Chapter 2

A Feedback-based Access Scheme for Cognitive Radio Networks Over Interference Channels with Primary Queue Guarantees

Providing wireless communication services is becoming more challenging due to spectrum scarcity problem; one technique to approach this problem is the cognitive radio technology in which the unlicensed user (or SU or cognitive user) is allowed to exploit unused spectrum by the licensed user (or PU) so that the spectrum utilization is improved and consequently the spectral efficiency increases [1], [4]. The primary user can use the channel at any time as long as it has a packet to transmit, while the coexistence of the secondary user with primary user is allowed provided that the secondary user does not violate some Quality of Service (QoS) requirements of the PU.

One of the cognitive radio scenarios is where the cognitive users sense the primary users activity and depending on the sensing information it takes the decision on whether to access the channel or not, and this is known as the *interweave model*. In this case, the cognitive users are not aware of their impact on the primary network which can be severe if there are large number of sensing errors. One solution, that can alleviate the sensing errors consequences to some extent, is to allow the secondary users to exploit the feedback sent from the primary receiver to primary transmitter and secondary users act based on the overheard feedback as introduced in [14], where the secondary users overhear the automatic repeat request (ARQ) [15]. In the model considered in [14] the PU has the authority to access the channel whenever it has a packet to transmit. The primary receiver sends an ACK over the feedback channel if the primary packet is successfully received. If the packet transmission fails, the primary receiver sends a NACK over the feedback link and consequently the primary user retransmits the packet again. In [17], a secondary transmission technique was introduced when the primary user retransmits the packet in order to manage the interference in retransmission-based wireless network.

In [18], a collision-based model with feedback exploitation was considered where the secondary user backs off completely from accessing the channel upon hearing a NACK to allow for collision-free primary retransmission while the secondary user attempts to access the channel if an ACK/no feedback is overheard. In [19], the secondary user power is controlled on the basis of the primary user feedback link.

In this chapter, we consider an interference-based model which can be thought of as a midway between the interweave model and the underlay model, in which the primary and secondary users coexist. It can also be thought of as a "hybrid", i.e., interweave/underlay [41]. The cognitive radio (CR) link inherits the channel sensing process from the interweave model, while co-existing with the PU is inherited from the underlay model. In our model, we allow the secondary user(s) to access the channel even if the primary user is either sensed to be active or known to be active through the overheard primary feedback, assuming NACK messages are received by the transmitting user with certainty. The secondary user(s) will have different access probabilities that depend on the PU state. The protection for the primary user will be provided by optimizing the secondary user access probabilities. We optimize the selection of the access probabilities based on maximizing the secondary network throughput subject to some PUs quality of service (QoS) constraints.

In the chapter, we will consider two PU QoS constraints, namely, the PU queue stability and the PU average delay. In the PU queue stability constraint the secondary network throughput is maximized subject to the constraint that the PU queue is stable, i.e., the queue length does not grow to infinity; while in the PU average delay constraint, we set a maximum on the average PU packet delay. Note that if an average PU delay constraint is employed this will guarantee that the PU queue is stable. We show that the interference-based model provides higher SU throughput than the collision-based model. Under the constraint of stable PU queues, the average delay is larger, however, in our interference-based model, when we fix a limit on the average PU delay for both systems, we observe an improvement in the SU throughput.

2.1 System Model

We consider a system consisting of one primary user (PU) and one secondary user (SU) as shown in Fig. 2.1. The SU accesses the channel with different access probabilities that depend on whether the primary is sensed to be active or idle as well as the overheard feedback. Clearly, the access probability when the PU is sensed to be idle will be higher than the access probability when the PU is sensed to be active or if the feedback indicates that the PU is active, i.e., if a NACK is overheard over the primary feedback channel.

In our model, the time is slotted and the slot duration is normalized to equal the time of one packet transmission. The primary user is assumed to have a buffer with infinite length to store the incoming packets. The packet arrival process at the primary user queue is Bernoulli with probability λ_p, where $0 \leq \lambda_p \leq 1$ due to slot time normalization[1]. This corresponds to the unslotted system with Poisson arrivals having mean λ_p packets/time slot when the slot

[1] λ_p must be less than or equal to one otherwise the primary queue will not be stable.

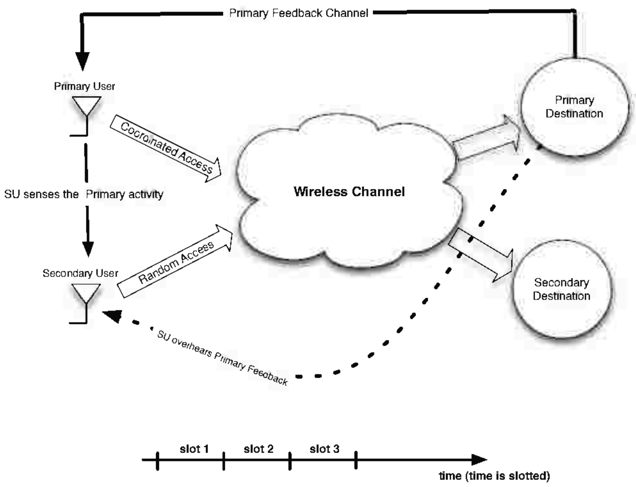

FIGURE 2.1: The system model.

duration is very small [51]. Moreover, in our system, we assume that the secondary user always has a packet to send [14].

The channel is modeled as a Rayleigh flat fading channel and is assumed to be stationary and independent from slot to another with additive white Gaussian noise (AWGN). Thus, the received signal at the intended receiver is given, in general, by

$$y = \sqrt{Gr^{-\gamma}}hx + n + I, \tag{2.1}$$

where G is the transmitted power, r is the distance between the two nodes, and γ is the path loss exponent. x is the transmitted signal, which is result from any constellation, M-ary PSK for instance, with zero mean and unit variance. h is the channel coefficient between the two nodes, modeled as circularly symmetric complex Gaussian random variable with zero mean and unit variance. The noise term n is also modeled as circularly symmetric complex Gaussian random variable with zero mean and variance N_0. The term I denotes the interference term that results from the possible transmission of the unintended transmitter, for example, the interference at the PU receiver if the SU was transmitting in the same time slot. If only one transmitter is active at a given time slot then $I = 0$.

The transmission is successful if the channel is not in outage, i.e. the received SNR (or SNIR) is greater than a pre-defined threshold ζ. From the signal model in (2.1) the outage probabilities are given by

$$
\begin{aligned}
P_p^o &= \Pr\left\{|h|^2 < \frac{\zeta N_0 r^\gamma}{G}\right\} = 1 - \exp\left(-\frac{\zeta N_0 r^\gamma}{G}\right) \\
P_p^{o'} &= \Pr\left\{|h|^2 < \frac{\zeta (N_0 + N_I) r^\gamma}{G}\right\} \\
&= 1 - \exp\left(-\frac{\zeta (N_0 + N_I) r^\gamma}{G}\right),
\end{aligned}
\tag{2.2}
$$

where $P_p^{o'}$ and P_p^o are the outage probability with and without interference, respectively. N_I is the variance of the interference which is approximated to be Gaussian[2].

In our model, we assume that the SU applies a sensing energy detector to detect the activity of the PU by setting an energy threshold; if the received energy is below that threshold the PU is detected to be idle otherwise the PU is detected to be active.

In our model, we assume that a primary feedback channel exists via which the primary receiver sends a feedback to acknowledge the reception of packets. So an ACK is sent if a packet is correctly received, and a NACK is sent if a packet is lost. The transmission failure is attributed to primary channel outage. In case of an idle slot, no feedback is sent. Secondary user is assumed to exploit this primary feedback perfectly and to act as follows: if an ACK/no feedback is heard, the secondary user starts sensing the channel in the next time slot. If the primary user is sensed to be idle, secondary user accesses the channel with access probability a_1. If the primary user is sensed to be active the secondary user has to decrease its access probability and it accesses the channel with access probability $a_1^{'}$. In our analysis, we optimize the selection of the access probabilities a_1 and $a_1^{'}$ and as we will show later $a_1 \geq a_1^{'}$, which is expected since if the primary user is sensed to be active more protection should be guaranteed for the PU and this can be achieved by lowering the SU access probability.

On the other hand, if a NACK is heard, secondary user does not need to sense the channel since it knows that the PU will be active to retransmit the lost primary packet. In this case, we assume that the SU accesses the channel with access probability a_2, which is less than the previous access probabilities a_1 and $a_1^{'}$. These access probabilities are chosen so as to maximize the secondary user throughput without violating some QoS constraint for the

[2]Our analysis will not be affected by the model used to get the outage probabilities since we care only about their values.

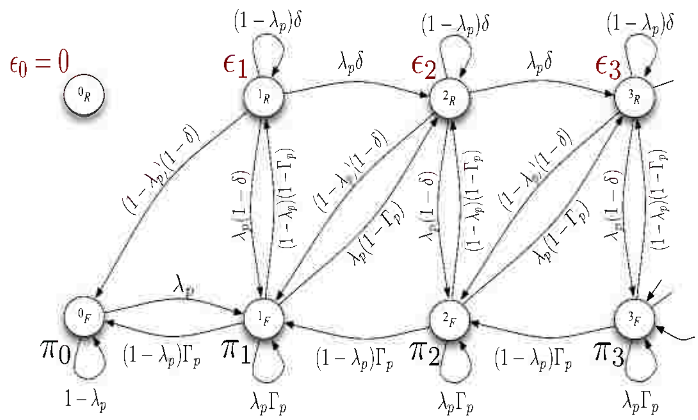

FIGURE 2.2: Markov Chain model of the PU queue evolution.

primary user. We will consider two QoS constraints, namely, the PU queue stability and the PU average delay constraints.

In the next section, we present the analysis of the primary user queue for our proposed system.

2.2 System Analysis

The Markov chain describing the primary user queue evolution is shown in Fig. 2.2. There are two types of states k_F and k_R denoting the case where PU has k packets and sending for the first time ("First" transmission) or a "Retransmission", respectively. π_k and ϵ_k are the stationary probabilities of the states k_F and k_R, respectively.

The transition from state $(k+1)_F$ and k_F occurs when the PU does not receive any packets, which occurs with probability $1-\lambda_p$, and succeeds in transmitting the packet, which occurs with probability

$$\begin{aligned}\Gamma_p =& \Pr\big(\text{(PU succeeds in transmission)} \ \cap \text{(SU detects PU to be active)}\big)\\ &+ \Pr\big(\text{(PU succeeds in transmission)} \ \cap \text{(SU detects PU to be idle)}\big)\\ =&(1-P_p^{\circ})(1-P^1)(1-a_1^{'}) + (1-P_p^{\circ'})(1-P^1)a_1^{'}\\ &+ (1-P_p^{\circ'})P^1 a_1 + (1-P_p^{\circ})P^1(1-a_1),\end{aligned} \tag{2.3}$$

where P^1 is probability that the energy detector's output of the received signal falls below the threshold when the PU is present. We will use P^0 to denote the probability that the energy detector's output of the received signal falls below the threshold when no PU is present. The last expression has four different terms that correspond to the combination of the decision of the SU of whether the PU is active or idle and the SU random access decision of whether to access the channel or not. If the SU decides to access the channel then the PU outage probability will be $P_p^{\circ'}$. If the SU decides not to access the channel then the primary outage probability will be P_p°. Note that the two events of no packet arrival, which occurs with probability $1 - \lambda_p$, and a packet transmission success in the first transmission, which has a probability, Γ_p, are independent so the joint probability is their product. Also, the PU stays in the k_F if a new packet arrives at the primary queue and the PU succeeds in transmitting a packet, which occur with probability $\lambda_p \Gamma_p$.

For the second type of states (Retransmission states), the transition from state k_R to $(k+1)_R$ occurs if the PU receives a packet with probability λ_p and fails in transmitting its packet, which occurs with probability

$$\begin{aligned} \delta =& \Pr(\text{(PU fails in transmission)} \cap \text{(SU decides not to access)}) \\ &+ \Pr(\text{(PU fails in transmission)} \cap \text{(SU decides to access the channel)}) \\ =& P_p^{\circ}(1-a_2) + P_p^{\circ'} a_2. \end{aligned} \tag{2.4}$$

Note that the last expression has no sensing errors since in the case of a NACK the SU knows perfectly the activity of the PU. The probability that the PU stays in the same state k_R is $(1 - \lambda_p)\delta$, which corresponds to primary packet transmission failure with no new primary packet arrival.

2.2.1 Maximizing the SU Throughput under Primary Queue Stability QoS Constraint

As mentioned above, one of our targets is to maximize the SU throughput without violating the primary QoS represented by the primary queue stability. Stability can be loosely defined as keeping a certain quantity, which we care about, bounded; in our case, it is the queue size. For a more general definition of stability see [48] and [49]. If the arrival and service processes of a queuing system are strictly stationary, Loynes' theorem can be applied to check the stability of the queue [52]. This theorem states that if the average arrival rate is less than the average service rate of a queuing system, whose arrival and service processes are strictly stationary, then the queue is stable. Otherwise, it is unstable. By this definition we can conclude that the queue is stable if there is a non-zero probability for the zero-state, i.e., there is a non zero probability that the queue will be empty. To derive the criteria for the primary queue in our system we will derive the stationary distribution of the primary queue Markov chain.

Referring to the Markov chain in Fig. 2.2, we can write the global balance equation around state 0_F as follows

$$\pi_0 \lambda_p = \pi_1 \bar{\lambda}_p \Gamma_p + \epsilon_1 \bar{\lambda}_p \bar{\delta}, \tag{2.5}$$

where the notation $\bar{x} = 1 - x$ is used throughout the chapter. Writing the balance equation around state 1_R we get

$$\epsilon_1 (1 - \delta \bar{\lambda}_p) = \pi_1 \bar{\lambda}_p \bar{\Gamma}_p,$$

therefore, we have

$$\pi_1 = \epsilon_1 \frac{1 - \delta \bar{\lambda}_p}{\bar{\lambda}_p \bar{\Gamma}_p}. \tag{2.6}$$

Substituting by (2.6) in (2.5), we get

$$\epsilon_1 = \frac{\lambda_p \bar{\Gamma}_p}{\chi} \pi_0, \tag{2.7}$$

where $\chi = \lambda_p \Gamma_p + \bar{\lambda}_p \bar{\delta}$. Now using (2.7) in (2.6) yields

$$\pi_1 = \frac{\lambda_p (1 - \delta \bar{\lambda}_p)}{\bar{\lambda}_p \chi} \pi_0. \tag{2.8}$$

Writing the balance equation around state 1_F, we have

$$\pi_1 (1 - \lambda_p \Gamma_p) = \pi_0 \lambda_p + \epsilon_1 \lambda_p \bar{\delta} + \pi_2 \bar{\lambda}_p \Gamma_p + \epsilon_2 \bar{\lambda}_p \bar{\delta}.$$

Using (2.5) to substitute for the term $\pi_0 \lambda_p$, we get

$$\pi_1 \bar{\Gamma}_p = \epsilon_1 \bar{\delta} + \pi_2 \bar{\lambda}_p \Gamma_p + \epsilon_2 \bar{\lambda}_p \bar{\delta}. \tag{2.9}$$

Using (2.7) and (2.8) into (2.9), we now have

$$\pi_2 \bar{\lambda}_p \Gamma_p + \epsilon_2 \bar{\lambda}_p \bar{\delta} = \frac{\lambda_p^2 \bar{\Gamma}_p}{\bar{\lambda}_p \chi} \pi_0. \tag{2.10}$$

Writing the balance equation around state 2_R, we get

$$\epsilon_2(1-\delta\bar{\lambda}_p) = \epsilon_1\lambda_p\delta + \pi_1\lambda_p\bar{\Gamma}_p + \pi_2\bar{\lambda}_p\bar{\Gamma}_p.$$

But since from (2.7) and (2.8) we have

$$\epsilon_1\lambda_p\delta + \pi_1\lambda_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0,$$

therefore,

$$\epsilon_2(1-\delta\bar{\lambda}_p) - \pi_2\bar{\lambda}_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0. \tag{2.11}$$

From (2.10) and (2.11) we can get the following

$$\epsilon_2 = \frac{\bar{\lambda}_p}{\lambda_p}\pi_2. \tag{2.12}$$

Therefore, using (2.12) in (2.10) we get

$$\epsilon_2 = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2 . \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0, \quad \text{and} \quad \pi_2 = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2 . \frac{\lambda_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0. \tag{2.13}$$

From the symmetry of the upcoming states in the Markov chain, one can expect that equation (2.12) can be generalized for any ϵ_k and π_k with $k \geq 2$, since all the upcoming balance equations will give the same result. Also this applies for the results in (2.13); verification of this is straight forward.

Therefore, we can now write the following results:

- $\epsilon_0 = 0.$
- $\epsilon_1 = \frac{\lambda_p\bar{\Gamma}_p}{\chi}\pi_0.$
- $\pi_1 = \frac{\lambda_p(1-\delta\bar{\lambda}_p)}{\bar{\lambda}_p\chi}\pi_0.$

And for $k \geq 2$ we have:

- $\epsilon_k = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^k . \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0.$
- $\pi_k = \frac{\lambda_p}{\bar{\lambda}_p}\epsilon_k.$

We can now use the normalization condition, $\sum_{k=0}^{\infty}(\pi_k + \epsilon_k) = 1$, to get the value of π_0. First, we will divide the summation as follows

$$\sum_{k=0}^{\infty}(\pi_k + \epsilon_k) = \pi_0 + \underbrace{(\pi_1 + \epsilon_1)}_{A} + \underbrace{\sum_{k=2}^{\infty}(\pi_k + \epsilon_k)}_{B} = 1. \tag{2.14}$$

Simplifying the term B: since, for $k \geq 2$, we have

$$\pi_k + \epsilon_k = \psi^k \frac{\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0, \quad \text{where} \quad \psi = \frac{\lambda_p \bar{\chi}}{\bar{\lambda}_p \chi}.$$

Hence,

$$B = \frac{\bar{\Gamma}_p \pi_0}{\bar{\chi}^2}\sum_{k=2}^{\infty}\psi^k = \left(\frac{\lambda_p \bar{\Gamma}_p}{\bar{\lambda}_p \chi}\right)\left(\frac{\lambda_p}{\chi - \lambda_p}\right)\pi_0. \tag{2.15}$$

The last summation converges only if $\psi < 1$, that is equivalent to $\lambda_p < \chi$. This is actually the stability condition for the PU queue. After some manipulations, the term A can be written as:

$$A = \left(\frac{\lambda_p \bar{\Gamma}_p}{\bar{\lambda}_p \chi}\right)\left(\frac{\chi + \bar{\Gamma}_p}{\bar{\Gamma}_p}\right)\pi_0. \tag{2.16}$$

From (2.15) and (2.16), and after some involved manipulations, the final result becomes

$$A + B = \frac{\lambda_p(\bar{\Gamma}_p + \bar{\delta})}{\chi - \lambda_p}\pi_0. \tag{2.17}$$

Using this final result of (2.17) in (2.14), we can write the value of π_0 as

$$\pi_0 = \frac{\chi - \lambda_p}{\bar{\delta}}, \tag{2.18}$$

which can be checked to satisfy the balance equation given in (2.5). This shows that if the stability condition is satisfied, i.e. $\lambda_p < \chi$, this is equivalent to having a non-zero probability for the queue length going down to zero.

Since our target is to maximize the SU throughput, we derive its expression. For the SU to succeed in transmission the SU link must not be in outage whether the PU is idle or active. Note that the SU decides to access the channel with an access probability that depends on the PU sensing decision in the case of a PU "ACK" or no feedback or if a "NACK" is overheard over the primary feedback channel. Thus the SU service process can be characterized as

$$
\begin{aligned}
Y = 1 \big(&\{Q=0\} \cap \bar{O}_s \cap Acs\big) \cup \big(\{Q=0\} \cap \bar{O}_s \cap Acs^*\big) \\
&\cup \big(\{Q\neq 0\} \cap \bar{O}_s' \cap Acs\big) \cup \big(\{Q\neq 0\} \cap \bar{O}_s' \cap Acs^*\big) \\
&\cup \Big(\{Q\neq 0\} \cap \bar{O}_s' \cap Acs'\Big), \quad (2.19)
\end{aligned}
$$

where $\bar{O}_s$ is the event that the SU link is not in outage when the PU is idle and $\bar{O}_s'$ is the event that the SU link is not in outage when the PU is accessing the channel. $\{Q = 0\}$ is the event that PU queue is empty (PU in idle state). Acs is the event that the SU accesses the channel when the PU is in the idle state using the a_1 access probability. Acs^* is the event that the SU accesses the channel with probability a_1' when the PU is sensed to be active. Acs' is the event that the SU accesses the channel with probability a_2 when the PU is in the retransmission state (a NACK is overheard over the primary feedback channel).

Therefore, the SU throughput, denoted by μ_s, is given by

$$
\begin{aligned}
\mu_s &= \pi_o(1-P_s^o)P^0 a_1 + \pi_o(1-P_s^o)(1-P^0)a_1' \\
&+ \left(\sum_{k=1}^{\infty}\pi_k\right)(1-P_s^{o'})P^1 a_1 + \left(\sum_{k=1}^{\infty}\pi_k\right)(1-P_s^{o'})(1-P^1)a_1' \\
&+ \left(\sum_{k=1}^{\infty}\epsilon_k\right)(1-P_s^{o'})a_2
\end{aligned} \tag{2.20}
$$

where $(\sum_{k=1}^{\infty}\epsilon_k)$ is the probability that the PU queue is not empty when a NACK is overheard over the primary feedback channel and it be calculated as follows.

$$\sum_{k=1}^{\infty}\epsilon_k = \epsilon_1 + \sum_{k=2}^{\infty}\epsilon_k$$

$$\sum_{k=2}^{\infty}\epsilon_k = \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_o\sum_{k=2}^{\infty}\left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^k = \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_o\frac{\left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2}{1-\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}}$$

$$\sum_{k=1}^{\infty}\epsilon_k = \left[\frac{\lambda_p\bar{\Gamma}_p}{\chi} + \frac{\lambda_p^2\bar{\Gamma}_p}{\chi(\chi-\lambda_p)}\right]\pi_o.$$

$$\sum_{k=1}^{\infty} \epsilon_k = \frac{\lambda_p \bar{\Gamma}_p}{\chi} \left[1 + \frac{\lambda_p}{\chi - \lambda_p} \right] \pi_o. \tag{2.21}$$

Also ($\sum_{k=1}^{\infty} \pi_k$) is probability that the PU queue is not empty when an ACK is overheard (or no feedback is sent) and this sum can be calculated as follows.

$$\sum_{k=1}^{\infty} \pi_k = \pi_1 + \sum_{k=2}^{\infty} \pi_k$$

$$\sum_{k=1}^{\infty} \pi_k = \frac{\lambda_p (1 - \delta \bar{\lambda}_p)}{\bar{\lambda}_p \chi} \pi_0 + \frac{\lambda_p}{\bar{\lambda}_p} \sum_{k=2}^{\infty} \epsilon_k$$

$$\sum_{k=1}^{\infty} \pi_k = \left[\frac{\lambda_p (1 - \delta \bar{\lambda}_p)}{\bar{\lambda}_p \chi} + \frac{\lambda_p}{\bar{\lambda}_p} \frac{\lambda_p^2 \bar{\Gamma}_p}{\chi (\chi - \lambda_p)} \right] \pi_o$$

$$\sum_{k=1}^{\infty} \pi_k = \frac{\lambda_p}{\bar{\lambda}_p \chi} \left[1 - \delta \bar{\lambda}_p + \frac{\lambda_p^2 \bar{\Gamma}_p}{\chi - \lambda_p} \right] \pi_o. \tag{2.22}$$

As mentioned above for the PU queue to be stable we must have $\pi_o > 0$, which leads to the condition $\lambda_p < \chi$. So the optimization problem of our model can be written as

$$\max_{a_1, a_1', a_2} \quad \mu_s, \quad \text{subject to} \quad \lambda_p < \chi, \tag{2.23}$$

which can be easily solved by a numerical search over the three unknown access probabilities (since the access probabilities are bounded between 0 and 1 and this simplifies the numerical search).

2.2.2 PU Average Packet Delay QoS Constraint

In this subsection, we maximize the secondary user throughput under the condition that the average PU packet delay does not exceed a pre-specified value D_o. In this case, the secondary user is allowed to gain access to the channel as long as it does not violate a PU delay constraint to guarantee the PU quality of service. The only change in our optimization problem is that the constraint on the maximization of the SU throughput is that the PU average delay is upper-bounded by D_o.

To get the PU average packet delay we apply Little's formula [51],

$$D_p = \frac{E\{Q\}}{\lambda_p}, \tag{2.24}$$

where D_p is the average PU packet delay, and Q is the number of packets in the PU queue. The average number of packets in the PU queue can be obtained as

$$E\{Q\} = \sum_{k=1}^{\infty} k\left(\epsilon_k + \pi_k\right). \tag{2.25}$$

Therefore, the PU average packet delay can be calculated to be

$$D_p = \frac{(\Gamma_p - \chi)(\chi - \lambda_p)^2 + (1-\lambda_p)^2(1-\Gamma_p)\chi}{(1-\lambda_p)(1-\chi)(1-\delta)(\chi-\lambda_p)}. \tag{2.26}$$

So in this case our optimization problem can be written as

$$\max_{a_1, a_1^{'}, a_2} \quad \mu_s, \quad \text{subject to} \quad D_p \leq D_o. \tag{2.27}$$

It is worth noting that the condition on the average PU packet delay ensures that the PU queue is stable. Upper-bounding the average packet delay ensures stability since if the queue is unstable then the average packet delay will grow to infinity, which can be easily seen from the expression in (2.26).

2.3 Numerical Results

In this section, we compare the performance of our proposed interference-based system and the collision-based system. In the collision-based system, a packet is assumed lost whenever two nodes attempt transmitting at the same time and a collision occurs. This is different from our, more realistic, interference-based model where simultaneous transmissions do not always result in packet errors; but of course there will be a higher probability of packet loss in case of simultaneous transmissions due to interference. In our simulations, we use the following values: $P_p^o = P_s^o = 0.024$, $P^0 = 0.9$ and $P^1 = 0.1$.

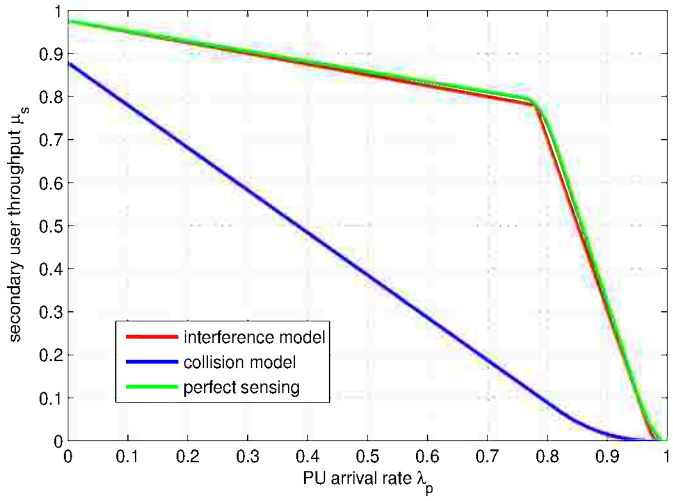

FIGURE 2.3: The SU throughput for the interference-based model and the collision-based model for $P_p^{o'} = P_s^{o'} = 0.22$ under PU queue stability constraint.

In Fig. 2.3, the SU throughput is plotted against the PU arrival rate, λ_p, for $P_p^{o'} = P_s^{o'} = 0.22$ under primary queue stability constraint. From Fig. 2.3 we can see that the interference-based model results in a significant gain in terms of the SU throughput that is very close to the secondary throughput of the genie-aided (perfect sensing) system, where the SU is assumed to perfectly know the activity of the PU. These gains are attributed to accessing the channel even when the PU is active since this does not always result in a packet loss. In collision-based models, the assumption that simultaneous transmissions will always results in packets loss will significantly decrease the system throughput.

In Fig. 2.4, the SU throughput is plotted against the PU arrival rate but for $P_p^{o'} = P_s^{o'} = 0.8$ under primary queue stability constraint. The SU throughput gains are less than the gains shown in Fig. 2.3 because in the case of $P_p^{o'} = P_s^{o'} = 0.8$, simultaneous transmissions cause high interference and result in higher outage probabilities. So, the gains of the interference-based model greatly depend on the outage probability when simultaneous transmissions occur; if the outage probability with interference approaches 1, then these SU throughput gains of the interference-based model will diminish since in this case we approach the collision-based model.

The SU access probabilities against the PU arrival rate are shown in Fig. 2.5 and Fig. 2.6 under primary queue stability constraint, in the case of low and high interference, respectively. For the low interference case, shown in Fig. 2.5, it clear that the access probabilities will be 1 for low primary arrival rates and start to decrease as the primary arrival rate increases to

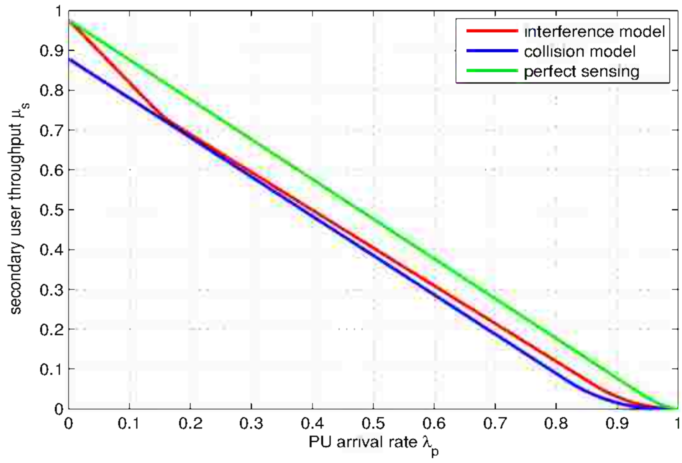

FIGURE 2.4: The SU throughput for the interference-based model and the collision-based model for $P_p^{o'} = P_s^{o'} = 0.8$ under PU queue stability constraint.

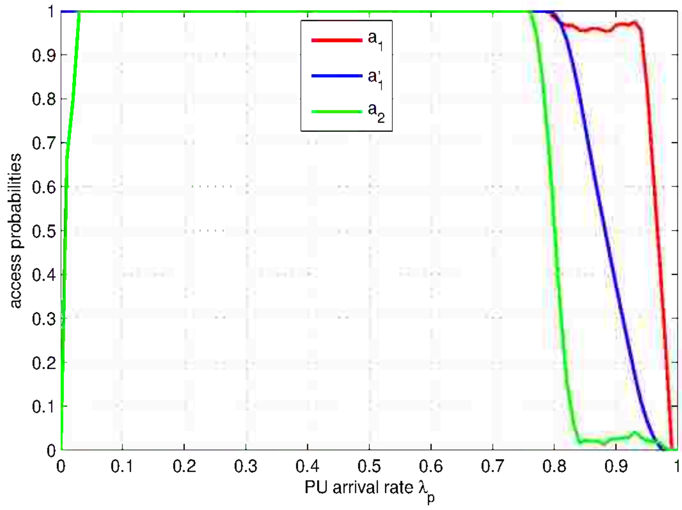

FIGURE 2.5: The SU access probabilities for the interference-based model for $P_p^{o'} = P_s^{o'} = 0.22$ under PU queue stability constraint.

ensure the stability of the primary queue. However, for the high interference case, shown in Fig. 2.6, we can see that the access probability when a NACK is received is always 0 since in this case the system is close to the collision limited system and in the case that the PU is perfectly known to be active it is better for the SU to back off to allow for interference free transmission of the PU.

In Fig. 2.7 and Fig. 2.8, we show the SU throughput against the PU arrival rate with an average PU packet delay constraint in the case of low and high interference, respectively. In Fig. 2.7 and Fig. 2.8, the maximum PU average packet delay is set to two slots. Also, the

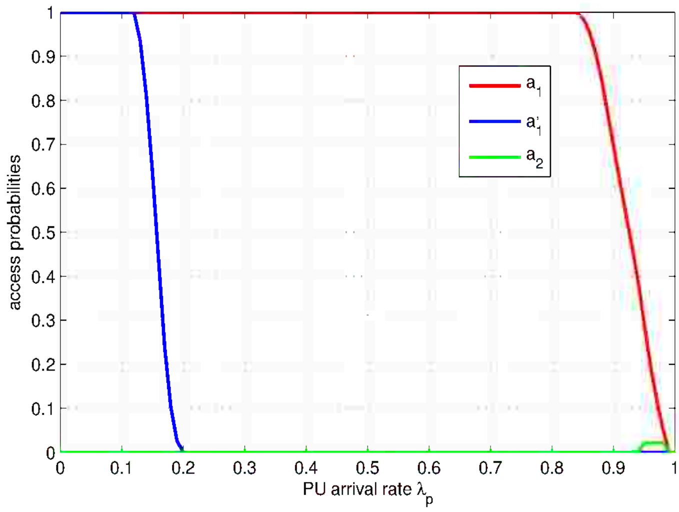

FIGURE 2.6: The SU access probabilities for the interference-based model for $P_p^{o'} = P_s^{o'} = 0.8$ under PU queue stability constraint.

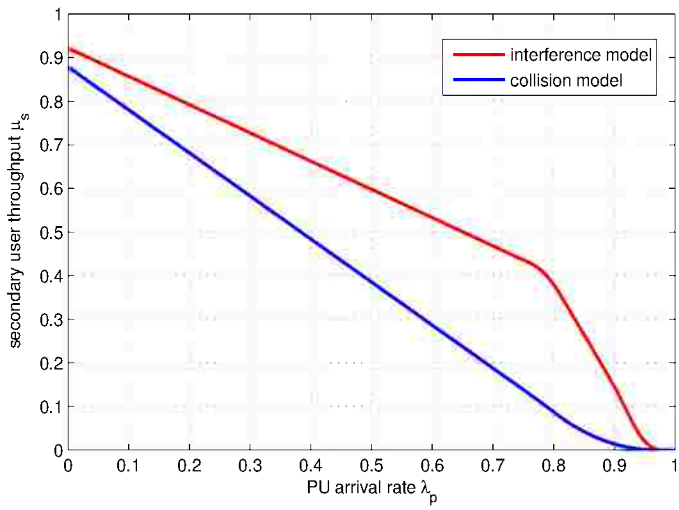

FIGURE 2.7: The SU throughput for the interference-based model and the collision-based model for $P_p^{o'} = P_s^{o'} = 0.22$ under average PU packet delay constraint.

average PU packet delay for $P_p^{o'} = P_s^{o'} = 0.1$ is plotted in Fig. 2.9. From these figures, we can draw the same conclusions as for the PU queue stability constraints; for low interference, higher gains of our interference-based model are expected as compared to the collision-based model and as the interference effect increases these gains will be lower. Also, we can see that comparing the results in Fig. 2.3 and Fig. 2.7 that the SU throughput will be lower for the delay-constrained system since the delay constraint is a more stringent constraint and it implies the stability constraint.

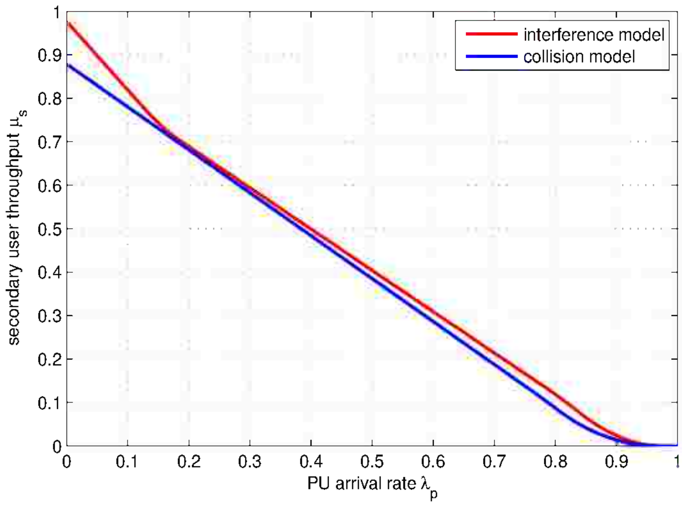

FIGURE 2.8: The SU throughput for the interference-based model and the collision-based model for $P_p^{o'} = P_s^{o'} = 0.8$ under average PU packet delay constraint.

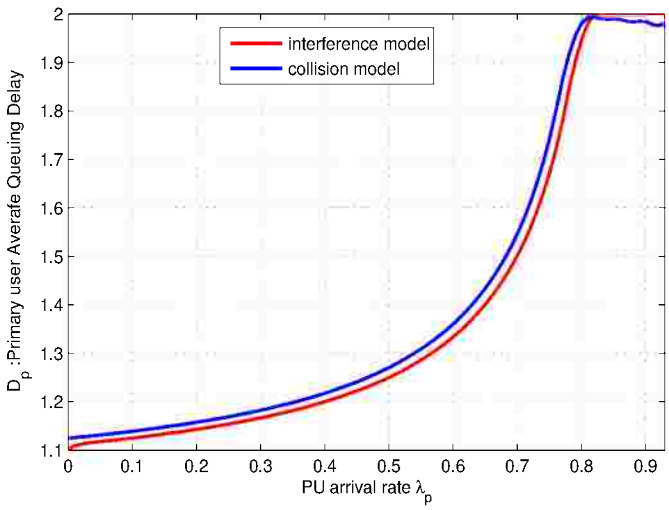

FIGURE 2.9: The average PU packet delay for $P_p^{o'} = P_s^{o'} = 0.1$ when the average PU packet delay constraint is set to 2 slots.

In Fig. 2.10, the SU access probabilities are plotted against the PU arrival rate when the PU average packet delay is limited to two time slots, which can be explained in the same way as for the case of PU queue stability constrained system.

In Fig. 2.11, the SU throughputs in case of the interference-based and collision-based models are plotted against the outage probability with interference $P_p^{'}$ for a fixed arrival rate of $\lambda_p = 0.4$. This plot shows that our proposed system provides gains along the whole range of the outage probability with interference and these gains decrease as $P_p^{'}$ approaches 1 as we move

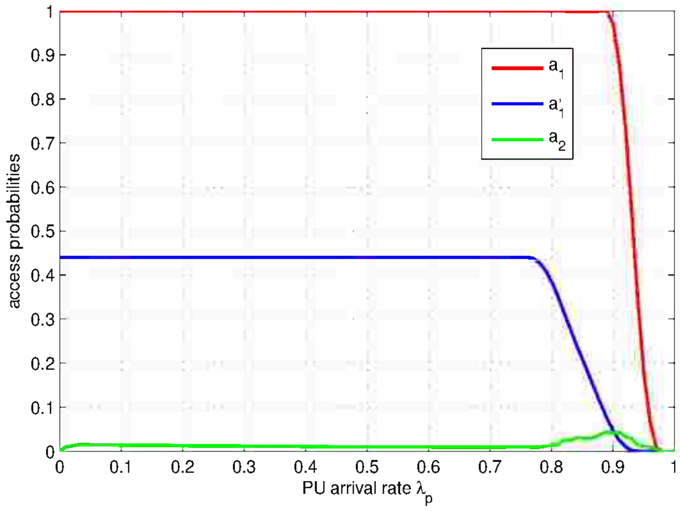

FIGURE 2.10: The SU access probabilities for the interference-based model for $P_p^{o'} = P_s^{o'} = 0.22$ under average PU packet delay constraint of two slots.

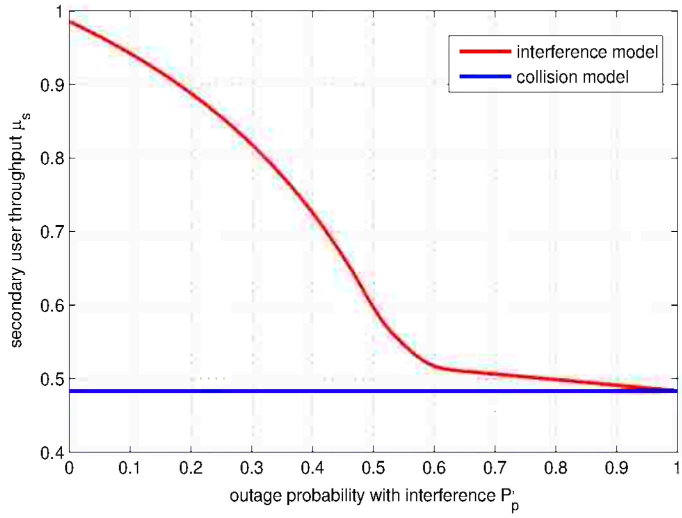

FIGURE 2.11: The SU throughput for the interference-based model and the collision-based model for $\lambda_p = 0.4$.

closer to the collision-based model. It worth noting that the throughput in case of collision-based model is constant independent of P_p' since in this model a collision will always be considered to cause packet loss.