A QUADRATIC PROGRAMMING APPROACH TO THE PROBLEM OF OPTIMAL PRICING AND USE OF COTTON IN EGYPT

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I

INTRODUCTION

This paper presents a multi-sectoral, multi-period quadratic programming model of an economy which depends on the production and export of one primary product and in which foreign trade plays a major role. The model is intended to answer the following questions :

— What is the optimal allocation of primary factors between the main raw material production and the other alternative uses ?

— If there are different varieties of this main crop, what is the optimal production of each variety ?

— How should the crop be distributed between exports and domestic industry, which eventually will lead to promoting exports of manufactured or semi-manufactured goods or will substitute for initial imports ?

— Finally, assuming the country has a large share of the world market for its main crop and that it could affect the world price, what are the prices which would maximize the net foreign exchange earnings of the economy ?

The model will be developed to fit the structural features of the Egyptian economy. However, it may be more generally applicable to the group of raw material producing countries which have a large share in the total world production and trade of the raw materials they produce. The model will be fitted with nine producing sectors involved in the growing and manufacturing of cotton either as direct producers or as suppliers of intermediate inputs and capital goods to the cotton producing sectors. The nine sectors produce respectively the following commodities :

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1—Long-staple cotton (1)

2—Medium-staple cotton (2)

3—Short-staple cotton (3)

4—Cereals — namely wheat, rice, barley, maize, millet.

5—Cotton yarn — this sector corresponds to a 4 — digit industry.

6—Cotton fabrics — this sector corresponds also to a 4 — digit industry.

7—Fuel and power — this sector corresponds to a 2 — digit industry, it does not include electricity.

8—Chemicals — this sector corresponds to a 2 — digit industry and is introduced in the model mainly because it supplies fertilizers to agriculture.

9—Engineering and metallurgical products — this sector is the aggregate of three 2 — digit industries, namely : basic metallic industry, engineering and metallic industry and means of transportation industry.

Following the classification applied by the Ministry of Planning, the exogenous sectors are Irrigation and Drainage — the High Dam — Electricity — Transport, Communications and Storage — the Suez Canal — Housing — Public Utilities — Services. The endogenous sectors of the model belong to Industry and to Agriculture. We should note that some of the more important industrial sectors are not included within the model : extractive industries, food processing, wood industries, rubber, non-metallic industries, rural industries ... Demands from these excluded sectors are regarded as exogenous. Also, we should note that an important portion of agriculture has been excluded from the model : cotton and cereals together represent only about half of the net agricultural production; the production of fruit, vegetables and animal products constitute the other half.

II

THE MODEL

A.—*Main Features of the Model*

The model is dynamic in the sense that for any one optimal solution, the variables relate to different periods of time. The periods considered are not of the same length. We limit our analysis to two periods. One extending from 1966 to 1970, the other from 1970 to 1980. During the first phase, the productive capacity is assumed to be fixed and investments during this period do not increase the productive capacity until the second phase. The second period is long enough to allow us to ignore investment-output lags, so that investments in this phase are assumed to increase output within the same phase.

The objective of the study is to maximize foreign exchange earnings (or alternatively to minimize the required inflow of foreign loans and aid) in the terminal year of the plan, namely 1980, subject to a predetermined set of final demands. This approach is thought to be more meaningful than that of maximizing consumption or output in the context of our model. This model is not an economy-wide model, but rather it is concerned with a portion of the economy, and minimizing costs seemed to us to be more suited to this partial equilibrium analysis than maximizing consumption of cotton goods which does not correspond to any acceptable index of welfare. Maximizing foreign exchange earnings (or minimizing the inflow of foreign loans) appears to be a more appropriate choice, for it contributes to solving the main problem of the economy under investigation, namely the balance-of-payments deficit.

The objective function is quadratic rather than linear, allowing for the possibility that the country is able to influence world market prices of the primary product of its specialization; this follows directly from the large share of the country in total world trade of this commodity. An assumption which would have the same effect on the solution as that of a falling demand price is that of rising supply price of exports, explained by increasing costs of production or the necessity of shipping to more distant markets additional production and of incurring more selling costs. Whenever the export price of a product is found to be constant, an upper bound to exports will be imposed to prevent indefinite expansion of output and exports in this direction. Import substitution is allowed at the margin — in the sense that any increase in supply would be realised through imports or domestic production depending on the comparative cost of the two activities.

Finally, investment over the plan period is assumed to grow exponentially. Investment in the terminal year is related to the change in capital stock between the initial and terminal years of the plan as well as to the change in the rate of growth of capital over the same period. This formulation is similar to the Sandee-Manne approach[(4)] in that it relates investment in the terminal year to cumulative investment over the plan period; it differs however in that it accounts for the effect of the change in the investment growth rate on investment. This aspect of the model will be considered further in the Appendix.

Other assumptions underlying the analysis are :

1) The marginal rather than the average input-output coefficients are constant. This assumption allows a certain flexibility in the average coefficients and implies that the elasticity of output with respect to any given input (measured as the ratio of the average to the marginal input-output coefficient) will be closer to unity in the terminal year of the plan than it was in the initial period.

2) The incremental capital-output ratios in all sectors are constant.

3) Additional output in most sectors, particularly the manufacturing industries requires imported capital equipment in fixed proportions to output increments.

4) Inventory investment is a fixed proportion of output increase over the period.

5) For each period, an investment balance must be satisfied which restricts investments to total domestic savings plus the annual inflow of foreign exchange resulting from a positive trade balance or from a net inflow of foreign loans and gifts or both.

6) Land, fixed capital stock and foreign exchange are the only scarce factors. Actually, in the case of Egypt, water rather than land is the scarce factor; but no satisfactory measure to this factor was found, therefore, cultivable land was taken instead as the scarce factor, cultivable land itself being primarily determined by the extent of irrigation which is exogenous to the model. Foreign exchange is not a primary factor in the same sense as land, for it could be produced within the model through excess exports over imports. Finally, labor supply is assumed to be adequate.

B.—*Model Formulation :*

The main variable for which optimal values for the various phases of the plan are being sought are the level of output, investment, exports and imports by sector and variety of goods and the export prices — Domestic consumption is assumed to be given exogenously.

In what follows, subscripts 1, 2, 3, 4 refer respectively to long staples, medium staples, short staples and cereals, 5 refers to cotton yarn, 6 to cotton fabrics, 7 to fuel and power, 8 to chemicals and 9 to engineering and metallurgical products.

The constraints of the initial programme can be read from Table 1 in the Appendix.

The thirteen first constraints hold for the first phase of the plan, the other constraints are for the second phase. Constraints (1) through (6) are balance equations for the three cotton varieties — long — medium — and short — staple cotton —, grain, chemicals and machinery respectively. — The following general restriction covers the material balance for the 1969 - 70 incremental flows of commodity i over the initial magnitudes :

[4 — year increase in domestic production of i net of current endogenous additional input demand and inventory accumulation]

—

[absolute level of imports of i in 1969 - 70]

+

[absolute level of exports of i in 1969 - 70]

+

[investment demand for i by the endogenous sectors in 1969 - 70]

$\leqslant$

$$
-\left[\text{4 — year increase in household, government and exogenous sectors demand for } i\right]
$$

$$
-\left[\text{imports of } i \text{ in 1965 - 66}\right]
$$

$$
+\left[\text{exports of } i \text{ in 1965 - 66}\right]
$$

$$
+\left[\text{investment demand for } i \text{ by the endogenous sectors in 1965 - 66}\right]
$$

Note that each sector is either an exporter or an importer. No sector is allowed to be both. Furthermore, the variables do not exist for all sectors — e.g. only machinery contributes to fixed investments; capacity increase in the chemicals sector and in machinery is exogenous over this phase of the plan and is therefore shifted to the right-hand side of the inequality. This formulation of the commodity balances in incremental form implies either that the productive capacity in all sectors is fully utilized in the initial period or that the rate of capacity utilization remains the same as it was in the initial year of the plan. This assumption might not be justified empirically but it had to be made since data on the degree of capacity utilization in different sectors are scanty.

Constraints (7) through (9) restrict the possibility of increasing land productivity in individual crops by setting a maximum to the growth rate of output of agricultural commodity i (i = 1, 2, 4) due to productivity improvement. These restrictions say that given the acreage of crop i, production of i cannot rise at more than 3.5% per annum, of its initial level, due to vertical expansion and to the use of chemicals. Over this phase of the plan, a 4-year lag is required to put additional land in cultivation. These constraints are of the form :

$$\left[\text{4 - year increase in output of } i \text{ due to productivity improvement}\right] \leqslant$$

$$[(1.035)^4 - 1]$$

$$\left[\text{initial 1965 - 66 absolute level of production of } i\right]$$

where $i = 1, 2, 4.$

Constraint (10) defines intermediate consumption of raw cotton by the spinning industry. The cotton yarn coefficient is assumed to be the same for all cotton varieties implying that all varieties of cotton are equally suited to produce any kind of yarn and that either all varieties of yarn require for their production the same quantity of cotton or else that the output of each kind of yarn changes in the same proportion as total yarn production. This assumption was imposed by the unavailability of statistics. Constraint (10) says that:

$$\left[\text{Sum of additional cotton of different varieties (long, medium, short staples) available to the spinning industry}\right] \leqslant$$

$$\left[\text{exogenously determined additional input requirement of cotton fibres by the spinning industry}\right]$$

(11) and (12) define respectively the increase in agricultural output due to the use of fertilizers and to vertical expansion and the allocation of additional land to different crops. This latter constraint says that the demand for additional land should be at most equal to the new supply of agricultural land.

Finally, (13) is a saving constraint asserting that :

[Sum of investments in the endogenous sectors in 1969 - 70 including planned inventory accumulation]

+

[Total exports by the endogenous sectors in 1969-70]

—

[Total imports by the endogenous sectors in 1969/70]

$\leqslant$

[Total savings available to the endogenous sectors in 1969 - 70.]

For the second phase, the following general constraint covers commodity i's balance :

[10-year increase in domestic production of *i* net of current endogenous input demand and of inventory accumulation]

—

[Additional imports of *i* over the period 1969 - 70 to 1979 - 80]

+

[Incremental exports of *i* over the period 1969-70 to 1979 - 80]

+

[Increase in the rate of investment demand for *i* by the endogenous sectors in 1979 - 80 over the prevailing rate in 1969 - 70] $\leqslant$

— [10-year increase in household, government and exogenous sectors demand for *i*]

These material balances are represented by restrictions (14) to (22).

Constraints (23), (24), (25) set the maximum annual rate of increase of various crops due to productivity improvement at 3.5%. Implicit in this formulation is the assumption that during the second phase of the plan, the area cultivated with individual crops increase linearly each year, whenever it changes. These constraints are of the following general form :

[10-year increase in crop *i* due to productivity improvement]

$$— [1.035^{10} — 1]$$

[Sum of the 4-year increase in crop *i* (over the first phase) due to both productivity improvement and to acreage expansion.]

$$- \left[\frac{1}{10} \sum_{\theta=0}^{9} \left\{ (1.035)^{\theta} - 1 \right\} \right]$$

$$\left[\begin{array}{c} \text{10-year increase in crop } i \text{ due to acreage} \\ \text{expansion} \end{array} \right] \leqslant [(1.035)^{10} - 1] \left[\text{initial 1965 - 66 absolute level of crop } i \right]$$

where i = 1, 2, 4.

(26) like (10) expresses the fact that the additional supply of cotton fibres of all varieties to the spinning industry should be at least equal to the increase in current input requirement of this industry.

$$- \left[\begin{array}{c} \text{Total increase in supply of different cotton varie-} \\ \text{ties to the spinning industry over the period} \\ \text{1969 - 70 to 1979 - 80} \end{array} \right] + \left[\begin{array}{c} \text{Additional current input requirement of cotton} \\ \text{fibres by the spinning industry over the period} \\ \text{1969 - 70 to 1979 - 80.} \end{array} \right] \leqslant 0$$

(27) and (28) set an upper bound to exports of yarn and of fuel and power, namely that :

$$\left[\text{exports of } i \text{ in 1979 - 80} \right]$$

$$-e_i \left[\text{10-year increase in domestic production of } i \right] \leqslant \left[\text{exports of } i \text{ in 1969 - 70} \right]$$

where i = 5,7 for yarn and for fuel and power respectively In the initial run e_i, the export promotion coefficient has been set at 10%, but this is a policy parameter which was changed in subsequent runs. This upper bound is intended to prevent solutions with unrealistically high values for the production and exports of the sectors which produce commodities with an infinitely elastic demand for their exports. The sectors with demand for their exports less than infinitely elastic — namely sectors 1, 2, 6 — will be constrained by demand, and the tendency for expansion in this direction will be dampened by price changes — the other sectors — i = 3, 4, 8, 9 — are net importers.

(29) through (34) express the relationship between sectoral investment and capacity increase, its general form is the following :

$$(k_j (1 - e_j) + k^*_j e_j) \left[\begin{array}{c} \text{10-year increase in domestic production of} \\ \text{commodity } i \end{array} \right]$$

$$- (\zeta_j + \eta_j) \left[\begin{array}{c} \text{Rate of investment demand for machinery} \\ \text{by j in 1969 - 70} \end{array} \right]$$

$$-\eta_j \left[\begin{array}{c} \text{increase in the rate of investment in machinery} \\ \text{by j over the 10-year period 1969 - 70 to 1979 - 80} \end{array} \right]$$

$$\leqslant [k_j (1 - e_j) + k^*_j e_j]$$

[Arbitrary value for the 10-year increase in domestic production of commodity j]

$$- (\zeta_j + \eta_j)$$

[corresponding arbitrary rate of investment demand for machinery by j in 1969-70]

$$- \eta_j$$

[corresponding arbitrary increase in the rate of investment in machinery by j over the 10-year period 1969-70 to 1979-80]

where k_j is the incremental machinery/ output ratio for commodity j, k^*_j is the corresponding ratio for export promotion output — e_j is the export promotion coefficient $e_j = 0.10$ for j = 5,7 in the initial run and is zero for the other commodities, ζ_j, η_j are defined in the Appendix.

The model, as formulated, allows for endogenous generation of demand for domestic and imported capital equipment. For domestic capital equipment, any change in the annual flow of output j over the planning period requires $k_j X_j$ change of capital equipment delivered by the domestic sector producing machinery. In our formulation, special allowance has been made for capital costs of export promotion. Normally import substitution and export promotion will be embodied in the plan itself as productive capacity is created and substitutes for imports or promotes exports at normal rates of return given the existing level of protection. However, the increase in import substitution and export promotion forthcoming in this way is not always sufficient to cover the gap between import requirements on the one hand and export earnings and the flow of foreign loans and gifts on the other. Therefore, it may be necessary to shift part of the investment resources from more to less profitable uses to close this gap. In the model we shall assume that the incremental capital-output ratio for capital shifted to additional exports is higher than the normal coefficient prevailing in the sector manufacturing those commodities, i.e. $k^*j > kj$ where k^*j is arbitrarily assumed to be 5% higher than kj .

Capacity change is given exogenously in the first phase of the plan, investment in this phase results in an increment in productive capacity in the second phase of the plan. This implies that there is a gestation lag of τ_1 between capital increase and the addition to capacity, where τ_1 is the duration of the first phase of the plan.

Finally, investment is assumed to grow at different rates λ_1, λ_2 in each phase, these growth rates are not specified exogenously but are determined optimally by the model. The algebraic derivation of the relation between capacity increase and fixed investment is worked out in the Appendix.

The assumptions applying to fixed domestic investment are also valid for imported capital equipment and a similar relationship with capacity increase holds. The assumptions of a constant incremental capacity-output ratio and of a constant incremental imported capital equipment-output, relation imply that over each period of the plan, and within each sector, invesment in imported capital equipment is a fixed proportion of total fixed investment. This permits to simplify the formulation of the capacity increase equations further by combining the two sets of equations determining the relation between capacity increase and domestic and imported capital equipment respectively into one set of equations relating output increase to total investment in capital equipment in each sector.

In order to make sure that the economy does not exceed the availability of raw materials supplied by mines and quarries and used as current inputs, constraint (35) is added namely :

$$\left[\begin{array}{c}\text{10-year increase in endogenous sectors current} \\ \text{demand for raw materials}\end{array}\right]$$

$$-\left[\text{absolute level of imports of raw materials in 1979-80}\right] \leqslant \left[\text{maximum increase in domestic rate of exploitation of raw materials over the period 1969-70 to 1979-80}\right] - \left[\text{absolute level of imports of raw materials in 1969-70}\right]$$

These raw materials are mainly composed of crude oil and minerals, supply of non-competitive primary products i.e. raw materials which are not available domestically and have to be all imported — is assumed to be adequate.

Like (12), constraint (36) states that the increase in demand for cultivable land cannot exceed the additional supply of land.

Finally, (37) is a savings constraint.

$$\left[\text{Total endogenous investment in 1979-80}\right] + \left[\text{Total exports by the endogenous sectors in 1979-80}\right] - \left[\text{Total endogenous imports in 1979-80}\right]$$

$$\leq$$

Total domestic savings available to the endogenous sectors in 1979 - 80

III

DATA REQUIRED

In order to apply the model just described, information is required to determine the demand functions, to define the technology of production and also to specify the various exogenous demands as well as the initial conditions describing the economy at the beginning of the planning period (5).

On the demand side, demand functions for different exports and imports have to be estimated. These functions are required for the formulation of the objective criterion. Various relations were fitted to specify the demand functions. Reliable estimates were obtained for export demand of long-staple and medium-staple cotton, as well as for cotton fabrics. For the initial run of the programme the following set of demand functions was used :

$$\bar{E}_1 (t) = \underset{(2.139)}{2.961} - \underset{(0.090)}{0.255} P_1 (t) + \underset{(0.220)}{0.342} P_2 (t) \quad R^2=0.535$$

$$\bar{E}_2 (t) = \underset{(2.596)}{9.663} + \underset{(0.109)}{0.420} P_1 (t) - \underset{(0.267)}{0.970} P_2 (t) \quad R^2=0.577$$

$$\bar{E}_6 (t) = \underset{(4.436)}{29.458} - \underset{(5.44)}{28.885} P_6 (t) \quad R^2=0.668$$

where $\bar{E}_i (t)$ is the quantity exported of commodity i in year t — $\bar{E}_1 (t)$, $\bar{E}_2 (t)$ are expressed in million cantars, $\bar{E}_6 (t)$ is in million kgs — and $P_i (t)$ is the corresponding price expressed in domestic currency with fixed, given exchange rates — $P_1 (t)$, $P_2 (t)$ are in L.E. per cantar and $P_6 (t)$ is in L.E. per kg.

The results for exports and imports of other sectors were extremely unsatisfactory — accordingly, it was decided to consider the prices for these exports and imports as exogenously given.

To determine the demands for output, the following have to be estimated for each phase :

— Vector of change of final demands, these are exogenously determined variables which include final consumption by households and by the government, and the demand of the exogenous production sectors.

— Vector of incremental stock-output coefficients.

On the production side, the following structural data are needed for each phase of the plan :

— matrix of incremental inter-industry flow coefficients.

— matrix of incremental capital-output ratios.

— Vector of incremental non-competitive investment imports coefficients.

— Vector of land-output coefficients.

Other exogenous variables are the increase in cultivated land over the plan and the annual flow of savings in the terminal year of each phase of the plan. The policy variables to be decided upon by the planner are :

— export promotion coefficients.

— maximum growth rate of savings over the plan period.

Finally, the initial conditions are prescribed by the following values of the pre-plan year.

— Vector of domestic output levels.

— Vector of imports.

— Vector of exports.

— Annual rate of investment in each sector.

— Total savings.

The effort required to obtain the quantity and quality of information necessary to implement the model in a way which would make it readily applicable to actual planning issues exceeded the time available to us : furthermore, the purpose of this study is not to develop a policy — making model, but rather to explore how the problem of resource allocation and pricing could be approached using programming techniques. Accordingly, some of the data used are hypothetical but based on Egyptian statistics where these are available. The main statistical sources are the publications of the Ministry of Planning and of the Department of Statistics and Census (now, the Department of Public Mobilization and Census). These sources provided a starting point for the estimation of coefficients which seemed reasonable and which it was thought would provide useful results. Broadly speaking, we believe that the model, although hypothetical, reflects the structure of Egypt's industry and represents an economy resembling the Egyptian economy in the sixties.

IV

USE OF THE MODEL AND DISCUSSION OF RESULTS

Various individual runs were programmed with the model formulated previously. In what follows, different uses of the model will be discussed and the main computational results will be examined [6].

A.—*The Optimal Solution*

One obvious type of information the model can give is that embodied in the optimal solution, namely the set of activity levels which will maximize net foreign exchange earnings under the assumptions of the model.

There is a drawback to the optimal solution; because of the linearity of the constraints, if X_i, M_i (where X_i is domestic production and M_i is importation) are two activities with constant prices and producing the same commodity, the model will tend to operate either X_i or M_i at the maximum level rather than a combination of both. This is to be expected under the assumptions of the model. The reason is that no direct restrictions have been placed on the amount of new investment in any domestic production sector or on sectoral imports. If desired any sectoral activity could be restricted by an upper bound namely $X_i \leqslant C_X$ or $M_i \leqslant C_M$ where C_X, C_M are arbitrary constants [7]. Another type of restriction would be to express various activities in incremental form in order to avoid that these activities be operated at less than the initial level.

The constraints of the initial run are reproduced in Table 1 in the Appendix. The optimal solution to this programme is shown in Table 2.

The model chooses to increase output of both long and medium staple cotton through investment in vertical expansion and productivity improvement. Additional land is all allocated to cereals production which is increased through investment in vertical expansion as well. Exports of long staple cotton are restricted below their average level over the period 1952-1965. The export price of this variety is accordingly raised. Exports of medium staple cotton increase over the second phase of the plan above their pre-plan level. The optimal solution does not allow imports of short-staple cotton.

Industrial capacity over the first phase of the plan has been assumed to be exogenous. The optimal growth rates of capacity in various industrial sectors over the second phase are shown in Table 3 of the Appendix.

Exports of cotton yarn and fuel and power grow over the second phase at the rates of 1.2% and 8.5% respectively while exports of fabrics decline at 2.8% per year, reflecting the fact that domestic consumption of cotton fabrics is growing faster than its production. The increase in cotton exports in the form of yarn rather than in the form of fabrics may be explained by prevailing market conditions.

On the import side, imported machinery grows over the second phase at the annual rate of 12.1% which exceeds the growth rate of domestic production of machinery. This shows that, with the assumed cost structure and the given resource availability, there is no tendency for substituting locally produced machinery for imported equipment, the tendency is rather in the opposite direction. Imports of chemicals do not change over the second phase of the plan and those of raw materials drop to zero; cereals imports increase at about 2.8% over the second phase of the plan, but domestic consumption is increasingly supplied with local grain production.

Finally, all increase in sectoral investment occurs over the first phase of the plan and then investment remains constant, indicating that it is more profitable — in terms of foreign exchange — to raise the investment rate early, concentrating additions to investment at the beginning of the plan rather than spreading them over the whole planning period(8). The algebraic formulation of investment demand within the endogenous sectors relates capacity increase to the total change in sectoral capital stock over the planning period, which, in turn, depends on the annual investment rate in each sector. In this formulation, investment at the end of each phase of the plan as well as its rate of growth over the two phases are variable. Investments over the two phases of the plan are substitutable in building up productive capacity. This explains the tendency of the model to concentrate additions to investment at the beginning of the plan especially since non-competitive capital imports necessary for investment over the first phase do not enter the maximand, unlike imported capital equipment over the second phase of the plan.

The macro-economic implication of the solution is that there is no shortage of savings supply to the endogenous sectors. In fact, savings are not fully utilized. This seems to be rather peculiar, but if we remember that imported capital equipment was assumed to be in fixed proportion with sectoral investment and that the objective was to maximize net foreign exchange earnings, we understand why savings are not all invested. The effective restriction in this case is the foreign exchange availability. Foreign resources and domestic savings are not substitutable in the model and foreign resources are not sufficient to allow the use of all available domestic savings. The way out of this situation is either to reduce savings to match the foreign exchange availability or else to increase foreign exchange resources through foreign loans and gifts. An interesting question to investigate empirically, in connection with this result, would be to try to determine the amount of inflow of foreign resources required to allow full utilization of domestic savings.

Note that with a downward falling demand for cotton the cotton acreage is smaller than its optimal level with infinitely elastic demand for cotton. This result has been tested further by solving our problem for a linear objective function characterized by fixed prices for different cotton varieties and for fabrics. The main difference between the linear programming results and the quadratic programming solution is that in the former case it appears that additional land is all allocated to growing long-staple cotton — rather than cereals as in the latter case. Also, more investments are devoted to improve productivity of the additional cotton area. Exports of long-staple cotton are intensely increased. The results concerning production of medium staples remain unaffected; however, exports of this variety of fibres are lower than in the initial optimum. Intermediate consumption of medium staples increases over the second phase of the plan and that of long staples remains unchanged after an initial increment over the first phase.

Finally, demand for grain is predominantly satisfied by imports. Output and exports of cotton manufactures in the linear case are below their optimal level of the quadratic case, the explanation being that in the case of an infinite elasticity of demand for cotton fibres, it seems to be more profitable to export raw cotton than cotton manufactures. The results concerning other sectors are not significantly affected by the change of the objective function from quadratic to linear.

The conclusion we may draw from these runs is that if the government aims at maximizing foreign exchange earnings on the assumption that all prices are given — i.e. if the government emulates the free behaviour of the farmers — the area cultivated with cotton will be larger and cotton exports will be higher than in the case where cotton prices are assumed to be a decreasing function of the volume of exports and where the government tries to exploit the power Egypt may have to affect world cotton prices. In addition to this conclusion we may also say that under the assumption of a falling export price for raw cotton, exports of long staple cotton are restricted more than exports of medium staple; the reason is that if Egypt has any monopoly power in the world market for cotton it is likely to be more intense for the long staple varieties than for the medium staple ones — due to the greater number of producers of the latter varieties and to the smaller share of Egypt in the market of these fibres. In both the linear and the quadratic cases, it seems to be more profitable to export cotton manufactures in the form of yarn rather than fabrics.

B.—*The Dual Solution* :

While the primal solution shows which commodities form bottlenecks for the economy — i.e. those commodities whose slack variable has value zero in the optimal solution — the dual gives a quantitative measure of the seriousness of each bottleneck. The primal solution gives the optimal allocation of resources for a given set of restrictions of the economy. Assuming this resource allocation, the dual indicates the directions in which expansion would most profitably take place.

For each phase of the plan, the dual solution determines the following shadow prices :

— Shadow prices of produced commodities,

— Shadow price of new capital in each producing sector — i.e. the interest rate paid on new capital in each producing sector over the period considered,

— Shadow price of land : this price is rent,

— Shadow price of domestic savings,

— Prices associated with the export promotion constraints,

— Prices introduced by the limitation on productivity increase in the cultivation of individual agricultural commodities.

Examination of the dual problem and its solution provides more insight into the structure of the model. The dual solution could be read off Table 4 of the Appendix.

C.—*Changes in Data* :

Another use of the model is to find different optimal solutions corresponding to changes in the constraints. Such changes are of three types :

1) Changes in the requirements vector — e.g. changing the cultivable area, the amount of savings available to the endogenous sectors...

2) Changes in the coefficients of the maximand. The implications of such changes are discussed in connection with the analysis of the sensitivity of the model to changes in the coefficients of the demand functions.

3) Changes in the input — output coefficients and the policy parameters such as the export promotion coefficients.

This use of the model is perhaps the most important one : since some restrictions of the model can be changed, it is useful to have a series of optimal solutions for different restrictions, allowing comparisons to be made of the maximum benefit attainable from varying economic policies.

1.—*Sensitivity of the Solution to Changes in Demand Coefficients.*

The demand relations entering the maximand are of the general form :

$$P_1\ (t) = \delta_1 + \epsilon_{11}\ \bar{\bar{E}}_1\ (t) + \epsilon_{12}\ \bar{\bar{E}}_2\ (t)$$

$$P_2\ (t) = \delta_2 + \epsilon_{21}\ \bar{\bar{E}}_1\ (t) + \epsilon_{22}\ \bar{\bar{E}}_2\ (t)$$

$$P_6\ (t) = \delta_6 + \epsilon_{66}\ \bar{\bar{E}}_6\ (t)$$

where the most probable values of the coefficients are: (9)

$\delta_1 = 59.50$	$\delta_2 = 35.69$	$\delta_6 = 1.019$
$\epsilon_{11} = -9.360$	$\epsilon_{21} = -4.053$	$\epsilon_{66} = -0.035$
$\epsilon_{12} = -3.300$	$\epsilon_{22} = -2.460$	

We are, however, very much uncertain about the true values of these coefficients and the elasticities. The effect of such uncertainty may be assessed using the simulation technique which consists essentially in drawing randomly a large number of possible sets of coefficients and computing the resulting influence on the solution vector for each of these sets. If this is done on a relatively large scale it is likely to show the influence on the solution vector of the fact that the demand coefficients are uncertain.

Using a table of random numbers, thirty sets of coefficients were drawn within predetermined ranges of variations from most probable values of these coefficients. The changes considered here are not infinitesimal. Hence the solution set may have changed, affecting the resulting calculation. Inspection of different solutions has shown that the solution set is affected by changes in demand coefficients. The equations which are sensitive to these variations are primarily the balance equation for long staples in the first phase — constraint (1) in Table 1 of the Appendix —, the constraints for productivity improvement in long staple cotton in both phases of the plan — (7), (23) —; much less sensitive to these variations are : the constraints for medium staples over the first phase (2), (8), the intermediate consumption of raw cotton constraint (10) and the savings constraint (13) over the first phase of the plan. The other constraints are robust and remain unaffected by changes in demand coefficients. Once the sensitivity of the solution set to changes in demand coefficients has been appraised, the magnitudes for different variables in the solution for the thirty sets of demand coefficients were compared to the original solution. The average, standard deviation as well as the extreme values were computed to permit the evaluation of the results. The average was then compared to the original case. (See Table 5 of the Appendix).

The conclusion that output of cotton should be increased through vertical expansion and that additional land should be allocated mainly to cereals production is still valid — in a few cases, however, it appears profitable to cultivate newly reclaimed land with medium staple cotton rather than cereals or to increase the acreage of both cereals and medium staple cotton; these cases are characterized by a high constant term in the price equation of medium staple cotton as well as a high price and marginal revenue of medium staples exports. Exports of long staples are restricted on the average, over the first phase of the plan, below their pre-plan level, then they are slightly increased over the second phase. Exports of medium staples are increased over the first phase and are further raised over the second phase. The result that cotton prices should be increased is confirmed by examination of various solutions.

The conclusions concerning activities in other sectors remain unshaken. In particular, cotton appears to be increasingly exported as yarn rather than fabrics. Like in the initial optimal solution, sectoral investment grows during the first phase and remains constant thereafter. Finally, the macro-economic implication that savings are not a limiting factor in the second phase of the plan, still holds. However, over the first phase, the result is less certain, because in some cases, the savings constraint is binding.

In general, it would appear safe to conclude that the results represented by the initial optimal solution are at least qualitatively correct.

2. — *Other Experiments: Increased Competition*

Various experiments have been performed with the model. They consisted in changing some of the coefficients or the parameters keeping the others constant at the level determined for the initial run, and then considering the effect of such changes on the optimal solution. The main changes involved the demand coefficients and the degree of competition faced by Egypt in the cotton world market, the export promotion coefficients e_i, the sectoral growth rates of final consumption demands and finally, the non-competitive imports coefficients.

In what follows, we shall only examine the effect of increased competition.

It is often argued that increased competition, from synthetic fibres or from new producers of long staple cotton, makes it less likely for Egyptian export restrictions to benefit the country, this argument relies on the assumption that increased competition implies increased elasticity of demand. Such an argument is defective. Increased competition does not necessarily mean higher demand elasticity.

Consider first the case of *innovations* which at a given price of Egyptian cotton shifts the demand curve for exports to the left. Assume that through the introduction of a new fibre, each variety of Egyptian cotton suddenly looses a proportion x % of its foreign markets in the sense that no matter what the Egyptian price is, the foreign demand for exports of each cotton variety is x% smaller than before. In this case, the new demand curves are given by :

$$P_1(t) = 59.469 - \frac{9.36}{1-x}\bar{E}_1(t) - \frac{3.300}{1-x}\bar{E}_2(t)$$

$$P_2(t) = 35.693 - \frac{4.053}{1-x}\bar{E}_1(t) - \frac{2.460}{1-x}\bar{E}_2(t)$$

Any one of these new demand curves has at each price the same elasticity as the old one; the elasticity is however lower for each quantity. Four alternative assumptions concerning x were made. It was first assumed that x = 0.10, then the more pessimistic assumptions that x = 0.25, 0.40 and 0.50 were successively made. These changes of the demand coefficients affected the optimal solution as shown in Table 6 of the Appendix. This table shows the optimal values of the variables affected by varying the demand coefficients. All other variables remained at the level determined by the initial optimal solution.

With constant marginal costs of producing raw cotton, the optimal volume of exports of long staple cotton declines with the narrowing of the world market for Egyptian exports. Exports of medium staples are less sensitive to increased competition; the volume of exports of these varieties remains constant when the Egyptian share of the market is reduced by 10% or even 25% but it starts declining over the second period of the plan when the loss of market rises to 40% and 50% of its original level. The change in prices due to increased competition appears to be small and irregular. Under the assumptions of the model, it is not possible to predict whether the demand elasticities for long staples rise or fall as a result of increased competition. For medium staples however, they appear to be lower the narrower the market gets. Note that with linear demand functions, demand elasticities take on all values between 0 and $-\infty$, and consequently determining the demand elasticity is not meaningful unless we specify the price or the quantity at which we measure this elasticity. By demand elasticity here, we mean that corresponding to the optimal solution.

The model does not allow the area cultivated with cotton to fall below its base-year level — accordingy, the need for restricting exports — and production — of cotton of both varieties is reflected into declining shadow prices of raw cotton. Productivity improvement in long staples is restricted; production of medium staples does not change, but these fibres are increasingly used in spinning over the second phase of the plan. Over the first phase, intermediate consumption of cotton fibres remains unchanged since the capacity of the spinning industry is fixed at an exogenously determined level. The loss of market of raw cotton and the resulting restriction of exports is partially offset by an increase in the volume of exports of cotton manufactures.

With this simple kind of shift of demand curves, it is possible to conclude unambiguously that exports of cotton should be restricted. Such simple shifts are not expected to occur in real life however, and it is possible that in some cases — with non-linear demand curves — the optimal volume of exports increases instead of falling.

These runs show that increased competition in the sense of a downward shift of the demand curve and increased demand elasticity are two different things.

The other type of increased competition is due to the *entry of new producers* of long staples in the world market, capturing a portion of Egypt's share of the market, which is larger the higher is the Egyptian price. If the supply curve of these new producers is linear, the new demand curve will for any particular price have a higher elasticity than the old one. Assume that due to the entry of new competitors in the world market, the cotton demand functions become :

$$P_1(t) = (1\text{-}x)\ 59.496 - (1\text{-}x)\ 9.360\ E_1(t) - (1\text{-}x)\ 3.300\bar{E}\ (t)$$

$$P_2(t) = (1\text{-}x)\ 35.693 - (1\text{-}x)\ 4.053\ E_1(t) - (1\text{-}x)\ 2.460\bar{E}_2\ (t)$$

where, as in the previous instance of increased competition x was assumed to be 0.10, 0.25, 0.40. 0.50 successively.

It follows from consideration of these two typical cases of increased competition that it is not possible to say a priori that more competition means a higher elasticity of demand for cotton and that export restrictions will not pay. The outcome concerning the optimum volume of cotton exports depends on other considerations besides demand conditions — among which are the shape of the Egyptian supply curves, the costs of production and the supply conditions of other prospective competitors, the costs of producing new substitutes for cotton.

V

CONCLUSIONS

Although the model is not directly applicable to actual planning issues, it shows however that it is possible to formulate and solve the problem of foreign exchange maximization for the Egyptian economy using programming techniques and it permits to draw some conclusions which, if not quantitatively reliable, seem to be at least qualitatively justified.

It was first shown that the ability of the Egyptian economy to affect the world price for certain cotton varieties is an important factor in the problem of foreign exchange maximization; moreover it has predictable and measurable effects which should not be neglected. Export restrictions seem to pay for certain kinds of cotton, even if increased competition from new substitutes or from the entry of new producers reduces the Egyptian share of the world market.

Expanding the area cultivated with cereals and supplying domestic consumption with local grain production rather than imports seem to be justified.

Also justified is to increasingly export cotton in the form of manufactures. On the basis of the prevailing demand conditions exports of yarn appear to be more profitable than those of fabrics. Another result concerning the cotton sectors is that, in optimum, long staple cotton varieties are more used to satisfy input requirements for spinning than other kinds of cotton. Furthermore, imports of lower grades of cotton to be used for intermediate consumption was not justified.

Certain general statements about production, imports and exports of other commodities are indicated. Increasing export promotive activity in fuel and power is desirable.

Chemicals appear to be a candidate for import substitution in contrast with machinery for which imports increase relatively to domestic production.

The macro-economic implications of the model have not been fully investigated. The main conclusion reached is that domestic savings do not provide a bottleneck for the economy; the real restriction is the availability of foreign exchange resources. An interesting question to ask is what amount of inflow of foreign resources is necessary to permit full employment of domestic savings. To answer such a question in any significant way requires however the extension of the model to embody the other sectors contributing to Egypt's foreign trade.

More extensive work to narrow down the uncertainty in the estimates of the demand coefficients seems to be worth undertaking. Other coefficients requiring improvement are mainly the input coefficients and the non-competitive capital imports coefficients.

We should also keep in mind that our results depend on the formulation of the objective function and that, they are expected to vary according to the choice of the maximand; maximizing net foreign exchange earnings may not be equivalent to mainly the input coefficients and the non-competitive capital come or consumption.

Finally, the present analysis, although it did not tell us to do anything that the development forecasters and planners would not have said, we believe that it has helped understanding some of the agricultural and export policy problems of an economy relying on one major crop as a source of foreign exchange receipts.

FOOTNOTES

(1) We define the long-staple varieties as these with fibres longer than 1 ⅜". These varieties are sometimes referred to as extra-long staples, they are mainly : Ménoufi, Giza 45, Giza 68, Karnak which is not being produced starting 1962 and has been replaced by the Menoufi variety.

(2) The medium-staple varieties are those with fibre length over 1 ⅛". These are sometimes referred to as long staple varieties. They are : Giza 47, Dandara, Giza 67, Giza 30, Ashmouni, Giza 66. The variery Ashmouni is almost equivalent to the American Upland.

(3) Egypt does not produce short-staple cotton — i.e. cotton with fibre length less than 1 ⅛" however, we introduce this commodity into the model to make allowance for the possibility of importing cheap short-staple cotton to be used for local industry and exporting more of the long and medium-staple varieties.

(4) Sandee, J., «**A Demonstration Planning model for India**» Asia Publishing House, London, and Statistical Publishing Society, Calcutta, 1960, p. 20 — Manne, A. «Key Sectors of the Mexican Economy 1960-1970» in **Studies in Process Analysis : Economy wide Production Capabilities,** Manne and Markowitz eds. pp. 383 - 5.

(5) For a detailed discussion of the sources of data and the statistical estimation of various coefficients and initial conditions see Kheir-El-Dine, H.A., **A Quadratic Programming Approach to the Problem of Optimal Pricing and Use of Cotton in Egypt,** Unpublished Ph.D. Thesis, Economics Department, Massachusetts Institute of Technology, 1967.

(6) The Fortran Program used for the solution was written by R.B. Wilson, it is based on a generalization to concave programming of Dantzig's simplex algorithm for linear programming. The computations were carried out with the use of the 360/model 65 computer available at the M.I.T. Computation Center.

(7) This type of restriction has been imposed on agricultural vertical expansion due to the use of chemicals and investment in improving land productivity. The model without such a restraint tended to produce some additional agricultural commodities using land only and some others using chemicals and other capital only. The cons-

traint imposed to avoid this solution was to assume that land productivity grows at a maximum rate of 3.5% a year; consequently, agricultural output due to vertical expansion could not exceed a certain fraction of output due to the cultivated area increase.

(8) In an iniial run where investment in the second phase was allowed to drop to zero, this point was more forcefully stressed by the optimal solution which showed a bigger increase in sectoral investment over the first phase, the investment in the terminal year was reduced to zero. This solution was thought to be unrealistic and accordingly we added the constraint that sectoral annual investment in the second phase cannot be lower than in the first phase.

(9) These coefficients are obtained by solving the demand equations formulated on p. 19 for the prices P_1 (t), P_2 (t), P_6 (t).

APPENDIX

Algebraic Derivation of the Capacity Increase Equations :

For simplicity, we shall ignore subscripts referring to goods or sectors, and import substituting or export promoting activities. The following symbols are used :

X = increase in output capacity in the second phase of the plan.

N_0 = The exogenously given rate of investment in the initial period or the last year prior to the beginning of the first phase.

N_1 = rate of investment at the end of the first phase.

N_2 = rate of investment at the end of the second phase.

K = addition to capital over the planning period.

k = incremental capital — output ratio.

λ_1 = rate of growth of investment over the first phase.

λ_2 = rate of growth of investment over the second phase.

τ_1 = length of the first phase in years.

τ_2 = length of the second phase in years.

Time is assumed to be a continuous variable.

We have :

$$K = k\ X = f \quad (N_1,\ N_2,\ \lambda_1,\ \lambda_2) \tag{1a}$$

where

$$N_1 = (1+\lambda_1)^{\tau_1} N_0 \tag{1b}$$

$$N_2 = (1+\lambda_2)^{\tau_2} N_1 \tag{1c}$$

λ_1, λ_2 are not specified growth rates, but are unknowns to be determined by the solution of the model.

$$k\ x = \left[\frac{(1+\lambda_1)^{\tau 1} - 1}{\lambda_1}\right] (1+\lambda_1)\ \mathcal{N}_0 + \left[\frac{(1+\lambda_2)^{\tau^2} - 1}{\lambda_2}\right] (1+\lambda_2)\ \mathcal{N}_1 \tag{1}$$

Expanding in Taylor's series, ignoring the terms higher than degree 1, we have :

$$k\ X = k\ X^0 + k\ d\ X \tag{2}$$

where X^0 = an arbitrary value of the increase in output capacity X,

$$d\ X = X - X^0$$

From (1)

$$kdX = k \left[\frac{\delta x}{\delta \mathcal{N}_1}\, d\, \mathcal{N}_1 + \frac{\delta x}{\delta \lambda_1}\, d\, \lambda_1 + \frac{\delta x}{\delta \lambda_2}\, d\, \lambda_2\right] \tag{3}$$

where from (1) :

$$k\, \frac{\delta x}{\delta \mathcal{N}_1} = \frac{(1+\lambda_2)^{\tau_2+1} - (1+\lambda_2)}{\lambda_2} \tag{3a}$$

$$k\, \frac{\delta x}{\delta \lambda_1} = \left[\frac{(1+\lambda_1)^{\tau_1}[\tau_1\lambda_1 - 1] + 1}{\lambda_1^2}\right] \mathcal{N}_0 \tag{3b}$$

$$k\, \frac{\delta x}{\delta \lambda_2} = \left[\frac{(1+\lambda_2)^{\tau_2}[\tau_2\lambda_2 - 1] + 1}{\lambda_2^2}\right] \mathcal{N}_1 \tag{3c}$$

Substituting from (3a), (3b), (3c) in (3) :

$$k\,d\,x = \frac{(1+\lambda_2)^{\tau_2+1}-(1+\lambda_2)}{\lambda_2}\,dN_1+N_0\left[\frac{(1+\lambda_1)^{\tau_1}[\tau_1\lambda_1-1]+1}{\lambda_1^2}\right]d\lambda_1$$

$$+N_1\left[\frac{(1+\lambda_2)^{\tau_2}[\tau_2\lambda_2-1]+1}{\lambda_2^2}\right]d\lambda_2 \qquad (4)$$

From (1b) and (1c) we have :

$$d\ N_1 = \tau_1 N_0(1+\lambda_1)^{\tau_1-1}\ d\lambda_1 \qquad (4a)$$

$$d\ N_2 = (1+\lambda_2)\tau_2 dN_1+\tau_2 N_1(1+\lambda_2)^{\tau_2-1}\ d\lambda_2 \qquad (4b)$$

(4a) shows that any change in the rate of investment in the final year of the first plan phase depends only on changes in the rate of growth of investment λ_1 ; (4b) shows that in the final year of the second phase of the plan, variations in investment depend both on how high the starting point is at the beginning of this phase (or the end of the preceding phase) and on the change in the rate of growth λ_2 over the second phase.

From (4a) we get :

$$d\lambda_1=\frac{d\ N_1}{N_0(\ 1+\lambda_1)^{\tau_1-1}} \qquad (5\ a)$$

From (4b) :

$$d\,\lambda_2=\frac{d\ N_2-(1+\lambda_2)^{\tau_2}dN_1}{\tau_2 N_1(1+\lambda_2)^{\tau_2-1}} \qquad (5b)$$

Substituting from (5a), (5b) in (4) and rearranging:

$$kdx = \left[\frac{(1+\lambda_2)^{\tau_2+1}-(1+\lambda_2)}{\lambda_2}+\frac{(1+\lambda_1)^{\tau_1}[\tau_1\lambda_1-1]+1}{\tau_1\lambda_1^2\,(1+\lambda_1)^{\tau_1-1}}\right.$$

$$\left.-\frac{(1+\lambda_2)^{\tau_2+1}[\tau_2\lambda_2-1]+(1+\lambda_2)}{\tau_2\lambda_2^2}\right]d\ N_1+\left[\frac{(1+\lambda_2)^{\tau_2}[\tau_2\lambda_2-1]+1}{\tau_2\lambda_2^2\ (1+\lambda_2)^{\tau_2-1}}\right]dN_2$$

$$=\zeta dN_1+\eta dN_2 \qquad (6)$$

Therefore :

$$k\,X = k\,X_0 + \zeta d\,N_1 + \eta\, d\,N_2 \qquad (6')$$

where $d\,N_1 = N_1 - N_1^0$ and $d\,N_2 = N_2 - N_2^0$: N_1^0, N_2^0 being arbitrary values for N_1 , N_2 corresponding to the arbitrary value X_0 for capacity increment. The parameters ζ , η are evaluated using arbitrary values for λ_1 , λ_2 corresponding to N_1^0, N_2^0.

Statistical Tables and Computational Results

As mentioned in the text, subscripts 1, 2, 3, 4, refer respectively to long staples, medium staples, short staples and cereals, 5 refers to cotton yarn, 6 to cotton fabrics, 7 to fuel and power, 8 to chemicals and 9 to engineering and metallurgical products.

In what follows, the barred variables refer to the absolute level of the endogenous magnitudes, the non-barred variables refer to incremental magnitudes. For simplicity, the time index t will be used for variables relating to both periods of the plan (it will be understood that t = 1, 2 for the periods 1966/67 to 1969/70 and 1970/71 to 1979/80 respectively — unless otherwise defined). The endogenous variables figuring in the optimal solution vector are :

$X_i'(t)$ = Change in output of agricultural commodity i (i = 1, 2, 4) due to productivity improvement over phase t of the plan (t = 1, 2).

$X_i''(t)$ = Change in output of agricultural commodity i (i = 1, 2, 4) due to acreage expansion over phase t of the plan (t = 1, 2).

$X_i(2)$ = Change in output of commodity i (i= 5, ..., 9) over the planning phase 1970/80.

$X_{ij}(t)$ = Change in output of commodity i used in sector j.

$\bar{M}_i(t)$ = imports of commodity i in year t where t = 1, 2 for 1970 and 1980 respectively.

$M_i(t)$ = change in imports of commodity i over phases 1 and 2 of the plan.

$M_{ij}(t)$ = change in imports of commodity i used in sector j.

$\bar{E}_i(t)$ = exports of commodity i in year t where t = 1, 2 for 1970 and 1980 respectively.

$E_i(t)$ = change in exports of commodity i over phases 1 and 2 of the plan.

$N_j(1)$ = rate of investment in 1970 originating in the machinery sector and used in sector j.

$I_j^n(2)$ = increase in the rate of investment in machinery by j over the 10-year period 1969/70 to 1979/80.

Finally subscripts α , RM refer respectively to total endogenous agricultural sector and to raw materials.

The primal problem has 53 variables and 37 constraints, therefore it has 37 prices attached to it, namely : 15 commodity prices $U_j(t)$ associated with the commodity balance constraints over the two periods of the plan; the prices of sectoral investment : $U_\alpha^I(t)$, $(t = 1, 2)$, for agricultural vertical expansion and $U_j^I(2)$, $(j = 5,\ldots, 9)$ for the other industrial sectors; the prices associated with the restrictions on productivity improvement in individual agricultural commodities $U_i^{ve}(t)$; $(i = 1, 2, 4;\ t = 1, 2)$; the price of intermediate consumption of raw cotton in the spinning industry $U_{\omega 5}(t)$, $(t = 1, 2)$; the price of the export promotion constraints $U_i^e(2)$, $(i = 5, 7)$; finally the prices of raw materials $U_{rm}(2)$, land $U_A(t)$, $(t = 1, 2)$ and domestic savings $U_S(t)$, $(t = 1, 2)$. Each price corresponds to one equation of the model as follows :

PHASE I			PHASE II		
Equation	Shadow Price	Subscript	Equation	Shadow Price	Subscript
(1) to (6)	U_j (1)	$j = 1,2,3,4,8,9$	(14) to (22)	U_j (2)	$j = 1, \ldots\ldots, 9$
(7) to (9)	U_i^{ve} (1)	$i = 1,2,4$	(23) to (25)	U_i^{ve} (2)	$i = 1, 2, 4$
(10)	$U_{\wp 5}$(1)	$\wp = 1+2+3$	(26)	$U_{\wp 5}$(2)	$\wp = 1 + 2 + 3$
(11)	U_α^i (1)	$\alpha = 1+2+4$	(27), (28)	U_i^e (2)	$i = 5, 7$
(12)	U_A (1)		(29)	U_α^I (2)	$\alpha = 1 + 2 + 4$
(13)	U_S (1)		(30) to (34)	U_j^I (2)	$j = 5, \ldots\ldots, 9$
			(35)	U_{rm} (2)	
			(36)	U_A (2)	
			(37)	U_S (2)	

The Constraints Matrix

	$X'_1(1)$	$X''_1(1)$	$X'_2(1)$	$X''_2(1)$	$X'_4(1)$	$X''_4(1)$	$\overline{M}_3(1)$	$\overline{M}_4(1)$	$\overline{M}_8(1)$	$\overline{M}_9(1)$
1.	—0.618	—0.618								
2.			—0.618	—0.618						
3.							—0.618			
4.					—0.743	—0.743		—1.000		
5.									—1.000	
6.										—1.000
7.	1.000									
8.			1.000							
9.					1.000					
10.										
11.	16.500		15.420		1.000					
12.		0.226		0.210		0.022				
13.	6.303	6.303	5.890	5.890	0.257	0.257	—0.618	—1.000	—1.000	—1.000
14.										
15.										
16.										
17.										
18.										
19.										
20.										
21.										
22.										
23.	—0.411	—0.411								
24.			—0.411	—0.411						
25.					—0.411	—0.411				
26.										
27.										
28.										
29.										
30.										
31.										
32.										
33.										
34.										
35.										
36.										

Table 1 (contd.)

	$E_1(1)$	$\bar{E}_2(1)$	$X_{15}(1)$	$X_{25}(1)$	$M_{35}(1)$	$X_{8\alpha}(1)$	$N_{\alpha}(1)$	$N_5(1)$	$N_6(1)$	$N_7(1)$
1.	1.000		1.000							
2.		1.000		1.000						
3.					1.000					
4.										
5.						1.000				
6.							0.711	0.327	0.327	0.295
7.										
8.										
9.										
10.			—16.500	—15.420	—1.000					
11.						—7.864	—6.159			
12.										
13.	59.500	35.690					0.711	0.327	0.327	0.295
14.										
15.										
16.										
17.										
18.										
19.										
20.										
21.										
22.							0.096	0.224	0.224	0.235
23.										
24.										
25.										
26.										
27.										
28.										
29.							—19.400			
30.								—12.520		
31.									—12.790	—12.850
32.										
33.										
34.										
35.										
36.										
37.	59.500	35.690					0.807	0.551	0.551	0.530

Table 1 (contd.)

	$N_8(1)$	$N_9(1)$	$X'_1(2)$	$X''_1(2)$	$X'_2(2)$	$X''_2(2)$	$X'_4(2)$	$X''_4(2)$	$X_5(2)$	$X_6(2)$
1.										
2.										
3.										
4.										
5.										
6.	0.319	0.413								
7.										
8.										
9.										
10.										
11.										
12.										
13.	0.319	0.413								
14.			—0.618	—0.618						
15.					—0.618	—0.618				
16.										
17.							—0.743	—0.743		
18.									—0.756	0.515
19.										—0.639
20.									0.035	0.004
21.									0.013	0.011
22.	0.227	0.190							0.020	0.002
23.			1.000	—0.176						
24.					1.000	—0.176				
25.							1.000	—0.176		
26.									0.698	
27.									—0.100	
28.										
29.			16.500		15.420		1.000			
30.									0.708	
31.										0.704
32.										
33.	—13.270									
34.		---13.790								
35.										
36.				0.226		0.210		0.022		
37.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

	$X_7(2)$	$X_8(2)$	$X_9(2)$	$M_3(2)$	$M_4(2)$	$M_8(2)$	$M_9(2)$	M_{RM} (2)	E_1 (2)	$E_2(2)$
1.										
2.										
3.										
4.										
5.										
6.										
7.										
8.										
9.										
10.										
11.										
12.										
13.										
14.									1.000	
15.										1.000
16.				—0.618						
17.					—1.000					
18.										
19.										
20.	—0.875	0.158	0.065							
21.	0.054	—0.432	0.080			—1.000				
22.	0.016	0.025	—0.146				—1.000			
23.										
24.										
25.										
26.										
27.										
28.	—0.100									
29.										
30.										
31.										
32.	1.779									
33.		1.750								
34.			1.310							
35.	0.474	0.035	0.086					—1.000		
36.										
37.		0.023	0.197	—0.618	—1.000	—1.000	—1.000	—1.000	59.500	35.690

Table 1 (cont'd)

	$E_5(2)$	$E_6(2)$	$E_7(2)$	$X_{15}(2)$	$X_{35}(2)$	$M_{85}(2)$	$X_{8\alpha}(2)$	$I_{\alpha}N(2)$	$I_5N(2)$	$I_6N(2)$
1.										
2.										
3.										
4.										
5.										
6.										
7.										
8.										
9.										
10.										
11.										
12.										
13.										
14.				1.000						
15.					1.000					
16.						1.000				
17.										
18.	1.000									
19.		1.000								
20.			1.000							
21.							1.000			
22.								0.807	0.551	0.551
23.										
24.										
25.										
26.				—16.500	—15.420	—1.000				
27.	1.000									
28.			1.000							
29.							—7.864	—9.762		
30.									—4.847	
31.										—5.340
32.										
33.										
34.										
35.										
36.										

	$I_7^N(2)$	$I_8^N(2)$	$I_9^N(2)$		
1.				≤	4.111
2.				≤	1.100
3.				≤	0.000
4.				≤	—97.970
5.				≤	—68.480
6.				≤	—102.300
7.				≤	0.697
8.				≤	0.528
9.				≤	22.110
10.				≤	—10.730
11.				≤	8.197
12.				≤	0.200
13.				≤	85.030
14.				≤	0.000
15.				≤	0.000
16.				≤	0.000
17.				≤	—100.100
18.				≤	26.860
19.				≤	—16.830
20.				≤	—44.570
21.				≤	—83.420
22.	0.530	0.546	0.609	≤	—316.700
23.				≤	1.935
24.				≤	1.464
25.				≤	61.400
26.				≤	0.000
27.				≤	26.890
28.				≤	12.910
29.				≤	0.032
30.				≤	3.544
31.				≤	1.124
32.	—4.418			≤	19.720
33.		—4.058		≤	2.203
34.			—3.863	≤	3.076
35.				≤	115.500
36.				≤	0.560
37.	0.530	0.546	0.609	≤	228.400

TABLE 2

INITIAL OPTIMAL SOLUTION

		Phase 1	Phase 2
Increase in Production	*Long Staple Cotton*		
	Due to productivity improvement X'_1	0.697	2.221
	Due to acreage expansion X'_1	0	0
	Medium Staple Cotton		
	Due to productivity improvement X'_2	0.528	1.681
	Due to acreage expansion X''_2	0	0
	Cereals		
	Due to productivity improvement X'_4	22.110	78.700
	Due to acreage expansion X''_4	9.091	25.450
	Cotton yarn X_5	—	32.450
	Cotton Fabrics X_6	—	41.280
	Fuel and Power X_7	—	161.200
	Chemicals X_8	—	278.400
	Machinery X_9	—	341.100
Competitive Imports	Short Staple Cotton $\overline{M}_3$	0	0
	Cereals $\overline{M}_4$	74.790	97.500
	Chemicals M_8	68.480	68.480
	Machinery $\overline{M}_9$	139.800	438.000
	Raw Materials M_{RM}	—	0
Exports	Long Staple Cotton $\overline{E}_1$	2.210	2.210
	Medium Staple Cotton $\overline{E}_2$	1.426	2.465
	Cotton Yarn E_5	—	30.140
	Cotton Fabrics $\overline{E}_6$	—	9.548
	Fuel and Power E_7	—	29.030
Change in Intermediate Consumption	Long Staple Cotton X_{15}	0.650	1.373
	Medium Staple Cotton X_{25}	0	0
	Short Staple Cotton M_{35}	0	0
	Cotton Yarn	—	21.580
	Cotton Fabrics	—	—
	Fuel and Power	—	86.500
	Chemicals :		
	Fertilizers $X_{8\alpha}$	0	0
	Other	—	188.600

Table 2 (cont'd)

		Phase 1	Phase 2
Sectoral Fixed Investments in Machinery and Means of Transport	Machinery	—	234.400
	Agriculture	7.281	7.281
	Cotton Yarn	1.552	1.552
	Cotton Fabrics	2.184	2.184
	Fuel and Power	20.790	20.790
	Chemicals	36.550	36.550
	Machinery	32.180	32.180
Sectoral Non-Competitive Machinery Imports	Agriculture	2.104	1.405
	Cotton Yarn	1.044	0.697
	Cotton Fabrics	1.470	0.981
	Fuel and Power	14.657	9.771
	Chemicals	24.891	16.594
	Machinery	18.890	12.582
Prices	Long Staple Cotton P_1	34.110	30.680
	Medium Staple Cotton P_2	23.230	20.670
	Cotton Fabrics P_6	—	0.685

Note that all magnitudes are expressed in constant 1959/60 in millions L.E. — except output, intermediate consumption and exports of long staple and medium staple cotton and of cotton fabrics which are expressed in million cantars and million kgs. Finally P_1, P_2 are in L.E. per cantar and P_6 is expressed in L.E. per kg.

TABLE 3

Optimal Sectoral Growth Rates in Industrial Sectors

Sector	Cotton Spinning	Cotton Weaving	Fuel and Power	Chemicals	Machinery
Optimal Annual Growth Rates	3.0%	3.7%	8.7%	10.7%	8.5%

TABLE 4

THE DUAL SOLUTION

Phase 1			*Phase 2*		
Shadow Price	*Unit*	*Magnitude of S.P.*	*Shadow Price*	*Unit*	*Magnitude of S.P.*
$U_1(1)$	L.E. per cantar	0	$U_1(2)$	L.E. per cantar	5.169
$U_2(1)$	»	7.310	$U_2(2)$	»	7.310
$U_3(1)$	L.E. per L.E.	0	$U_3(2)$	L.E. per L.E.	0.313
$U_4(1)$	»	1.000	$U_4(2)$	»	1.000
$U_8(1)$	»	1.000	$U_5(2)$	»	0.314
$U_9(1)$	»	1.000	$U_6(2)$	L.E. per kg.	0.351
			$U_7(2)$	L.E. per L.E.	0.151
			$U_8(2)$	»	0.425
			$U_9(2)$	»	1.000
$U_{\omega 5}(1)$	L.E. per L.E.	0	$U_{\omega 5}(2)$	L.E. per L.E.	0.313
$U_1^{VE}(1)$	L.E. per cantar	0.963	$U_1^{VE}(2)$	L.E. per cantar	2.344
$U_2^{VE}(1)$	»	6.048	$U_2^{VE}(2)$	»	3.723
$U_4^{VE}(1)$	L.E. per L.E.	1.027	$U_4^{VE}(2)$	L.E. per L.E.	0.692
$U_\alpha^I(1)$	»	0	$U_\alpha^I(2)$	»	0.0516
			$U_5^I(2)$	»	0.0799
			$U_6^I(2)$	»	0.0782
			$U_7^I(2)$	»	0.0778
			$U_8^I(2)$	»	0.0754
			$U_9^I(2)$	»	0.0725
			$U_5^E(2)$	L.E. per L.E.	0.686
			$U_7^E(2)$	»	0.849
			$U_{RM}(2)$	L.E. per L.E.	0.0835
$U_A(1)$	L.E. per feddan	46.69	$U_A(2)$	L.E. per feddan	39.30
$U_S(1)$	L.E. per L.E.	0	$U_S(2)$	L.E. per L.E.	0

TABLE 5
SUMMARY EFFECT OF CHANGES IN DEMAND COEFFICIENTS ON THE OPTIMAL SOLUTION VECTOR

Phase 1

Variable*	Initial Solution	Average of 30 sets of solutions	Standard Deviation	Highest Value	Lowest Value
$X'_1(1)$	0.697	0.591	0.043	0.710	0
$X''_1(1)$	0	0	0	0	0
$X'_2(1)$	0.528	0.517	0.012	0.553	0.172
$X''_2(1)$	0	0.064	0.043	0.952	0
$X'_4(1)$	22.110	22.110	0	22.110	22.110
$X''_4(1)$	9.091	8.485	0.423	9.091	0
$\bar{M}_3(1)$	0	0	0	0	0
$\bar{M}_4(1)$	74.790	75.240	0.307	81.540	74.790
$\bar{M}_8(1)$	68.48	68.48	0	68.48	68.48
$\bar{M}_9(1)$	139.8	139.8	0.251	140.1	138.8
$\bar{E}_1(1)$	2.210	2.324	0.154	4.496	0.981
$\bar{E}_2(1)$	1.426	1.328	0.083	2.015	0

Phase 2

Variable*	Initial Solution	Average of 30 sets of solutions	Standard Deviation	Highest Value	Lowest Value
$X'_1(2)$	2.221	2.045	0.076	2.239	0.450
$X''_1(2)$	0	0	0	0	0
$X'_2(2)$	1.681	1.765	0.041	2.542	1.535
$X''_2(2)$	0	0.346	0.145	2.667	0
$X'_4(2)$	78.70	77.87	0.039	78.70	70.48
$X''_4(2)$	25.45	22.15	1.390	25.45	0
$X_5(2)$	32.45	33.76	1.156	48.04	27.21
$X_6(2)$	41.28	43.36	0.983	61.14	34.60
$X_7(2)$	161.2	161.3	0.051	161.8	161.0
$X_8(2)$	278.4	278.5	0.060	278.9	278.3
$X_9(2)$	341.1	340.7	0.172	342.3	337.6
$M_3(2)$	0	0	0	0	0
$M_4(2)$	22.71	25.78	1.312	47.73	22.71
$M_8(2)$	0	0	0	0	0
$M_9(2)$	298.2	298.3	0.450	299.4	297.8
$M_{RM}(2)$	0	0	0	0	0
$E_1(2)$	0	0.053	0.041	1.193	0
$E_2(2)$	1.039	1.056	0.147	3.219	0
$\bar{E}_5(2)$	30.14	30.30	0.077	31.69	29.61
$\bar{E}_6(2)$	9.548	10.869	0.627	22.24	5.282
$\bar{E}_7(2)$	29.03	29.04	0.018	29.09	29.01

TABLE 5 (cont'd)

	Phase 1						*Phase 2*				
Variable*	Initial Solution	Average of 30 sets of solutions	Standard Deviation	Highest Value	Lowest Value	Variable*	Initial Solution	Average of 30 sets of solutions	Standard Deviation	Highest Value	Lowest Value
X_{15} (1)	0.650	0.682	0.054	1.455	0	X_{15} (2)	1.373	1.211	0.059	1.384	0.180
X_{25} (1)	0	0.090	0.060	1.426	0	X_{25} (2)	0	0.248	0.066	1.039	0
M_{35} (1)	0	0	0	0	0	M_{35} (2)	0	0	0	0	0
X_8 (1)	0	0	0	0	0	X_8(2)	0	0	0	0	0
N_α (1)	7.281	7.201	0.053	7.541	6.012	I_α^{n} (2)	0	0	0	0	0
N_5 (1)	1.552	1.645	0.045	2.434	1.256	I_5^{n} (2)	0	0	0	0	0
N_6 (1)	2.184	2.297	0.055	3.277	1.817	I_6^{n} (2)	0	0	0	0	0
N_7 (1)	20.79	20.80	0.031	20.87	20.76	I_7^{n} (2)	0	0	0	0	0
N_8 (1)	36.55	36.56	0.003	36.61	36.54	I_8^{n} (2)	0	0	0	0	0
N_9 (1)	32.18	32.15	0.049	32.29	31.85	I_9^{n} (2)	0	0	0	0	0
P_1 (1)	34.10	34.81	1.010	46.96	24.60	P_1(2)	30.68	31.09	0.930	40.45	21.56
P_2 (1)	23.23	22.03	1.408	38.78	11.16	P_2(2)	20.67	19.32	0.971	31.38	9.78
						P_6(2)	0.685	0.646	0.031	1.036	0.411

* Note that for imports and exports, the barred variables refer to absolute magnitudes in 1969/70 and 1979/80 for t = 1, 2 respectively. The non-barred variables refer to incremental magnitudes over phase 2 of the plan. In this table and thereafter I_j^{n} (2), j = , 5, ..., 9 refers to the increment in investment rate in machinery and equipment in Sector j over the second phase of the plan, and $I_j^{n}(2) = I_{9j}^{n}(2) + M_j^{I}(2)$ where I_{9j}^{n} refers to additional domestic machinery used for investment purposes in sector j over the 2nd phase of the plan and M_j^{I} is the additional non-competitive capital equipment used in j.

TABLE 6

OPTIMAL SOLUTIONS CORRESPONDING TO DIFFERENT DEGREES OF INCREASED COMPETITION DUE TO THE INTRODUCTION OF NEW SUBSTITUTES FOR COTTON

a) Primal Solutions

	Phase 1						*Phase 2*				
x Variable	Initial Case	0.10	0.25	0.40	0.50	x Variable	Initial Case	0.10	0.25	0.40	0.50
X'_1	0.697	0.697	0.697	0.697	0	X'_1	2.221	2.221	2.221	2.221	1.482
						X_5	32.450	32.450	32.450	33.180	35.910
						X_6	41.280	41.280	41.280	42.210	45.690
E_1	2.210	1.892	1.416	0.952	0.875	$\bar{E}_1$	2.210	1.892	1.416	0.952	0.875
E_2	1.426	1.426	1.426	1.426	1.426	$\bar{E}_2$	2.465	2.465	2.465	2.432	1.869
						$\bar{E}_5$	30.140	30.140	30.140	30.210	30.480
						$\bar{E}_6$	9.548	9.548	9.548	10.140	12.360
						X_{15}	1.373	1.373	1.373	1.373	0.916
						X_{25}	0	0	0	0.033	0.646
N^{α}	7.281	7.281	7.281	7.281	6.652	N^{α}	7.281	7.281	7.281	7.281	6.652
N_5	1.552	1.552	1.552	1.593	1.748	N_5	1.552	1.552	1.552	1.593	1.748
N_6	2.184	2.184	2.184	2.235	2.427	N_6	2.184	2.184	2.184	2.235	2.427
P_1	34.11	34.59	35.55	36.81	33.71	P_1	30.68	30.78	30.98	31.27	31.12
P_2	23.23	23.27	23.36	23.41	20.16	P_2	20.67	20.43	19.95	19.29	17.83
						P_6	0.685	0.685	0.685	0.664	0.586

TABLE 6 (cont'd)

b) Dual Solutions

Phase 1

S.P. \ x	0	0.10	0.25	0.40	0.50
U_1	0	0	0	0	0
U_2	7.310	6.755	5.641	4.083	1.289
$U\delta_5$	0	0	0	0	0
U_1^{VE}	0.963	0.963	0.963	0.760	0
U_2^{VE}	6.048	5.564	4.592	3.234	0.795

Phase 2

S.P. \ x	0.	0.10	0.25	0.40	0.50
U_1	5.169	5.169	5.169	4.369	1.376
U_2	7.310	6.755	5.641	4.083	1.286
U_5	0.314	0.314	0.314	0.263	0.070
U_6	0.351	0.351	0.351	0.309	0.154
$U\delta_5$	0.313	0.313	0.313	0.265	0.083
U_1^{VE}	2.344	2.344	2.344	1.849	0
U_2^{VE}	3.723	3.380	2.691	1.728	0
U_5^{E}	0.686	0.686	0.686	0.737	0.931

c) Demand Elasticities

Phase 1

Elasticity \ x	0	0.10	0.25	0.40	0.50
e_{11}	—3.94	—4.21	—4.80	—5.92	—3.97
e_{22}	—15.80	—14.25	—11.90	—9.56	—4.62

Phase 2

Elasticity \ x	0	0.10	0.25	0.40	0.50
e_{11}	—3.59	—3.74	—4.18	—5.03	—3.66
e_{22}	—8.13	—7.24	—5.89	—4.62	—3.21
e_{66}	—2.05	—2.05	—2.05	—1.87	—1.35