.7594 | 1.5085 | 1.37 | 0.980 | 0.199 | 1.8406 | 2.0947 |
| 0.98 | 0.831 | 0.557 | 0.7646 | 1.5200 | 1.38 | 0.982 | 0.190 | 1.8617 | 2.1132 |
| 0.99 | 0.836 | 0.549 | 0.7698 | 1.5314 | 1.39 | 0.984 | 0.180 | 1.8829 | 2.1320 |

| x    | sin x | cos x  | sinh x | cosh x | x    | sin x | cos x | sinh x | cosh x |
|------|-------|--------|--------|--------|------|-------|-------|--------|--------|
| 1.40 | 0.985 | 0.170  | 1.9043 | 2.1509 | 1.80 | 0.974 | -227  | 2.0422 | 3.1075 |
| 1.41 | 0.987 | 0.160  | 1.9259 | 2.1700 | 1.81 | 0.972 | -277  | 2.9734 | 3.1370 |
| 1.42 | 0.989 | 0.150  | 1.9477 | 2.1894 | 1.82 | 0.959 | -247  | 3.0049 | 3.1669 |
| 1.43 | 0.990 | 0.140  | 1.9597 | 2.2090 | 1.83 | 0.957 | -256  | 3.0357 | 3.1972 |
| 1.44 | 0.992 | 0.130  | 1.9919 | 2.2288 | 1.84 | 0.954 | -266  | 3.0689 | 3.2277 |
| 1.45 | 0.993 | 0.121  | 2.0143 | 2.2488 | 1.85 | 0.961 | -276  | 3.1013 | 3.2585 |
| 1.46 | 0.994 | 0.111  | 2.0369 | 2.2691 | 1.86 | 0.959 | -285  | 3.1340 | 3.2897 |
| 1.47 | 0.995 | 0.110  | 2.0593 | 2.2896 | 1.87 | 0.956 | -295  | 3.1671 | 3.3212 |
| 1.48 | 0.996 | 0.091  | 2.0826 | 2.3103 | 1.88 | 0.953 | -304  | 3.2005 | 3.3520 |
| 1.49 | 0.997 | 0.081  | 2.1059 | 2.3312 | 1.89 | 0.950 | -314  | 3.2342 | 3.3852 |
| 1.50 | 0.998 | 0.071  | 2.1293 | 2.3524 | 1.90 | 0.946 | -323  | 3.2682 | 3.4177 |
| 1.51 | 0.999 | 0.061  | 2.1529 | 2.3738 | 1.91 | 0.943 | -333  | 3.3025 | 3.4506 |
| 1.52 | 0.999 | 0.051  | 2.1768 | 2.3955 | 1.92 | 0.940 | -342  | 3.3372 | 3.4838 |
| 1.53 | 0.999 | 0.041  | 2.2008 | 2.4174 | 1.93 | 0.936 | -352  | 3.3722 | 3.5173 |
| 1.54 | 1.000 | 0.031  | 2.2251 | 2.4395 | 1.94 | 0.933 | -361  | 3.4075 | 3.5512 |
| 1.55 | 1.000 | 0.021  | 2.2496 | 2.4619 | 1.95 | 0.929 | -370  | 3.4432 | 3.5855 |
| 1.56 | 1.000 | 0.011  | 2.2744 | 2.4815 | 1.96 | 0.925 | -379  | 3.4792 | 3.6201 |
| 1.57 | 1.000 | 0.001  | 2.2993 | 2.5074 | 1.97 | 0.921 | -389  | 3.5156 | 3.6551 |
| 1.58 | 1.000 | -0.000 | 2.3245 | 2.5305 | 1.98 | 0.917 | -399  | 3.5523 | 3.6904 |
| 1.59 | 1.000 | -0.010 | 2.3499 | 2.5538 | 1.99 | 0.913 | -407  | 3.5894 | 3.7261 |
| 1.60 | 1.000 | -0.029 | 2.3756 | 2.5775 | 2.00 | 0.909 | -416  | 3.6269 | 3.7622 |
| 1.61 | 0.999 | -0.039 | 2.4015 | 2.6014 | 2.01 | 0.905 | -425  | 3.6647 | 3.7986 |
| 1.62 | 0.997 | -0.049 | 2.4276 | 2.6255 | 2.02 | 0.901 | -434  | 3.7028 | 3.8355 |
| 1.63 | 0.995 | -0.059 | 2.4540 | 2.6499 | 2.03 | 0.897 | -443  | 3.7414 | 3.8727 |
| 1.64 | 0.993 | -0.069 | 2.4806 | 2.6746 | 2.04 | 0.892 | -452  | 3.7803 | 3.9103 |
| 1.65 | 0.991 | -0.079 | 2.5075 | 2.6995 | 2.05 | 0.887 | -461  | 3.8195 | 3.9483 |
| 1.66 | 0.989 | -0.089 | 2.5346 | 2.7247 | 2.06 | 0.883 | -470  | 3.8593 | 3.9876 |
| 1.67 | 0.987 | -0.099 | 2.5620 | 2.7502 | 2.07 | 0.878 | -479  | 3.8993 | 4.0255 |
| 1.68 | 0.984 | -0.109 | 2.5896 | 2.7760 | 2.08 | 0.873 | -488  | 3.9398 | 4.0647 |
| 1.69 | 0.982 | -0.119 | 2.6175 | 2.8020 | 2.09 | 0.868 | -496  | 3.9806 | 4.1043 |
| 1.70 | 0.980 | -0.129 | 2.6456 | 2.8283 | 2.10 | 0.863 | -505  | 4.0219 | 4.1443 |
| 1.71 | 0.978 | -0.139 | 2.6740 | 2.8549 | 2.11 | 0.858 | -513  | 4.0635 | 4.1847 |
| 1.72 | 0.985 | -0.149 | 2.7027 | 2.8818 | 2.12 | 0.853 | -522  | 4.1056 | 4.2256 |
| 1.73 | 0.987 | -0.159 | 2.7317 | 2.9090 | 2.13 | 0.848 | -531  | 4.1480 | 4.2668 |
| 1.74 | 0.986 | -0.168 | 2.7609 | 2.9364 | 2.14 | 0.842 | -540  | 4.1909 | 4.3080 |
| 1.75 | 0.984 | -0.178 | 2.7904 | 2.9642 | 2.15 | 0.837 | -547  | 4.2342 | 4.3507 |
| 1.76 | 0.982 | -0.188 | 2.8202 | 2.9922 | 2.16 | 0.831 | -556  | 4.2779 | 4.3932 |
| 1.77 | 0.980 | -0.198 | 2.8503 | 3.0206 | 2.17 | 0.826 | -564  | 4.3220 | 4.4362 |
| 1.78 | 0.978 | -0.208 | 2.8806 | 3.0492 | 2.18 | 0.820 | -572  | 4.3666 | 4.4796 |
| 1.79 | 0.976 | -0.217 | 2.9112 | 3.0782 | 2.19 | 0.814 | -580  | 4.4116 | 4.5236 |

|      | SIN Z | COS Z | SIN Z  | COSH Z | Z    | SIN Z | COS Z | SIN Z  | COSH Z |
|------|-------|-------|--------|--------|------|-------|-------|--------|--------|
| 2 20 | 0 609 | - 539 | 4 4571 | 4 5679 | 2 60 | 0 516 | - 857 | 6 6947 | 6 7690 |
| 2 21 | 0 603 | - 507 | 4 4630 | 4 6127 | 2 61 | 0 507 | - 862 | 6 7628 | 6 8300 |
| 2 22 | 0 797 | - 605 | 4 5454 | 4 6580 | 2 62 | 0 498 | - 867 | 6 8315 | 6 9043 |
| 2 23 | 0 791 | - 613 | 4 5032 | 4 7037 | 2 63 | 0 490 | - 872 | 6 9008 | 6 9720 |
| 2 24 | 0 784 | - 620 | 4 6434 | 4 7409 | 2 64 | 0 481 | - 877 | 6 9709 | 7 0423 |
| 2 25 | 0 778 | - 628 | 4 6912 | 4 7966 | 2 65 | 0 472 | - 882 | 7 0417 | 7 1140 |
| 2 26 | 0 771 | - 636 | 4 7394 | 4 8437 | 2 66 | 0 463 | - 886 | 7 1132 | 7 1831 |
| 2 27 | 0 765 | - 644 | 4 7880 | 4 8914 | 2 67 | 0 454 | - 891 | 7 1854 | 7 2540 |
| 2 28 | 0 759 | - 651 | 4 8372 | 4 9395 | 2 68 | 0 445 | - 895 | 7 2583 | 7 3268 |
| 2 29 | 0 752 | - 659 | 4 8868 | 4 9881 | 2 69 | 0 436 | - 900 | 7 3319 | 7 3998 |
| 2 30 | 0 748 | - 666 | 4 9370 | 5 0372 | 2 70 | 0 427 | - 904 | 7 4063 | 7 4735 |
| 2 31 | 0 730 | - 674 | 4 9876 | 5 0868 | 2 71 | 0 418 | - 908 | 7 4814 | 7 5479 |
| 2 32 | 0 732 | - 681 | 5 0387 | 5 1370 | 2 72 | 0 409 | - 912 | 7 5572 | 7 6231 |
| 2 33 | 0 725 | - 688 | 5 0903 | 5 1876 | 2 73 | 0 400 | - 917 | 7 6338 | 7 6990 |
| 2 34 | 0 719 | - 696 | 5 1424 | 5 2388 | 2 74 | 0 391 | - 920 | 7 7112 | 7 7758 |
| 2 35 | 0 712 | - 703 | 5 1951 | 5 2905 | 2 75 | 0 382 | - 924 | 7 7894 | 7 8523 |
| 2 36 | 0 704 | - 710 | 5 2483 | 5 3427 | 2 76 | 0 372 | - 928 | 7 8680 | 7 9310 |
| 2 37 | 0 697 | - 717 | 5 3020 | 5 3954 | 2 77 | 0 363 | - 932 | 7 9480 | 8 0106 |
| 2 38 | 0 690 | - 724 | 5 3562 | 5 4487 | 2 78 | 0 354 | - 935 | 8 0285 | 8 0900 |
| 2 39 | 0 683 | - 731 | 5 4109 | 5 5026 | 2 79 | 0 344 | - 939 | 8 1098 | 8 1712 |
| 2 40 | 0 676 | - 737 | 5 4662 | 5 5570 | 2 80 | 0 335 | - 942 | 8 1910 | 8 2527 |
| 2 41 | 0 668 | - 744 | 5 5221 | 5 6119 | 2 81 | 0 326 | - 946 | 8 2749 | 8 3351 |
| 2 42 | 0 661 | - 751 | 5 5785 | 5 6674 | 2 82 | 0 316 | - 949 | 8 3586 | 8 4182 |
| 2 43 | 0 653 | - 757 | 5 6354 | 5 7235 | 2 83 | 0 307 | - 952 | 8 4432 | 8 5022 |
| 2 44 | 0 645 | - 764 | 5 6920 | 5 7801 | 2 84 | 0 297 | - 955 | 8 5287 | 8 5871 |
| 2 45 | 0 638 | - 770 | 5 7510 | 5 8373 | 2 85 | 0 288 | - 958 | 8 6150 | 8 6728 |
| 2 46 | 0 630 | - 777 | 5 8097 | 5 8951 | 2 86 | 0 278 | - 961 | 8 7021 | 8 7594 |
| 2 47 | 0 622 | - 783 | 5 8689 | 5 9535 | 2 87 | 0 268 | - 963 | 8 7902 | 8 8469 |
| 2 48 | 0 614 | - 789 | 5 9280 | 6 0125 | 2 88 | 0 259 | - 966 | 8 8791 | 8 9352 |
| 2 49 | 0 606 | - 795 | 5 9892 | 6 0721 | 2 89 | 0 249 | - 969 | 8 9689 | 9 0244 |
| 2 50 | 0 599 | - 801 | 6 0502 | 6 1320 | 2 90 | 0 239 | - 971 | 9 0596 | 9 1146 |
| 2 51 | 0 590 | - 807 | 6 1110 | 6 1931 | 2 91 | 0 230 | - 973 | 9 1512 | 9 2056 |
| 2 52 | 0 582 | - 813 | 6 1741 | 6 2545 | 2 92 | 0 220 | - 976 | 9 2437 | 9 2976 |
| 2 53 | 0 574 | - 819 | 6 2369 | 6 3166 | 2 93 | 0 210 | - 978 | 9 3371 | 9 3905 |
| 2 54 | 0 566 | - 824 | 6 3004 | 6 3793 | 2 94 | 0 200 | - 980 | 9 4315 | 9 4844 |
| 2 55 | 0 558 | - 830 | 6 3645 | 6 4426 | 2 95 | 0 190 | - 982 | 9 5268 | 9 5790 |
| 2 56 | 0 549 | - 836 | 6 4293 | 6 5066 | 2 96 | 0 181 | - 984 | 9 6231 | 9 6749 |
| 2 57 | 0 541 | - 841 | 6 4946 | 6 5712 | 2 97 | 0 171 | - 985 | 9 7200 | 9 7716 |
| 2 58 | 0 533 | - 846 | 6 5607 | 6 6365 | 2 98 | 0 161 | - 987 | 9 8185 | 9 8690 |
| 2 59 | 0 524 | - 852 | 6 6274 | 6 7024 | 2 99 | 0 151 | - 989 | 9 9177 | 9 9680 |

| $x$  | $\sin x$ | $\cos x$ | $\sinh x$ | $\cosh x$ | $x$  | $\sin x$ | $\cos x$ | $\sinh x$ | $\cosh x$ |
|------|----------|----------|-----------|-----------|------|----------|----------|-----------|-----------|
| 3 00 | 0 141    | - 990    | 10 018    | 10 068    | 3 40 | - 256    | - 957    | 14 965    | 14 999    |
| 3 01 | 0 131    | - 991    | 10 119    | 10 168    | 3 41 | - 265    | - 964    | 15 116    | 15 149    |
| 3 02 | 0 121    | - 993    | 10 221    | 10 270    | 3 42 | - 275    | - 962    | 15 268    | 15 301    |
| 3 03 | 0 111    | - 994    | 10 325    | 10 373    | 3 43 | - 284    | - 959    | 15 422    | 15 455    |
| 3 04 | 0 101    | - 995    | 10 429    | 10 477    | 3 44 | - 294    | - 956    | 15 577    | 15 610    |
| 3 05 | 0 092    | - 996    | 10 534    | 10 581    | 3 45 | - 304    | - 953    | 15 734    | 15 766    |
| 3 06 | 0 082    | - 997    | 10 640    | 10 687    | 3 46 | - 313    | - 950    | 15 893    | 15 924    |
| 3 07 | 0 072    | - 997    | 10 748    | 10 794    | 3 47 | - 323    | - 947    | 16 053    | 16 084    |
| 3 08 | 0 062    | - 998    | 10 856    | 10 902    | 3 48 | - 332    | - 943    | 16 215    | 16 245    |
| 3 09 | 0 052    | - 999    | 10 966    | 11 011    | 3 49 | - 341    | - 940    | 16 378    | 16 408    |
| 3 10 | 0 042    | - 999    | 11 076    | 11 122    | 3 50 | - 351    | - 937    | 16 543    | 16 573    |
| 3 11 | 0 032    | -1 00    | 11 188    | 11 233    | 3 51 | - 360    | - 933    | 16 709    | 16 739    |
| 3 12 | 0 022    | -1 00    | 11 301    | 11 345    | 3 52 | - 369    | - 930    | 16 877    | 16 907    |
| 3 13 | 0 012    | -1 00    | 11 415    | 11 459    | 3 53 | - 379    | - 926    | 17 047    | 17 077    |
| 3 14 | 0 002    | -1 00    | 11 530    | 11 574    | 3 54 | - 388    | - 922    | 17 219    | 17 248    |
| 3 15 | - 008    | -1 00    | 11 647    | 11 690    | 3 55 | - 397    | - 918    | 17 392    | 17 421    |
| 3 16 | - 018    | -1 00    | 11 764    | 11 807    | 3 56 | - 406    | - 914    | 17 567    | 17 596    |
| 3 17 | - 028    | -1 00    | 11 883    | 11 925    | 3 57 | - 415    | - 910    | 17 744    | 17 772    |
| 3 18 | - 038    | - 999    | 12 003    | 12 044    | 3 58 | - 426    | - 905    | 17 923    | 17 951    |
| 3 19 | - 048    | - 999    | 12 124    | 12 165    | 3 59 | - 434    | - 901    | 18 103    | 18 131    |
| 3 20 | - 058    | - 998    | 12 246    | 12 287    | 3 60 | - 443    | - 897    | 18 286    | 18 313    |
| 3 21 | - 068    | - 998    | 12 369    | 12 410    | 3 61 | - 452    | - 892    | 18 470    | 18 497    |
| 3 22 | - 078    | - 997    | 12 494    | 12 534    | 3 62 | - 460    | - 888    | 18 655    | 18 682    |
| 3 23 | - 088    | - 996    | 12 620    | 12 660    | 3 63 | - 469    | - 883    | 18 843    | 18 870    |
| 3 24 | - 098    | - 995    | 12 747    | 12 786    | 3 64 | - 478    | - 878    | 19 033    | 19 059    |
| 3 25 | - 108    | - 994    | 12 876    | 12 915    | 3 65 | - 487    | - 874    | 19 224    | 19 250    |
| 3 26 | - 118    | - 993    | 13 006    | 13 044    | 3 66 | - 496    | - 869    | 19 418    | 19 444    |
| 3 27 | - 128    | - 992    | 13 138    | 13 175    | 3 67 | - 504    | - 864    | 19 613    | 19 639    |
| 3 28 | - 138    | - 990    | 13 269    | 13 307    | 3 68 | - 513    | - 859    | 19 811    | 19 836    |
| 3 29 | - 148    | - 989    | 13 403    | 13 440    | 3 69 | - 521    | - 853    | 20 010    | 20 035    |
| 3 30 | - 158    | - 988    | 13 548    | 13 575    | 3 70 | - 530    | - 848    | 20 211    | 20 236    |
| 3 31 | - 168    | - 986    | 13 674    | 13 711    | 3 71 | - 538    | - 843    | 20 414    | 20 439    |
| 3 32 | - 178    | - 984    | 13 812    | 13 848    | 3 72 | - 547    | - 837    | 20 620    | 20 644    |
| 3 33 | - 187    | - 982    | 13 951    | 13 987    | 3 73 | - 555    | - 832    | 20 828    | 20 852    |
| 3 34 | - 197    | - 980    | 14 092    | 14 127    | 3 74 | - 563    | - 826    | 21 037    | 21 061    |
| 3 35 | - 207    | - 978    | 14 234    | 14 269    | 3 75 | - 572    | - 821    | 21 249    | 21 272    |
| 3 36 | - 217    | - 976    | 14 377    | 14 412    | 3 76 | - 580    | - 815    | 21 463    | 21 486    |
| 3 37 | - 226    | - 974    | 14 522    | 14 557    | 3 77 | - 588    | - 809    | 21 679    | 21 702    |
| 3 38 | - 236    | - 972    | 14 668    | 14 702    | 3 78 | - 596    | - 803    | 21 897    | 21 919    |
| 3 39 | - 246    | - 969    | 14 816    | 14 850    | 3 79 | - 604    | - 797    | 22 117    | 22 140    |

| x    | sinh x | cosh x | sinh x | cosh x | x                 | sin x | cos x   | sinh x | cosh x |
|------|--------|--------|--------|--------|-------------------|-------|---------|--------|--------|
| 3 80 | - 612  | - 791  | 22 339 | 22 362 | 6 00              | - 279 | 0 960   | 201 71 |        |
| 3 81 | - 620  | - 785  | 22 564 | 22 566 | 6 10              | - 182 | 0 963   | 222 93 |        |
| 3 32 | - 628  | - 779  | 22 791 | 22 813 | 6 20              | - 083 | 0 997   | 240 37 |        |
| 3 83 | - 635  | - 772  | 23 020 | 23 042 | 6 30              | 0 017 | 1 000   | 272 29 |        |
| 3 84 | - 643  | - 765  | 23 252 | 23 274 | 6 40              | 0 117 | 0 993   | 300 34 |        |
| 3 85 | - 651  | - 759  | 23 486 | 23 507 | 6 50              | 0 215 | 0 977   | 332 57 |        |
| 3 86 | - 650  | - 753  | 23 722 | 23 743 | 6 60              | 0 312 | 0 950   | 367 35 |        |
| 3 87 | - 656  | - 746  | 23 961 | 23 982 | 6 70              | 0 405 | 0 914   | 406 20 |        |
| 3 88 | - 673  | - 740  | 24 202 | 24 222 | 6 80              | 0 494 | 0 869   | 448 92 |        |
| 3 89 | - 681  | - 733  | 24 445 | 24 466 | 6 90              | 0 578 | 0 816   | 4 0 14 |        |
| 3 90 | - 688  | - 726  | 24 691 | 24 711 | 7 00              | 0 657 | 0 754   | 548 32 |        |
| 3 91 | - 695  | - 719  | 24 940 | 24 960 | 7 10              | 0 729 | 0 685   | 605 98 |        |
| 3 92 | - 702  | - 712  | 25 190 | 25 210 | 7 20              | 0 794 | 0 608   | 659 72 |        |
| 3 93 | - 709  | - 705  | 25 444 | 25 465 | 7 30              | 0 850 | 0 526   | 740 15 |        |
| 3 94 | - 716  | - 698  | 25 700 | 25 719 | 7 40              | 0 899 | 0 439   | 817 9  |        |
| 3 95 | - 723  | - 691  | 25 958 | 25 977 | 7 50              | 0 938 | 0 347   | 904 02 |        |
| 3 96 | - 730  | - 683  | 26 219 | 26 238 | 7 60              | 0 968 | 0 251   | 999 10 |        |
| 3 97 | - 737  | - 676  | 26 483 | 26 502 | 7 70              | 0 988 | 0 153   | 1104 2 |        |
| 3 98 | - 744  | - 669  | 26 749 | 26 768 | 7 80              | 0 999 | 0 054   | 1220 3 |        |
| 3 99 | - 750  | - 661  | 27 018 | 27 038 | 7 90              | 0 999 | - 0 046 | 1348 0 |        |
| 4 00 | - 757  | - 654  | 27 290 | 27 308 | 8 00              | 0 989 | - 1 46  | 1490 5 |        |
| 4 10 | - 818  | - 575  | 30 162 | 30 178 | 8 50              | 0 799 | - 602   | 2457 4 |        |
| 4 20 | - 872  | - 490  | 33 336 | 33 351 | 9 00              | 0 412 | - 911   | 4051 5 |        |
| 4 30 | - 916  | - 401  | 36 843 | 36 857 | 9 50              | - 075 | - 997   | 6679 9 |        |
| 4 40 | - 952  | - 307  | 40 720 | 40 732 | 10 0              | - 544 | - 839   | 11013  |        |
| 4 50 | - 978  | - 211  | 45 003 | 45 014 |                   |       |         |        |        |
| 4 60 | - 994  | - 112  | 49 737 | 49 747 |                   |       |         |        |        |
| 4 70 | - 1 00 | - 012  | 54 969 | 54 978 |                   |       |         |        |        |
| 4 80 | - 996  | 0 086  | 60 751 | 60 759 |                   |       |         |        |        |
| 4 90 | - 982  | 0 187  | 67 141 | 67 149 |                   |       |         |        |        |
| 5 00 | - 959  | 0 284  | 74 203 | 74 210 | $\frac{1}{2} \pi$ | 0 707 | 0 707   | 0 8687 | 1 3240 |
| 5 10 | - 926  | 0 378  | 82 007 | 82 014 | $\frac{1}{4}$     | 1,000 | 0 000   | 2 3013 | 2 5092 |
| 5 20 | - 883  | 0 469  | 90 633 | 90 639 | $\frac{3}{4}$     | 0 707 | - 707   | 5 2280 | 5 3223 |
| 5 30 | - 832  | 0 554  | 100 17 | 100 17 | $\pi$             | 0 000 | -1 00   | 11 540 | 11 392 |
| 5 40 | - 773  | 0 635  | 110 70 | 110 71 | $\frac{5}{4}$     | - 707 | - 707   | 25 367 | 25 387 |
| 5 50 | - 706  | 0 709  | 122 34 | 122 35 | $\frac{3}{2}$     | -1 00 | 0 000   | 55 654 | 55 603 |
| 5 60 | - 631  | 0 775  | 135 21 | 135 22 | $\frac{7}{4}$     | - 707 | - 707   | 122 07 | 122 08 |
| 5 70 | - 551  | 0 835  | 149 43 | 149 44 | $2 \pi$           | 0 000 | 1 000   | 267 74 |        |
| 5 80 | - 465  | 0 886  | 165 15 | 165 15 | $\frac{9}{4}$     | 0 707 | 0 707   | 587 24 |        |
| 5 90 | - 374  | 0 928  | 182 32 | 182 32 | $\frac{5}{2}$     | 1 000 | 0 000   | 1208 0 |        |







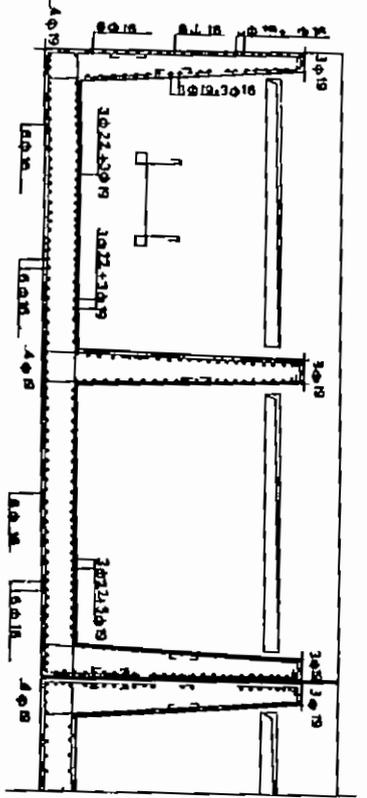
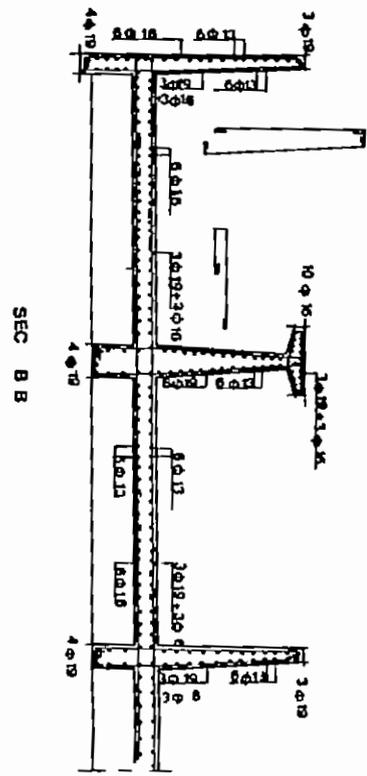
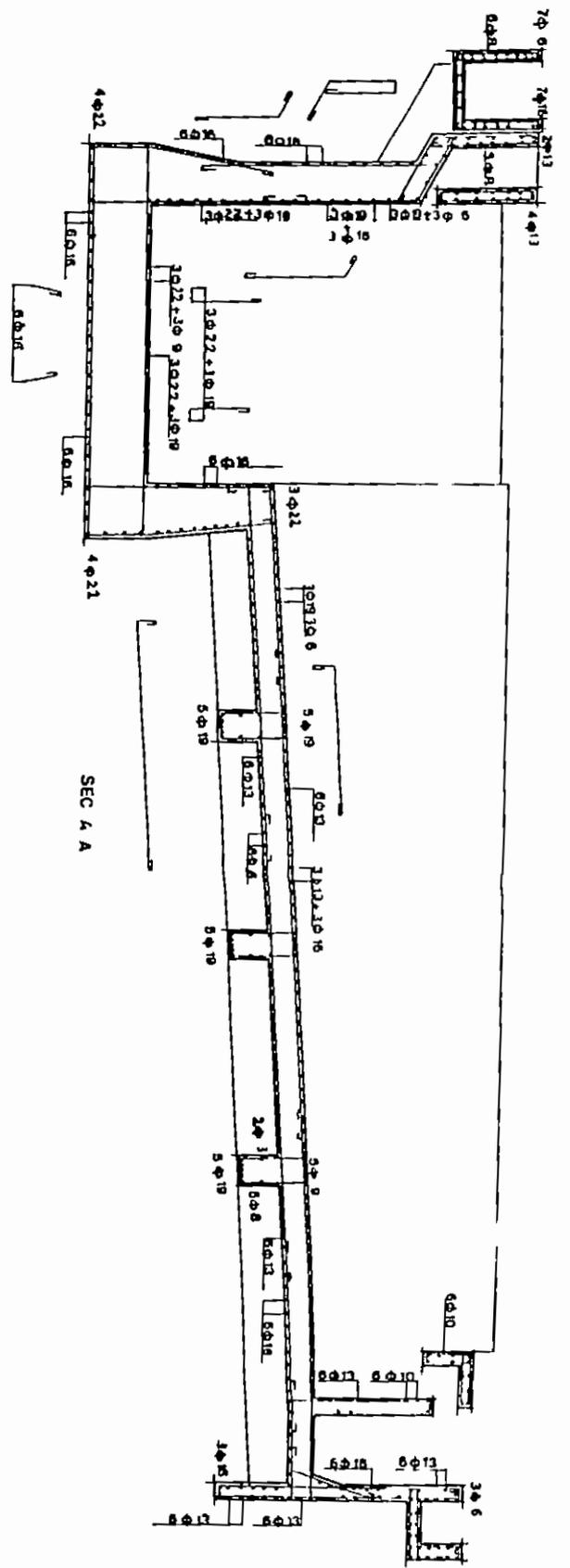


FIG X-4  
 EL NASR PIPE FACTORY  
 HELWAN  
 SETTLING TANK OF STEEL FILING  
 DETAILS OF REINFORCEMENTS

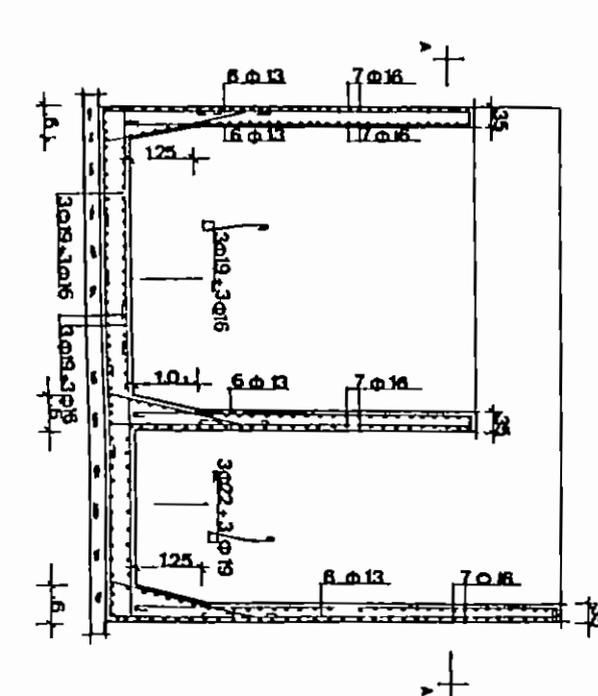
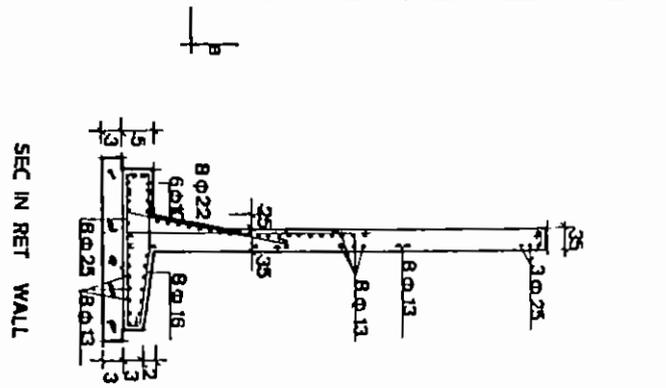
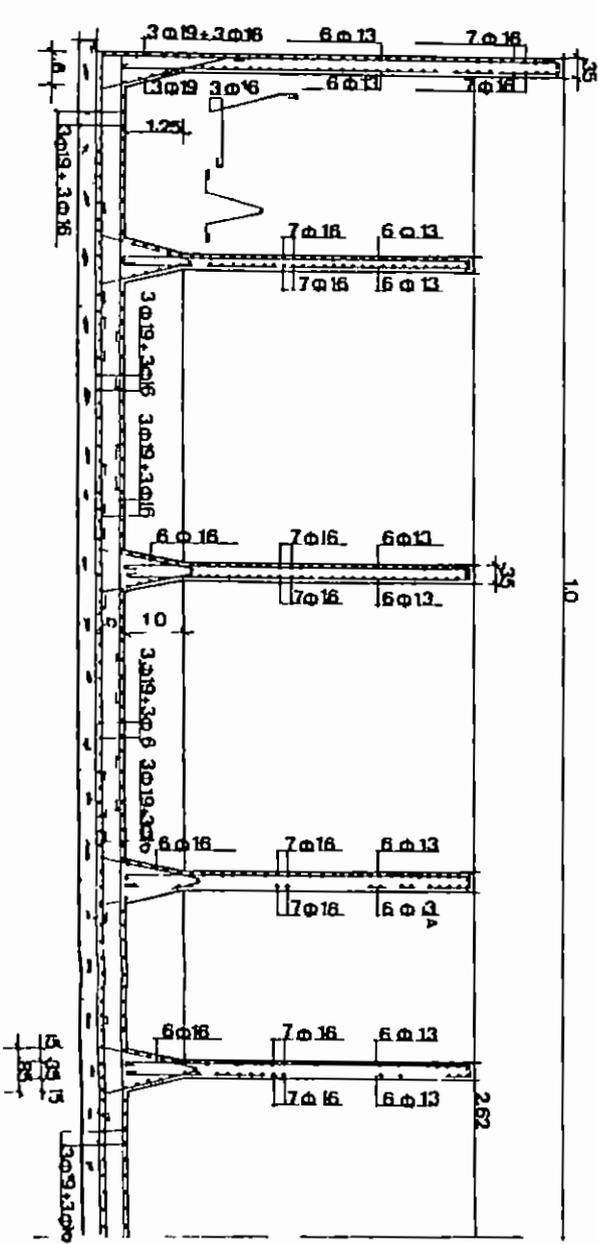
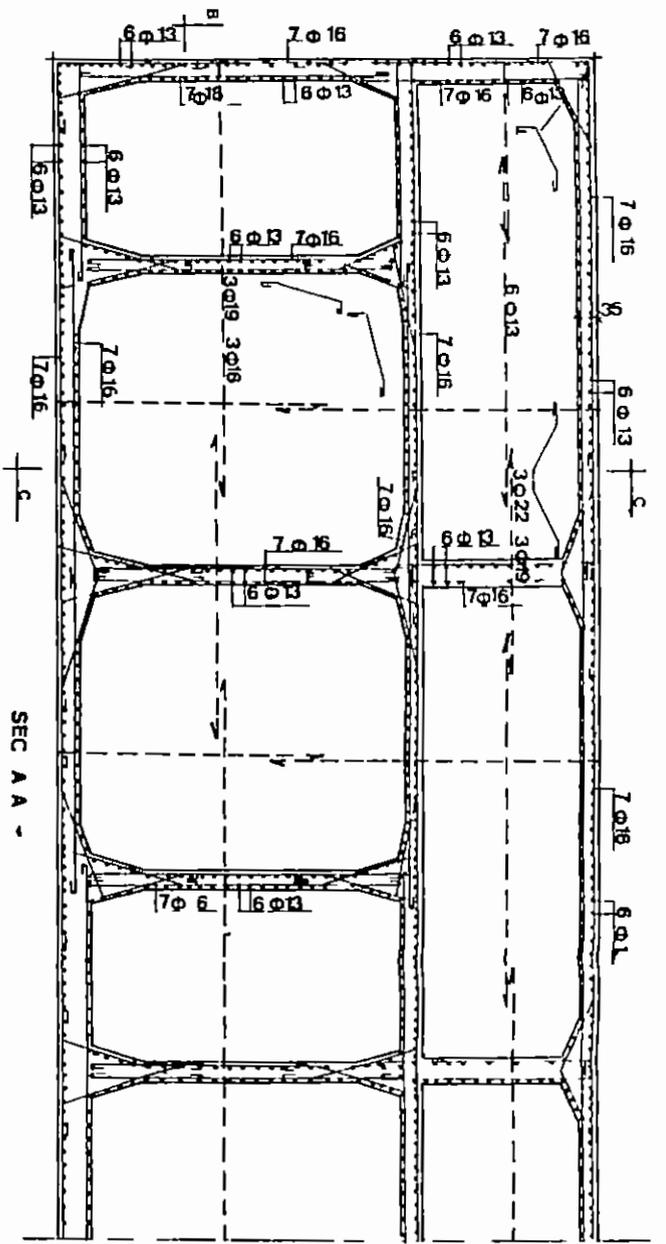


FIG X 7

EL NASR PIPE FACTORY  
HELMAN  
MAIN PUMP STATION  
DETAILS OF REINFORCEMENTS

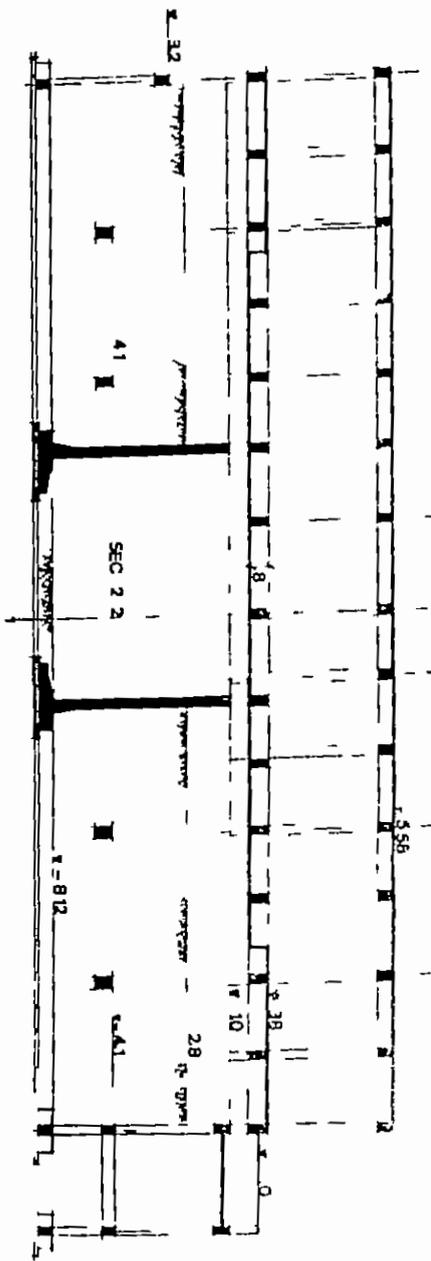
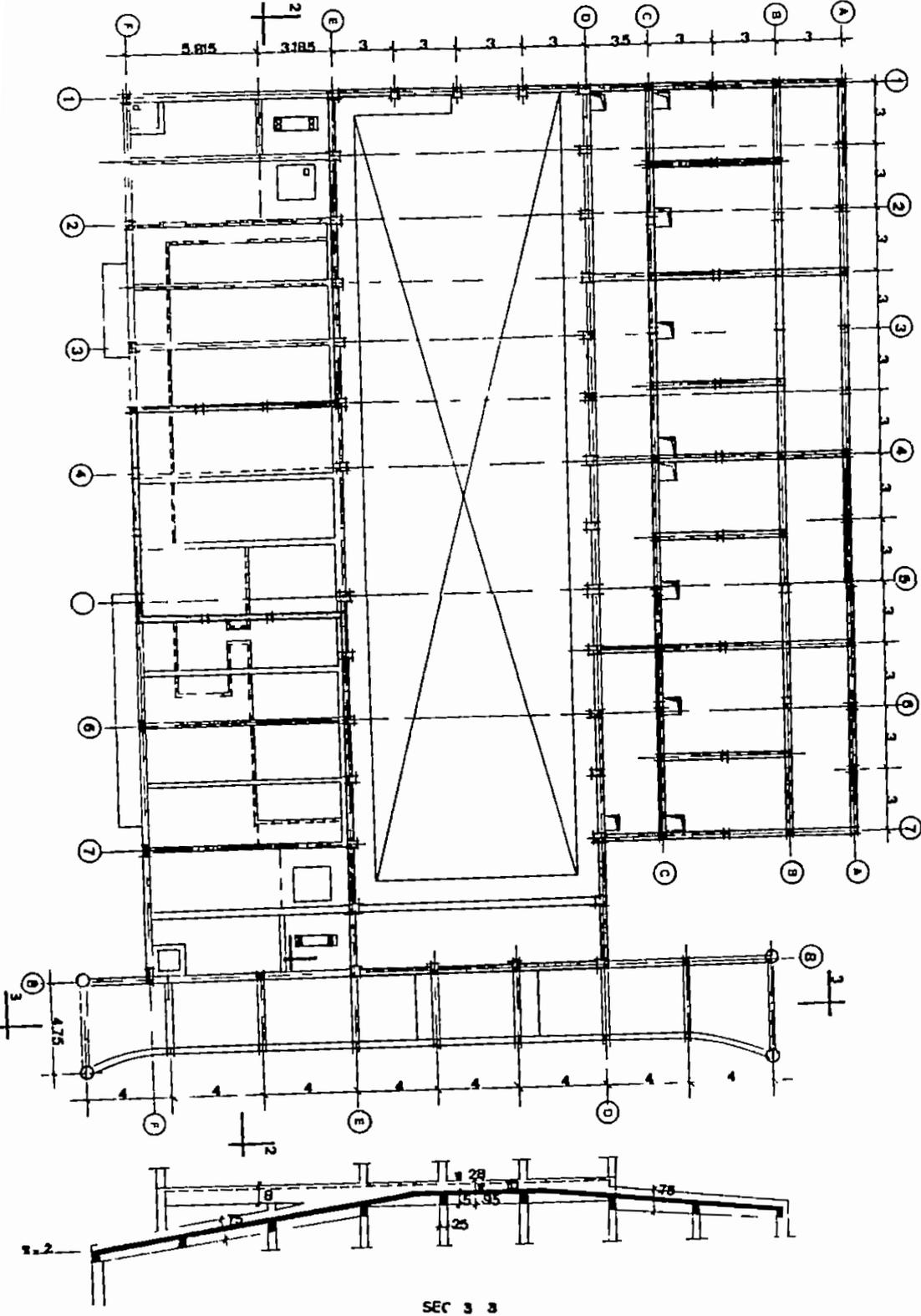


FIG X 6  
 EL NASR PIPE FACTORY  
 HELWAN  
 MAIN PUMP STATION  
 GENERAL LAYOUT

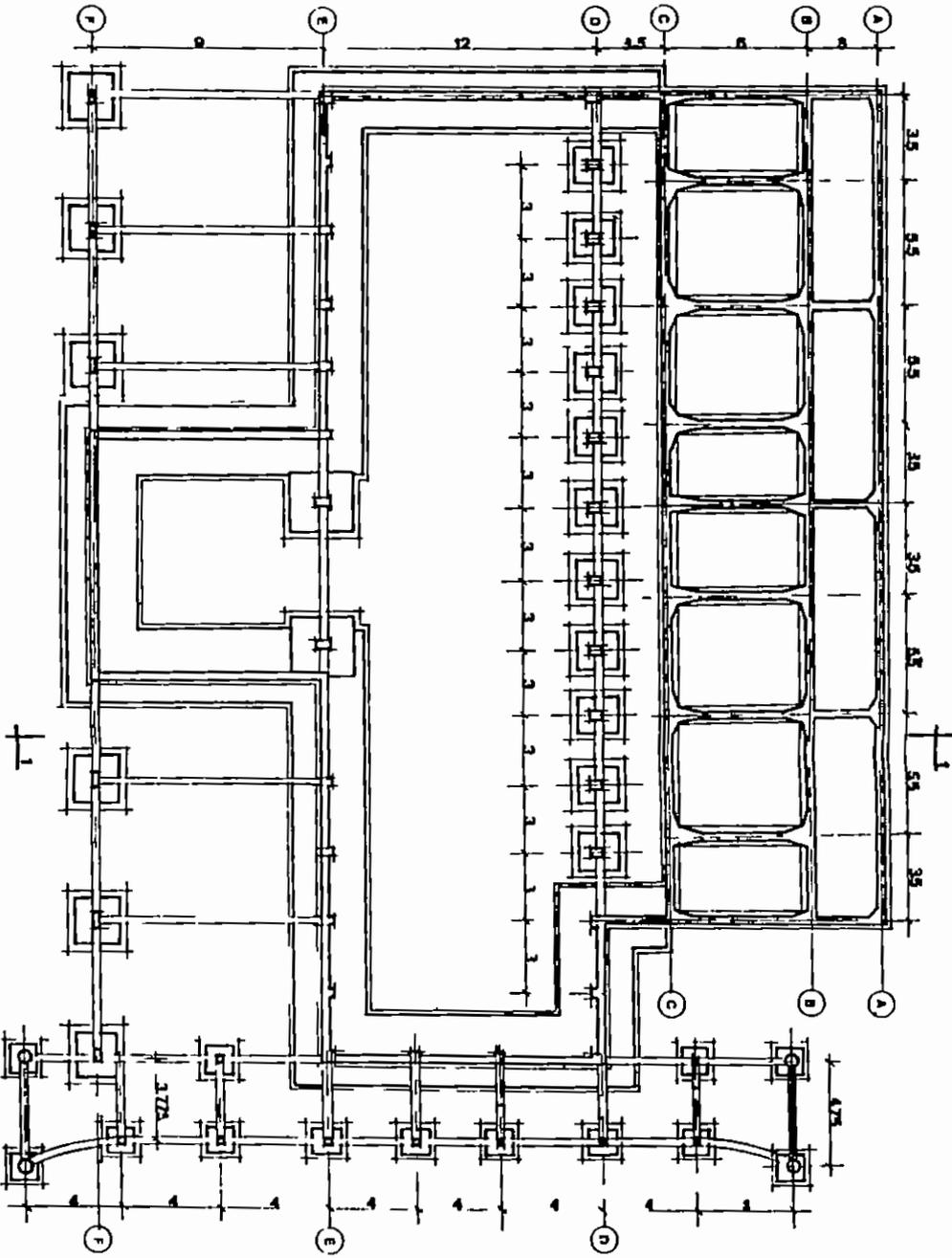
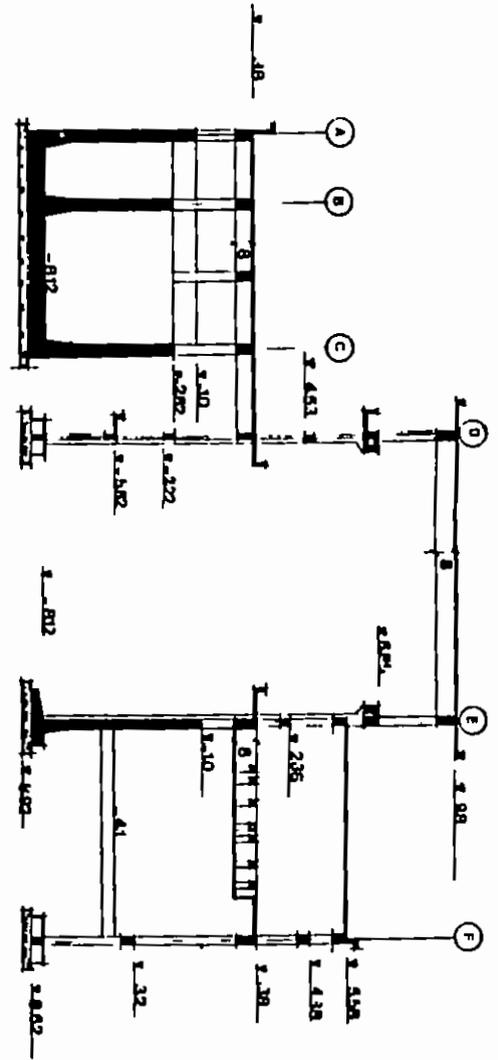
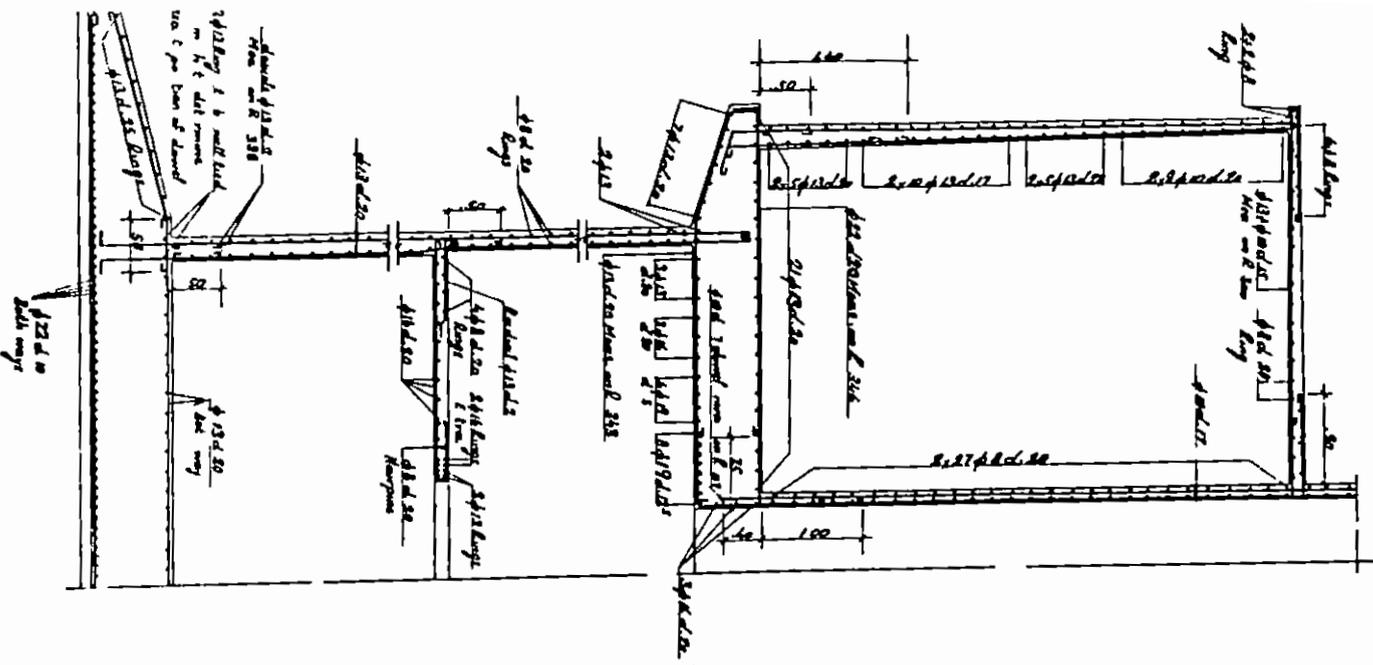


FIG X.5  
 EL NASR PIPE FACTORY  
 HELWAY  
 MAIN PUMP STATION  
 FOUNDATION

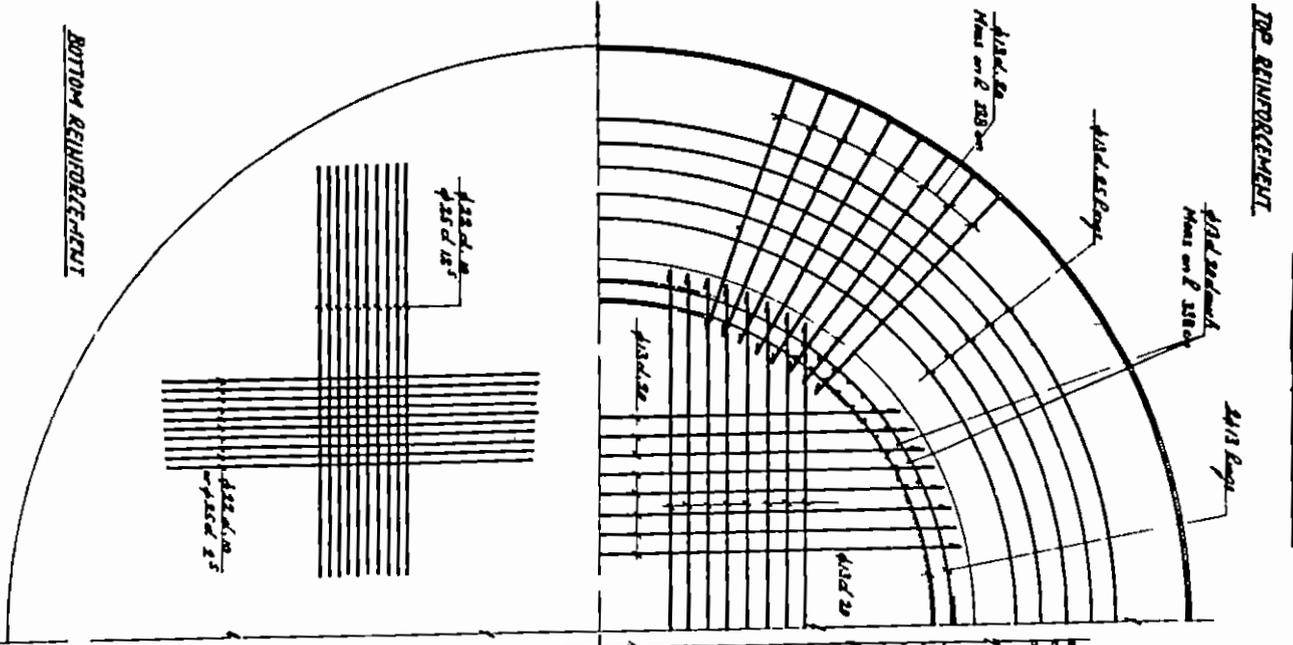


SEC L1

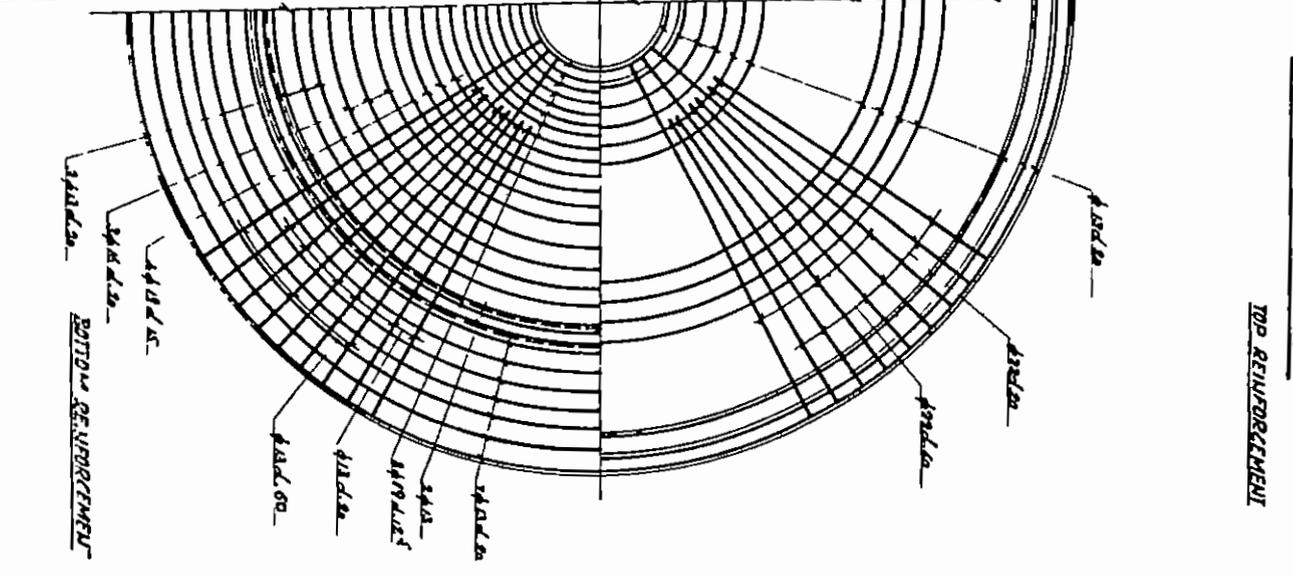
HALF SECTION OF WATER TOWER



REINFT OF FOUNDATION



REINFT OF TRUK FLOOR



BOTTOM REINFORCEMENT

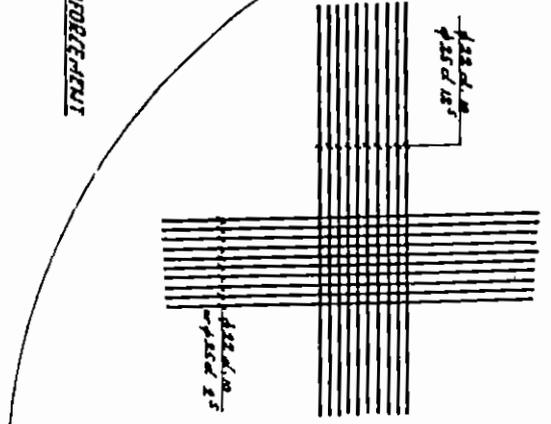
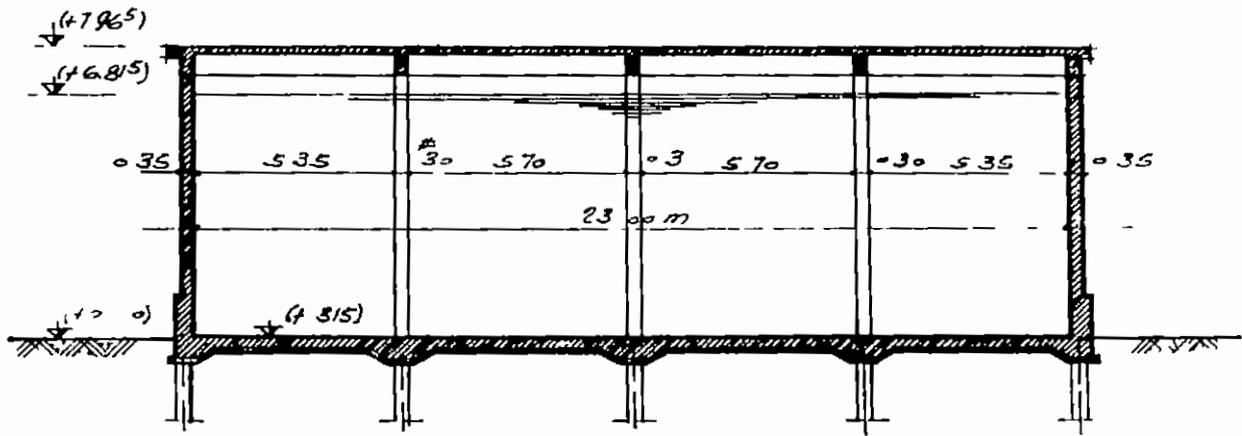


FIG VI-44

SAUDI ARABIA AT RIYADH  
WATER TOWER 1957  
CAPACITY 300 M<sup>3</sup>  
REINFORCEMENT DETAILS

# TREATED WATER TANK



SEC A A

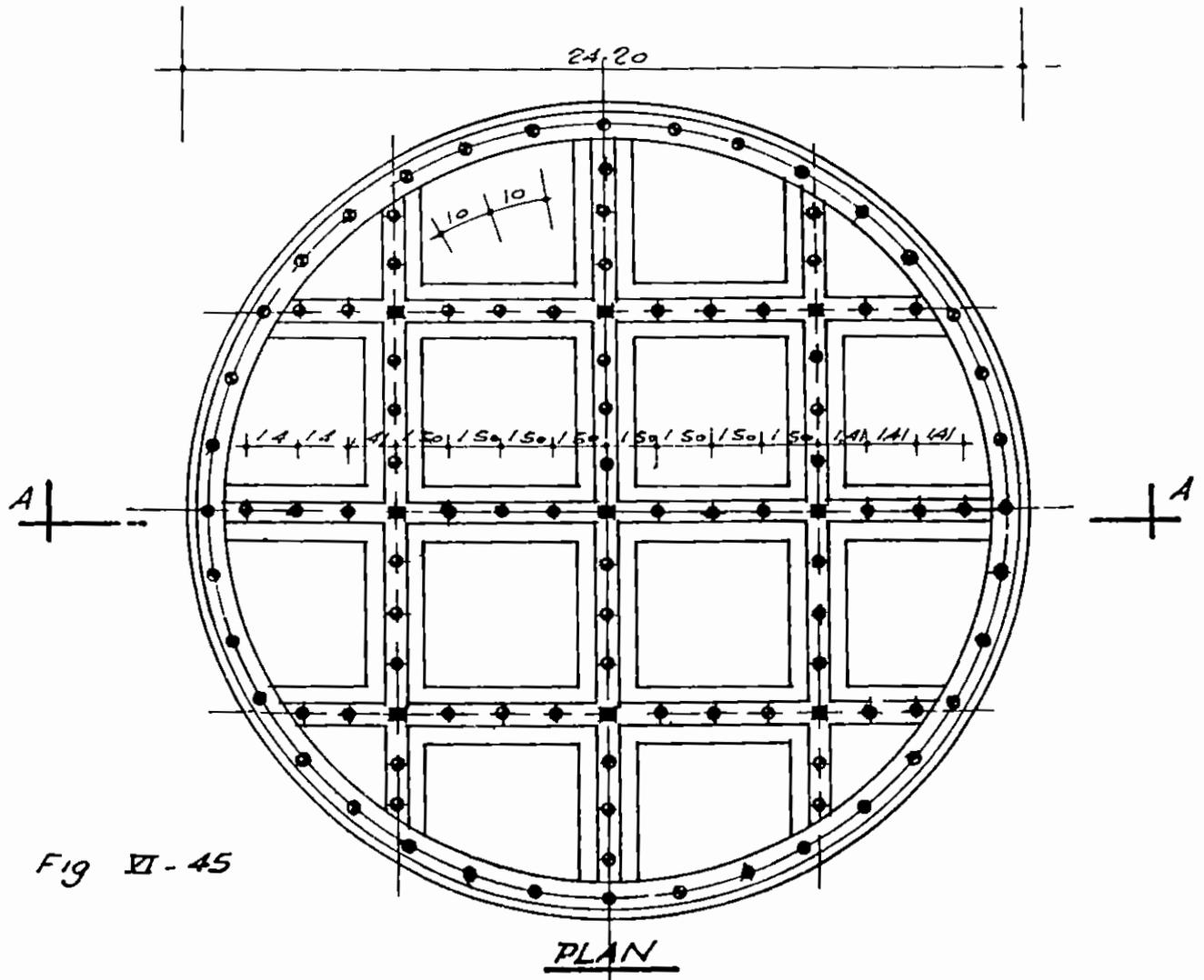


Fig VI-45

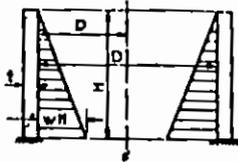
PLAN





TABLE I

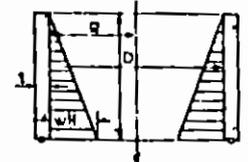
Tension in circular rings  
 Triangular load  
 Fixed base free top  
 $T = \text{coef} \times HR$   
 Poisson's ratio



| H/Dt | Coefficients at point |       |       |       |       |       |       |       |       |       |
|------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 00H                   | 01H   | 02H   | 03H   | 04H   | 05H   | 06H   | 07H   | 08H   | 09H   |
| 0.4  | 0.149                 | 0.134 | 0.120 | 0.101 | 0.082 | 0.066 | 0.049 | 0.029 | 0.014 | 0.004 |
| 0.8  | 0.263                 | 0.239 | 0.215 | 0.190 | 0.160 | 0.130 | 0.098 | 0.063 | 0.034 | 0.010 |
| 1.2  | 0.283                 | 0.271 | 0.254 | 0.234 | 0.209 | 0.180 | 0.142 | 0.099 | 0.054 | 0.016 |
| 1.6  | 0.285                 | 0.288 | 0.288 | 0.288 | 0.250 | 0.228 | 0.185 | 0.134 | 0.075 | 0.023 |
| 2.0  | 0.234                 | 0.251 | 0.273 | 0.285 | 0.285 | 0.274 | 0.232 | 0.172 | 0.104 | 0.031 |
| 3.0  | 0.134                 | 0.203 | 0.267 | 0.322 | 0.357 | 0.362 | 0.330 | 0.282 | 0.157 | 0.052 |
| 4.0  | 0.067                 | 0.164 | 0.258 | 0.339 | 0.403 | 0.429 | 0.409 | 0.334 | 0.210 | 0.073 |
| 5.0  | 0.025                 | 0.137 | 0.245 | 0.346 | 0.428 | 0.477 | 0.468 | 0.398 | 0.258 | 0.092 |
| 6.0  | 0.016                 | 0.119 | 0.234 | 0.344 | 0.441 | 0.504 | 0.514 | 0.447 | 0.301 | 0.112 |
| 8.0  | 0.011                 | 0.104 | 0.218 | 0.335 | 0.443 | 0.534 | 0.575 | 0.530 | 0.381 | 0.181 |
| 10.0 | -0.011                | 0.098 | 0.208 | 0.323 | 0.437 | 0.542 | 0.608 | 0.589 | 0.440 | 0.179 |
| 12.0 | 0.005                 | 0.097 | 0.202 | 0.312 | 0.429 | 0.543 | 0.628 | 0.633 | 0.494 | 0.211 |
| 14.0 | 0.002                 | 0.098 | 0.200 | 0.306 | 0.420 | 0.539 | 0.639 | 0.686 | 0.541 | 0.241 |
| 16.0 | 0.000                 | 0.098 | 0.199 | 0.304 | 0.412 | 0.531 | 0.641 | 0.687 | 0.582 | 0.265 |

Table II

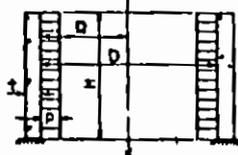
Tension in circular rings  
 Triangular load  
 Hinged base free top  
 $T = t \times HR$   
 Poisson's ratio



| H/Dt | Coefficients at point |       |       |       |       |       |       |       |       |       |
|------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 00H                   | 01H   | 02H   | 03H   | 04H   | 05H   | 06H   | 07H   | 08H   | 09H   |
| 0.4  | 0.474                 | 0.440 | 0.395 | 0.352 | 0.308 | 0.264 | 0.215 | 0.185 | 0.111 | 0.057 |
| 0.8  | 0.423                 | 0.402 | 0.381 | 0.358 | 0.330 | 0.297 | 0.249 | 0.202 | 0.145 | 0.076 |
| 1.2  | 0.350                 | 0.355 | 0.361 | 0.362 | 0.358 | 0.343 | 0.309 | 0.256 | 0.186 | 0.098 |
| 1.6  | 0.271                 | 0.303 | 0.341 | 0.389 | 0.395 | 0.385 | 0.362 | 0.314 | 0.233 | 0.124 |
| 2.0  | 0.205                 | 0.260 | 0.321 | 0.373 | 0.411 | 0.434 | 0.419 | 0.389 | 0.280 | 0.181 |
| 3.0  | 0.074                 | 0.179 | 0.281 | 0.376 | 0.449 | 0.508 | 0.519 | 0.479 | 0.375 | 0.210 |
| 4.0  | 0.017                 | 0.137 | 0.253 | 0.367 | 0.468 | 0.548 | 0.579 | 0.553 | 0.447 | 0.256 |
| 5.0  | 0.008                 | 0.114 | 0.235 | 0.358 | 0.469 | 0.562 | 0.617 | 0.608 | 0.503 | 0.29  |
| 6.0  | 0.011                 | 0.103 | 0.223 | 0.343 | 0.463 | 0.565 | 0.639 | 0.643 | 0.547 | 0.327 |
| 8.0  | 0.018                 | 0.096 | 0.208 | 0.324 | 0.443 | 0.564 | 0.681 | 0.697 | 0.621 | 0.386 |
| 10.0 | 0.008                 | 0.095 | 0.200 | 0.311 | 0.428 | 0.552 | 0.686 | 0.730 | 0.678 | 0.433 |
| 12.0 | 0.002                 | 0.097 | 0.197 | 0.302 | 0.417 | 0.541 | 0.664 | 0.750 | 0.720 | 0.477 |
| 14.0 | 0.000                 | 0.098 | 0.197 | 0.289 | 0.408 | 0.531 | 0.659 | 0.761 | 0.752 | 0.513 |
| 16.0 | 0.002                 | 0.100 | 0.198 | 0.299 | 0.403 | 0.521 | 0.650 | 0.764 | 0.776 | 0.536 |

Table III

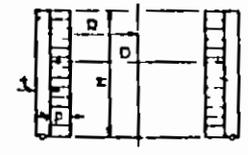
Tension in circular rings  
 Rectangular load  
 Fixed base free top  
 $T = \text{coef} \times pR$   
 Poisson's ratio



| H/Dt | Coefficients at point |       |       |       |       |       |       |       |       |       |
|------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 00H                   | 01H   | 02H   | 03H   | 04H   | 05H   | 06H   | 07H   | 08H   | 09H   |
| 0.4  | 0.582                 | 0.505 | 0.431 | 0.353 | 0.277 | 0.208 | 0.145 | 0.092 | 0.046 | 0.013 |
| 0.8  | 1.052                 | 0.921 | 0.796 | 0.669 | 0.542 | 0.415 | 0.289 | 0.179 | 0.089 | 0.024 |
| 1.2  | 1.218                 | 1.078 | 0.946 | 0.808 | 0.605 | 0.519 | 0.378 | 0.246 | 0.127 | 0.034 |
| 1.6  | 1.257                 | 1.141 | 1.009 | 0.881 | 0.742 | 0.600 | 0.449 | 0.294 | 0.153 | 0.045 |
| 2.0  | 1.253                 | 1.144 | 1.041 | 0.929 | 0.808 | 0.667 | 0.514 | 0.345 | 0.186 | 0.055 |
| 3.0  | 1.160                 | 1.112 | 1.061 | 0.998 | 0.912 | 0.798 | 0.646 | 0.459 | 0.258 | 0.081 |
| 4.0  | 1.085                 | 1.073 | 1.057 | 1.029 | 0.977 | 0.887 | 0.746 | 0.553 | 0.322 | 0.105 |
| 5.0  | 1.037                 | 1.044 | 1.047 | 1.042 | 1.015 | 0.949 | 0.825 | 0.629 | 0.379 | 0.128 |
| 6.0  | 1.010                 | 1.024 | 1.038 | 1.045 | 1.034 | 0.986 | 0.879 | 0.694 | 0.430 | 0.149 |
| 8.0  | 0.989                 | 1.005 | 1.022 | 1.036 | 1.044 | 1.026 | 0.953 | 0.788 | 0.519 | 0.189 |
| 10.0 | 0.989                 | 0.998 | 1.010 | 1.023 | 1.039 | 1.040 | 0.996 | 0.859 | 0.591 | 0.226 |
| 12.0 | 0.994                 | 0.997 | 1.003 | 1.014 | 1.031 | 1.043 | 1.022 | 0.911 | 0.652 | 0.262 |
| 14.0 | 0.997                 | 0.998 | 1.000 | 1.007 | 1.022 | 1.040 | 1.035 | 0.949 | 0.705 | 0.294 |
| 16.0 | 1.000                 | 0.999 | 0.999 | 1.003 | 1.015 | 1.032 | 1.040 | 0.975 | 0.750 | 0.321 |

Table IV

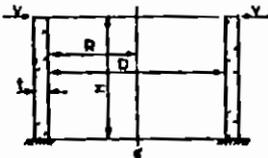
Tension in circular rings  
 Rectangular load  
 Hinged base free top  
 $T = \text{coef} \times pR$   
 Poisson's ratio



| H/Dt | Coefficients at point |       |       |       |       |       |       |       |       |       |
|------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 00H                   | 01H   | 02H   | 03H   | 04H   | 05H   | 06H   | 07H   | 08H   | 09H   |
| 0.4  | 1.474                 | 1.340 | 1.195 | 1.052 | 0.908 | 0.764 | 0.615 | 0.465 | 0.311 | 0.144 |
| 0.8  | 1.423                 | 1.302 | 1.181 | 1.058 | 0.930 | 0.797 | 0.649 | 0.502 | 0.345 | 0.181 |
| 1.2  | 1.350                 | 1.255 | 1.161 | 1.062 | 0.958 | 0.843 | 0.709 | 0.556 | 0.380 | 0.198 |
| 1.6  | 1.271                 | 1.203 | 1.141 | 1.069 | 0.985 | 0.885 | 0.758 | 0.61  | 0.433 | 0.22  |
| 2.0  | 1.205                 | 1.160 | 1.121 | 1.073 | 1.011 | 0.934 | 0.819 | 0.689 | 0.480 | 0.261 |
| 3.0  | 1.074                 | 1.079 | 1.081 | 1.075 | 1.049 | 1.006 | 0.919 | 0.779 | 0.575 | 0.310 |
| 4.0  | 1.017                 | 1.037 | 1.053 | 1.067 | 1.089 | 1.048 | 0.978 | 0.853 | 0.647 | 0.356 |
| 5.0  | 0.992                 | 1.014 | 1.035 | 1.056 | 1.069 | 1.062 | 1.017 | 0.908 | 0.704 | 0.394 |
| 6.0  | 0.989                 | 1.003 | 1.023 | 1.043 | 1.063 | 1.066 | 1.039 | 0.943 | 0.747 | 0.427 |
| 8.0  | 0.985                 | 0.996 | 1.008 | 1.024 | 1.043 | 1.064 | 1.081 | 0.997 | 0.821 | 0.486 |
| 10.0 | 0.992                 | 0.995 | 1.000 | 1.011 | 1.028 | 1.052 | 1.066 | 1.030 | 0.78  | 0.33  |
| 12.0 | 0.998                 | 0.997 | 0.997 | 1.002 | 1.017 | 1.041 | 1.064 | 1.050 | 0.920 | 0.577 |
| 14.0 | 1.000                 | 0.998 | 0.997 | 0.999 | 1.008 | 1.031 | 1.059 | 1.061 | 0.952 | 0.613 |
| 16.0 | 1.002                 | 1.000 | 0.998 | 0.999 | 1.003 | 1.021 | 1.050 | 1.064 | 0.976 | 0.636 |

Table V

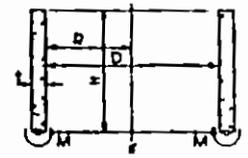
Tension in circular rings  
 Shear/m V applied at top  
 Fixed base free top  
 $T = \text{coef} \times VR/H$   
 Poisson's ratio



| H/Dt | Coefficients at point |       |       |       |      |      |      |      |      |      |
|------|-----------------------|-------|-------|-------|------|------|------|------|------|------|
|      | 00H                   | 01H   | 02H   | 03H   | 04H  | 05H  | 06H  | 07H  | 08H  | 09H  |
| 0.4  | 1.57                  | 1.32  | 1.08  | 0.86  | 0.65 | 0.47 | 0.31 | 0.18 | 0.08 | 0.02 |
| 0.8  | 3.09                  | -2.35 | -2.04 | -1.57 | 1.15 | 0.80 | 0.51 | 0.28 | 0.13 | 0.03 |
| 1.2  | 1.95                  | 3.17  | 2.44  | 1.79  | 1.25 | 0.81 | 0.48 | 0.25 | 0.10 | 0.02 |
| 1.6  | 4.57                  | 3.54  | 2.60  | 1.80  | 1.17 | 0.69 | 0.36 | 0.16 | 0.05 | 0.01 |
| 2.0  | 5.12                  | 3.83  | 2.68  | -1.74 | 1.02 | 0.52 | 0.21 | 0.05 | 0.01 | 0.01 |
| 3.0  | 6.32                  | 4.37  | 2.70  | 1.43  | 0.58 | 0.02 | 0.15 | 0.18 | 0.13 | 0.04 |
| 4.0  | 7.34                  | 4.73  | 2.60  | 1.10  | 0.19 | 0.26 | 0.38 | 0.33 | 0.19 | 0.06 |
| 5.0  | 8.22                  | 4.99  | 2.45  | 0.79  | 0.11 | 0.47 | 0.50 | 0.37 | 0.20 | 0.08 |
| 6.0  | 9.02                  | 5.17  | 2.27  | 0.50  | 0.34 | 0.59 | 0.53 | 0.35 | 0.17 | 0.01 |
| 8.0  | 10.42                 | 5.38  | 1.85  | 0.02  | 0.63 | 0.66 | 0.46 | 0.24 | 0.09 | 0.01 |
| 10.0 | 11.87                 | 5.43  | 1.43  | 0.38  | 0.78 | 0.62 | 0.33 | 0.12 | 0.02 | 0.00 |
| 12.0 | 12.76                 | 5.41  | -1.03 | 0.63  | 0.83 | 0.52 | 0.21 | 0.04 | 0.02 | 0.00 |
| 14.0 | 13.77                 | 5.34  | 0.68  | 0.80  | 0.81 | 0.42 | 0.13 | 0.00 | 0.03 | 0.01 |
| 16.0 | 14.74                 | 5.22  | 0.33  | 0.96  | 0.76 | 0.32 | 0.05 | 0.04 | 0.05 | 0.02 |

Table VI

Tension in circular rings  
 Moment/m M applied at base  
 Hinged base free top  
 $T = \text{coef} \times MR/H^2$   
 Poisson's ratio



| H/Dt | Coefficients at point |      |      |      |      |      |      |       |       |       |
|------|-----------------------|------|------|------|------|------|------|-------|-------|-------|
|      | 00H                   | 01H  | 02H  | 03H  | 04H  | 05H  | 06H  | 07H   | 08H   | 09H   |
| 0.4  | 2.70                  | 2.50 | 2.30 | 2.12 | 1.91 | 1.69 | 1.41 | 1.13  | 0.80  | 0.44  |
| 0.8  | 2.02                  | 2.06 | 2.10 | 2.14 | 2.10 | 2.02 | 1.95 | 1.75  | 1.39  | 0.80  |
| 1.2  | 1.08                  | 1.42 | 1.79 | 2.03 | 2.48 | 2.85 | 2.80 | 2.60  | 2.22  | 1.37  |
| 1.6  | 0.12                  | 0.79 | 1.43 | 2.04 | 2.72 | 3.25 | 3.56 | 3.59  | 3.13  | 2.01  |
| 2.0  | 0.88                  | 0.22 | 1.10 | 2.02 | 2.90 | 3.69 | 4.30 | 4.44  | 4.08  | 2.74  |
| 3.0  | 1.78                  | 0.71 | 0.43 | 1.60 | 2.95 | 4.9  | 5.66 | 6.48  | 6.55  | 4.7   |
| 4.0  | 1.67                  | 1.00 | 0.08 | 1.04 | 2.47 | 4.31 | 6.34 | 8.17  | 8.82  | 6.81  |
| 5.0  | 1.54                  | 1.03 | 0.42 | 0.45 | 1.86 | 3.93 | 6.60 | 9.41  | 11.03 | 9.02  |
| 6.0  | 1.04                  | 0.86 | 0.59 | 0.05 | 1.21 | 3.34 | 6.54 | 10.28 | 13.08 | 11.41 |
| 8.0  | 0.24                  | 0.53 | 0.73 | 0.87 | 0.02 | 2.05 | 5.87 | 11.32 | 16.57 | 10.06 |
| 10.0 | 0.21                  | 0.23 | 0.64 | 0.94 | 0.73 | 0.82 | 4.79 | 11.63 | 19.48 | 20.87 |
| 12.0 | 0.32                  | 0.05 | 0.46 | 0.96 | 1.15 | 0.18 | 3.52 | 11.27 | 21.80 | 2.73  |
| 14.0 | 0.28                  | 0.04 | 0.28 | 0.76 | 1.29 | 0.87 | 2.29 | 10.55 | 23.50 | 30.34 |
| 16.0 | 0.22                  | 0.07 | 0.08 | 0.64 | 1.28 | 1.30 | 1.12 | 9.67  | 24.53 | 34.65 |

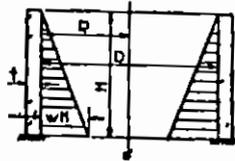
When this table is used for shear applied at the base while the top is fixed, the coefficient for 00H is the coefficient for the wall and 01H is the coefficient for the top. Shear acts in the direction of the arrow shown.

When this table is used for moment applied at the top while the top is hinged, the coefficient for 00H is the coefficient for the wall and 01H is the coefficient for the top. Moment is applied at the edge as shown, which causes outward rotation at the top.

Table VIII

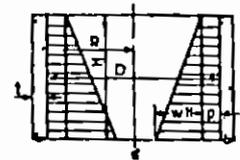
Moments in cylindrical wall  
Triangular load  
Fixed base free top

$M_m = \text{coef} \times w/H$   
Position indicated in the side



Moments in cylindrical wall  
Trapezoidal load  
Hinged base free top

$M_m = f \times (wH + pH)$   
Position indicated in the side



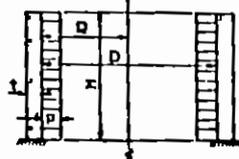
| H/D  | Coefficients |      |      |      |      |      |      |      |      |      |
|------|--------------|------|------|------|------|------|------|------|------|------|
|      | 0.1H         | 0.2H | 0.3H | 0.4H | 0.5H | 0.6H | 0.7H | 0.8H | 0.9H | 1.0H |
| 0.4  | 0005         | 0014 | 0021 | 0007 | 0042 | 0150 | 0302 | 0529 | 0818 | 1205 |
| 0.8  | 0011         | 0037 | 0063 | 0080 | 0070 | 0023 | 0068 | 0224 | 0465 | 0795 |
| 1.2  | 0012         | 0042 | 0077 | 0103 | 0112 | 0090 | 0022 | 0108 | 0311 | 0602 |
| 1.6  | 0011         | 0041 | 0075 | 0107 | 0121 | 0111 | 0058 | 0051 | 0232 | 0505 |
| 2.0  | 0010         | 0035 | 0068 | 0099 | 0120 | 0115 | 0075 | 0021 | 0185 | 0438 |
| 3.0  | 0006         | 0024 | 0047 | 0071 | 0090 | 0097 | 0077 | 0012 | 0119 | 0333 |
| 4.0  | 0003         | 0015 | 0028 | 0047 | 0066 | 0077 | 0069 | 0023 | 0080 | 0268 |
| 6.0  | 0002         | 0008 | 0016 | 0029 | 0046 | 0059 | 0059 | 0028 | 0058 | 0222 |
| 8.0  | 0001         | 0003 | 0008 | 0019 | 0032 | 0046 | 0051 | 0029 | 0041 | 0187 |
| 10.0 | 0000         | 0001 | 0002 | 0008 | 0018 | 0028 | 0038 | 0029 | 0022 | 0148 |
| 12.0 | 0000         | 0000 | 0001 | 0004 | 0007 | 0018 | 0029 | 0028 | 0012 | 0122 |
| 14.0 | 0000         | 0000 | 0000 | 0000 | 0001 | 0008 | 0019 | 0023 | 0001 | 0090 |
| 16.0 | 0000         | 0000 | 0001 | 0002 | 0001 | 0004 | 0013 | 0019 | 0001 | 0079 |

| H/D  | Coefficients |      |      |      |      |      |      |      |      |      |
|------|--------------|------|------|------|------|------|------|------|------|------|
|      | 0.1H         | 0.2H | 0.3H | 0.4H | 0.5H | 0.6H | 0.7H | 0.8H | 0.9H | 1.0H |
| 0.4  | 0020         | 0072 | 0181 | 0230 | 0301 | 0348 | 0357 | 0312 | 0197 | 0    |
| 0.8  | 0019         | 0064 | 0133 | 0207 | 0271 | 0319 | 0329 | 0292 | 0187 | 0    |
| 1.2  | 0018         | 0058 | 0111 | 0177 | 0237 | 0280 | 0298 | 0283 | 0171 | 0    |
| 1.6  | 0012         | 0044 | 0091 | 0145 | 0195 | 0236 | 0255 | 0232 | 0155 | 0    |
| 2.0  | 0009         | 0033 | 0073 | 0114 | 0158 | 0199 | 0219 | 0205 | 0145 | 0    |
| 3.0  | 0004         | 0018 | 0040 | 0083 | 0092 | 0127 | 0152 | 0153 | 0111 | 0    |
| 4.0  | 0001         | 0007 | 0018 | 0033 | 0057 | 0083 | 0109 | 0118 | 0092 | 0    |
| 6.0  | 0000         | 0001 | 0006 | 0016 | 0034 | 0057 | 0080 | 0094 | 0078 | 0    |
| 8.0  | 0000         | 0000 | 0002 | 0008 | 0019 | 0039 | 0062 | 0078 | 0068 | 0    |
| 10.0 | 0000         | 0000 | 0002 | 0000 | 0007 | 0020 | 0038 | 0057 | 0044 | 0    |
| 12.0 | 0000         | 0000 | 0001 | 0002 | 0000 | 0005 | 0017 | 0032 | 0039 | 0    |
| 14.0 | 0000         | 0000 | 0001 | 0001 | 0001 | 0000 | 0012 | 0026 | 0033 | 0    |
| 16.0 | 0000         | 0000 | 0000 | 0001 | 0002 | 0004 | 0008 | 0022 | 0029 | 0    |

Table IX

Moments in cylindrical wall  
Rectangular load  
Fixed base free top

$M_m = f \times pH$   
Position indicated in the side

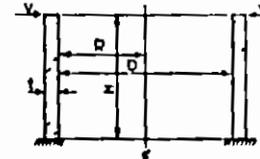


| H/D  | Coefficients |      |      |      |      |      |      |      |      |      |
|------|--------------|------|------|------|------|------|------|------|------|------|
|      | 0.1H         | 0.2H | 0.3H | 0.4H | 0.5H | 0.6H | 0.7H | 0.8H | 0.9H | 1.0H |
| 0.4  | 0023         | 0093 | 0227 | 0439 | 0710 | 1018 | 1455 | 2000 | 2593 | 3310 |
| 0.8  | 0000         | 0008 | 0025 | 0083 | 0185 | 0362 | 0594 | 0917 | 1325 | 1835 |
| 1.2  | 0008         | 0028 | 0037 | 0028 | 0009 | 0089 | 0227 | 0468 | 0815 | 1178 |
| 1.6  | 0011         | 0036 | 0062 | 0077 | 0068 | 0011 | 0093 | 0267 | 0529 | 0876 |
| 2.0  | 0010         | 0036 | 0066 | 0088 | 0089 | 0059 | 0019 | 0167 | 0389 | 0719 |
| 3.0  | 0007         | 0026 | 0051 | 0074 | 0091 | 0083 | 0042 | 0053 | 0223 | 0483 |
| 4.0  | 0004         | 0015 | 0033 | 0052 | 0068 | 0075 | 0053 | 0013 | 0145 | 0365 |
| 6.0  | 0002         | 0008 | 0019 | 0035 | 0051 | 0061 | 0052 | 0007 | 0101 | 0293 |
| 8.0  | 0001         | 0004 | 0011 | 0022 | 0036 | 0049 | 0048 | 0017 | 0073 | 0242 |
| 10.0 | 0000         | 0001 | 0003 | 0008 | 0018 | 0031 | 0038 | 0024 | 0040 | 0184 |
| 12.0 | 0000         | 0000 | 0001 | 0000 | 0004 | 0014 | 0024 | 0022 | 0012 | 0123 |
| 14.0 | 0000         | 0000 | 0000 | 0000 | 0002 | 0010 | 0018 | 0021 | 0007 | 0105 |
| 16.0 | 0000         | 0000 | 0000 | 0001 | 0001 | 0000 | 0012 | 0020 | 0005 | 0091 |

Table X

Moments in cylindrical wall  
Shear /m V applied at top  
Fixed base free top

$M_m = \text{coef} \times V$   
Position indicated in the side

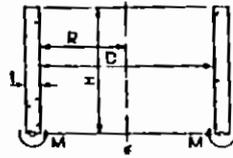


| H/D  | Coefficients |       |       |       |       |       |       |       |       |       |
|------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 0.1H         | 0.2H  | 0.3H  | 0.4H  | 0.5H  | 0.6H  | 0.7H  | 0.8H  | 0.9H  | 1.0H  |
| 0.4  | 0.093        | 0.172 | 0.240 | 0.300 | 0.354 | 0.402 | 0.448 | 0.492 | 0.535 | 0.578 |
| 0.8  | 0.085        | 0.145 | 0.185 | 0.208 | 0.220 | 0.224 | 0.223 | 0.219 | 0.214 | 0.208 |
| 1.2  | 0.082        | 0.132 | 0.157 | 0.164 | 0.159 | 0.148 | 0.127 | 0.108 | 0.084 | 0.067 |
| 1.6  | 0.079        | 0.122 | 0.139 | 0.138 | 0.125 | 0.105 | 0.081 | 0.058 | 0.039 | 0.024 |
| 2.0  | 0.077        | 0.115 | 0.126 | 0.119 | 0.103 | 0.080 | 0.056 | 0.031 | 0.008 | 0.019 |
| 3.0  | 0.072        | 0.100 | 0.100 | 0.086 | 0.068 | 0.044 | 0.025 | 0.008 | 0.010 | 0.024 |
| 4.0  | 0.068        | 0.088 | 0.081 | 0.063 | 0.043 | 0.025 | 0.010 | 0.001 | 0.010 | 0.019 |
| 6.0  | 0.064        | 0.078 | 0.067 | 0.047 | 0.028 | 0.013 | 0.003 | 0.003 | 0.007 | 0.011 |
| 8.0  | 0.062        | 0.070 | 0.056 | 0.038 | 0.018 | 0.006 | 0.000 | 0.003 | 0.005 | 0.006 |
| 10.0 | 0.057        | 0.058 | 0.041 | 0.021 | 0.007 | 0.000 | 0.002 | 0.003 | 0.002 | 0.001 |
| 12.0 | 0.053        | 0.049 | 0.029 | 0.012 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.000 |
| 14.0 | 0.049        | 0.042 | 0.022 | 0.007 | 0.000 | 0.002 | 0.002 | 0.001 | 0.000 | 0.000 |
| 16.0 | 0.046        | 0.036 | 0.017 | 0.004 | 0.001 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 |
| 16.0 | 0.044        | 0.031 | 0.012 | 0.001 | 0.002 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 |

Table XI

Moments in cylindrical wall  
Moment/m M applied at base  
Hinged base free top

$M_m = \text{coef} \times M$   
Position indicated in the side

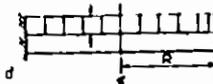


| H/D  | Coefficients |       |       |       |       |       |       |       |       |       |
|------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 0.1H         | 0.2H  | 0.3H  | 0.4H  | 0.5H  | 0.6H  | 0.7H  | 0.8H  | 0.9H  | 1.0H  |
| 0.4  | 0.013        | 0.051 | 0.109 | 0.195 | 0.296 | 0.414 | 0.547 | 0.692 | 0.843 | 1.000 |
| 0.8  | 0.009        | 0.040 | 0.090 | 0.164 | 0.253 | 0.375 | 0.503 | 0.659 | 0.824 | 1.000 |
| 1.2  | 0.006        | 0.027 | 0.063 | 0.125 | 0.206 | 0.316 | 0.454 | 0.616 | 0.802 | 1.000 |
| 1.6  | 0.003        | 0.011 | 0.035 | 0.078 | 0.152 | 0.253 | 0.333 | 0.570 | 0.775 | 1.000 |
| 2.0  | 0.002        | 0.002 | 0.012 | 0.034 | 0.096 | 0.193 | 0.340 | 0.519 | 0.748 | 1.000 |
| 3.0  | 0.007        | 0.022 | 0.030 | 0.029 | 0.010 | 0.087 | 0.227 | 0.426 | 0.692 | 1.000 |
| 4.0  | 0.008        | 0.026 | 0.044 | 0.051 | 0.034 | 0.073 | 0.150 | 0.354 | 0.645 | 1.000 |
| 5.0  | 0.007        | 0.024 | 0.045 | 0.061 | 0.057 | 0.015 | 0.095 | 0.296 | 0.606 | 1.000 |
| 6.0  | 0.005        | 0.018 | 0.040 | 0.0   | 0.06  | 0.037 | 0.057 | 0.252 | 0.572 | 1.000 |
| 8.0  | 0.001        | 0.009 | 0.022 | 0.044 | 0.068 | 0.062 | 0.002 | 0.178 | 0.515 | 1.000 |
| 10.0 | 0.000        | 0.002 | 0.009 | 0.028 | 0.053 | 0.06  | 0.031 | 0.123 | 0.467 | 1.000 |
| 12.0 | 0.000        | 0.000 | 0.003 | 0.010 | 0.040 | 0.064 | 0.049 | 0.081 | 0.424 | 1.000 |
| 14.0 | 0.000        | 0.000 | 0.000 | 0.008 | 0.029 | 0.059 | 0.060 | 0.048 | 0.387 | 1.000 |
| 16.0 | 0.000        | 0.000 | 0.002 | 0.003 | 0.021 | 0.051 | 0.066 | 0.045 | 0.354 | 1.000 |

Table XII

Moments in circular slab without center support  
Uniform load  
Fixed edge

$M_m = \text{coef} \times pR$   
Position indicated in the side



| Coefficients                      |       |       |       |       |       |       |       |       |       |       |
|-----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.00R                             | 0.10R | 0.20R | 0.30R | 0.40R | 0.50R | 0.60R | 0.70R | 0.80R | 0.90R | 1.00R |
| 0.75                              | 0.73  | 0.67  | 0.57  | 0.43  | 0.25  | 0.03  | 0.23  | 0.3   | 0.87  | 1.25  |
| Radial moments M <sub>r</sub>     |       |       |       |       |       |       |       |       |       |       |
| 0.75                              | 0.74  | 0.71  | 0.66  | 0.59  | 0.50  | 0.39  | 0.26  | 0.11  | 0.06  | 0.25  |
| Tangential moments M <sub>t</sub> |       |       |       |       |       |       |       |       |       |       |
| 0.75                              | 0.74  | 0.71  | 0.66  | 0.59  | 0.50  | 0.39  | 0.26  | 0.11  | 0.06  | 0.25  |

When the table is used for the applied load which is top hinged or fixed at the top of the wall, the top moment is applied at the top of the wall.

Table XIII

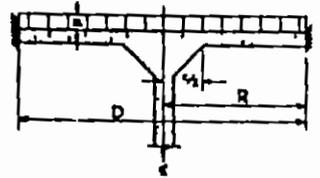
Moments in circular slab with center support

Uniform load

Fixed edge

Mom = coef X pR

Positive sign indicates compression in surface to dead



| r/D  | Coefficients at point    |         |         |         |         |         |        |        |         |        |        |         |        |
|------|--------------------------|---------|---------|---------|---------|---------|--------|--------|---------|--------|--------|---------|--------|
|      | 0.05R                    | 0.10R   | 0.15R   | 0.20R   | 0.25R   | 0.30R   | 0.40R  | 0.50R  | 0.60R   | 0.70R  | 0.80R  | 0.90R   | 1.00R  |
|      | Radial moments $M_r$     |         |         |         |         |         |        |        |         |        |        |         |        |
| 0.05 | -0.2100                  | 0.0729  | 0.0275  | 0.0026  | 0.0133  | 0.0238  | 0.0342 | 0.0347 | 0.0277  | 0.0142 | 0.0049 | 0.0294  | 0.0569 |
| 0.10 |                          | -0.1433 | -0.0624 | 0.0239  | 0.0011  | 0.0136  | 0.0290 | 0.0326 | +0.0276 | 0.0168 | 0.0021 | 0.0255  | 0.0541 |
| 0.15 |                          |         | -0.1089 | 0.0521  | 0.0200  | 0.0002  | 0.0220 | 0.0293 | 0.0289  | 0.0169 | 0.0006 | 0.0218  | 0.0490 |
| 0.20 |                          |         |         | -0.0862 | 0.0429  | -0.0161 | 0.0133 | 0.0249 | 0.0264  | 0.0176 | 0.0029 | -0.0178 | 0.0441 |
| 0.25 |                          |         |         |         | -0.0698 | -0.0351 | 0.0029 | 0.0194 | 0.0231  | 0.0177 | 0.0049 | -0.0143 | 0.0393 |
|      | Tangential moments $M_t$ |         |         |         |         |         |        |        |         |        |        |         |        |
| 0.05 | -0.0417                  | -0.0700 | -0.0541 | 0.0381  | 0.0251  | -0.0145 | 0.0002 | 0.0085 | 0.0118  | 0.0109 | 0.0065 | 0.0003  | 0.0118 |
| 0.10 |                          | 0.0287  | 0.0421  | 0.0354  | 0.0258  | -0.0168 | 0.0027 | 0.0059 | 0.0099  | 0.0098 | 0.0061 | 0.0007  | 0.0108 |
| 0.15 |                          |         | -0.0218 | 0.0284  | -0.0243 | -0.0177 | 0.0051 | 0.0031 | 0.0080  | 0.0086 | 0.0057 | 0.0006  | 0.0096 |
| 0.20 |                          |         |         | 0.0172  | 0.0203  | 0.0171  | 0.0070 | 0.0013 | 0.0063  | 0.0075 | 0.0052 | 0.0003  | 0.0088 |
| 0.25 |                          |         |         |         | -0.0140 | 0.0150  | 0.0083 | 0.0005 | 0.0046  | 0.0064 | 0.0048 | 0.0000  | 0.0078 |

Table XIV

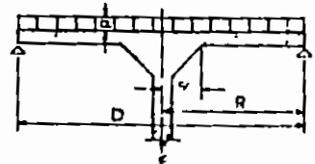
Moments in circular slab with center support

Uniform load

Hinged edge

Mom = coef X pR

Positive sign indicates compression in surface to dead



| r/D  | Coefficients at point    |         |         |         |         |         |        |        |        |        |        |        |        |
|------|--------------------------|---------|---------|---------|---------|---------|--------|--------|--------|--------|--------|--------|--------|
|      | 0.05R                    | 0.10R   | 0.15R   | 0.20R   | 0.25R   | 0.30R   | 0.40R  | 0.50R  | 0.60R  | 0.70R  | 0.80R  | 0.90R  | 1.00R  |
|      | Radial moments $M_r$     |         |         |         |         |         |        |        |        |        |        |        |        |
| 0.05 | -0.3658                  | 0.1388  | 0.0640  | 0.0221  | 0.0058  | 0.0255  | 0.0501 | 0.0614 | 0.0629 | 0.0560 | 0.0437 | 0.0247 | 0      |
| 0.10 |                          | -0.2487 | 0.1180  | 0.0557  | 0.0176  | 0.0081  | 0.0391 | 0.0539 | 0.0578 | 0.0532 | 0.0418 | 0.0237 | 0      |
| 0.15 |                          |         | 0.1869  | 0.0977  | -0.0467 | -0.0135 | 0.0258 | 0.0451 | 0.0518 | 0.0494 | 0.0393 | 0.0226 | 0      |
| 0.20 |                          |         |         | -0.1465 | 0.0800  | 0.0381  | 0.0109 | 0.0352 | 0.0452 | 0.0451 | 0.0368 | 0.0215 | 0      |
| 0.25 |                          |         |         |         | -0.1172 | 0.0645  | 0.0055 | 0.0245 | 0.0381 | 0.0404 | 0.0340 | 0.0230 | 0      |
|      | Tangential moments $M_t$ |         |         |         |         |         |        |        |        |        |        |        |        |
| 0.05 | -0.0731                  | 0.1277  | 0.1040  | 0.0788  | 0.0569  | 0.0391  | 0.0121 | 0.0061 | 0.0175 | 0.0234 | 0.0251 | 0.0228 | 0.0168 |
| 0.10 |                          | -0.0488 | 0.0768  | 0.0684  | 0.0539  | 0.0394  | 0.0153 | 0.0020 | 0.0134 | 0.0197 | 0.0218 | 0.0199 | 0.0145 |
| 0.15 |                          |         | -0.0374 | 0.0516  | -0.0470 | 0.0375  | 0.0175 | 0.0014 | 0.0097 | 0.0163 | 0.0186 | 0.0172 | 0.0123 |
| 0.20 |                          |         |         | 0.0293  | 0.0367  | 0.0333  | 0.0184 | 0.0042 | 0.0065 | 0.0132 | 0.0158 | 0.0148 | 0.0103 |
| 0.25 |                          |         |         |         | 0.0234  | 0.0263  | 0.0184 | 0.0062 | 0.0038 | 0.0103 | 0.0132 | 0.0127 | 0.008  |

Table XV

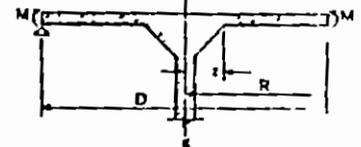
Moments in circular slab with center support

Moment/m M applied at edge

Hinged edge

M m = coef X M

Positive sign indicates compression in top surface



| r/D  | Coefficients at point    |        |        |       |       |       |       |       |       |       |       |       |       |
|------|--------------------------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | 0.05R                    | 0.10R  | 0.15R  | 0.20R | 0.25R | 0.30R | 0.40R | 0.50R | 0.60R | 0.70R | 0.80R | 0.90R | 1.00R |
|      | Radial moments $M_r$     |        |        |       |       |       |       |       |       |       |       |       |       |
| 0.05 | 2.650                    | -1.121 | -0.622 | 0.333 | 0.129 | 0.029 | 0.268 | 0.450 | 0.598 | 0.718 | 0.824 | 0.917 | 1.000 |
| 0.10 |                          | 1.950  | 1.026  | 0.584 | 0.305 | 0.103 | 0.187 | 0.394 | 0.558 | 0.692 | 0.808 | 0.907 | 1.000 |
| 0.15 |                          |        | 1.694  | 0.930 | 0.545 | 0.260 | 0.078 | 0.323 | 0.510 | 0.663 | 0.790 | 0.900 | 1.000 |
| 0.20 |                          |        |        | 1.366 | 0.842 | 0.499 | 0.057 | 0.236 | 0.451 | 0.624 | 0.768 | 0.871 | 1.000 |
| 0.25 |                          |        |        |       | 1.204 | 0.765 | 0.216 | 0.130 | 0.392 | 0.577 | 0.740 | 0.880 | 1.000 |
|      | Tangential moments $M_t$ |        |        |       |       |       |       |       |       |       |       |       |       |
| 0.05 | -0.530                   | 0.980  | 0.847  | 0.688 | 0.544 | 0.418 | 0.211 | 0.042 | 0.095 | 0.212 | 0.314 | 0.405 | 0.486 |
| 0.10 |                          | 0.388  | 0.641  | 0.608 | 0.518 | 0.419 | 0.233 | 0.072 | 0.066 | 0.185 | 0.290 | 0.384 | 0.469 |
| 0.15 |                          |        | -0.319 | 0.472 | 0.463 | 0.404 | 0.251 | 0.100 | 0.035 | 0.157 | 0.261 | 0.363 | 0.451 |
| 0.20 |                          |        |        | 0.272 | 0.372 | 0.368 | 0.281 | 0.123 | 0.007 | 0.129 | 0.240 | 0.340 | 0.433 |
| 0.25 |                          |        |        |       | 0.239 | 0.305 | 0.277 | 0.145 | 0.020 | 0.099 | 0.214 | 0.310 | 0.411 |

Table XVI

Shear at base of cylindrical wall

$$V = F \times \begin{cases} H/D & (\text{ft. g l.}) \\ M/H & (\text{mm lb.}) \end{cases}$$

Fixed edge

| H/D  | Fixed edge | Rectangular | Triangular | Moment |
|------|------------|-------------|------------|--------|
| 0.4  | 0.436      | 0.765       | 0.245      | 1.58   |
| 0.8  | 0.374      | 0.552       | 0.234      | 1.75   |
| 1.2  | 0.339      | 0.460       | 0.220      | 2.00   |
| 1.6  | 0.317      | 0.407       | 0.204      | 2.28   |
| 2.0  | 0.299      | 0.370       | 0.189      | 2.67   |
| 3.0  | 0.262      | 0.310       | 0.158      | 3.18   |
| 4.0  | 0.236      | 0.271       | 0.137      | 3.68   |
| 6.0  | 0.213      | 0.243       | 0.121      | 4.10   |
| 8.0  | 0.197      | 0.22        | 0.110      | 4.49   |
| 10.0 | 0.174      | 0.193       | 0.096      | 5.18   |
| 12.0 | 0.158      | 0.172       | 0.087      | 5.81   |
| 14.0 | 0.145      | 0.158       | 0.079      | 6.38   |
| 16.0 | 0.135      | 0.147       | 0.073      | 6.88   |
| 18.0 | 0.127      | 0.137       | 0.068      | 7.36   |

Table XVIII

Stiffness of cylindrical wall  
Near edge hinged far edge free

$$k = \text{coef} \times Et/H$$

| H/D | Coefficient | H/D | Coefficient |
|-----|-------------|-----|-------------|
| 0.4 | 0.139       | 5   | 0.713       |
| 0.8 | 0.270       | 6   | 0.783       |
| 1.2 | 0.346       | 8   | 0.903       |
| 1.6 | 0.399       | 10  | 1.010       |
| 2.0 | 0.445       | 12  | 1.108       |
| 3.0 | 0.548       | 14  | 1.198       |
| 4.0 | 0.635       | 16  | 1.281       |

Table XVII

Load on center support for circular slab

$$L = \text{coef} \times \begin{cases} H/D & (\text{hinged edge}) \\ M & (\text{mm lb.}) \end{cases}$$

| H/D  | 0.05  | 0.10  | 0.15  | 0.20  | 0.25  |
|------|-------|-------|-------|-------|-------|
| 0.05 | 1.30  | 1.387 | 1.463 | 1.542 | 1.625 |
| 0.10 | 0.839 | 0.919 | 1.007 | 1.101 | 1.200 |
| 0.15 | 0.616 | 0.666 | 0.729 | 0.799 | 0.861 |

Table XIX

Stiffness of circular plates  
With center support

$$k = \text{coef} \times Et/R$$

| D    | 0.05  | 0.10  | 0.15  | 0.20  | 0.25  |
|------|-------|-------|-------|-------|-------|
| 0.05 | 0.290 | 0.309 | 0.332 | 0.358 | 0.387 |

Without center support  
Coef = 0.104

Table XX Supplementary Coefficients for Values of H<sup>2</sup>/Dt Greater than 16 (Extension of Tables I to XI, XVI and XVIII)\*

| H/D                | TABLE I               |       |        |       |       | TABLE II              |       |       |       |        | TABLE III             |        |       |        |       | TABLE IV              |       |               |           |             |
|--------------------|-----------------------|-------|--------|-------|-------|-----------------------|-------|-------|-------|--------|-----------------------|--------|-------|--------|-------|-----------------------|-------|---------------|-----------|-------------|
|                    | Coefficients at point |       |        |       |       | Coefficients at point |       |       |       |        | Coefficients at point |        |       |        |       | Coefficients at point |       |               |           |             |
|                    | 75H                   | 80H   | 85H    | 90H   | 95H   | 75H                   | 80H   | 85H   | 90H   | 95H    | 75H                   | 80H    | 85H   | 90H    | 95H   | 75H                   | 80H   | 85H           | 90H       | 95H         |
| 20                 | 0.716                 | 0.654 | 0.520  | 0.325 | 0.115 | 0.812                 | 0.817 | 0.756 | 0.603 | 0.344  | 0.949                 | 0.825  | 0.629 | 0.379  | 0.128 | 1.062                 | 1.017 | 0.908         | 0.703     | 0.394       |
| 24                 | 0.746                 | 0.702 | 0.577  | 0.372 | 0.137 | 0.816                 | 0.839 | 0.793 | 0.647 | 0.377  | 0.986                 | 0.879  | 0.694 | 0.430  | 0.149 | 1.068                 | 1.039 | 0.943         | 0.747     | 0.427       |
| 32                 | 0.782                 | 0.768 | 0.663  | 0.459 | 0.182 | 0.814                 | 0.861 | 0.847 | 0.721 | 0.436  | 1.028                 | 0.953  | 0.788 | 0.519  | 0.189 | 1.064                 | 1.061 | 0.997         | 0.821     | 0.488       |
| 40                 | 0.800                 | 0.805 | 0.731  | 0.530 | 0.217 | 0.802                 | 0.868 | 0.880 | 0.778 | 0.483  | 1.040                 | 0.996  | 0.859 | 0.591  | 0.228 | 1.062                 | 1.068 | 1.030         | 0.878     | 0.533       |
| 48                 | 0.791                 | 0.828 | 0.785  | 0.593 | 0.254 | 0.791                 | 0.864 | 0.900 | 0.820 | 0.527  | 1.043                 | 1.022  | 0.911 | 0.652  | 0.262 | 1.041                 | 1.064 | 1.050         | 0.920     | 0.577       |
| 56                 | 0.763                 | 0.838 | 0.824  | 0.636 | 0.285 | 0.781                 | 0.859 | 0.911 | 0.852 | 0.563  | 1.040                 | 1.035  | 0.949 | 0.705  | 0.294 | 1.021                 | 1.059 | 1.061         | 0.952     | 0.613       |
| H/D                | TABLE V               |       |        |       |       | TABLE VI              |       |       |       |        | TABLE VII             |        |       |        |       | TABLE VIII            |       |               |           |             |
|                    | Coefficients at point |       |        |       |       | Coefficients at point |       |       |       |        | Coefficients at point |        |       |        |       | Coefficients at point |       |               |           |             |
|                    | 00H                   | 05H   | 10H    | 15H   | 20H   | 75H                   | 80H   | 85H   | 90H   | 95H    | 80H                   | 85H    | 90H   | 95H    | 100H  | 75H                   | 80H   | 85H           | 90H       | 95H         |
| 20                 | 18.44                 | 9.98  | 4.90   | 1.59  | 0.22  | 15.30                 | 25.9  | 36.9  | 43.3  | 35.3   | 0.015                 | 0.014  | 0.005 | 0.018  | 0.083 | 0.008                 | 0.014 | 0.020         | 0.024     | 0.020       |
| 24                 | 18.04                 | 10.34 | 4.54   | 1.00  | 0.68  | 13.20                 | 25.9  | 40.7  | 81.8  | 45.3   | 0.012                 | 0.012  | 0.007 | 0.013  | 0.053 | 0.005                 | 0.010 | 0.015         | 0.020     | 0.017       |
| 32                 | 20.84                 | 10.72 | 3.70   | 0.04  | 1.26  | 8.10                  | 23.2  | 45.9  | 85.4  | 83.8   | 0.007                 | 0.009  | 0.007 | 0.008  | 0.040 | 0.000                 | 0.005 | 0.009         | 0.014     | 0.013       |
| 40                 | 23.34                 | 10.86 | 2.86   | 0.72  | 1.66  | 3.28                  | 19.2  | 46.5  | 77.9  | 83.5   | 0.002                 | 0.005  | 0.006 | 0.005  | 0.032 | 0.000                 | 0.003 | 0.006         | 0.011     | 0.011       |
| 48                 | 25.52                 | 10.82 | 2.06   | 1.26  | 1.68  | 0.70                  | 14.1  | 45.1  | 87.2  | 103.0  | 0.000                 | 0.001  | 0.006 | 0.003  | 0.026 | 0.000                 | 0.001 | 0.004         | 0.008     | 0.010       |
| 56                 | 27.54                 | 10.68 | 1.36   | 1.60  | 1.82  | 3.40                  | 8.2   | 42.2  | 94.0  | 121.0  | 0.000                 | 0.000  | 0.004 | 0.001  | 0.023 | 0.000                 | 0.000 | 0.003         | 0.007     | 0.008       |
| H <sup>2</sup> /Dt | TABLE IX              |       |        |       |       | TABLE X               |       |       |       |        | TABLE XI              |        |       |        |       | TABLE XVI             |       |               |           | TABLE XVIII |
|                    | Coefficients at point |       |        |       |       | Coefficients at point |       |       |       |        | Coefficients at point |        |       |        |       | T/F                   | R/F   | T or R Hinged | M m t Edg | Stiffness   |
|                    | 00H                   | 05H   | 10H    | 15H   | 100H  | 05H                   | 10H   | 15H   | 20H   | 25H    | 80H                   | 85H    | 90H   | 95H    | 100H  | 0.114                 | 0.122 | 0.062         | 8.20      | 1.430       |
| 20                 | 0.015                 | 0.013 | 0.002  | 0.024 | 0.073 | 0.032                 | 0.039 | 0.033 | 0.023 | 0.014  | -0.015                | 0.035  | 0.296 | 0.606  | 1.000 | 0.114                 | 0.122 | 0.062         | 8.20      | 1.430       |
| 24                 | 0.012                 | 0.012 | 0.004  | 0.018 | 0.061 | 0.031                 | 0.035 | 0.028 | 0.018 | 0.009  | -0.037                | 0.057  | 0.250 | 0.572  | 1.000 | 0.102                 | 0.111 | 0.055         | 8.94      | 1.568       |
| 32                 | 0.008                 | 0.009 | 0.006  | 0.010 | 0.048 | 0.028                 | 0.029 | 0.020 | 0.011 | +0.004 | 0.062                 | 0.062  | 0.178 | +0.515 | 1.000 | 0.089                 | 0.098 | 0.048         | 10.36     | 1.810       |
| 40                 | 0.005                 | 0.007 | 0.007  | 0.005 | 0.037 | 0.026                 | 0.025 | 0.015 | 0.006 | 0.001  | 0.067                 | 0.031  | 0.123 | 0.467  | 1.000 | 0.080                 | 0.086 | 0.043         | 11.62     | 2.025       |
| 48                 | 0.004                 | 0.006 | +0.006 | 0.003 | 0.031 | 0.024                 | 0.021 | 0.011 | 0.003 | 0.000  | 0.064                 | 0.049  | 0.081 | 0.424  | 1.000 | 0.072                 | 0.079 | 0.039         | 12.76     | 2.220       |
| 56                 | 0.002                 | 0.004 | 0.005  | 0.001 | 0.026 | 0.023                 | 0.018 | 0.008 | 0.002 | 0.000  | -0.059                | -0.060 | 0.048 | 0.387  | 1.000 | 0.067                 | 0.074 | 0.038         | 13.76     | 2.400       |

\* Point to show the plate may be determined by sketching curves with the help of