Chapter 2

D-Q Model of Induction Machine

2.1 Introduction

Usually, when an electrical machine is simulated in circuit simulators like PSpice, its steady state model is used, but for electrical drive studies, the transient behavior is also important. One advantage of Simulink over circuit simulators is the ease in modeling the transients of electrical machines and drives and to include drive controls in the simulation. As long as the equations are known, any drive or control algorithm can be modeled in Simulink. However, the equations by themselves are not always enough; some experience with differential equation solving is required [12].

2.2 Induction Machine Model

2.2.1 Equations

The induction machine d-q or dynamic equivalent circuit in synchronously rotating reference frame is shown in Figure 2.1.

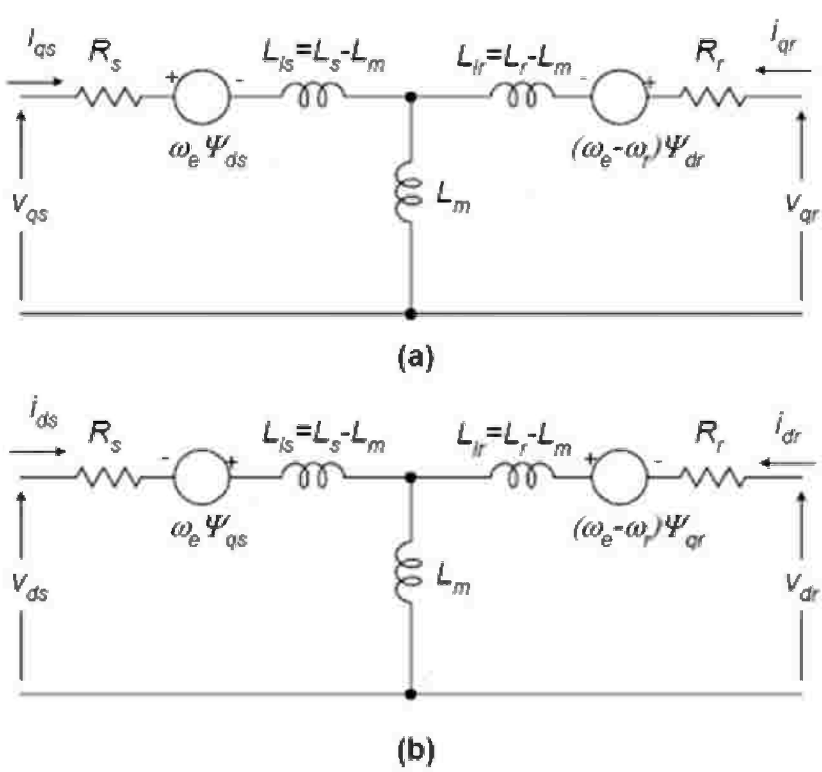

Figure 2.1 Induction machine model

One of the most popular induction motor models derived from this equivalent circuit is Krause's model detailed in [13-14]. According to his model, the modeling equations in flux linkage form are as follows:

$$\frac{dF_{qs}}{dt} = \omega_b \left[v_{qs} - \frac{\omega_e}{\omega_b} F_{ds} + \frac{R_s}{X_{ls}}\left(F_{mq} + F_{qs}\right)\right] \quad (2.1)$$

$$\frac{dF_{ds}}{dt} = \omega_b \left[v_{ds} + \frac{\omega_e}{\omega_b} F_{qs} + \frac{R_s}{X_{ls}}\left(F_{md} + F_{ds}\right)\right] \quad (2.2)$$

$$\frac{dF_{qr}}{dt} = \omega_b \left[v_{qr} - \frac{(\omega_e - \omega_r)}{\omega_b} F_{dr} + \frac{R_r}{X_{lr}}\left(F_{mq} - F_{qr}\right)\right] \quad (2.3)$$

$$\frac{dF_{dr}}{dt} = \omega_b \left[v_{dr} + \frac{(\omega_e - \omega_r)}{\omega_b} F_{qr} + \frac{R_r}{X_{lr}}\left(F_{md} - F_{dr}\right)\right] \quad (2.4)$$

$$F_{mq} = X_{ml}^{*}\left[\frac{F_{qs}}{X_{ls}} + \frac{F_{qr}}{X_{lr}}\right] \quad (2.5)$$

$$F_{md} = X_{ml}^{*}\left[\frac{F_{ds}}{X_{ls}} + \frac{F_{dr}}{X_{lr}}\right] \quad (2.6)$$

$$i_{qs} = \frac{1}{X_{ls}}\left(F_{qs} - F_{mq}\right) \quad (2.7)$$

$$i_{ds} = \frac{1}{X_{ls}}\left(F_{ds} - F_{md}\right) \quad (2.8)$$

$$i_{qr} = \frac{1}{X_{lr}}\left(F_{qr} - F_{mq}\right) \quad (2.9)$$

$$i_{dr} = \frac{1}{X_{lr}}\left(F_{dr} - F_{md}\right) \quad (2.10)$$

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)\frac{1}{\omega_b}\left(F_{qs} i_{qs} - F_{qs} i_{ds}\right) \quad (2.11)$$

$$T_e - T_L = J\left(\frac{2}{P}\right)\frac{d\,\omega_r}{dt} \quad (2.12)$$

Where

d: Direct axis,

q: Quadrature axis,

s: Stator variable,

r: Rotor variable,

F_{ij}: is the flux linkage (i =q or d and j=s or r),

v_{qs}, v_{ds}: q and d–axis stator voltages,

v_{qr}, v_{dr}: q and d–axis rotor voltages,

F_{mq}, F_{md}: q and d–axis magnetizing flux linkages,

R_r: Rotor resistance,

R_s: Stator resistance,

X_{ls}: Stator leakage reactance,

X_{lr}: Rotor leakage reactance,

$X_{ml}{}^{*} = \frac{1}{\frac{1}{X_m}+\frac{1}{X_{ls}}+\frac{1}{X_{lr}}}$

i_{qs}, i_{ds}: q and d–axis stator currents,
i_{qr}, i_{dr}: q and d–axis rotor currents,
J: Moment of inertia,
T_e: Electrical output torque,
T_L: Load torque,
ω_e: Stator angular electrical frequency,
ω_b: Motor angular electrical base frequency, and
ω_r: Rotor angular electrical speed.

For the induction machine represented with these equations, v_{qr} and v_{dr} in (2.3) and (2.4) are set to zero. An induction machine model can be represented with five differential equations. To solve these equations, they have to be rearranged in the state-space form, $\dot{x} = Ax + b$ where $x = [F_{qs}\ F_{ds}\ F_{qr}\ F_{dr}\ \omega_r]^t$ is the state vector. Note that $F_{ij} = \psi_{ij}\,\omega_b$, where ψ_{ij} is the flux. In this case, state-space form can be achieved by inserting (2.5) and (2.6) in (2.1→2.4) and collecting the similar terms together so that each state derivative is a function of only other state variables and model inputs[15]. Then, the modeling equations (2.1→2.4 and 2.12) of a squirrel cage induction motor in state-space become:

$$\frac{dF_{qs}}{dt} = \omega_b\left[v_{qs} - \frac{\omega_e}{\omega_b}F_{ds} + \frac{R_s}{X_{ls}}\left(\frac{X_{ml}{}^{*}}{X_{lr}}F_{qr} + \left(\frac{X_{ml}{}^{*}}{X_{ls}} - 1\right)F_{qs}\right)\right] \tag{2.13}$$

$$\frac{dF_{ds}}{dt} = \omega_b\left[v_{ds} + \frac{\omega_e}{\omega_b}F_{qs} + \frac{R_s}{X_{ls}}\left(\frac{X_{ml}{}^{*}}{X_{lr}}F_{dr} + \left(\frac{X_{ml}{}^{*}}{X_{ls}} - 1\right)F_{ds}\right)\right] \tag{2.14}$$

$$\frac{dF_{qr}}{dt} = \omega_b\left[-\frac{(\omega_e-\omega_r)}{\omega_b}F_{dr} + \frac{R_r}{X_{lr}}\left(\frac{X_{ml}{}^{*}}{X_{ls}}F_{qs} + \left(\frac{X_{ml}{}^{*}}{X_{lr}} - 1\right)F_{qr}\right)\right] \tag{2.15}$$

$$\frac{dF_{dr}}{dt} = \omega_b\left[\frac{(\omega_e-\omega_r)}{\omega_b}F_{qr} + \frac{R_r}{X_{lr}}\left(\frac{X_{ml}{}^{*}}{X_{ls}}F_{ds} + \left(\frac{X_{ml}{}^{*}}{X_{lr}} - 1\right)F_{dr}\right)\right] \tag{2.16}$$

$$\frac{d\omega_r}{dt} = \frac{P}{2J}\left(T_e - T_L\right) \tag{2.17}$$

2.2.2 Simulation

The inputs of the induction machine are the three-phase voltages, their fundamental frequency, and the load torque. The outputs, on the other hand, are the three phase currents, the electrical torque, and the rotor speed. The d-q model requires that the entire three-phase variables have to be transformed to the two-phase synchronously rotating frame. Consequently, the induction machine model will have blocks transforming the three-phase voltages to the d-q frame and the d-q currents back to three-phase. The induction machine model implemented in this chapter is shown in Figure 2.2 [13-14]. It consists of: abc-syn conversion, syn-abc conversion, and the induction machine d-q model blocks. The following subsections will explain each block.

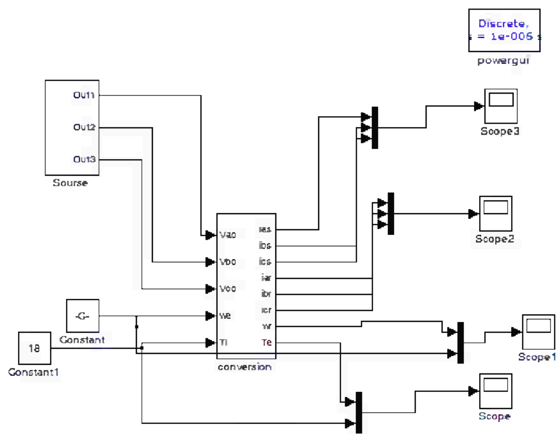

Figure 2.2 Induction machine simulations

2.2.2.1 Conversion Block

The block is shown in Figure 2.3. It contains three different blocks. It will be explained one after the other.

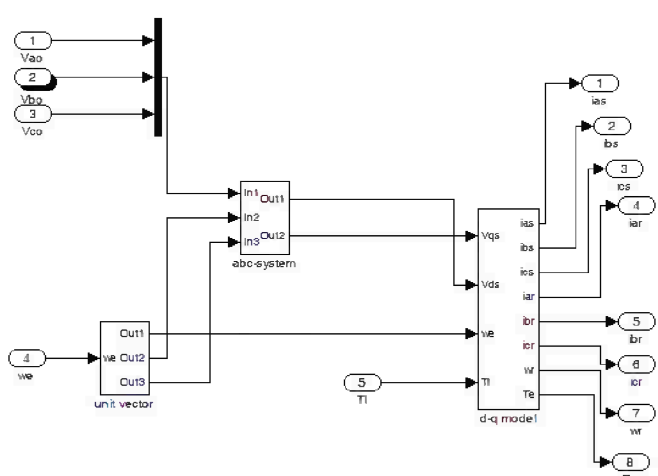

Figure 2.3 Conversion block

2.2.2.2 abc System Block

Unit vectors $\cos\theta_e$ and $\sin\theta_e$ are used in vector abc system block and d-q model block. The angle θ_e is calculated directly by integrating the frequency of the input three-phase voltages ω_e

$$\theta_e = \int \omega_e dt \qquad (2.18)$$

Also the rotor angle θ_r is calculated directly by:

$$\theta_r = \int \omega_r dt \tag{2.19}$$

To convert three-phase voltages to voltages in the two phase synchronously rotating frame, they are first converted to two-phase stationary frame using (2.20) and then from the stationary frame to the synchronously rotating frame using (2.21)

$$\begin{bmatrix} V^s{}_{qs} \\ V^s{}_{ds} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \tag{2.20}$$

Where the superscript "s" refers to stationary frame.

$$\begin{Bmatrix} V_{qs} = V^s{}_{qs} * \cos\theta_e - V^s{}_{ds} * \sin\theta_e \\ V_{ds} = V^s{}_{ds} * \cos\theta_e + V^s{}_{ds} * \sin\theta_e \end{Bmatrix} \tag{2.21}$$

Also the stator and rotor currents in stationary reference frame are given by:

$$\begin{Bmatrix} i^s{}_{qs} = i_{qs} * \cos\theta_e + i_{ds} * \sin\theta_e \\ i^s{}_{ds} = -i_{qs} * \cos\theta_e + i_{ds} * \sin\theta_e \end{Bmatrix} \tag{2.22}$$

$$\begin{Bmatrix} i^s{}_{qr} = i_{qr} * \cos(\theta_e - \theta_r) + i_{dr} * \sin(\theta_e - \theta_r) \\ i^s{}_{dr} = -i_{qr} * \cos(\theta_e - \theta_r) + i_{dr} * \sin(\theta_e - \theta_r) \end{Bmatrix} \tag{2.23}$$

The three-phase stator currents $[i_A\ i_B\ i_C]$ and three-phase rotor currents $[i_a\ i_b\ i_c]$ are given by:

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i^s{}_{qs} \\ i^s{}_{ds} \end{bmatrix} \tag{2.24}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i^s{}_{qr} \\ i^s{}_{dr} \end{bmatrix} \tag{2.25}$$

2.2.2.3 D-Q Model Block

Figure 2.4 shows the inside of this block where each equation from the induction machine model is implemented in a different block. First consider the flux linkage state equations because flux linkages are required to calculate all the other variables. These equations could be implemented using Simulink "State-space" block, but to have access to each point of the model, implementation using discrete blocks is preferred.

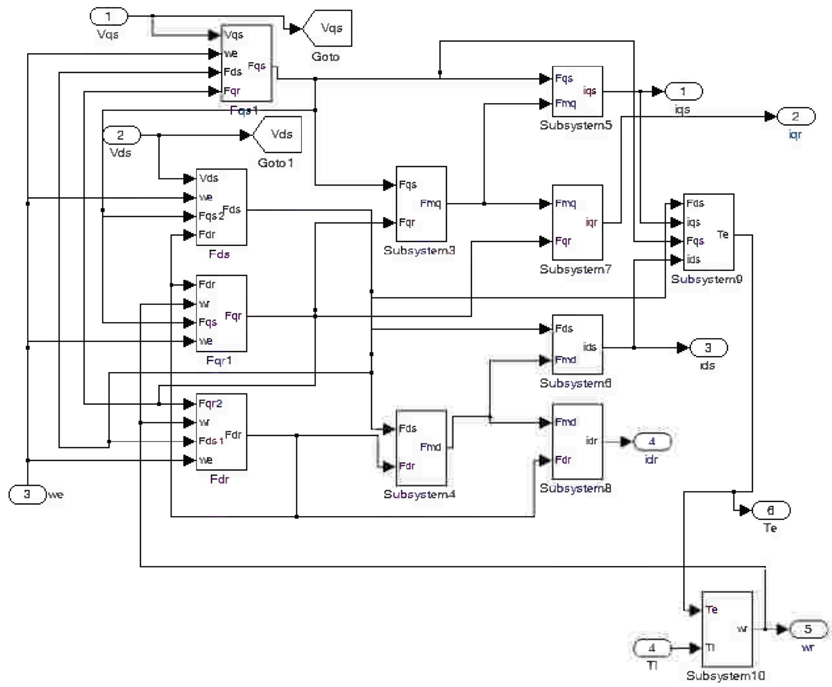

Figure 2.4 Dynamic model of Induction machine

2.2.3 Results

2.2.3.1 Initialization

To simulate the machine in Simulink, the Simulink model has to be initialized first so that it will know all the machine parameters. For this reason, an initialization file containing all the machine parameters is formed. This file assigns values to the machine parameter variables in the Simulink model. The following machine parameters are calculated from blocked rotor test, open circuit test and dc test for a 5-kW, 4-pole induction machine:

Rotor resistance R_r = 1.395-Ω Rotor leakage inductance L_{lr} = 0.005839-H

Stator resistance R_s = 1.0405-Ω Stator leakage inductance L_{ls} = 0.005839-H

Mutual inductance L_m=0.1722-H Inertia J = 0.0131-Kg.m^2

Stator to rotor effective turns ratio = 1.1/1.0

2.2.3.2 Scopes

The induction motor having the previous parameters is simulated by applying 400-V 50-Hz with load torque T_L = 18-Nm and $\omega_e = \omega_b$ = 314.16-rad/sec. Figure 2.5 show the waveforms of the three-phase currents of the stator and the rotor, electric torque, and rotor speed.

Calculations based on standard equivalent circuit for T_L = 18-Nm gives the following results:
Stator rms current = 5.92-A, Rotor rms current = 4.18-A, Rotor speed = 1460-rpm.
These calculations when compared with Figure 2.5 indicate complete similarity.

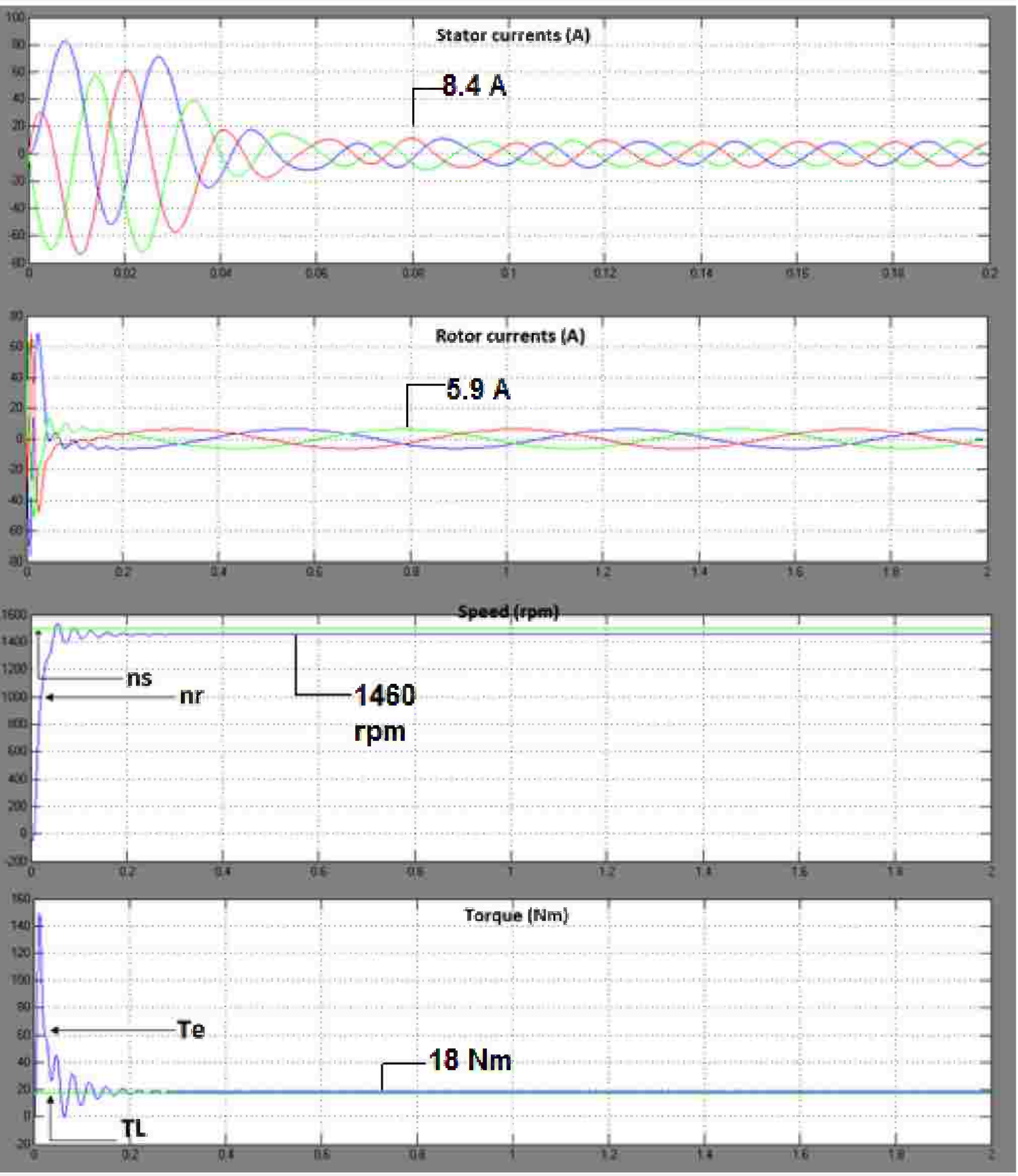

Figure 2.5 Performance Characteristics