EXPLICIT ORTHOGONAL SYMMETRISED SPIN STATES : FOR GENERAL INTERNAL CASES (Letter)[6]

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ABSTRACT

In a previous work by the author (ref. 6) a simple explicit expansion is given for the states of n spin 1/2 particles characterised by the Young tableaux for the function

$$\left| \begin{matrix} 1 & \cdots\cdots\cdots & n-1 & \\ P_1 P_2 & \cdots\cdots\cdots & P_{m-1} & n \end{matrix} \right| \quad \text{and} \quad \left| \begin{matrix} & \cdots\cdots\cdots & & n \\ P_1 P_2 & \cdots\cdots\cdots & P_{m-1} & n-1 \end{matrix} \right|$$

the expansions given were proved to satisfy Young theorem for external symmetry and internal symmetry in three special casec.

In this paper the expansions given before (ref. 6) were jroved to satisfy Young theorm for the general internal cases : $n' - 1$ and n' in different rows and columns.

§ 1. The general internal case : in $n' - 1$ and n' in different rows and columns.

The proof in this case is very similar to the general external case treated in sc. 4. (6) As before we have variables $x_1 x_2 \ldots\ldots x' \, h - 1'$ where row

$$h' = n' - 2m' + 1 \tag{1}$$

and (with external type ψ_n for example)

$$\psi_{(n')n} = \left| \begin{matrix} 1 & \boxed{h' - 1 = h' - 2m'} & n' - 1 \ldots\ldots n - 1 \\ P_1 P_2 \cdots P_{m'-1} n' & P_{m'+1} P_{m'+2} \cdots P_{m'+k} & m \quad n \end{matrix} \right| SS\rangle, \tag{2}$$

$$\psi_{(n'-1)n} = \left| \begin{matrix} & \boxed{1 \leftarrow h' - 1 = n' - 2m' \rightarrow n'} & \ldots\ldots \quad n - 1 \\ P_1 P_2 \cdots P_{m'-1} n'-1 & P_{m'+1} P_{m'+2} \cdots P_{m'+k} & P_m \quad n \end{matrix} \right| SS\rangle, \tag{3}$$

there being $h' - 1 = n' - 2m'$ (4)

numbers in the top row of each row of each symbol appeaving just above **n′** and ending just before $n_2 - 1$ in (n') n and vice versa in φ $(n'-1)n$. It is the satne over these these $h'-1$ variables using the previous type relation

$$\begin{vmatrix} x1 \\ \\ n'-1 \end{vmatrix} n' + \begin{vmatrix} n'-1 \\ \\ n' \end{vmatrix} xi + \begin{vmatrix} n' \\ \\ xi \end{vmatrix} \qquad (n'-1) = o$$

which produces the required

$$P_{n',n'-1} \;{}^{+}(n')\,n = -\frac{1}{h'}\quad (n')n^{+}\; \frac{[(h')^2 - 1]}{h''} \;{}^{+}(n'-1)n \quad (6$$

$$P_{n',\,n'-1} \;{}^{+}(n'-1)n = \frac{1}{n'}\quad (n'-1)n + \frac{[(h')^2 - 1]}{}\;{}^{+}(n')n \quad (7$$

there is however now the difference that in some cases the second forms in (5) may form part of a two particle determinant the relationship holds however in the modified form

$$\begin{vmatrix} xi \\ n'-1 \end{vmatrix} \begin{vmatrix} n' \\ P' \end{vmatrix} + \begin{vmatrix} n'-1 \\ n' \end{vmatrix} \begin{vmatrix} xi \\ P \end{vmatrix} + \begin{vmatrix} n' \\ xi \end{vmatrix} \begin{vmatrix} n' \\ P' \end{vmatrix} = o$$

for any fixed number n′ and the proof proceeds then exactly as before, We find it simplest to illstroke this with an example.

In the following example $n' = 7$, $n'\ n'-1 = 6$ and $h' = 4$

$$\left| \begin{array}{l} {}^{+}n' = \begin{vmatrix} 1 & 2 & 4 & 5 & (n'-1) & 9 \\ 3n' & 8 & 10 \end{vmatrix}\ 11 \quad \rangle \\ {}^{+}n'-1 = \begin{vmatrix} 1 & 2 & 4 & 5 & n'9 \\ 3 & (n'-1) & 8 & 10 \end{vmatrix}\ 11 \quad \rangle \end{array} \right.$$

In full

$$\Psi_{n'} = \frac{1}{3'2(n'+1)\ n'4.3.43} \sum_{q_1} \left|\begin{matrix} q_1 \\ 3 \end{matrix}\right| \sum_{q_2} \left|\begin{matrix} q_2 \\ n'-1 \end{matrix}\right| \sum_{q_3} \left|\begin{matrix} q_3 \\ 8 \end{matrix}\right| \sum_{q_4} \left|\begin{matrix} q_4 \\ 10 \end{matrix}\right|_{a_1 a_2}$$

$$\Psi_{n'-1} = \frac{1}{3,2\ n'\ (n'-1)4.3.4.3} \sum_{q_1} \left|\begin{matrix} q_1 \\ 3 \end{matrix}\right| \sum_{q_2} \left|\begin{matrix} q \\ n'-1 \end{matrix}\right| \sum_{q_3} \left|\begin{matrix} q_3 \\ 8 \end{matrix}\right| \sum_{q_4} \left|\begin{matrix} q_4 \\ 10 \end{matrix}\right|_{\overline{a_1 a_2}}$$

$a_1 a_2$ (11)

$a_1 a_2$ (11)'

The expansion of (11) contains 72 terms which inthe abbreviated notation of the thesis of N.G. El-Sharkawy (p 267) may be written as

$$(12)^2\ (45\ (n'-1)\ 9) + (12)\ (45\ (n'-1)\ (1245\ (n'-1)\ )\ (1245\ (n'-1)\ 9) \qquad (12)$$

the expansion of (12)' contains 54 terms which may be written as

$$(12)^2\ (45n')\ (45n'9) + (12)\ (45)\ (1245n')\ (12\ 45n'9) \qquad (12)'$$

The nature of this is such that all the allowed numerators in the product (in the form of two-particle determinants, with denominators 3, n', 8, 10 for n' and 3, n'-1, 8, 10 for n'- 1) occur in the expansion by taking one number from each of the four reduced brackets avoiding repetition. Note that $(12)^2$ means (1) (2) + (2) (1).

n' we have thus 72 numerations.

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