AN INTERNAL INTERFERENCE METHOD FOR THE SEPARATION OF LONGITUDINAL VIBRATIONS FROM COUPLED FLEXURAL MODES

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SUMMARY

When a z-cut circular quartz wedge, coated on both surfaces with a highly dielectric reflecting surface and excited electrically by a certain type of electrodes, the internal interference fringes of considerable dispersion are disturbed and blurred upon those portions of the wedge specimen showing vibrations of longitudinal type in the form of nodes and antinodes. The dispersion should not be high, otherwise, there is always the very real possibility of missing nodal regions, but this is not the case when a considerable number of fringes cover the whole field of view.

It has been proved experimentally that our previous use of multiple beam Fizeau fringes in which the fringes are disturbed and blurred upon those portions of the faces which enjoy a normal displacement, i.e. motion in the up and down direction at oscillation, proved to be more complicated than those predicted theoretically. Experimentally, those vibrations were coupled to flexural modes, while the oscillating internal interference mode have proved to agree very well with the modes predicted theoretically and the coupled flexural vibrations do not show any effect in those patterns. So the method can be taken as a much more satisfactory way for the separation of the longitudinal modes under studies from the coupled flexural modes.

Interference of light effected from the two crystal faces :

For such interference, owing to the great path difference between successive beams, the monochromatism and parallelism of the incidental illumination are critical, necessitating a low pressure mercury arc source and a very small aperture at the focus of the collimater lens. For nonnormal incidence the fringes are localized in a plane through the intersection of the interferometer surfaces perpendicular to the light beam. For small gaps any slight deviation from normality has a negligible effect on the plane of localization, but in the present case the line of intersection of the surfaces may be a few meters away from the specimen, so that a very small deviation from normality shifts the plane of localization by several centimeters, thus preventing the fringes and the specimen from being focussed upon simultaneously since the crystal must have a finite wedge angle to secure a suitable fringe dispersion, the crystal used for most of the experiments did not fulfil the critical conditions for obtaining sharp fringes, owing to its large thickness (2 mm.).

For internal interference, the optical path difference between successive beams depends on both the thickness and refractive index. On vibration, the change in this path difference is ,

$$\delta(2\mu t) = 2t\delta\mu + 2\mu\delta t$$

μ being the refractive index.

Considering first an isotopic body, the effect of stress is to make it birefringent. Hence the fringes split into two components for which the refractive indices are

$$\mu_1 = K(P+Q) - C(P-Q) + \mu \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

$$\mu_2 = K(P+Q) + C(P-Q) + \mu \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

where P and Q being the principal stresses and K and C are stress-optic coefficients.

For the type of vibration considered in this work, the change of thickness is

$$2\,hz = h(P+Q) \times \text{const.}$$

so that the changes of the path difference between successive beams are

$$\delta(2\,\mu t)_1 = 2h\ K'(P+Q) - C(P-Q)$$

$$\delta(2\,\mu t)_2 = 2h\ K'(P+Q) + C(P-Q)$$

The motion of the fringes may be visualized as a bodily displacement proportional to P + Q (and to the surface displacement) with an additional splitting of the fringe proportional to P — Q.

If $P + Q = o$, the fringe is split symmetrically while if $P - Q = o$, the fringe does not split but is merely displaced. Since the effects due to both the change of thickness and the sum of the principal stresses are additive, the bodily displacement of the fringe will usually be large compared with the splitting.

Consider now the particular case of quartz cut normal to the optic axis, certain difficulties arise. When the quartz is stressed, an analysis of the intensity distribution of the broadened fringes is complicated, due to the combination of rotatory power and birefringence.

In an unstressed z-cut plate, each plane polarized element of an incident unpolarized beam suffers a rotation on each passage through the crystal which is annulled whenever it is reflected in the reverse direction, so that the multiple beam due to each element, are in a condition to interfere in the normal way. If the plate is now subjected to a static isotropic stress, so that $P - Q = o$, in the xy-plane, the mean refractive index alters, but due to the relation between the stress-optic coefficients in quartz, it does not become birefringent in the z-direction and so the fringes are unaltered except for a shift proportional to $P + Q$. If now the alternative state is considered in which $P - Q$ is finite but $P + Q$ is zero, whatever the effect of this stress, the resultant fringes will be symmetrical about their original positions.

The general case, in which P and Q have arbitrary values is a superposition of these two effects. It seems likely therefore, that the general appearance of the fringe pattern wonld be somewhat similar to the isotropic case, although the polarization state of the emergent light is uncertain.

The effects due to the magnitude of the sum of the principal stresses will be regarded as the primary cause of the observed fringe shift and other effects will be regarded as modifying influence.

Symmetrical modes ($n = o$)

(a) Type (A) the nodal system is that summarized in table (1). The radii given there are those at which the sum of the principal stresses, the areal dilatation and the normal displacement all vanish. Now any fringe splitting effects are due to the difference between the principal stresses, and the radii at which this is zero have been given in Table (2). Since the roots of $J_0(kr)$, apart from the first are near to those of $J_2(kr)$, it follows that the nodes, apart from the first, should be fairly clearly defined.

Fig. (1) shows the first modes of the specially worked 2.0 mm. crystal. There is, in fact, very little evidence of any fringe splitting, even for the first node or indeed anywhere on the whole surface. The nodal radii agree well with the expected values.

We compare the fringe displacement observed by simple interference with that due to internal interference, however, as the former fringe displacement is extremely small compared with the latter.

Fig. (2) shows the two types of interference simultaneously for the A.1 mode. This was achieved by resting the doubly coated crystal on the similarly coated reference flat. It is seen that for a substantial displacement of the internal interference fringes, the displacement of the simple interference fringes is negligible.

The change of mean path difference between successively reflected beams, due to the stress-optic effect is found to be about 2.5 time that due to the change of thickness of the crystal. The change of the optical thickness of the crystal is about 3 times the displacement of either surface relative to the median plane. Hence it follows, that the displacement of the internal interference fringes is more than 10 times as great region.

(b) Type (B)

Here (P + Q) = zero, so the movement of the fringes is entirely due to the birefringent effect. The fringe splitting is symmetricall about the rest position and the fringes should be completely undisturbed where P — Q = 0 which occurs in the isotropic region, where nodes are thus found from the roots of J_2 (k,r) and coincide with the zero order stress fringes.

Figs. (3 and 4) show the experimental pattern for the B.0.3 modes of the 2.0 mm. crystal. The observed nodal radii agree well with the theory. There is however, some evidence of fringe doubling showing that the fringe splitting is not quite symmetrical and implying that (P + Q) is not zero.

The general appearance of the oscillating fringe envelopes for his modes is little different from that of simple interference fringe envelopes.

Type C modes :

(a) n = 1 modes. The nodes of (P + Q) consist of a single diameter and a number of circles. Figs. (5 and 6) show the observed pattern for 2.0 mm. crystal. These were are the only modes strong enough to show any noticable fringe displacement. The values of m are o, 1 and 4, respectively (The nodal radius for m = 4 agrees well with theory).

(b) n = 2 modes, Here there are two diametrical nodes and various circular nodes, the radii having already been given in Table (3). These modes were more readily excited than the n = 1 modes. Figs. (7 and 8) show the modes for which m = o, 2, 3, 5 and 7 respectively. The nodal radii fit the theoretical values very wel.

TABLE 1

Type (A) Modes
Radii of modes of normal displacement

m	0	1	2	3	4
(ka)	1.939	5.362	8.552	11.717	14.873
r_1		0.449	0.281	0.209	0.162
r_2			0.645	0.471	0.371
r_3				0.739	0.582
r_4					0.793

TABLE 2

Type (A) Modes
Radii of zero order stress fringes

m	0	1	2	3	4
(ka)	1.939	5.362	8.552	11.717	14.873
r_1	0	0	0	0	0
r_2		0.958	0.600	0.438	0.345
r_3			0.984	0.718	0.566
r_4				0.992	0.781
r_5					0.995

TABLE 3

Type C, $n = 1$ Modes
Radii of the Circular Modes of the Normal Displacement

m	0	1	2	3	4	5	6	7
(ka)	1.713	3.537	4.452	6.512	6.936	8.668	10.080	10.769
r_1	0	0	0	0	0	0	0	0
r_2			0.861	0.588	0.552	0.442	0.380	0.356
r_3						0.809	0.686	0.651
r_4								0.945

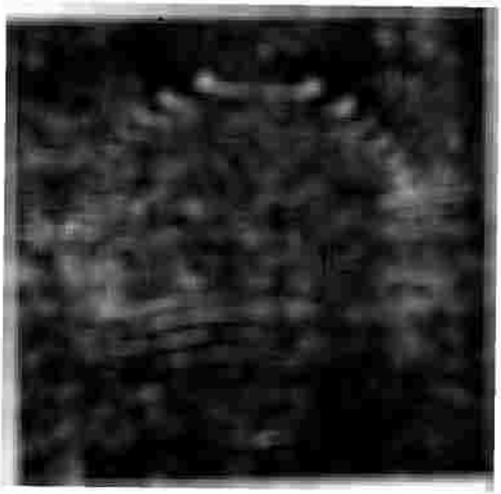

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

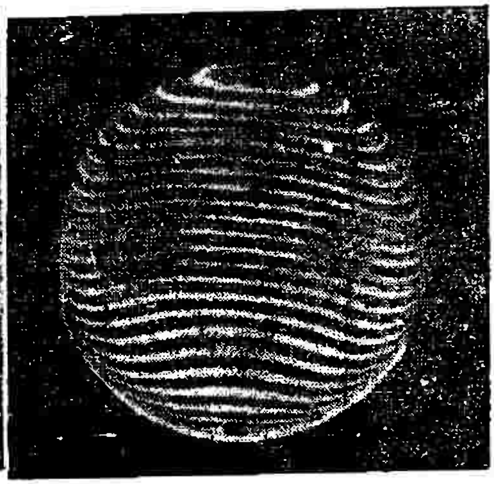

Fig. 6

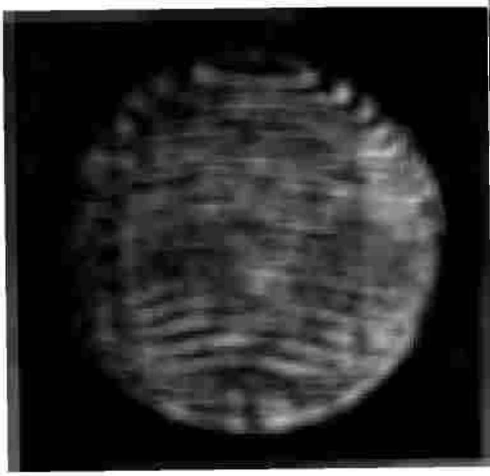

Fig. 7

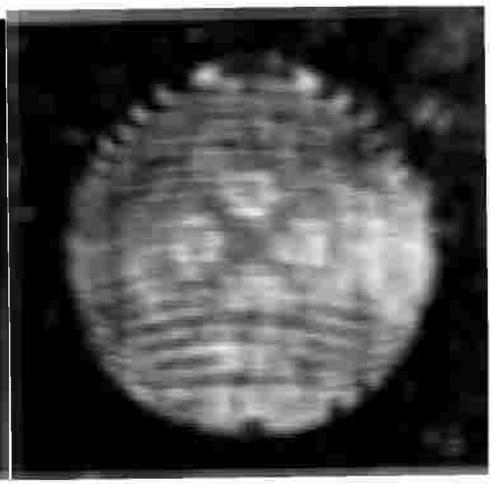

Fig. 8