Using curvature to infer COVID-19 fractal epidemic network fragility and systemic risk

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The damage of the novel Coronavirus disease (COVID-19) is reaching unprecedented scales. There are numerous classical epidemiology models trying to quantify epidemiology metrics. Usually, to forecast the epidemics, these classical approaches need parameter estimations, such as the contagion rate or the basic reproduction number. Here, we propose a data-driven, parameter-free approach to access the fragility and systemic risk of epidemic networks by studying the Forman-Ricci curvature. Network curvature has been used successfully to forecast risk in financial networks and we suggest that those results can be translated for COVID-19 epidemic time series as well. We first show that our hypothesis is true in a toy-model of epidemic time series with delays, which generates epidemic networks. By doing so, we are able to verify that the Forman-Ricci curvature can be a parameter-free estimate for the fragility and risk of the network at each stage of the simulated pandemic. On this basis, we then compute the Forman-Ricci curvature for real epidemic networks built from epidemic time series available from the World Health Organization (WHO). The Forman-Ricci curvature allow us to detect early warning signs of the emergence of the pandemic. The advantage of the method lies in providing an early geometrical data marker for epidemics, without the need of parameter estimation and stochastic modeling. The strategy above, together with other data-driven tools for investigating epidemic network dynamics, can be readily implemented on a daily basis to quickly estimate the growth, risk and fragility of real COVID-19 epidemic networks at different scales.

Epidemic outbreaks represent a significant concern for global health. Currently, the COVID-19 outbreak has caught the attention of researchers worldwide due to its rapid spread, high fluctuation in the incubation time and uncertain health and economic outcomes. One of the most urgent challenges of this outbreak concerns the development of a coordinated and continuous data-driven feedback system that could quantify the spread and the risk of the epidemic, without strongly depending on parameter estimation and even when data is heterogeneous or subject to noise. Such a data-driven system would allow to develop adequate responses at different scales (global, national or local) and allocate limited resources in the most effective ways.

Recent developments in topological and geometric data analysis [1–5] offer useful perspectives regarding real data treatment, having yielded outstanding results over the past years across many fields [6–9]. As an emerging and promising approach in network science and complex systems more generally [10], topological and geometric data analysis describes the shape of the data by associating high dimensional objects [1, 8, 11].

Among the numerous successful interdisciplinary ap-

plications of applied geometry and topology, ranging from differentiating cancer networks [12] to modeling phase transitions in brain networks [13], one idea in particular can be beneficial to measure the systemic risk and fragility of COVID-19 epidemic networks in a data-driven way: Using network curvature to infer the network fragility and systemic risk. This could give insights into the current aspects of the pandemics without the need for parameter estimations.

Our idea is inspired by earlier results obtained for financial networks [14], where the authors showed that it was possible to relate financial network fragility with the Ollivier-Ricci curvature of a network. Most importantly, the Ollivier-Ricci curvature emerged as a data-driven "crash hallmark" for major changes in stock markets over the past 15 years. In their study of market fragility, they used these geometric tools to analyse and characterize the interaction between the economic agents (the nodes of a financial network) and its correlation levels (which defines the edges' weights). In addition, these tools also allowed them to track the curvature of the financial network as a function of time, i.e. how the shape of the financial network changed according to a dynamic

economic scenario.

As a result, the Ollivier-Ricci curvature emerged as a strong quantitative indicator of the systemic risk in financial networks. From a implementation perspective, [15] proved that there is an alternative, simpler discretization for computing the Ricci curvature, namely the Forman-Ricci curvature, which has analogous properties to the Ollivier-Ricci curvature, with the added value that the Forman-Ricci curvature has a faster computation time in large-scale, real-world networks. Therefore, this paper will use the Forman-Ricci curvature as an estimator of fragility in an epidemic network, which is simply defined as follows[16]:

$$\begin{aligned} \mathrm{F}(e) = &\#\{triangles\ containing\ e\} + 2 \\ &- \#\{edges\ parallel\ to\ e\}, \end{aligned} \quad (1)$$

where the *parallel edges* to e are the edges that are sharing a node or a triangle with e, but not both. Here, we refer to triangles as the simultaneous connection between three nodes.

Given that both epidemic and financial networks are built on correlations between time series, we use geometric tools analogous to [15] to provide a novel application of the Forman-Ricci curvature: Inferring the fragility and systemic risk of epidemic networks, in particular, the COVID-19 network.

In this paper, we create an epidemic network consisting of edges and links, based on the reported epidemic time series. We define each spatial domain of the epidemic as the node of a network, and the links between two locations are based on the Pearson correlation coefficient (or any similarity measure) between their epidemic time-series. We chose the links of the network according to the Pearson correlation coefficient between two locations in descending order, which means that we include the strongest links first in the network, until the network reaches the Giant components (the state in which we have a single cluster of connected nodes).

Given that we are still in the early stages of the COVID-19 pandemic, we need to show that the Forman-Ricci curvature suffices to detect fragility and risk in a toy-model, i.e., a simulated epidemic network obtained from simulated time series for the epidemic. We will first build this simple model heuristically and in second step move towards the analysis of real COVID-19 data.

A simple way to access the number of cases in an epidemic network is to use the fractal growth hypothesis, as observed in [17], where the daily number of cases $n(t)$ in an epidemic follows a power-law distribution with an exponential cutoff:

$$n(t) = Kt^x \exp(-t/t_0), \quad (2)$$

where, K, x and t_0 are fitting parameters. In Fig. 1, we show examples of the fit between (2) and the number of
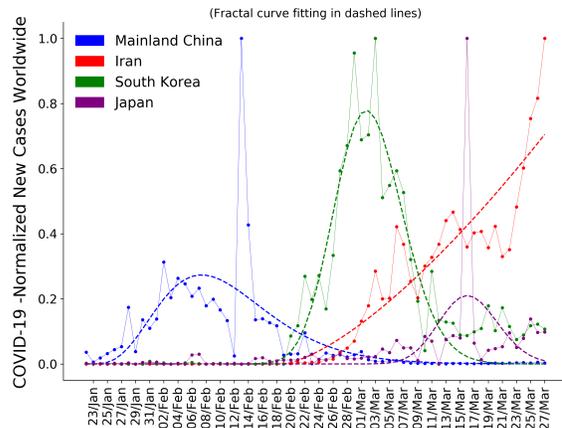

FIG. 1: COVID-19 per country. Illustration of the number of cases and fitting through fractal growth (dashed lines), Eq. (2), for a representative number of countries.

reported COVID-19 new cases for four countries, namely, China, Iran, South Korea and Japan. This fit suggests that (2) paves a simple way for building a toy-model for epidemic time series. We stress that our aim here is not to find whether the best fit for the pandemic is exponential or power law, which was already addressed in [17, 18], but to build a simple toy-model that allows us to test our hypothesis relating Forman-Ricci curvatures to epidemic networks.

Inspired by this equation, we can suggest a phenomenological toy-model for generating epidemic time series with noise that can capture the growth of an epidemic network. We assume that in each node i of the epidemic network, the daily number of cases follows a fractal epidemic growth with Gaussian noise $w_i(t)$ and a time delay d_i in relation to the epicenter:

$$n_i(t) = \begin{cases} w_i(t) & \text{if } t \le d_i \\ K_i(t - d_i)_i^x \exp\left(-\frac{(t-d_i)}{t_0^i}\right) + w_i(t) & \text{if } t > d_i \end{cases} . \quad (3)$$

We now show that the Forman-Ricci curvature suffices to detect fragility and risk for the simulated epidemic network. The starting point for creating a fractal epidemic network is based on simulating epidemic time series with delays from (3). In a second step, we define the weights of the epidemic network through the Pearson correlation coefficient between time series $n_i(t)$ and $n_j(t)$. The temporal epidemic network is computed for a given time window, and the process is repeated for the next time window, thus obtaining an evolving network. This approach is inspired by network analysis in other fields, such as neuroscience [19] or finance [20]. We illustrate the delayed epidemic time series, its Pearson correlation matrix and its corresponding network for a given time
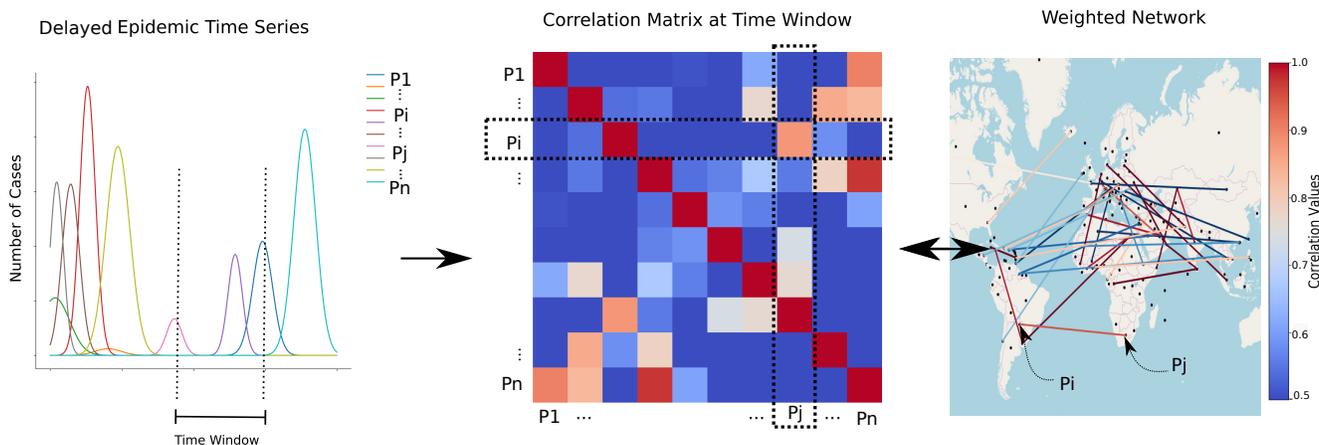
FIG. 2: Illustration of the creation of epidemic networks based on the correlations between epidemic time series across spatial domains for a given time window. This approach allow us to infer network signatures for epidemic outbreaks without relying on parameter estimation of classic stochastic epidemic approaches.
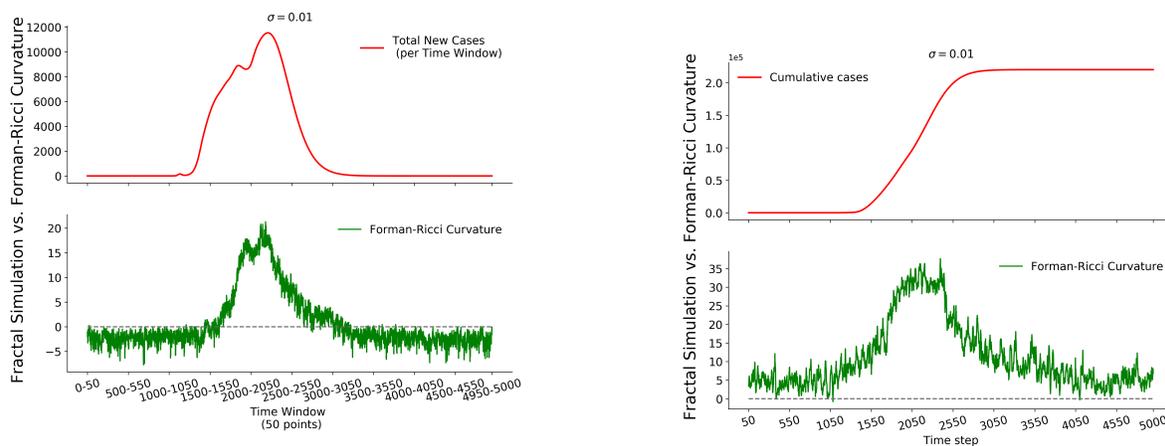

FIG. 3: (Top) Illustration of toy-model epidemic curves, both new cases and cumulative cases, according to (3) and its respective Forman-Ricci curvature at the bottom, with white noise parameter $\sigma = 0.01$.

point in Fig. 2, resulting in a time evolving network.

The third step is to infer the fragility of the time evolving epidemic network by tracking geometric changes in this network as a function of time. More specifically, we observe the mean changes in the discrete version of the Forman-Ricci curvature [21] for a selected moving window for each location affected by the epidemic and use the network curvature as a indicator for its fragility and risk. Thus, we assume that the application to epidemic time series follows an analogous behaviour to the one observed for stock markets in [14].

As a proof of concept, we then investigate a simulated time series with delays in (3). We generated 50 time series with parameters K_i, x_i, d_i, and t_0^i randomly chosen in the interval $K_i \in [0, 20]$, $x_i \in [0, 5]$, $d_i \in [10, 21]$, and $t_0^i \in [0, 1]$. We also included a small white noise with zero mean and variance of $\sigma = 0.01$.

Fig. 3 shows that the epidemic curve generated from our toy-model in Eq. (3) is compatible with an epidemic outbreak and contrasts the simulated epidemic curve with its Forman-Ricci curvature. We observe that the curvature is constant before the starting of the simulated epidemic, grows during its progression and reaches its maximum during the peak of the simulated outbreak. After the end of the simulated epidemic, the curvature comes back to its initial level. We emphasize that the inclusion of white noise $w_i(t)$ in our model was very important to destroy spurious deterministic correlations that appear at the end of the outbreak.

Having proven that our hypothesis is true for a toy-model of epidemic networks, we are now ready to test whether the Forman-Ricci curvature is a reliable network fragility metric for real COVID-19 data available from the World Heath Organization (WHO). In Fig. 4 we illus-
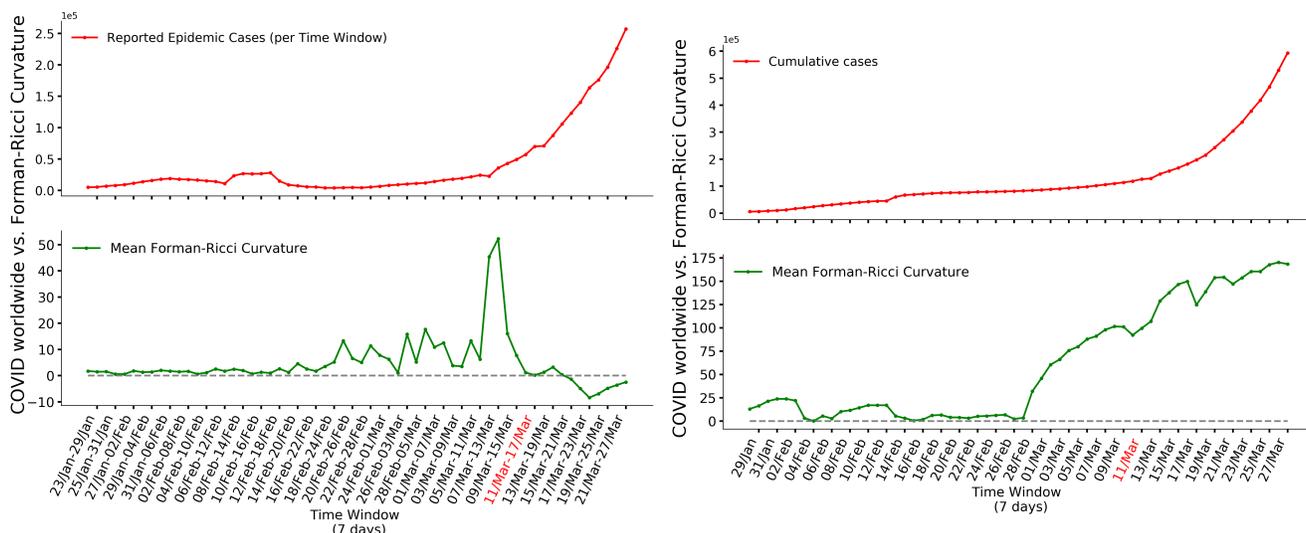
FIG. 4: (Top) Reported epidemic cases per time window, both new cases and cumulative cases, vs. its Forman-Ricci curvature (bottom) for the same time period. In red, we indicate the moment when the WHO declared COVID-19 as a pandemic.
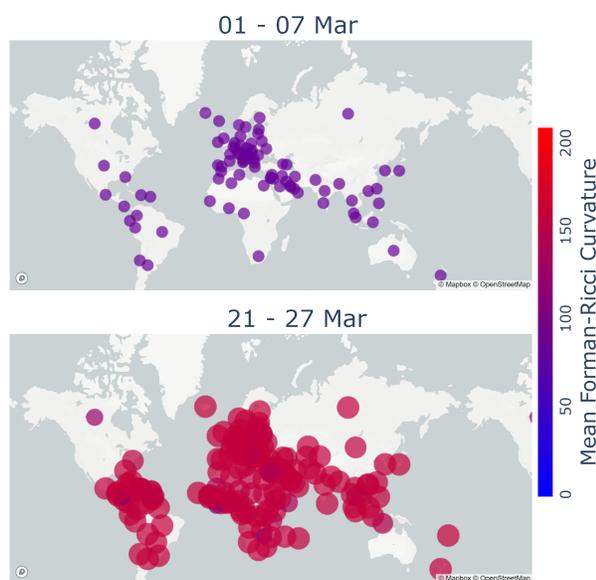

FIG. 5: Illustration of the ditribution of the Forman-Ricci curvature for two different time windows based in cumulative cases.

trate both the epidemic curve (top) and the Forman-Ricci curvature (bottom) for the COVID-19 database [22]. As in the simulated data, the curvature was stable at the beginning and grows over time, signaling increasing risk and fragility of the epidemic network. Remarkably, we observe that the curvature of the epidemic network gives an early warning sign for the emergence of the pandemics,

as the curvature starts to increase weeks before the exponential growth in number of cases is observed and the WHO declares COVID-19 as a pandemic (see Fig. 4, in red). Fig. 5 provides an additional geographical illustration of the distribution of the Forman-Ricci curvature across countries for two time windows in March.

We conclude that the Forman-Ricci curvature metric used in this paper might be a strong indicator for the fragility and systemic risk in the COVID-19 epidemic and, consequently, a data-driven approach to epidemic outbreaks more generally. Another added value of this geometric approach, in contrast to the classical stochastic and modelling simulations, is that the results emerge intrinsically and empirically independent of parameter estimations for the pandemic, e. g. its contagion rate or basic reproduction number. This paves the way for predicting and tracking the risk of the epidemic in the absence of reliable parameter estimations. More generally, geometric and topological methods seem to emerge as promising support tools for future epidemic control policies.

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[1] Herbert. Edelsbrunner and J. (John) Harer. *Computa-*

tional topology : an introduction. American Mathematical Society, 2010.

[2] Valerio Pascucci, Xavier Tricoche, Hans Hagen, and Julien Tierny. *Topological Methods in Data Analysis and Visualization: Theory, Algorithms, and Applications.* Springer Science & Business Media, 2010.

[3] Dane Taylor, Florian Klimm, Heather A Harrington, Miroslav Kramár, Konstantin Mischaikow, Mason A Porter, and Peter J Mucha. Topological data analysis of contagion maps for examining spreading processes on networks. *Nature communications*, 6:7723, 2015.

[4] Larry Wasserman. Topological data analysis. *Annual Review of Statistics and Its Application*, 5:501–532, 2018.

[5] Afra Zomorodian. Topological data analysis. *Advances in applied and computational topology*, 70:1–39, 2012.

[6] Nina Otter, Mason A Porter, Ulrike Tillmann, Peter Grindrod, and Heather A Harrington. A roadmap for the computation of persistent homology. *EPJ Data Science*, 6(1):17, 2017.

[7] Peter Bubenik. Statistical topological data analysis using persistence landscapes. *The Journal of Machine Learning Research*, 16(1):77–102, 2015.

[8] Manish Saggar, Olaf Sporns, Javier Gonzalez-Castillo, Peter A Bandettini, Gunnar Carlsson, Gary Glover, and Allan L Reiss. Towards a new approach to reveal dynamical organization of the brain using topological data analysis. *Nature communications*, 9(1):1–14, 2018.

[9] Joan T Matamalas, Sergio Gómez, and Alex Arenas. Abrupt phase transition of epidemic spreading in simplicial complexes. *Phys. Rev. Research*, 2, 012049(R) 2020.

[10] Giovanni Petri, Martina Scolamiero, Irene Donato, Francesco Vaccarino, and Renaud Lambiotte. Topological Strata of Weighted Complex Networks. *PLoS ONE*, 8(6), 2013.

[11] Csaba D Toth, Joseph O'Rourke, and Jacob E Goodman. *Handbook of discrete and computational geometry.* Chapman and Hall/CRC, 2017.

[12] Romeil Sandhu, Tryphon Georgiou, Ed Reznik, Liangjia Zhu, Ivan Kolesov, Yasin Senbabaoglu, and Allen Tannenbaum. Graph curvature for differentiating cancer networks. *Scientific reports*, 5:12323, 2015.

[13] Fernando AN Santos, Ernesto P Raposo, Maurício D Coutinho-Filho, Mauro Copelli, Cornelis J Stam, and Linda Douw. Topological phase transitions in functional brain networks. *Physical Rev. E*, page 100, 032414, 2019.

[14] Romeil S Sandhu, Tryphon T Georgiou, and Allen R Tannenbaum. Ricci curvature: An economic indicator for market fragility and systemic risk. *Science advances*, 2(5):e1501495, 2016.

[15] Areejit Samal, RP Sreejith, Jiao Gu, Shiping Liu, Emil Saucan, and Jürgen Jost. Comparative analysis of two discretizations of ricci curvature for complex networks. *Scientific reports*, 8(1):1–16, 2018.

[16] Maryam Pouryahya, James Mathews, and Allen Tannenbaum. Comparing three notions of discrete ricci curvature on biological networks. *arXiv preprint arXiv:1712.02943*, 2017.

[17] Anna L Ziff and Robert M Ziff. Fractal kinetics of covid-19 pandemic. *medRxiv*, 2020.

[18] Dominik Wodarz, and Natalia L Komarova. Patterns of the COVID19 epidemic spread around the world: exponential vs power laws. *medRxiv*, 2020.

[19] Alex Fornito, Andrew Zalesky, and Edward T. Bullmore. *Fundamentals of brain network analysis.*

[20] Kyungsik Kim, Soo Yong Kim, and Deock-Ho Ha. Characteristics of networks in financial markets. *Computer physics communications*, 177(1-2):184–185, 2007.

[21] RP Sreejith, Karthikeyan Mohanraj, Jürgen Jost, Emil Saucan, and Areejit Samal. Forman curvature for complex networks. *Journal of Statistical Mechanics: Theory and Experiment*, 2016(6):063206, 2016.

[22] World health organization, *novel coronavirus (2019-ncov) situation reports.* technical report 1-66, who, 2020.