CHAPTER 1

SPACE TRAVEL

"The first step made on the moon"
"Declared our freedom from"
"Our eternal jailer"
"The Earth's"
"Gravity"

A. INTRODUCTION

A.1. Introduction to space travel

Human beings are made of stardust. We are, literally made of atoms created and blown into space since the Big Bang and the creation of the Universe. This fact forms an important link that connects us with the whole universe. We try to understand the natural world around us through science. Scientific activity is one of the main features of the contemporary world and perhaps more than any other, distinguishes the present time from earlier ones. Many dangers nowadays threaten our world, e.g. pollution, the heating of the Earth by the greenhouse effect, radiation effects due to nuclear uses, etc. The salvations that may save the world will, everyone of them, be traced back to science. We need people who can see straight ahead and deep into the problems. Those are the experts. But we also need peripheral vision to our problems even to the layman and particularly to those people who are going to be professionals in different fields and who are going to make use of the new and modern technologies which might bring global destruction if not used with extreme care and caution.

Science confers great power. We use this power daily, for instance, switch on a light, a television set, an automobile, or a computer. Every such device has powerful effects on the world, both good effects (light to read by) and bad effects (pollution from electric generating plants). If we accept technology's power without also accepting the responsibility of using it wisely, we invite death, devastation and pollution, which form the monster's retaliation against its maker.

This book deals with that part of science called "physics", and its human connections. Physics studied show the most general features of the natural world. Other sciences focus only on particular facets of nature: biologists study living organisms, geologists study Earth's and Earth's structure, etc. But physicists search for principles that apply to living organisms and to Earth's structure and to all other parts of the universe. For example, most objects fall when you drop them, whether this object is a frog or a rock or otherwise. Such idea that objects fall when you drop them is a principle of physics.

A.2. Humans and the fascination by the heavens

Long time ago, at least 3000 years B.C., the Egyptians and the Arabs were fascinated by the heavens. Ancient priests looked to the stars in the sky and found that they move in orderly, predictable patterns. There, they considered, a place perhaps of gods. Although astrology, the belief that the positions of the stars significantly influence human affairs, has been an exploded superstition for two centuries, yet today, human scientific, religious, and aesthetic fascination with stars may be greater than ever.

A.3. The early belief that the Earth is at the center of the universe

Observing the motions of planets and stars in the sky made ancient people to assume that all celestial bodies move in circles around the Earth, with their axis of rotation fixed in the direction of the North star. The polar north star could be easily seen in the sky if one observes the two constellations, the big Dipper and the little Dipper (see Fig. 1.1). The brightest star of the little Dipper that lies on the extension of the two stars forming the trail of the Big Bear constellation, is the polar north star.

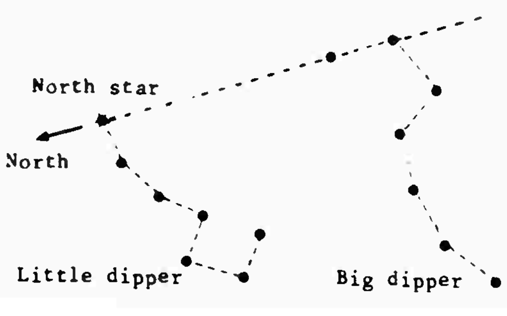


Fig. 1.1

A.4. The man who stopped the Sun and moved the Earth

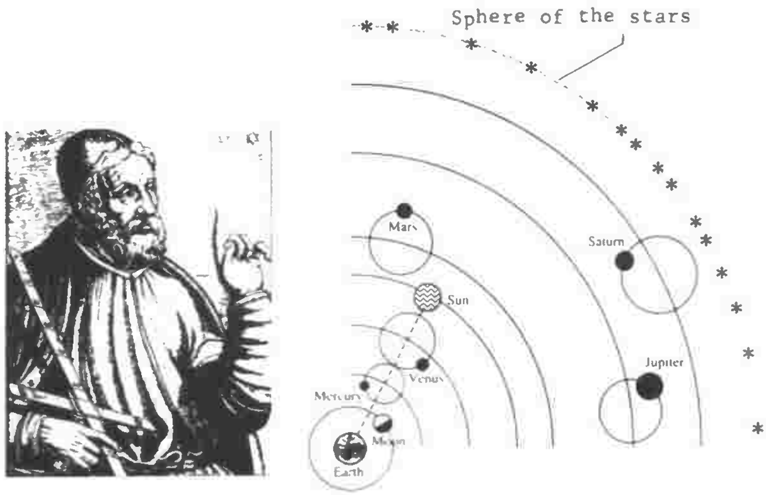
The year 1543 is considered as the birth year of modern science. Copernicus contradicted Ptolomy's theory about the Earth being at the center of the universe. He looked beyond Earth itself to imagine Earth as an object in space similar to other objects in space. He discovered that the visible planets and the Earth move in uniform motion in circles around the Sun, and only the moon circles the Earth, see Fig. 1.2.

Despite the agreement of Copernicus theory with astronomical data, there were many questions without answer, e.g. How could Earth move? What keeps it moving? Why aren't birds and clouds left behind? etc. Nevertheless, the new revolutionary view that the Earth is moving around the Sun has been widely accepted. Copernican astronomy started science moving toward the physics of Isaac Newton which arose partly because of such questions as "what keeps Earth moving?"

A.5. Kepler and the Sun-focused universe

Tycho Brahe and Kepler came after Copernicus. They spent many years pursuing the structure of the heavens. Brahe deviced excellent sighting devices from which accurate data were obtained for the planetary orbits known at that time. Kepler tried to translate Brahe's Earth-based data to Sun-based data. He was thinking how would the planetary positions appear if seen from the Sun. For the planet Mars, Kepler found that Brahe's data did not fit in the Copernican circular orbit. It was off by 8 arc-minutes. Accordingly, he rejected the Copernican theory concerning the uniform circular motion of the planets around the Sun.

Years later, Kepler presented his theory about the motion of planets in a Sun-focused ellipses, i.e. having the Sun in one of the two foci of the ellipse. This theory explained all Tycho Brahe's data and unified this data in a few principles such as the principle of elliptical orbits, Fig. 1.3.



Ptolemy's conception for the Earth-centered universe.

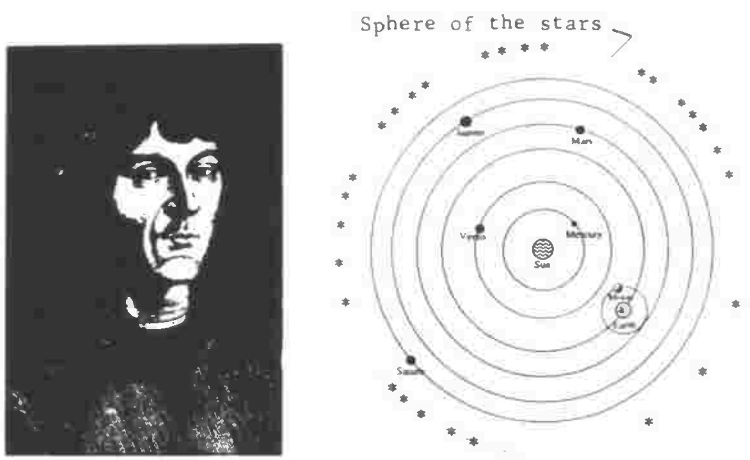


Fig. 1.2. Copernicus conception for the Sun-centered universe.

Isaac Newton, born a few years after Kepler's death, made important use of his theories to reach Newton's great success in the theories of force, motion and gravity.

A.6. Kepler's laws of motion

After Kepler discovered that the orbit of planet Mars could be accurately described by an ellipse with the Sun at one focus, he generalized this analysis to include the motion of all planets. The complete analysis was summarized in three statements, now known as Kepler's laws:

1: All planets move in elliptical orbits with the Sun at one of the focal points, Fig. 1.4.

2. The line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.

3. The square of the orbital period of any planet is proportional to the cube of the semi-major axis of the elliptical orbit.

Newton afterwards, demonstrated that these laws were the consequence of a simple force that exists between any two masses. Newton's law of universal gravitation, together with his development of the laws of motion, provides the basis of a full mathematical solution of the motion of planets and satellites. More important, Newton's universal law of gravity correctly describes the gravitational attractive force between any two masses.

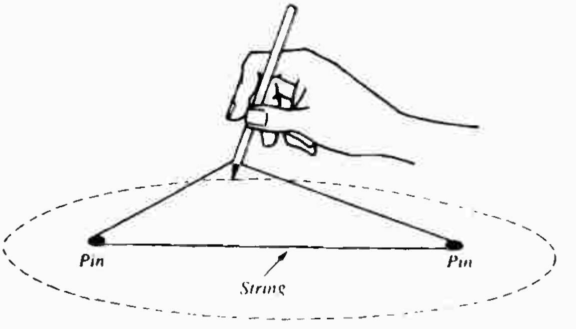


Fig. 1.3. How to construct an ellipse with pins and string. Each pin acts as a focus of the ellipse and its shape can be changed by changing the distance between the pins.

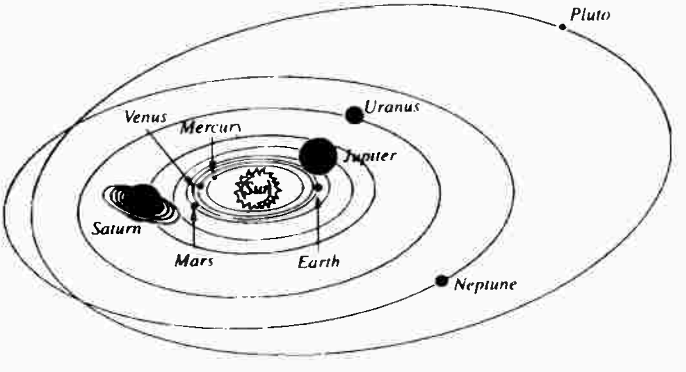


Fig. 1.4. Sketch showing the elliptical paths of the planets as they orbit the Sun (not to scale). This is a view of the orbits seen from an observer which is not perpendicularly centered above the Sun. Most of the planet ellipses are very close to circles.

A.7. Summary of the early history of astronomy

Observations	Typical dates	Theories
Stars, Sun, moon, planets	3000 B.C.	
	500	Phytagorian theory, Earth-centered universe
	↓	
Each planet moves in a varying rate	300 B.C.	
Earth seems motionless		
Planets are brighter during retrograde motion	100 B.C.	Theory of Earth-centered epicycles
	↓	
	00	Ptolomy's theory: Earth-centered epicycles
	↓	
Detailed quantitative measurements	100 A.D.	
	1500 A.D.	Copernicus theory: Sun-centered universe
Tycho Brahe's accurate measurements disprove Ptolomy's and Copernicus theories	↓	
	1600 A.D.	Kepler's theory: Sun-focused ellipses
Galileo's telescopic observations disprove Earth's centered theories	↓	

A.8. Newton and the apple

It has always been considered as a general principle in life the "what goes up must come down". All objects fell back to the ground when thrown. Aristotlian physics stated that this phenomena is the nature of things. The ancient idea of a flat Earth was supported by the remaining of ocean waters on the ground. The idea of gravitation was not known until a famous incident of an apple dropping in front of Newton. What makes the apple fall to the ground? Let's follow Newton's thinking, Fig. 1.5.

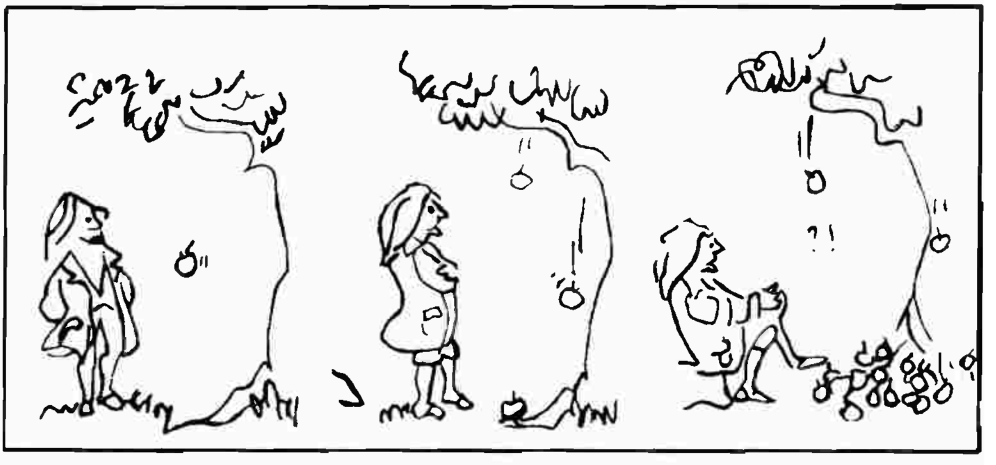


Fig. 1.5. Newton and the apple, thinking about gravity.

The falling apple has velocity, acceleration directed downwards. There must be a pull in the same direction forcing the apple towards the ground. As Newton recounted it, he invented the central idea of his law of gravity from seeing the similarity of the apple on the tree and the moon in the sky. How come these two objects are similar? Both are more-or-less round but the apple is on Earth and the moon is in the heavens; one quickly rots, the other seems eternal; one drops to the ground, and the other stays high in the sky. The unifying insight of Newton led him to consider one and the same phenomenon of mutual attraction to act on both objects causing one to fall on the ground and the other to orbit around the Earth.

Aided by the scientific foundations laid by Copernicus, Brahe, Kepler, Descartes, and Galileo, Newton arrived to discover the well-known laws of motion. The ideas surrounding the laws of motion, combined with the astronomical ideas of Copernicus and Kepler, then led Newton to the law of gravity.

A.9. Summary of Newton's laws of motion

Concept of inertia: A body that is subject to no external forces will stay at rest if it was at rest to begin with, and will keep moving if it was moving to begin with. All bodies have inertia. Inertia is the property of matter that relates to the tendency of an object to remain at rest or in uniform motion.

Newton's first law: States that if a body is at rest it will remain at rest unless acted upon by an external force. Similarly, a body in uniform motion in a straight line will maintain that motion unless acted upon by an external force, like sliding motion on ice.

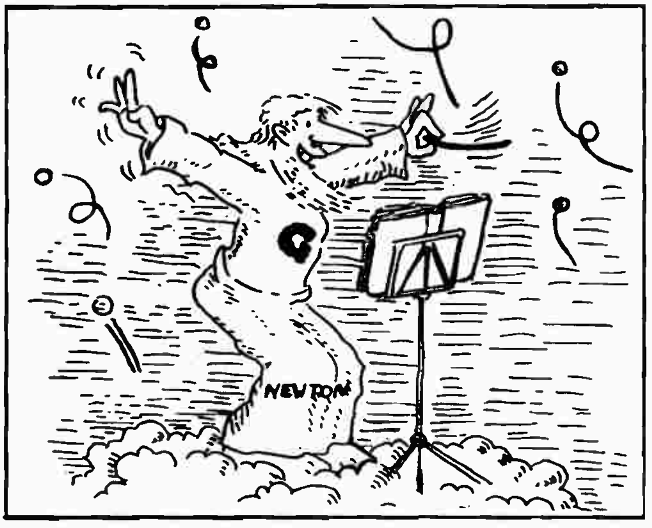
Newton's second law: States that the time rate of change of momentum of a body is equal to the resultant force acting on the body. If the mass of the body is constant, the net force equals the product of the mass into acceleration, like the case of accelerating car.

Newton's third law: Is called the law of force pairs, i.e. one cannot do just one thing. It states that: whenever one object exerts a force on a second object, the second object exerts a force on the first one: "forces always come in pairs". Furthermore, the two forces are equal in strength but opposite in direction, as in a compressed spring.

A.10. The decline of the Newtonian universe

Newton's extraordinary success in accounting for the movements of the planets and their satellites, made scientists to believe that any physical science could in time evolve to a state where predictions with the same precision would become routine. More than 300 years, Newtonian mechanics dominated the minds of all physicists by his harmonic oscillator with the deterministic solutions to all dynamical systems. It is until very recently, that modern culture still assumed that the mechanical Newtonian world view is the science's view of reality. This was the materialistic world view that leaves little room for freedom, chance, or creativity. It leaves little or no room for spiritual values. Newtonian determinism implied that the universe's future behavior is entirely determined by the present state, and it will go on like the workings of a perfect clock.

May be, because of this materialistic view point, Maxwell and Hertz after the discovery of electromagnetic (EM) waves, could not contemplate the idea that the energy of the EM wave might exist apart from any material particles. For this reason, they developed, wrongly, the idea that there exists a medium called "ether" filling all space. Afterwards, Einstein proved that the propagation of electromagnetic waves needs no material substance.



So, the discovery of the electromagnetic field turned out to be philosophically revolutionary. The two post-Newtonian theories, relativity theory and quantum theory, contradict both the specific predictions and conceptual underpinnings of Newtonian physics. A more clear evidence that Newtonian assumptions have broken down is the discovery of chaotic behavior of dynamical systems.

Perhaps the first clear hint that the tidy planetary system of Newton was not a model of wide applicability came with the discovery by Reynolds of vortex formation in turbulent fluid flow. Although this was more than a century and a half ago, it was only recently that we accepted that precise predictability is the characteristic of only a very few physical systems, and precise laws may generate randomness. Lorentz presented a model describing thermal convection in the atmosphere. Using a computer to solve his differential equations, he found that for some range of parameters, the system possessed a periodic solutions which show irregular variations that seem to be unpredictable as a random process.

Observations in many other fields proved that probabilistic behavior can emerge from Newtonian mechanics which is totally deterministic. Heneri Poincaré pointed out early this century that stochastic behavior is inherent in Newtonian mechanics. Physicists did not approach the problem of non-linear behavior and chaos in dynamical systems except very recently. In the beginning, they took chaos for randomness, until chaotic behavior was observed every where.

Newton's harmonic oscillator.

A.11. Range of applicability of Newtonian mechanics

Newtonian mechanics is used only to common phenomena on Earth. It is a simplifying special case of a much complicated behavior, applied to our everydays orders of magnitudes, e.g. it is not valid in case of very small objects as well as very large objects, such as galaxies and celestial bodies. It is also not valid when the moving bodies have speeds greater than about 1% of the speed of light. Fig. 1.6 shows the range of applicability of Newtonian mechanics. Outside this range, quantum mechanics and relativity theories should be applied.

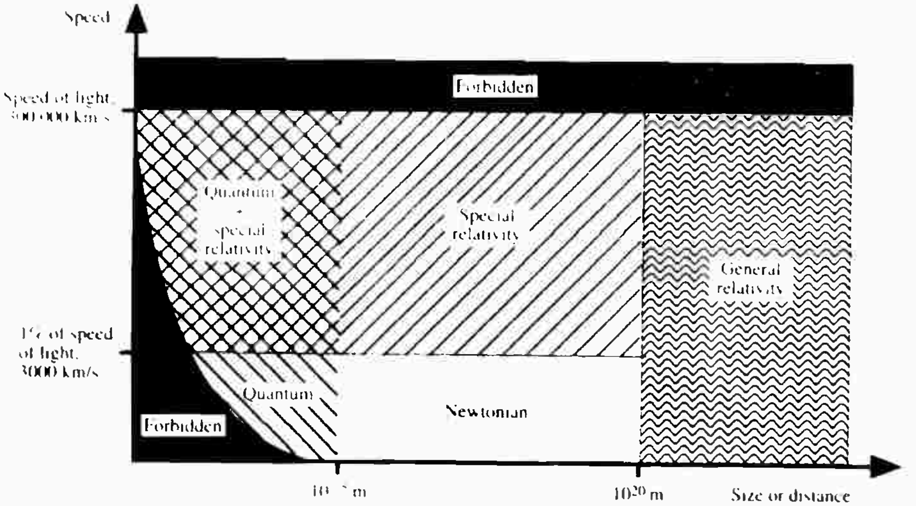


Fig. 1.6. It should be rekoned that Newtonian physics apply only to common phenomena on Earth, but breaks down for phenomena that are far from this normal range, i.e. it breaks down for very small objects, very large objects, and very fast objects. It also breaks down for strong gravitational forces, such as those near a black hole. The quantum and relativity theories apply throughout the entire range of phenomena observed to date.

A.12. Newton's universal law of gravity

Newton made the bold statement that the law of force governing the motion of planets has the same mathematical form as the force that attracts a falling apple to the Earth. He stated that:

> Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, Fig. 1.7.

If the particles have masses m_1 and m_2 and are separated by a distance r, the magnitude of this gravitational force is:

$$F = G\frac{m_1 m_2}{r_{12}^2} = F_{12} = F_{21}$$

and is directed to the centers of the two masses, where G is a universal constant called the gravitational constant, which was first determined by Cavendish in 1798. The value of G in SI units is:

$$G = 6.672 \times 10^{-11} \quad N.m^2/kg^2$$

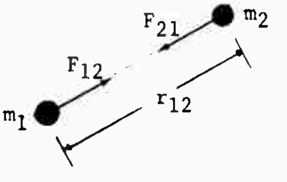


Fig. 1.7. The gravitational force between two particles is attractive. Note that $F_{12} = F_{21}$.

A.13. The law of gravitational and the acceleration of motion

According to the law of universal gravitation, the force on a mass m at the surface of the Earth is given by:

$$F = G \frac{m\,M_e}{R_e^2}$$

where M_e is the mass of the Earth, and R_e is the radius of the Earth. However, we have a more familiar expression for this force:

$$F = m\,g$$

where g is the acceleration of gravity at the surface, taken to be 9.8 m/s^2. Putting the two expressions together:

$$m\,g = G \frac{m\,M_e}{R_e^2}$$

and so,

$$g = G \frac{M_e}{R_e^2}$$

R_e is measured from the center of the Earth to the surface. If this distance is changed, e.g. on the top of a high mountain or in a deep ocean trench, g will change too. The acceleration of gravity, g, at the Earth's surface decreases with height, d, by an amount: $(2d/R_e).g$; but if we go below the Earth's surface, g decreases by only one half the above value (the student might prove that), taking into account that the attracting forces exerted by all mass elements at distances larger than the distance to the earth-center cancel each other.

A.14. Loosing weight

Usually we associate weight with gravity, but it is likely to have a rather different view of how these two concepts relate to each other. Is the reading of a bathroom weight scale for a person the same as the reading while riding down in a fast lift in a city building? The change in the reading of the scales as the lift speeds up and slows down means that the scale does not just measure the force of gravity on the person. What the scale reads actually is the APPARENT WEIGHT of the person, namely, his weight plus F or minus F where F is the force provided by the lift. The plus or minus signs depend on whether the gravitational force is in the same or in opposite direction to that provided by the lift. According to Newton's third law of motion, this force is the amount of force causing the acceleration of the lift.

Now consider that this person is an astronaut in a space shuttle accelerated with, e.g. 1.5 g's (that is, at 1.5 x 9.8 m/s^2) while lifted off in a rocket from a launch-pad on Earth. His apparent weight will be: $m\,g + F = m\,g + 1.5\,m\,g = 2.5\ m\,g$. Which means that if his body weight on Earth is 60 kg-wt, then his apparent weight under these conditions will be 150 kg-wt (Newton).

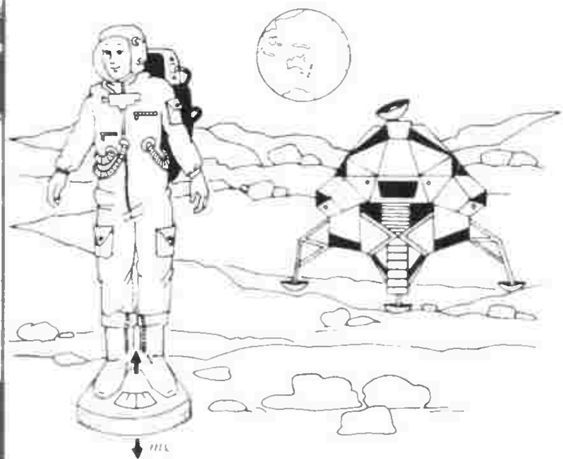
Figure 1.8 shows the effect of increased acceleration on an astronaut during his journey to the moon, and under the low gravity of the moon after he launched there.

A.15. Apparent weight and mass

If the astronaut is made to launch on the moon having with him his body weight scale, what will be the reading of the scale? On the moon the gravitational field, g', say is 1.61 N/kg, which is approximately one-sixth of its magnitude on Earth. Objects are lighter on the moon, so the weight of the astronaut will now read less than the reading on Earth. Note that the readings are different for his body weight on Earth and on the moon although his body mass did not change.

If the space ship with the astronaut in it is coasting (that is, the engines are off) in space, far from any stars or other massive bodies, in a region where the gravitational field is zero, then the effective weight of the astronaut will become zero. He will float weightlessly around the space ship. The satellite and the astronaut are actually free-falling bodies and so they behave as weightless bodies.

The effect of acceleration on Lt. Col. John L. Stapp while riding in a rocket propelled test sled at Holloman Air Force Base, USA. The photos show the first 5 seconds of acceleration in which the speed of the sled increased to more than 670 km/h, subjecting Col. Stapp to a force of 12 g's. His face shows the effect of the increasing apparent weight.



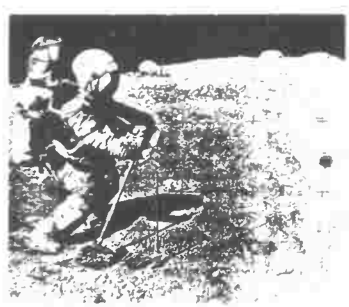


Fig. 1.8. Journey to the moon.

A.16. Weightlessness

Weightlessness is not exclusively an experience for astronauts; we have all experienced it, even if only for very short periods. Any body experiencing a jump from a relatively high place will undergo a free fall. During this free fall he will feel weightlessness, because his apparent weight will be zero.

A weightless environment has advantages and disadvantages for astronauts. They can move and lift very heavy equipment across the room. On the other hand, its annoyance appear when you try to write something, e.g. besides, there are physiological effects associated with weightlessness. On Earth, gravity is pulling the fluids in the body toward the feet. In orbit, the equilibrium distribution of fluid is different, tending to shift toward the upper parts of the body. An interesting observation is that the astronaut grows about 3 cms in height while in orbit. Since there is no downward force on the spine, the spongy discs in the spinal column are no longer compressed. As the discs relax, the astronauts grow. But after coming back to Earth they shrink back to their normal height.

In a weightless environment, the cardiovascular system does not need to work very hard to pump blood around the body. It is easier to get the blood back up from the legs, or up to the brain. The cardiovascular muscles become deconditioned. This is not a problem as long as astronauts are in orbit, but when they return back to Earth the cardiovascular system will once again be called to pump blood against gravity. This might not be a problem if the astronaut remains in orbit for a little period of time. But, it will become considerably serious for extended flights in a space station.

A.17. Measurement of the gravitational constant

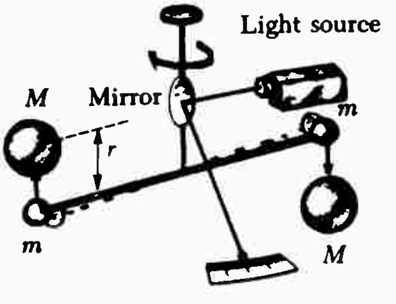
Cavendish was the first to measure the gravitational constant, G. He used two small lead spheres each of mass m, fixed to the ends of a light horizontal rod suspended by a fine fiber as shown in Fig. 1.9. Two large spheres each of mass M are then placed near the smaller spheres. The attractive force between the smaller and the larger spheres causes the rod to rotate and twist the fiber suspension. The angle through which the suspended rod rotates is measured by the deflection of a beam of light reflected from a mirror attached to the vertical suspension. Cavendish studied the effect of using different masses, and found that the force of attraction is proportional to Mm. He also studied the effect of distance between the centres of the two spheres, and found that the force was proportional to the inverse square of the distance, r, between the spheres.

The gravitational constant, G, was found from the equation:

$$F = G \frac{M\,m}{r^2}$$

from which $G = 6.67 \times 10^{-11}$ $N.m^2/kg^2$.

Fig. 1.9. Schematic diagram of Cavendish apparatus for measuring G. The small spheres of mass m are attracted to the large spheres of mass M, and the bar rotates through an angle from which the gravitational force is found.



B. THE BUILDING OF THE ROCKET AND THE HEAT ENGINE FOR SPACE FLIGHT

B.1. Conservation of energy

Energy is based on the notion of work. An object A does work on an object B if A exerts a force on B while B moves. Both force and motion are needed in order for work to be done. Work is proportional to force and distance:

$$\text{Work} = \text{Force} \times \text{Distance}$$

We do work when we lift an object on the ground. The object then has the ability of doing work when it is left to drop. Moving objects also have the ability to do work. Energy is the given word for the ability to do work. Gravitational energy is given by the weight into height above the ground. It is due to gravitational forces. There are other types of energy such as thermal, elastic, kinetic, etc.

Any system that experiences only gravitational forces conserves its total energy. The gain or loss of kinetic energy is exactly balanced by the loss or gain in gravitational energy. It is also possible to prove that any system that obeys Newton's laws and that possesses only kinetic, gravitational, and elastic energy must conserve its total energy.

The law of conservation of energy states that the total energy of all participants in any process must be unchanged throughout that process. That is, energy cannot be created or destroyed. Energy can be transformed from one form to another, and it can be transferred from one place to another, but the total amount always stays the same.

Physicists have discovered several other natural quantities that do not change, i.e. that are conserved. It is interesting to list them, beginning with energy conservation.

1. The total energy is conserved in a closed system.
2. The total motion through space, or the total linear momentum.

3. The total rotational motion, or the total angular momentum.
4. Total electric charge.

B.2. The work-energy principle

Work is an energy transfer. Heating is another worklike process by which energy can be transferred. Thermal energy transfer can be thought of as microscopic work. The work-energy principle when expanded to include not only ordinary work but also microscopic work (heating), is called **the first law of thermodynamics**.

Presently, there are several experiments proving the law of conservation of energy. The equivalence of work and heat was first proved by the famous Joule's paddle wheel experiment.

B.3. Work and heat "The Joule paddle wheel experiment"

In any real mechanical system there is always some mechanical energy lost by frictional forces. Various experiments show that this mechanical energy is transformed into thermal energy. Joule was the first to establish the equivalence of work and heat.

A schematic diagram of Joule's famous experiment is shown in Fig. 1.10. Water is contained in a thermally insulated container. Work is done on the water by a rotating paddle wheel, which is driven by weights falling at a constant speed. The water, which is stirred by the paddles, is warmed due to the friction between it and the paddles. Joule calculated the loss in potential energy of the two falling weights (of mass m each) through a distance h. This energy is used to heat the water. If W is the water equivalent (mass x specific heat) of the water and container, and if ΔT is the increase in the temperature of the water, then the heat gained is equal to $W.\Delta T$. By varying the conditions of the experiment, Joule found that the loss in mechanical energy, 2 mgh, is proportional to the increase in temperature of the water. The constant of proportionality was named after Joule as "Joule's equivalent of heat (J)". Therefore,

$$\text{Work} = J \times \text{heat gained}$$

The mechanical equivalent of heat (J) was found to be equal to 4.186 J/cal.

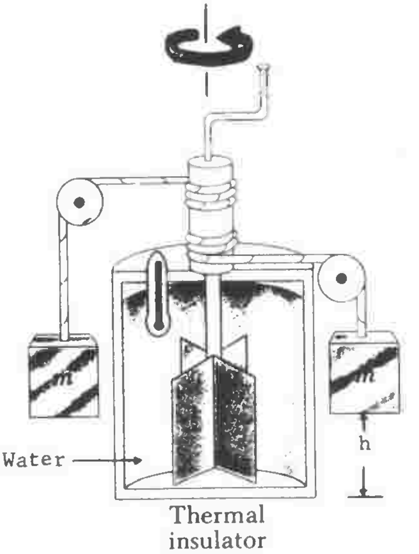


Fig. 1.10. An illustration of Joule's experiment for measuring the mechanical equivalent of heat.

The transfer of mechanical energy into heat energy is very important in case of rocket propulsion through the air forming the Earth's atmosphere. If the rocket is not protected by insulating shields from outside, the rocket and all the bodies and equipment carried in will be totally burnt by the intense heat due to friction. We consider this problem of protecting the rockets from excessive heating later when we deal with the insulating properties of materials and how thermal conductivity is measured.

B.4. Thermodynamics

Thermodynamics is concerned with the general relations between the quantities of a system in which the temperature plays a role. It is based on two laws derived from experience, but it does not deal with the atomic explanation of these quantities. For example, consider a system such as a gas enclosed in a cylinder closed by a piston kept in a definite position, and is placed in a heat bath of a definite temperature. After some time a state of equilibrium is reached. This equilibrium does not mean that the motion of the constituent particles ceased. On the contrary, the gas molecules remain in permanent motion, colliding with each other and exchanging energy. But these motions are not perceptible on a macroscopic scale, and it is not the task of thermodynamics to describe them. The functions of state describing a thermodynamic system are therefore macroscopic variables.

B.5. The first law of thermodynamics "WORK AND HEAT"

Consider a system the state of which is changed a little by supplying a small quantity of heat dQ. Let dW be the amount of work done by the system during this process. The law of conservation of energy requires that the difference between the quantity of heat supplied to the system and the quantity of work done by the system is equal to the increase of the internal energy of the system, dU. Thus,

$$dQ = dU + dW$$

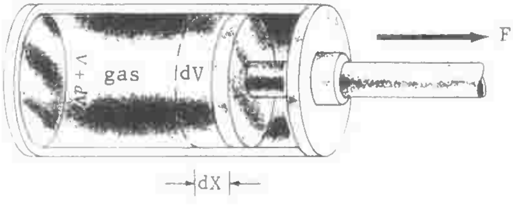
The increase in the internal energy, dU, of the system is partly potential arising from intermolecular attractions, and partly kinetic energy from the motion of the molecule forming the system.

B.5.1. Work done by a gas during expansion

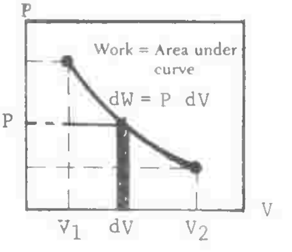
Consider a one gram of a gas enclosed in a cylinder closed by a movable piston with cross-sectional area A (cm^2). Let the piston moves a distance dx outwards. The force exerted by the gas F = P.A , where P is the initial pressure of the gas when its volume was V.

The work done = F.dx = P.A dx . But A.dx = dV , the elemental change of volume,. Thus for a finite expansion from V_1 to V_2 as shown in Fig. 1.11, the total work done during the process might be found from the area under the curve, i.e.

$$W = \int_{V_1}^{V_2} P\, dV$$



Cylinder closed by a movable piston



P-V diagram

Fig. 1.11

B.5.2. Application of the first law

From the first law we shall now derive some general laws.

1. When heat is supplied to a system at constant volume, the external work done is zero, so the quantity of heat provided at constant volume $Q_V = U_2 - U_1$, is equal to the increase of internal energy of the system.

2. If we supply the heat at constant pressure, the external work done by the system is equal to $P(V_2 - V_1)$, when the volume increased from V_1 to V_2. Thus according to the first law:

$$\begin{aligned} Q_P &= U_2 - U_1 + P(V_2 - V_1) \\ &= (U_2 + P V_2) - (U_1 + P V_1) \\ &= H_2 - H_1 \end{aligned}$$

where $H = U + PV$ is called the enthalpy or the heat content in the system. It is equal to the quantity of heat supplied to a system at constant pressure.

When we evaporate a liquid, we supply the latent heat at the boiling temperature while the vapor pressure remains constant. Thus the latent heat of steam represents the difference in enthalpy of 1 gram of liquid and 1 gram of water vapor at 100 °C.

The latent heat of steam = 2.26×10^5 Joules/kg (J/kg)

The external work done during evaporation = P dV

= 1.7×10^4 J/kg.

The energy difference between liquid and vapor = 2.1×10^5 J/kg.

B.5.3. Adiabatic change

If a change in the state of a system is done in such a way that no heat enters or leaves it, the change is said to be adiabatic. A gas expanding adiabatically will do external work on the expense of the internal energy and so its temperature drops.

The perfect gas equation (PV = RT) does not hold in the case of adiabatic change. To derive the equation of adiabatic change, consider 1 gram of a gas contained in an isolated cylinder the state of which is represented by P, V, T.

From the definition of specific heat: it is the change in internal energy of this gram of gas by a change of temperature of 1 °C. If this change is done under constant volume, no external work is done, and thus we get C_V . If the change is done under constant pressure we get the specific heat C_P which is greater than C_V by the amount of work done to maintain the pressure constant, which is equal to $PV_2 - PV_1$ in heat units. Thus,

$$C_P - C_V = RT_2 - RT_1$$

Since $T_2 - T_1 = 1$ °C. Therefore,

$$C_P - C_V = R$$

But, we have from the first law of thermodynamics $dQ = dU + dW$ and $dU = C_V\, dT$, thus,

$$dQ = C_V\, dT + P\, dV$$

But since the system is isolated and no heat is given or taken from it, therefore, $dQ = 0$.

Also, from the perfect gas equation we have by differentiation,

$$P\,dV + V\,dP = R\,dT$$

Using the above equations we get:

$$\frac{C_V}{R}(P\,dV + V\,dP) + P\,dV = 0$$

But, $C_P - C_V = R$, thus,

$$C_P\,P\,dV + C_V\,V\,dP = 0$$

Therefore,

$$\frac{C_P}{C_V}\frac{dV}{V} + \frac{dP}{P} = 0$$

Integrating we get:

$$\frac{C_P}{C_V}\log V + \log P = \text{constant}$$

or $$\log P.V^{(C_P/C_V)} = \text{constant}$$

or $$P\,.\,V^{\gamma} = \text{constant}$$

where $\gamma = C_P/C_V$.

This is the equation replacing Boyle's law in an adiabatic change in a gas.

B.5.4. Equation for temperature change in an adiabatic

In order to introduce temperature in the equation of an adiabatic change, we use the gas equation $PV = RT$, and substitute in the equation of the adiabatic change. Thus:

$$P\,.\,V^{\gamma} = (RT/V)\,.\,V^{\gamma} = RT\,.\,V^{\gamma-1} = \text{constant}$$

But R is the gas constant, therefore,

$$T \cdot V^{\gamma - 1} = \text{constant}$$

This gives the equation for an adiabatic temperature change. The value of g for a polyatomic gas is of the order of 1.3 .

Example 1.1:

An ideal gas at 17 °C has a pressure of 76 cm.Hg, and is compressed i) isothermally and ii) adiabatically until its volume is halved, in each case reversibly. Calculate in each case the final pressure and temperature of the gas, assuming $C_P = 2100$ and $C_V = 1500$ J/kg.K .

Solution:

i) Isothermally: PV = constant . Therefore,

$$P\,(V/2) = 76 \times V$$
$$P = 152 \text{ cm.Hg}$$

The temperature is constant at 17 °C.

ii) Adiabatically: PV^{γ} = constant, and $C_p/C_V = \gamma = 2100/1500 = 1.4$.

Therefore,

$$P\,(V/2)^{1.4} = 76 \times V^{1.4}$$

$$P = 76 \times 2^{1.4} = 201 \text{ cm.Hg}$$

Since $TV^{\gamma - 1}$ = constant, thus:

$$T \times (V/2)^{0.4} = (273 + 17) \times V^{0.4}$$

T = 383 K (T = 110 °C).

B.6. The second law of thermodynamics

B.6.1. Ordinary heat engines

The first law of thermodynamics is merely the law of conservation of energy generalized to include heat as a form of energy transfer. This law tells us that an increase in one form of energy must be accompanied by a decrease in some other form of energy. No restrictions are put to the types of energy conservation.

There is an important difference between heat and work. It is impossible to convert heat completely into work without changing the surroundings. Accordingly, these processes are called irreversible, denoting those processes that occur naturally in only one direction. The second law of thermodynamics establishes which processes in nature may or may not occur.

Probably the most important application to the second law of thermodynamics is the efficiency of the heat engine. A heat engine is a device that converts thermal energy into other useful forms of energy. A heat engines is a device that carries a substance through a cycle during which:

1. heat is absorbed from a source at a high temperature,
2. work is done by the engine, and
3. heat is expelled by the engine to a sink whose temperature is lower than the source.

An example is the heat engine of the automobile which takes heat from fuel combustion and converts a fraction of this energy to mechanical energy for motion, the rest of the energy is expelled out as exhaust.

The automobile heat engine works by burning fuel-air mixture. The combustion of this mixture gives it a high pressure, thus enabling hot gases to push strongly on a piston connected by a rod to the drive wheels. The combustion here is internal because it occurs directly inside the gases that do the work. There is an external combustion engine in which the fuel provides thermal energy to a second substance such as steam which then does the actual work, like steam turbines.

These mechanisms differ completely from that required for rocket propulsion engines in ballistic missiles in which the ejection of the hot gases from the nozzle of the rocket gives its push forward.

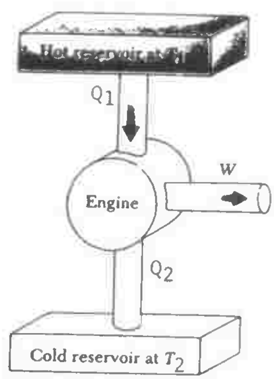
Figure 1.12 shows a schematic diagram representation of a heat engine. The engine in the circular area receives heat Q_1 from the reservoir at temperature T_1, expels heat Q_2 to the cold reservoir at temperature T_2, and does work W. The work generated is the difference between Q_1 and Q_2:

$$W = Q_1 - Q_2$$

The efficiency of the engine is defined as the ratio of this work to the heat absorbed from the high temperature source. The efficiency, h is given by:

$$\eta = W/Q = (Q_1 - Q_2)/Q_1$$
$$= 1 - (Q_2/Q_1)$$

If $Q_2 = 0$, i.e. no heat wasted, then the efficiency would be: $\eta = 1$, or 100%. This ultimate efficiency is unattainable.



Heat engine

Fig. 1.12. Flow chart of an ideal heat. engine.

An ideal, reversible, engine is the engine that can be operated in a reverse direction. It can convert work into heat at the same rate as it converts heat into work. Such engine is called **Carnot Engine**. The operation of Carnot's heat engine requires that the working material, a gas in a cylinder closed by a piston, be taken over a sequence of four steps in a cycle called **Carnot cycle**.

B.6.2. Carnot cycle

Carnot's engine is represented by a gas filled cylinder fitted with a piston. The cylinder can be put on either the source or the sink whose temperatures are T_1 and T_2 respectively. To describe the operation of this engine, it is best to use a P-V diagram (see Fig. 1.13).

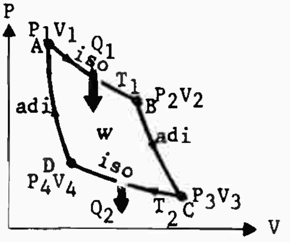


Fig. 1.13. The Carnot cycle shown on a P-V diagram.

The initial volume and pressure of the gas in the cylinder are V_1 and P_1. We take the gas through a sequence of four steps. The last step brings the gas to its initial volume and pressure.

Step 1: Place the cylinder in contact with the source. The temperature of the gas is maintained at a temperature T_1. The gas is allowed to expand from an initial volume V_1 to a new volume V_2. During this expansion the gas does work on the piston, i.e. the engine absorbs heat from the source and converts it into work.

Step 2: Remove the piston from the source, then let the gas expand adiabatically to a volume V_3, the pressure becomes P_3. The temperature will fall during this expansion.

Step 3: When the temperature has fallen to T_2 we put the cylinder on the sink of temperature T_2 and compress the gas until its state becomes P_4, V_4. Work is converted into heat which is ejected into the sink.

Step 4: The gas is removed from the sink and them compressed adiabatically until its volume and pressure return to their initial values P_1 and V_1 .

B.6.3. Calculation of the work done during Carnot cycle

The isothermal lines AB and CD shown in Fig. 1.13 are:

$$P_1 V_1 = P_2 V_2 \quad \text{and} \quad P_3 V_3 = P_4 V_4$$

The equations of the adiabatic lines BC and DA are:

$$P_2 V_2^{\gamma} = P_3 V_3^{\gamma} \quad \text{and} \quad P_1 V_1^{\gamma} = P_4 V_4^{\gamma}$$

The work done during the isothermal process AB is:

$$W_{A-B} = \int_1^2 P\, dV = \int_1^2 (RT_1/V)\, dV = RT_1 \ \log (V_2/V_1)$$

Similarly,

$$W_{C-D} = RT_2 \ \log (V_4/V_3)$$

The total isothermal work done in the process

$$= RT_1 \ \log (V_2/V_1) + RT_2 \ \log (V_4/V_3)$$

But, we have:

$$\frac{P_1 V_1}{P_4 V_4} = \frac{P_2 V_2}{P_3 V_3} = \frac{RT_1}{RT_2} = \frac{T_1}{T_2}$$

Also,

$$\frac{P_1 \ V_1^{\gamma}}{P_4 \ V_4^{\gamma}} = 1 = \frac{P_2 \ V_2^{\gamma}}{P_3 \ V_3^{\gamma}}$$

From the above equations we get:

$$V_1/V_2 = V_4/V_3$$

Therefore, the isothermal work becomes:

$$W(iso) = R\,(T_1 - T_2)\log(V_2/V_1)$$

The work done during the adiabatic process BC is equal to the work done during the adiabatic process DA, but with negative sign (the student is asked to prove that using the adiabatic equation for the change). Thus the total work done during the adiabatic changes is zero.

Therefore, the whole work done during Carnot cycle is equal to the area of the cycle which equals:

$$\Sigma\ W = R\,(T_1 - T_2)\log(V_2/V_1)$$

Since $(Q_1 - Q_2)$ is equal to the useful work done, therefore,

$$Q_1 - Q_2 = R\,(T_1 - T_2)\log(V_2/V_1)$$

Thus,

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

From which:

$$Q_1/Q_2 = T_1/T_2$$

or:

$$Q_1/T_1 - Q_2/T_2 = 0$$

Therefore,

$$\oint dQ/T = 0$$

Putting

$$dS = dQ/T$$

The function S is called **ENTROPY**, and it is a function of state.

B.7. Disorder in a physical system

B.7.1. Physical meaning of entropy

The entropy is defined as the function which is applied to that thermal property of a substance which remains constant as long as heat is not communicated or abstracted from it by external bodies. The entropy change has the same dimensions as a specific heat.

For a reversible heat cycle the entropy does not change. It remains constant. We do not have any instrument that could give us a measure of entropy for a system as we do in the case of the other functions of state such as temperature, pressure, and volume. We can only calculate it indirectly from dQ/T.

From a kinetic point of view the entropy can be taken as a measure of the degree of disorder in a system. When we cool a system at constant volume we continuously withdraw heat, and hence entropy from the system, and at the same time the order increases more and more. As a gas condenses into a liquid, the molecules will take up certain preferred positions with respect to each other, not like the case of a gas. Similarly, when we solidify a liquid it becomes more ordered in a crystal lattice. As the temperature decreases more and more, thermal agitation energy becomes less and less, and finally at the absolute zero there is no thermal motion of any kind and thus no disorder could be found and the entropy will be zero.

B.7.2. The increase of entropy and degradation of energy

We have seen for a reversible cycle $dS = 0$. But in reality we never find a reversible cycle as that described by Carnot. All heat engines are irreversible. Part of the heat energy is used to overcome resistive agents during operation. The efficiency of the heat engine defined as the useful work divided by heat intake is equal to 1 for a reversible engine, but it is less than one for an irreversible one.

For a reversible cycle:

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

For an irreversible cycle:

$$\frac{Q_1 - Q_2}{Q_1} < \frac{T_1 - T_2}{T_1}$$

Therefore,
$$1 - Q_2/Q_1 < 1 - T_2/T_1$$

$$Q_2/T_2 - Q_1/T_1 > 0$$

$$S_2 - S_1 > 0$$

Therefore,
$$dS > 0$$

Thus, although the irreversible Carnot cycle has returned to its initial state at the end of the cycle, the entropy of the system taken as a whole increases.

Knowing that all heat processes taking place in the world are irreversible processes, we find that "The entropy of the world tends to a maximum". This would lead to the conclusion that: The available energy of the universe is tending to zero. When this state is reached not a single heat engine could be operated.

B.7.3. Cosmological implications - concerning entropy

We have seen that the entropy of any irreversible process increases, and since all processes involved in our universe are irreversible, the whole entropy of the universe tends toward a maximum value. It is obvious that maximum entropy will be attained when the whole world reaches a state of uniform temperature and density. All physical, and chemical, and biological processes would cease. A state of perfect disorder implies no energy available from doing work. This gloomy state of affairs is sometimes referred to as the heat death of the world when every living organism will die in the whole universe.

It was also argued, similarly, that since we are still living, then the entropy of the world did not reach its maximum value yet. Accordingly, only a finite number of years have passed since the creation of the world. If the actual laws of nature had been valid through an infinite number of years, the maximum of entropy would have already been attained. Since this is obviously not the case and the world is still alive, the laws of nature and our world itself cannot have existed for an infinite number of years. It must have been created only a finite number of years ago.

Many philosophers envisaged some kind of ending to our world. This opinion has been cheered as a spiritual factor. But this running towards the death of the world has been considered in contrast with the evolution theory of organisms which shows a tendency towards greater and greater differentiation. Two conflicting tendencies must therefore be recognized in the universe: the first, the increase of entropy, is used to predict a gloomy feature for our world; the second causes life to be flourishing as the evolution theory predicts. Sometimes this conflict has been interpreted as a scientific background for the external struggle between good and evil.

As a matter of fact the term entropy of the universe is a pure paper and pencil affair. It was recently stated that the entropy of the universe might not be the sum of the entropies of all parts of the universe. Since the sum of the entropies of a system can be regarded only under very restricted conditions as the entropy of the whole system. We do not even know whether the sum is convergent from the purely mathematical view-point. Therefore, all these conclusions concerning the creation and death of the world are results of a loose way of thinking. Besides, since no statement of thermodynamics contained the expressions "velocity" and "time", thus we are given not the slightest estimation of how long it may take to establish a certain state of the "world". If a theory does not contain any statement about time, it has no operational meaning to say that it predicts a certain future of the world.

B.7.4. Entropy and disorder

Nature generally implies to us that a disorderly arrangement in any physical process is much more probable than an orderly one if the laws of nature are allowed to act without the interference of mankind. One main result of statistical mechanics is that isolated systems tend toward disorder and entropy is a measure of disorder. In the light of this view, Boltzmann put an alternative method for calculating entropy or the degree of disorder in terms of the probability of occurrence of a particular event. Boltzmann's relation is:

$$S = k \ln W$$

where W is the probability of having the event, and k is Boltzmann's constant.

In order to understand the meaning of this equation, imagine that we have a bag containing 100 small balls, where 50 of them are white, T, and the other 50 are black, B. Draw one ball, record its color, return it to the bag and draw again. Continue this process until four balls have been drawn. Obviously that the probability of drawing a white ball is the same as that of a black one. The results we find are given in the following table.

Possible results of drawing four balls from the bag.

End result	Possible findings						Total number of same results
All white, T	TTTT						1
1 B,3T	TTTB	TTBT	TBTT	BTTT			4
2 B, 2T	TTBB	TBTB	TBBT	BTTB	BTBT	BBTT	6
3 B, 1T	BBBT	BBTB	BTBB	TBBB			4
All black, B	BBBB						1

The table shows that although there is only one possible way to draw all the balls either black or white, there are six sequences that could give two whites and two blacks. Four sequences are required to produce three from one color and one from the other color. This indicates that there is much lower probability for producing the ordered states (all white, or all black). Thus, we can regard entropy as an index of how far a system has changed from an ordered to a disordered state.

The increase of entropy is associated with a degradation of energy taking on a form that is less useful for doing work. The conversion of high-grade energy to thermal energy is an important source of thermal pollution on Earth.

Entropy and disorder
"The Department of Entropy"

B.8. The rocket engine

B.8.1. Drive forces

An interesting dilemma arises when we consider any "self-propelled" object. Can an animal or a person accelerate without a reacting force by the environment? No. Things cannot accelerate themselves. What happens when we walk? If you stand up and take a step forward, the bottom of your foot accelerates you. The foot pushes backward against the floor. The law of force pairs tells us that, if you push on the ground, the ground pushes on you. And since your foot pushes backward on the ground, the ground must push forward on your foot. This is the force that propels you forward. It is one example of a **DRIVING FORCE**.

B.8.2. Rocket drive

If a person is standing on a perfectly smooth ground, e.g. ice pond, and if he throws ahead an object in his hand. While throwing, he pushes on the object, so it pushes in the opposite direction and the person acquires velocity and he will slide along the pond. If he likes to move faster he has to throw the body faster or he might throw another object.

This is the principle of the rocket drive. Rockets take along their own material just to have something to push against. The rocket fuel in the space shuttle engines is usually hydrogen and oxygen, stored in liquid form. When combined, their combustion produces steam which is ejected out of the nozzle of the rocket engine, pushing the shuttle forward, Fig. 1.14.

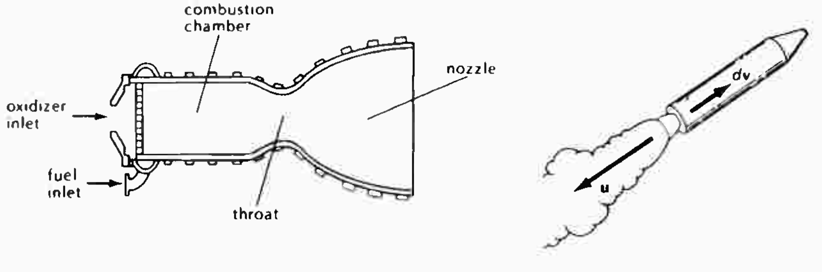


Fig. 1.14. The rocket drive.

B.8.3. Motion of a body (the rocket) with variable mass

Rocket propulsion depends on reaction forces. The machine exerts a backward push against its environment (air), and the reaction of the environment pushes the rocket forward. In empty space the propulsion of the spacecraft is more difficult since there is nothing in the environment to push against. Thus it is necessary for the machinery to supply its own medium on which to push. In the operation of a rocket engine, this medium consists of the exhaust gas. The rocket engine produces a large quantity of hot, high pressure gas from the combustion of the fuel in the combustion chamber. This gas is ejected at high speed at the tail of the rocket (from its nozzle). The rocket pushes on the gas and the reaction force of the gas propels the rocket forward.

To obtain a simple equation of motion for a rocket, let us assume that the gas particles ejected by the rocket engine all have the same exhaust velocity, u, relative to the rocket, and move in an exactly backward direction. We can use the law of conservation of momentum to obtain the equation of motion.

During motion, and the ejection of gases from nozzle of the rocket, the mass remaining in the rocket decreases. At a certain moment of flight:

The mass of rocket and fuel $= m_o$,
Its vertical velocity at this moment $= u$, and
Its momentum $= mu$

After an interval of time dt, the rocket mass decreased by dm, the mass of the fuel used. The relative velocity of the ejected gases is u_n relative to the rocket.

The velocity of gas molecules relative to Earth is:

$$u' = u - u_n$$

The momentum of the gas molecules in the time dt is:

$$u' \, dm = (u - u_n) \, dm$$

At the end of the time dt the rocket mass is (m-dm), and its velocity (u+du). The momentum of the rocket at the end of the interval is (m-dm)(u+du). The total momentum of the rocket and the ejected gases at the end of the time dt is $(m-dm)(u+du) + dm\,(u-u_n)$. But, from

Newton's second law: The force is equal to the rate of change of momentum, thus:

$$d(mu)/dt = - m g$$

The force balances the gravitational attraction of the Earth. Therefore,

$$- m g dt = [(m - dm)(u + du) + dm (u - u_n) - m u]$$

Thus: $$m (du/dt) = u_n (dm/dt) - m g$$

The rocket acceleration is:

$$du/dt = (u_n/m) (dm/dt) - g$$

The more the rocket gets upwards the less become the acceleration of gravity, g, and its mass decreases, but the rate (dm/dt) remains constant since it is the combustion rate.

In order to find the rocket velocity at any amount, we integrate the above equation, putting negative sign to dm/dt, because the mass decreases with time. Therefore,

$$m (du/dt) = - u_n (dm/dt) - m g$$

$$\int du = - u_n \int dm/m - g \int dt$$

Therefore, $$u = u_n \ln (m_o/m) - g t$$

$$[\int_{m_o}^{m} dm = \ln (m/m_o) = - \ln (m_o/m)]$$

C. OFF THE PLANET AROUND THE SOLAR SYSTEM

C.1. Gravitational field and gravitational potential

Consider an object of mass m near the Earth's surface, a body experiences a gravitational force (mg) directed toward the center of the Earth. The gravitational field equals the gravitational force acting on a unit mass placed at that point. Consequently, the gravitational field that the object experiences at some point has a magnitude equal to the acceleration of gravity at that point, i.e.

$$g = \frac{F}{m} = G\,\frac{M_e}{r^2}$$

This expression is valid at all points outside the Earth's surface.

The potential energy of the object near the Earth's surface is usually denoted by

$$U = m\,g\,d$$

where d is the height of the object. This is only valid near the surface since the gravitational force between two bodies varies as $(1/r^2)$, thus the correct potential energy will depend on the amount of separation between the two objects.

The gravitational potential energy associated with a given displacement is defined as the negative of the work done by the gravitational force during that displacement, or

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\,dr$$

Thus, if a mass m is moved between two points above the Earth's surface then the gravitational potential will be:

$$U_f - U_i = -G M_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

If we consider that the body was brought from infinity ($r_i = \infty$) we obtain the important result

$$U(r) = -G \frac{M_e m}{r}$$

This equation applies to the Earth-particle system separated by a distance r (provided that r is greater than the Earth's radius). In general any two objects in space of masses m_1 and m_2 separated by a distance r will have a gravitational potential energy given by:

$$U = -G \frac{m_1 m_2}{r}$$

The potential energy is negative since the force is attractive and we have a zero potential energy when the particle separation is infinity (see Fig. 1.15).

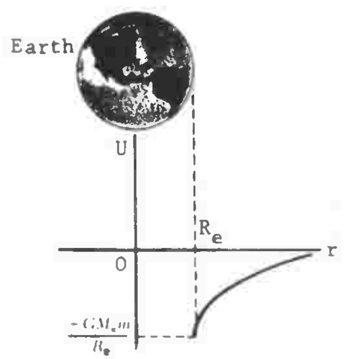


Fig. 1.15. Gravitational potential energy U versus distance r for a particle above the Earth's surface. The potential goes to zero as r approaches infinity.

C.2. Energy consideration in planetary and satellite motion

A satellite of mass m moving with a speed v in the vicinity of Earth of mass M_e will form a two-body system having a total energy equal to the sum of the kinetic energy of the mass of the satellite and the potential energy of the system, i.e.

$$E = K + U$$

If the satellite is orbiting Earth in a circular path of radius r then the energy of the system is:

$$E = \frac{1}{2} m v^2 - G \frac{M_e m}{r}$$

For a stationary orbit the total energy is necessarily negative, i.e. $E < 0$ for any system consisting of a mass m moving in a circular orbit about a body of big mass $M >> m$, like the case of Earth and Sun. Newton's second law applied to the body of mass m, gives:

$$G \frac{M_e m}{r^2} = \frac{m v^2}{r}$$

Therefore,
$$\frac{1}{2} m v^2 = G \frac{M_e m}{2 r}$$

The total energy of the bound system is thus

$$E = - G \frac{M_e m}{2 r}$$

Note that the kinetic energy is positive and equal to one half the magnitude of the potential energy. The absolute value of E is equal to the binding energy of the system.

C.3. Changing the orbit of a satellite

If we want to move an Earth satellite of mass m from a circular orbit of radius $2R_e$ (R_e is the Earth's radius) to one of radius $3R_e$, we use the equation of the total energy of the system. Apply it for the total initial and final energies:

$$E_i = - G \frac{M_e m}{4R_e} \quad ; \quad E_f = G \frac{M_e m}{6R_e}$$

The work required to increase the energy of the system is

$$W = E_f - E_i = -G\frac{M_e m}{6R_e} - [-G\frac{M_e m}{4R_e}] = G\frac{M_e m}{12R_e}$$

For example, if the satellite is of mass 1000 kg, then the work required will be $W = 5.2 \times 10^9$ joules, which is the energy equivalent of about 40 gallons of gasoline.

It should be noted that part of the work done goes into increasing the potential energy and part into decreasing the kinetic energy.

C.4. Projectiles and escape velocity

Gravity always pulls us down to the ground. However, if we consider the trajectory of a ballistic missile launched with a speed of about 8 km/s, the missile will not come back to the ground but will go in orbit around the Earth, see Fig. 1.16. If the speed is less than that, the trajectory will be a portion of an elliptical orbit cut short by impact on Earth.

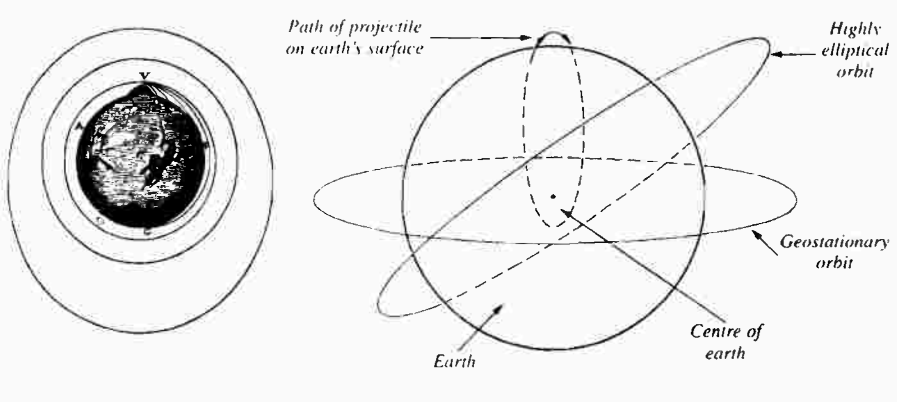


Fig. 1.16. Example of (elliptical) Earth orbits.

The connection between projectile motion and orbital motion could be visualized if we imagine that we fire a projectile from a launching place on a high mountain. If the firing velocity is fairly low, the projectile will arc toward the Earth and strike the near base of the mountain. The trajectory is usually considered as a segment of a parabola, but more precisely it is a segment off an ellipse. If we increase the firing velocity, the projectile will describe larger and larger arc. Finally, reaching a velocity of 8 km/s, the rate at which the trajectory curves down due to gravitational forces is precisely matched by the curvature of the surface of the Earth. The projectile never hits the Earth and keeps forever moving in a circular orbit. This example makes it clear that orbital motion is a free fall motion. The minimum value of the initial speed with which an object will escape the Earth's gravitational field is called the escape velocity.

We have see before that the energy of a two body system formed of a satellite orbiting the Earth is:

$$E = \frac{1}{2} m v_i^2 - G \frac{M_e m}{r_i} = \frac{1}{2} m v_f^2 - G \frac{M_e m}{r_f}$$

At the surface of the Earth, the firing velocity $v_i = v$ and $r_i = R_e$. When the object reaches its maximum altitude, then $v_f = 0$ and $r_f = r_{max}$.

Solving to get the initial firing velocity v_i,

$$v_i^2 = 2G M_e \left[\frac{1}{R_e} - \frac{1}{r_{max}} \right]$$

This expression gives the maximum altitude of the projectile, h, from the knowledge of $h = r_{max} - R_e$.

If is now possible to calculate the minimum speed the object must have at the Earth's surface in order to escape from the influence of the Earth's gravity. This corresponds to the situation where the object can just reach infinity with a final speed of zero.

Putting: r_{max} = infinity and $v_i = v_{esc}$

We get $v = (2 G M_e / R_e)^{1/2}$

Note that the escape velocity is independent of the mass of the object projected from the Earth. Besides, if an object is given the escape velocity, then the total energy of the system formed of the object and the Earth will be zero. If v_i was greater than v_{esc} , the total energy will be greater than zero, and the object will have some residual energy.

C.5. Orbital velocity

When a satellite is put in a stationary orbit, the centripetal force on the satellite due to its circular motion will be exactly balanced by the gravitational pull by the Earth. Thus,

$$\frac{m\, v_{orb}^2}{R} = G\,\frac{M_e\, m}{R^2}$$

R being the radius of the orbital. Therefore,

$$V_{orb} = (G\, M_e / R)^{1/2}$$

We usually take the radius of the orbit as the radius of Earth, since the altitude of the orbiting satellite is usually very small compared to the Earth's radius.

Substituting with numbers in the escape and orbital velocities in the Earth's gravitational field, we get:

v_{esc} = 11.2 km/s and

v_{orb} = 8 km/s

The energy needed to launch a spacecraft of weight 5,000 kg:

1. in an orbit around the Earth:

$$E = \frac{1}{2}\, m\, v_{orb}^2 = 1.6 \times 10^{11} \text{ Joules}$$

2. to escape from Earth's gravity:

$$E = \frac{1}{2}\, m\, v_{esc}^2 = 3.14 \times 10^{11} \text{ Joules}$$

C.6. Masses and escape velocities for planets, moon and Sun

Planet	Mass (kg)	Escape velocity (km/s)
Mecury	3.18×10^{23}	4.3
Venus	4.88×10^{24}	10.3
Earth	5.98×10^{24}	11.2
Mars	6.42×10^{23}	5.0
Jupiter	1.9×10^{27}	60.0
Saturn	5.68×10^{26}	36.0
Uranus	8.68×10^{25}	22.0
Neptune	8.68×10^{26}	24.0
Pluto	1.4×10^{22}	1.1
Moon of Earth	7.36×10^{22}	2.3
Sun	1.99×10^{30}	618.0

C.7. The acceleration on the moon

The orbital period of the moon around the Earth is 27.32 days (the day = 2.36×10^6 s), and the mean distance of the moon from Earth, r_m, is $r_m = 3.84 \times 10^8$ m. Due to the circular motion of the moon around the Earth, there exists a centripetal acceleration, a_m, equal to:

$$a_m = v^2 / r_m$$

where v is the moon's velocity in its orbital and is equal to:

$$v = 2 p r_m / T$$

T being the period of moon's motion around the Earth. Thus,

$$a_m = \frac{(2 p r_m / T)^2}{r_m} = 2.72 \times 10^{-3} \quad m/s^2$$

C.8. Mass of the Sun

Kepler's third law predicted that the square of the orbital period of any planet is proportional to the cube of the semi-major axis of the elliptical orbit of the planet around the Sun. Consider planet Earth of mass M_e which is moving around the Sun in nearly a circular orbit of radius R_e (see Fig. 1.17).

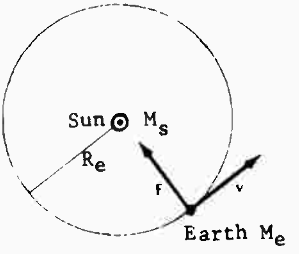


Fig. 1.17

Since the gravitational force on the planet is equal to the centripetal force needed to keep it moving in a stationary orbit, thus:

$$G \frac{M_s M_e}{R_e^2} = \frac{M_e v^2}{R_e}$$

M_s is the Sun's mass, and v is the orbital velocity of the Earth, and is given by:

$$v = 2 p R_e / T$$

where T is the period of Earth around the Sun which is equal to 3.156×10^7 seconds. Substituting in the above equation we get:

$$G \frac{M_s}{R_e} = \left(\frac{2 p R_e}{T}\right)^2$$

Therefore, $$T^2 = \left(\frac{4 p^2}{G M_s}\right) R_e^3$$

The mass of the Sun could thus be obtained knowing that the distance of Earth from the Sun $R_e = 1.496 \times 10^{11}$ m:

$$M_s = 4 p^2 (1.496 \times 10^{11} \text{ m})^3 / (6.67 \times 10^{-11}) (3.156 \times 10^7 \text{ s})^2$$

Therefore, $M_s = 1.99 \times 10^{30}$ kg

C.9. The heating of rockets by air friction

It is well-known that heat passes from one body to another by one or more of the following ways:

i. Convection
ii. Conduction
iii. Radiation

Convection is only known in liquids and gases because of the loose bonding of molecules together which allows the streaming motion of molecules caused by the difference in density of the parts which are at different temperatures.

In the process of conduction the heat energy diffuses through the body by the action of molecules possessing greater kinetic energy on those possessing less. This action takes place by means of the elastic binding forces between the atoms in the case of solids. Materials differ in their thermal conductivities, good conductors and insulators.

The motion on Earth of any kind of vehicle-automobile, ship, aircraft, etc. - is met always with frictional forces that opposes the motion. These frictional forces transform part of the mechanical energy of motion to heat energy. This effect becomes considerable and serious when the velocity of motion is greatly increased, as in the case of a moving rocket. The heat generated by air friction with the moving rocket in the dense parts of the Earth's atmosphere might completely damage the whole rocket. It is therefore necessary to shield thermally the outside surface of the rocket from the atmosphere. Highly insulating materials are used to cover the outer surface of the rocket to prevent it from being burnt out during its crossing of the air layer forming the Earth's atmosphere.

In order to decide which insulating material is best to shield the rocket, one has to determine the thermal properties of materials, particularly its thermal conductivity.

C.9.1. Thermal conductivity

If one part of a solid is heated, then a heat current will flow from the hot part to the cold part, defined as the amount of heat that passes by some given place in the solid per unit time. The temperature will drop as we go away from the hot part. The temperature distribution inside is determined by the temperature gradient, defined as the difference in temperature per unit length in any one direction along heat flow.

If we imagine a cube of material situated along a heat current, the faces of the cube are one meter apart, and have a temperature difference of one degree Kelvin, and if heat flows in a steady state through the cube at right angles to its faces, and none is lost from its sides, then the heat flow per unit area per second is numerically equal to the thermal conductivity of the material.

This definition leads to a general equation for the flow of heat through any parallel-sided slab of material, when no heat is lost from the sides of the slab. If the cross-sectional area of the slab is A, its thickness is L, and the temperature of its faces are T_1 and T_2, then the heat Q flowing through it per second is:

$$Q = K A (T_2 - T_1)/L$$

In terms of calculus this equation might be written as:

$$dQ/dt = - K A \, dT/dx$$

The temperature gradient dT/dx is negative since T decreases as x increases.

C.9.2. Measurement of the thermal conductivity of rocket shields

In measuring the thermal conductivity of an insulator, the difficulty is to get an adequate heat flow. The shield insulator is usually thin sheet, in order not to put much weight to the rocket. The insulator is heated by a steam chamber C, whose bottom is thick enough to contain a hole for a thermometer (Fig. 1.18). The specimen rests on a thick brass slab B, also containing a thermometer. To ensure good thermal contact, the adjoining faces C, D and B must be flat and clean. When the temperature have become steady, the heat passing from C through the insulator D escapes from B by radiation and convection. Its rate of escape from B is roughly proportional to the excess temperature of B over the room temperature, according to Newton's law of cooling. Thus at steady state, the rate of, heat loss is just equal to the rate of heat conduction through the insulating plate D.

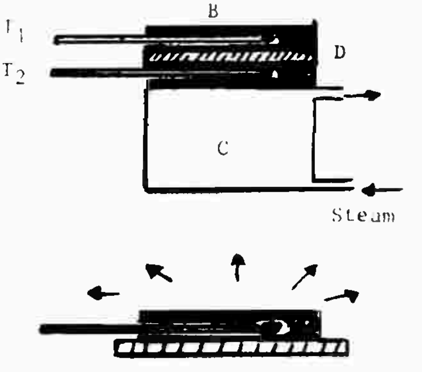


Fig. 1.18.

After the steady state is reached, the temperatures of the two thermometers T_1 and T_2 are taken. If x is the thickness of the insulating slab, then the temperature gradient in the material is: $(T_2 - T_1)/x$.

In order to find the rate of flow of heat, which is equal to the rate of loss of heat to the surroundings, we take away the specimen D and heat B directly from C until its temperature has risen by about 10 degrees above T_1. We then remove C and put it on a thick layer of insulating material like felt F. At intervals of a minute or less, we measure the temperature of B, and afterwards plot it against the time. Knowing the mass and specific heat of the brass slab B and the rate at which the temperature drops, we find the rate of heat lost which is the same as the rate of heat conducted through the insulator.

$$\text{Rate of heat flow} = dQ/dt = M\, c\, dT/dt$$

where M is the mass of brass disc, c is its specific heat and dT/dt is the temperature fall per second. If A is the cross-sectional area of the specimen, and K is its thermal conductivity, then

$$K\, A\, (T_2 - T_1)/x = M\, c\, dT/dt$$

Thus K can be calculated.

Problem:

Find the rate of heat transfer through the bottom of a kettle from a hot plate to the water in it, given that the water is boiling at 100 °C and the temperature of the hot plate is 101.2 °C. The bottom of the kettle consists of an inner layer of stainless steel 0.05 cm thick weld to an out layer of copper 0.03 cm thick. The area of the bottom of the kettle is 300 cm^2. K_{Cu} = 92 cal/s.m.°C and K_{Fe} = 11 cal/s.m.°C.

Hint: Note that heat flow in the copper and in the steel must be the same.

(Answer: 7.4 x 10^2 cal/s).

C.9.3. Radiation of heat in free space

After crossing the air atmosphere and reaching the free space, the rocket will not be heated by air friction but only by the heat radiation coming from the Sun. In heat radiation the heat is carried from place to place by electromagnetic waves. Heat radiation, which is essentially infrared waves, is transferred through vacuum and does not need any medium for its propagation as the case of conduction or convection.

Radiant heat coming from the Sun will heat extensively that side of the rocket facing the Sun. So, if one face of the rocket is left facing the Sun always, the heating effect might damage the rocket. Accordingly, the rocket is made to spin about its axis in order to cool the hot face while being opposite to the Sun.

C.9.4. The black body radiation

When we heat a body to high temperature, it first becomes red in color, then it glows with white light when the temperature is sufficiently raised. If we analyse the emitted light with a prism, we find that the spectrum of thermal radiation emitted is continuous and the energy is smoothly distributed over all wavelengths. Fig. 1.19 shows the distribution of intensity in a hot body heated at the given temperature.

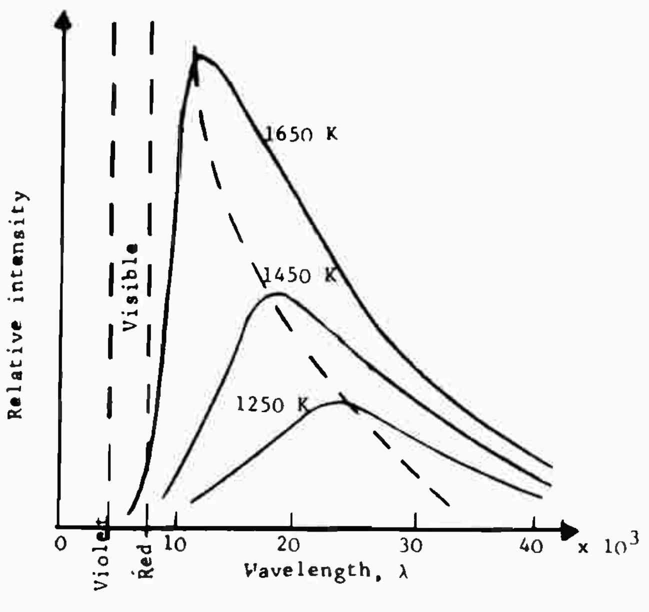


Fig. 1.19.

The thermal radiation emerging from the surface of a glowing body is generated within the volume of the body by the random thermal motions of the atoms and electrons maintained in equilibrium at any particular temperature. This equilibrium shapes the continuous spectrum of radiation.

The flux of thermal radiation emerging from the surface of a glowing body depends to some extent on the characteristics of the surface. A good absorber is a good emitter; and a poor absorber is a poor emitter. The body with a perfectly absorbing (and emitting) surface is called a black body.

C.9.5. Energy quanta

The theoretical derivation of the black body spectrum was first tried by Lord Rayleigh on classical grounds, but with no success. The correct formula for the spectral emittance of a black body radiator was proposed by Max Plank in 1900 as an empirical law that fits in the experimentally measured points. But afterwards he gave theoretical justification of this law. Plank stated that the spectral emittance S_l, namely, the energy flux (or power per unit area) emitted by the surface of the glowing body per unit wavelength interval, is given by:

$$S_\lambda = (2\, p\, c^2\, h / \lambda^5)\, (1 / e^{hc/kT\lambda} - 1)$$

where h is Plank's constant ($h = 6.63 \times 10^{-34}$ J/s) and l is the wavelength.

Calculated values of spectral emittance agreed very precisely with the measured values.

In proving this theory, Plank made two bold assumptions concerning the nature of atomic vibrations as harmonic oscillators:

1. An oscillator of frequency f will have discrete values of energy, namely, E = 0, hf, 2hf, 3hf, ··· All other values of energy are forbidden. h is Plank's constant.

2. A vibrating atom or molecule emits or absorbs energy in discrete units of energy called quanta, or photons, as they are now called. The energy is therefore:

$$E_n = n\, h\, f$$

where n is a positive integer called Quantum Number. The energy of the harmonic oscillator is said to be quantized.

For an oscillator of frequency $f = 10^{15}$ c/s, which is typical for atomic vibrations, the energy quantum is $hf = 6.6 \times 10^{-19}$ J. Since this is a very small amount of energy, quantization does not make itself felt at a macroscopic level. But quantization plays a very important role at the atomic level.

Quantization of energy makes no sense in classical physics. There is nothing in the laws of Newton that would prevent an oscillator from acquiring energy in any amount whatsoever. Although Plank could not justify his postulates, yet it formed a good start for the development of quantum mechanics.

C.9.6. Wien's displacement law

From Plank's formula one can show that the spectral emittance of a black body has a maximum at a wavelength given by

$$\lambda_{max} . T = 2.9 \times 10^{-3} \text{ m.K}$$

Experimental results showed that the radiant energy varies with wavelength and temperature as shown in Fig. 1.19. As the temperature of the black body increases, the total amount of energy it emits increases. Also, with increasing temperatures, the peak of the distribution shifts to shorter wavelengths. This shift was found to obey the above relationship, called Wien's displacement law. Any body of unknown temperature could be tested in this way. If its spectral distribution curve is known and the peak position determined, then using Wien's displacement law its temperature could be determined.

This is the same method with which we predicted that the universe has a temperature 3 Kelvin. Recently, the large telescopes deviced for picking any messages coming from the outer space, and are sent by intelligent beings like human beings, these telescopes plotted spectral distribution of long wireless waves coming from everywhere around us. The peak of this spectral distribution predicted that the thermal radiation that is filling the universe. and which still remained after the Big Bang and the creation of the universe, this radiation has a temperature of three degrees Kelvin.

C.9.7. Stefan's radiation law

By integrating Plank's formula one obtains the total flux or power per unit area, emitted by the black body at all wavelengths. One can show that the total energy radiated per meter square per second from a body of temperature T is proportional to the fourth power of the temperature,

$$E = \sigma T^4$$

where s is called Stefan's constant (= 5.67×10^{-8} $W/m^2.K^2$). This law is Stefan-Boltzmann radiation law. Astronomers sometimes determine the size or the temperature of a star by a method that relies on this law.

Example 1.2:

Determine the radius of the star Capella from the following data: Radiant energy reaching Earth from the star is 1.2×10^{-8} W/m^2,.the distance of the star is 4.3×10^{17} m, and its surface temperature is 5200 K.

Solution:

Applying Stefan-Boltzmann's law, the energy radiated from the surface of the star is (σT^4). The surface area of the star is ($4 \pi R^2$).

$$\text{The total emitted power} = (4 \pi R^2)(\sigma T^4)$$

This power is spread over a sphere of radius 4.3×10^{17} m centered on the star. Since at each point of this sphere the power per unit area is 1.2×10^{-8} W/m^2 . Then, the total power is

$$4 \pi (4.3 \times 10^{17})^2 (1.2 \times 10^{-8}) = 2.8 \times 10^{28} \text{ W}$$

From the above equality we obtain

$$(4 \pi R^2)(\sigma T^4) = 2.8 \times 10^{28}$$

Therefore,

$$R = 7.3 \times 10^9 \text{ m}$$

This is about 10 times the radius of the Sun.

Example 1.3:

Thermal radiation from the human body

The temperature of skin is about 35 °C. What is the wavelength at which the peak occurs in the radiation emitted from the skin?

Solution:

From Wien's displacement law:

$$\lambda_{max} \cdot T = 2.9 \times 10^{-3} \text{ m.K}$$

Since T for skin is 308 K, then $\lambda_{max} = 940\ \mu m$.

Example 1.4:

What is the energy carried by a quantum of light (yellow) whose frequency is 6.0×10^{14} Hz?

Solution:

The energy E is given by:

$$E = hf = 6.6 \times 10^{-34} \times 6.0 \times 10^{14} = 3.98 \times 10^{-19} \text{ J} = 2.45 \text{ eV}$$

C.10. Motion at high speeds

C.10.1. Theory of special relativity

Newton's laws of motion are equally valid in every inertial reference frame. Newtonian mechanics were formulated to describe the motion of objects in space and time. This formulation works very well at low speeds but fails when applied to bodies that have speeds approaching that of light.

In 1905, Einstein published his special theory of relativity which was based on two basic postulates:

1. The laws of physics are the same in all inertial systems.
2. The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s, in all inertial reference frames. The value of c is independent of the motion of the observer or the motion of the light source.

Special theory of relativity covers phenomena such as the time dilation and the slowing down of clocks, and the contraction of length in moving reference frames as detected by a stationary observer. This theory also discusses the relativistic forms of momentum and energy providing the famous mass-energy equivalence relation, namely,

$$E = m c^2$$

C.10.2. The contradiction with Galilean transformation

According to the principle of Newtonian relativity, the laws of mechanics are the same in all inertial frames of reference. Consider two inertial systems S and S' (see Fig. 1.20). The system S' (the rocket for instance) is moving with a constant velocity v along the xx' axis, where v is measured relative to the system S. Assume that an event occurs at the point P and that the origins of S and S' coincide at t = 0, (the rocket is on the Earth), the frame S is represented by the Earth.

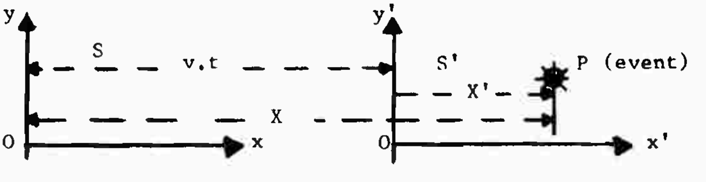


Fig. 1.20. An event occurs at P in the moving frame S'.

An observer on the Earth would describe the event with space-time coordinates (x,y,z,t), while the astronaut on the spaceship would use (x',y',z',t') to describe the same event. As we can see from the figure, these coordinates are related by the equations:

$$x = x - vt \quad ; \quad y = y \quad ; \quad z' = z \quad ; \quad t' = t$$

These equations are known as the Galilean transformations of coordinates. Note that the time is assumed to be the same in both coordinates, i.e. clocks are universal and time is absolute.

If the astronaut in the frame of reference (rocket) S' sent a pulse of light from a light battery, then according to the Newtonian relativity, the speed of the light pulse should be (c + v) relative to the observer on Earth (frame S). A paradox concerning the constancy of velocity of light immediately arises. Maxwell's equations in electromagnetic theory predicts that the velocity of light has always the fixed value of:

$$c = (\mu_0 \varepsilon_0)^{-1/2} = 3.0 \times 10^8 \text{ m/s}$$

μ_0 and ε_0 being the permeability and permittivity of space. This was the same postulate given by Einstein in his theory. This is in contradiction with what one would expect based on the Galilean addition law for velocities. According to this law the velocity of light should not be the same in all inertial frames.

In order to resolve this paradox, we have to consider that either:

1. the Galilean addition law for velocities is incorrect, or
2. the laws of electricity and magnetism, Maxwell's laws, are not the same in all inertial frames.

If the Galilean transformations were incorrect, then we should abound on the absoluteness of time and of length that form the basis of the Galilean transformations. This should be the case because Maxwell's equations are derived for electromagnetic waves that propagate in free space and do not need any medium for its propagation.

We thus conclude that the invariance of the speed of light conflicts with the Galilean law for the addition of velocities. We should throw this law and change our ideas about space and time, by considering the new Lorentzian transformations as predicted by the theory of relativity.

C.10.3. The Lorentz transformations

In relativistic physics we must construct a new set of transformation equations which take into account the relativity of time and length. The Lorentz transformations express the fundamental characteristics of relativistic space and time, i.e. the geometry of space time diagram which is called Minkowski world. These transformations have the property of keeping the speed of light the same in all reference frames, on Earth as well as on the spaceship.

If we return back to the two frames S and S' representing a rocket moving with speed v above the Earth, then an observer on Earth[7] will report the coordinates of an event on the rocket as (x,y,z,t), while the observer on the spaceship will report the same event using the coordinates (x',y',z',t') according to the following Lorentz transformations:

$$x' = \gamma(x - vt) \quad ; \quad y' = y \quad ; \quad z' = z \quad ; \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

where
$$\gamma = 1/\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

C.10.4. Time dilation

In order to show the relativity of time, we consider a spaceship moving with a velocity v above the Earth. An astronaut in the ship, observer O', operates a laser source of light to reflect on a plane mirror fixed on the ceiling distant, d, from him. The time taken by the light pulse to travel to the mirror and come back is (see Fig. 1.21):

$$\Delta t' = 2\, d/c$$

where c is the velocity of light.

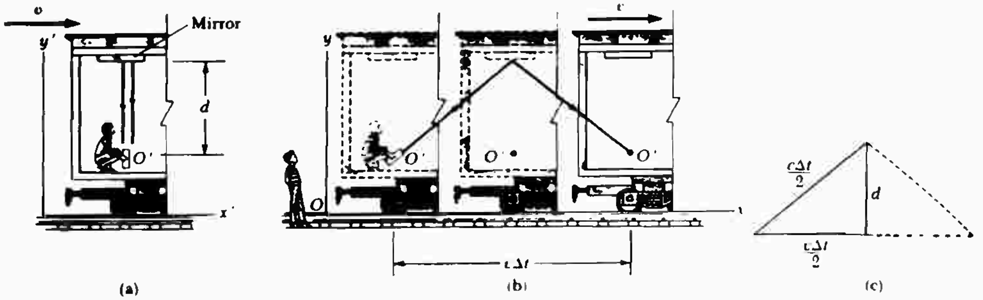


Fig. 1.21

Now, consider an observer, O, on Earth who is able to see the same incident of light pulse coming out of the light battery and coming back to the astronaut. The observer on Earth will see the path of light as shown in the figure, because of the motion of the spaceship.

The distance moved by light going and back $= c \cdot \Delta t$

To the observer on Earth the distance moved by the spaceship
$= v \cdot \Delta t$

Using the right triangle in the figure to get the relation between Δt and $\Delta t'$, we find from simple geometry:

$$(c \cdot \Delta t/2)^2 = (v \cdot \Delta t)^2 + d^2$$

Solving for Δt:

$$\Delta t = \frac{2d}{c\,(1 - v^2/c^2)^{1/2}}$$

But $\Delta t' = 2\,d/c$

Thus: $$\Delta t = \frac{\Delta t'}{(1 - v^2/c^2)^{1/2}} = \gamma \cdot \Delta t'$$

This equation shows that the time of the same event as measured on Earth, Δt, is not equal to the time measured on another frame of reference, $\Delta t'$, i.e. on the spaceship. The time measured in a stationary frame is longer than that measured in the moving frame.

It is thus concluded that according to a stationary observer, a moving clock runs slower than an identical stationary clock. This effect is called **time dilation**.

Time dilation is now a very real phenomenon that has been verified by various expeiments using high energy elementary particles.

C.10.5. Length contraction

We have seen that time is not absolute, that is, the time interval between two events depends on the frame of reference in which it is measured. Similarly, the length of an object measured in a reference frame in which the object is moving is always less than its length measured in the reference frame in which the object is at rest. This effect is called **length contraction.**

Consider a stick viewed by an observer in a frame attached to the stick, i.e. both have the same velocity. Let this stick be seen by an observer in a frame in which the stick has a velocity v relative to the frame. The length of the stick, L, is shorter than L', as shown in Fig. 1.22.

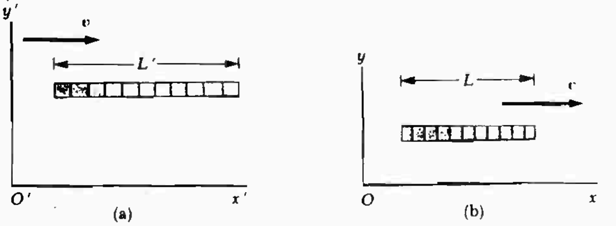


Fig. 1.22. The stick is seen shorter relative to a rest frame.

The real length is obtained when both stick and frame are moving with the same velocity. Therefore,

$$L_{real} = v \cdot \Delta t$$

Now, if only the stick is moving and the frame of reference is at rest, then its new length will be:

$$L' = v \cdot \Delta t'$$

But we have see that:

$$\Delta t = \gamma \cdot \Delta t'$$

Thus:

$$L_{real} = L' (1 - v^2/c^2)^{1/2}$$

The stick will appear shorter than its rest length by the factor:

$$1/\gamma = (1 - v^2/c^2)^{1/2}$$

It should be noted that the length contraction takes place only along the direction of motion.

C.10.6. Velocity of celestial bodies and the red shift

The velocity of distant galaxies and stars relative to us can be determined by the red shift of the light coming to us from them. The red shift is based on the well known Doppler effect that occurs when a source of waves (sound or light) is moving away from an observer. A typical example is a train sounding its siren and is moving away from an observer. The frequency of the siren sound appears decreased by the Doppler effect. Correspondingly, the wave length increases. The magnitude of the wavelength shift toward the larger wavelengths depends on the recession velocity of the sounding body.

If c is the velocity of waves, and v is the velocity of the receding sounding body, then the frequency, f_o, of the radiating source will be observed, f, according to the equation:

$$f = f_o / (1 + v/c)$$

The increase in wavelength will thus be:

$$\Delta\lambda/\lambda_o = -v/c$$

This equation is valid on condition that the velocity of the body is small compared to the velocity of light in case of light sources, and to the velocity of sound in case of sound sources.

This Doppler effect applies in the case of light sources as in sound. But, because the velocities of the celestial bodies emitting light waves are very large, so relativistic effects must be considered. According to the relativity theory, the emitter is subject to a relativistic time dilation effect. This in itself would reduce the frequency of the emitter by a factor β, where

$$\beta = (1 - v^2/c^2)^{1/2}$$

Inserting this factor in the above equation we get:

$$f = f_o (1 - v^2/c^2)^{1/2} / (1 + v/c)$$

This is exact formula for the Doppler shift of light, and it could be simplified as follows:

$$f = f_o \left[\frac{1 - v/c}{1 + v/c}\right]^{1/2}$$

For low speeds ($v << c$), this formula could be approximated to the equation:

$$\Delta\lambda/\lambda_o = -v/c$$

<u>**Example 1.5**</u>:

The Doppler shift in the light coming from the Quasar 3C 147 amounts to a factor of 1.55 . We can thus get the recession velocity as follows:

$$\frac{f}{f_o} = \frac{1}{1.55} = \left[\frac{1 - v/c}{1 + v/c}\right]^{1/2}$$

From which the velocity of the Quasar is:

$$v = 0.41 \, c$$

i.e. 41% of the velocity of light.

C.11. The man stepping on the moon

Satellites launched in circular orbits are very useful for communications and Earth monitoring purposes. In such cases the kinetic energy remains constant. But if we want to use the spacecraft to probe space, e.g. to visit the moon or another planet.

As the spacecraft recedes from Earth, traveling against the pull of the Earth's gravity, the gravitational potential energy will be increasing. When the spacecraft engines are turned off (coasting), its total energy must remain constant. Thus the kinetic energy will continually decrease until it comes close to the moon or another planet, the reverse will happen, its kinetic energy will start to increase again at the expense of the gravitational potential energy, i.e. the loss in kinetic energy (K_a - K_b) is equal to the gain in potential energy (U_b - U_a) between the two points a and b.

In the case of Apollo's (11) flight carrying astronauts to the surface of the moon, during the coasting phase with the engines off, Apollo's distance from the Earth's center increased from 2.63×10^7 m to 2.09×10^8 m while its speed decreased from 5374 m/s to 1532 m/s. Its mass was about 5.6×10^3 kg and so the kinetic energy decreased by:

$$\frac{1}{2} (v_a^2 - v_b^2) = 7.4 \times 10^{10} \text{ Joules}$$

Coasting is not entirely free from external influences. Although there is no significant resistance in space, there is still significant gravity unless the space craft is extremely far from all large bodies such as Earth, Sun and moon. Gravity has a very long range effect. The spacecraft to the moon actually slowed during the first part of its journey because of Earth's gravity, and then speed up during the last part because of the pull of the moon's gravity.

As Apollo (11) proceeded toward the moon, its speed increased but safe landing on the moon's surface was established by using the retro-rocket engines to oppose the moon's gravity and at last the first man stepped on the moon's surface, (see Fig. 1.23).

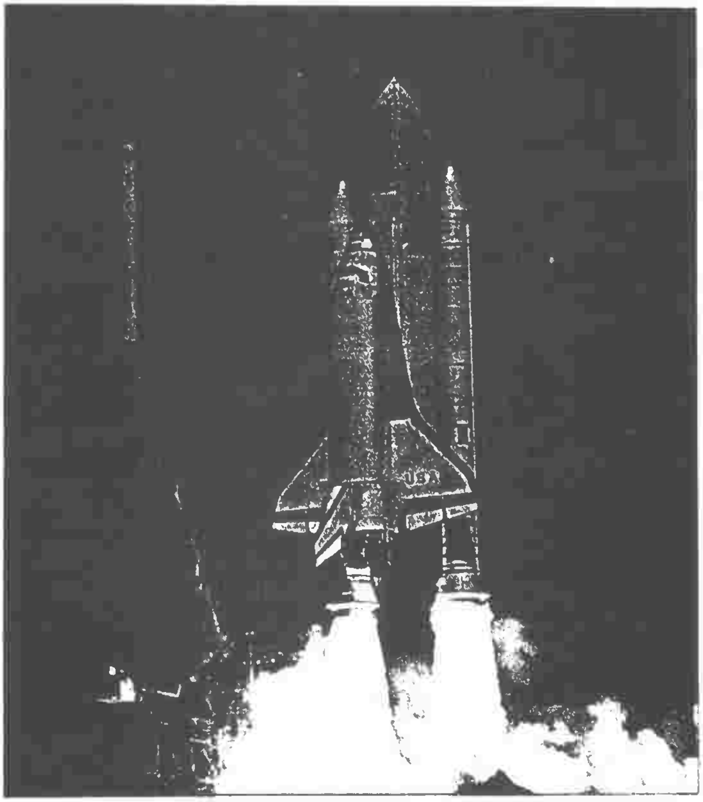


Fig. 1.23. shows a US shuttle after being launched vertically.

C.12. Testing the moon rocks by alpha-particle scattering

In his search for the atomic structure, Thomson proposed the following picture: An atom consists of a number of electrons, Z, embedded in a cloud of positive charge. The positive charge in the cloud is Ze, so that it exactly neutralizes the negative charge -Ze of the electrons. This model was a failure because it could not explain the observed spectral lines and the spectral series.

In 1910, Rutherford presented the nuclear atomic model. The mass of the atom is not spread out over a cloud, but it is concentrated in a small nucleus at the center of the atom. The electrons move around this nucleus. Rutherford arrived at this picture of the atom from his famous alpha-particle scattering experiment.

Thin foils of gold or silver were used as targets for a beam of alpha particles from a radioactive source. After passing through the foil the particles were detected on a zinc sulphide screen which registers the impact of each particle by a faint scintillation (see Fig. 1.24). Some of the alpha particles were deflected by such a large angle that they came out backwards. Rutherford recognized that this rebound backward could not happen unless the particle has struck a very small massive nucleus inside the atom. Thus he proposed his nuclear model implying that an atom consists of a small nucleus of charge Ze containing almost all the mass of the atom; this nucleus is surrounded by a negatively charged cloud of Z electrons.]

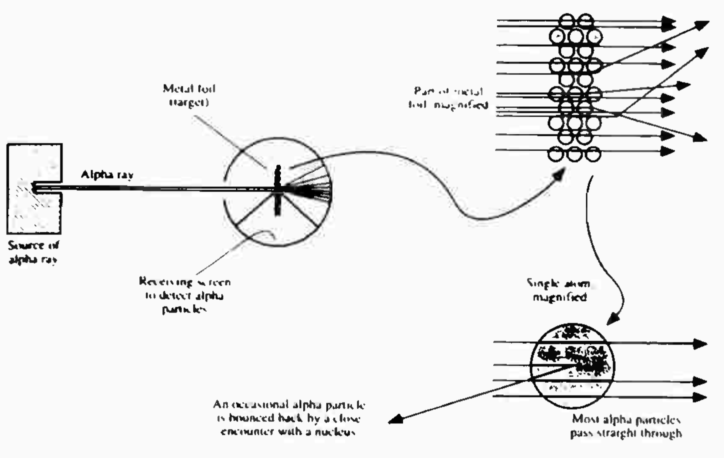


Fig. 1.24. Trajectory of alpha particles.

Based on his nuclear model of the atom, Rutherford calculated what fraction of the beam of alpha partiles should be deflected, and through what angle. If an alpha particle passes close to the nucleus, it will experience a large electric repulsion and it will be deflected by a large angle. If it passes far from the nucleus, it will be defleced by a small angle (see Fig. 1.24). The number of particles rebounding in a certain direction depends on the atomic number Z of the atoms causing this rebound.

This method was used to investigate the composition of the moon rocks. Rutherford's apparatus was launched smoothly on the moon surface, and the number of rebound particles was determined by counters. Using Rutherford's formula, the atomic number Z of the atoms responsible for the observed scattering was found, and consequently the elements of the moon rocks were identified.