# On The Spectral Light Current Of An Optical Resonator

By

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## ABSTRACT

A theoretical derivation for the transmitted spectral light current distribution from an optical resonator is given taking into account the radiating solid angle of the point light source. From the energy law, the transmitted light current, in case of no interference, was calculated to test this derived spectral distribution.

## (I) INTRODUCTION

Most of the previous work in the field of interference of light deals with the spectral light current density distribution of the resulting interference fringes. The calculations are based on the assumption that the incident wave of a certain amplitude[1] does not suffer any change due to the inverse-square law during its multi-reflections; i.e. the incident waves intensity is considered to be equal to its optical density. This assumption requires that the light source is illuminating the optical resonator with a unit solid angle.

From the experimental point of view, the optical set-up system illuminates the resonator in a given solid angle, which is not, by necessity, equal to a unit solid angle.

In the literature[2] although it has been assumed that this angle is very small, but the optical resonator still

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transmitts within the solid angle a light current and not light current density. In the present work, the transmitted spectral light current from an optical resonator when illuminated within a given solid angle is theoretically calculated.

## (II) THEORY

As seen in Fig. (1), we consider a deformed point light source illuminating an optical Fabry-Perot resonator with a current density ($I_0$) in a solid angle ( $\omega$ ) such that :

$$\omega > \frac{A}{\left[a + 2\,(N - 1)\,D\right]^2} \quad ,$$

where : (A) is the area of the collimating lens,

(a) is the distance between the deformed point light source and the collimating lens.

(N) is the effective number of the transmitted light currents from the resonator [3]

The value of (N) can be calculated from :

$$N = 1/(1 - R^2) \quad ,$$

where R is the reflection coefficient of the mirrors of the resonator.

For the given radiating solid angle ($\omega$), the transmitted light current from the resonator can be calculated for two cases :

1) The optical path length (D) of the resonator is larger than the coherence length of the incident light. In this case, no interference occurs [4 & 5] and the detector will be affected by a light current ($L_c$) which can be derived from a sum of partial light currents of different radiating solid angles and of different light current densities; i.e. :

$$L_c = (1-R)^2 \left[ \frac{I_o A}{a^2} + \frac{I_o A\, R^2}{(a+2D)^2} + \frac{I_o A\, R^4}{(a+4D)^2} + \dots + \frac{I_o A\, R^{2(n-1)}}{\left[a + 2(n-1)\, D\right]^2} \right]$$

$$\text{or} \quad L_c = I_o (1-R)^2 \sum_{n=1}^{\infty} \frac{A\, R^{2(n-1)}}{\left[a + 2(n-1)\, D\right]^2} \qquad (1)$$

2) The optical path length (D) of the resonator is smaller than the coherence length of the light.In this case the condition for producing interference is satisfied and the detector will be affected by a light current of interference fringes which are produced from the interference of partial currents of wave amplitudes of different densities and of different solid angles, i.e. ,

$$L_c(\theta) = (1)R)^2 \left[ \left(\frac{I_o A}{a^2}\right)^{1/2} + \left(\frac{I_o A}{(a+2D)^2}\right)^{1/2} \cdot R.e^{i\theta} + \left(\frac{I_\theta\, A}{(a+4D)^2}\right)^{1/2} x \right.$$
$$\left. R^2.e^{2i\theta} + \dots + \left(\frac{I_o\, A}{\left[a + 2\,(n-1)\, D\right]^2}\right)^{1/2} .R^{n-1} e^{(n-1)i\theta} \right]^2, \qquad (2)$$

where $\theta/2$ is the phase shift of the wave per path. Eq. (2) can be rewritten as :

$$L_c(\theta) = (1-R)^2 I_o \left[ \left( \sum_{n=1}^{\infty} \frac{A^{1/2}\, R^{n-1} \cos\,(n-1)\,\theta}{a + 2\,(n-1)\, D} \right)^2 + \left( \sum_{n=1}^{\infty} \frac{A^{1/2} R^{n-1} \sin\,(n-1)\,\theta}{a + 2\,(n-1)\, D} \right)^2 \right] \qquad (3)$$

For testing eq. (3) we calculate the average energy quantity $\overline{L_c(\theta)}$ ) of the spectral light current $L_c$ ($\theta$) from the energy law. Integrating $L_c(\theta)$ over $\theta$ and calculating the average energy quantity, we get :

$$\overline{L_c}(\theta) = \frac{1}{2\pi} \int_0^{2\pi} L_c(\theta)\, d\theta = I_o (1-R)^2 \sum_{n=1}^{\infty} \frac{A\, R^{2(n-1)}}{\left[a+2(n-1)D\right]^2} = L_c \qquad (4)$$

Thus the average energy quantity $\overline{L_c}(\theta)$ gives the same energy quantity of the light current ($L_c$) as in eq. (1) [6].

Let us now consider that the deformed point light source illuminates the resonator in a solid angle such that

$$w \leq \frac{A}{\left[a + 2\,(N-1)\,D\right]^2}$$

In this solid angle, all effective light currents transmitted from the resonator are collimated by the lens. As mentioned before, if we assume the case of no interference, then the detector will be affected by the sum of partial light currents of different current densities but of the same radiating solid angle ( w ) :

$$L_c = \frac{I_o\,A\,(1 - R)^2}{\left[a + 2\,(n-1)\,D\right]^2} \sum_{n=1}^{\infty} R^{2(n-1)}$$

$$\text{or}\quad L_c = \frac{I_o\,A}{\left[a + 2\,(N-1)D\right]^2} \times \frac{(1 - R)^2}{1 - R^2} \qquad (5)$$

If now the condition of interference excists, the detector will be affected by the spectral light current arizing from the sum of partial currents of wave amplitudes of different currents densities but of the same solid angle (w) :

$$L_c(\theta) = \left[\sum_{n=1}^{\infty} I_o^{1/2}\,(1-R)\,\frac{A^{1/2}\,R^{n-1}\cdot e^{i(n-1)\,\theta}}{a + 2\,(N-1)\,D}\right]^2$$

$$= \frac{I_o A\,(1 - R)^2}{\left[a + 2(n-1)\,D\right]^2}\cdot\left[\sum_{n=1}^{\infty} R^{n-1}\cdot e^{i(n-1)\,\theta}\right]^2$$

$$\text{or}\quad L_c(\theta) = \frac{I_o\,A}{\left[a + 2(N-1)D\right]^2} \times \frac{(1 - R)^2}{1-2\,R\cos\theta + R^2} \qquad (6)$$

In this case, it is seen that the spectral light current has the same distribution of the well-known Airy-formula (1& 2) which deals with the spectral light current density distribution of the transmitted interference fringes of a Fabry - Perot resonator. In his calculations, Airy assumed that the light source is deformed to illuminate the optical resonator with a unit solid angles.

From the point of view of the energy law , we have :

$$\overline{L_c}(\theta) = \frac{1}{2\pi} \int_0^{2\pi} L_c(\theta) d\theta = \frac{1}{2\pi} \frac{I_o \quad A}{\left[a+2(N-1)D\right]^2} \int_0^{2\pi} \frac{(1-R)^2}{(1-2R\cos\theta+R^2)} d\theta$$

$$\text{or } \overline{L_c}(\theta) = \frac{I_o \ A}{\left[a + 2\ (n-1)\ D\right]^2} \times \frac{(1-R)^2}{1-R^2} , \qquad (7)$$

which is equal to $L_c$ in eq. (5).

The spectral light current distributions for the two cases according to eqs. 3 & 6 are illustrated in Fig. (2).

Fig. (3) shows the variation of the average transmitted spectral light current with the distance (a) of the point source in the two cases when (w) is i) larger and ii) smaller or equal to $A/\left[a + 2\ (N-1)\ D\right]^2$. It is seen that the average values of light current calcula-ted from our theoretical derivation are in good agreement with that of Airy type at larger values of the distance (a). where the solid angle (w) is smaller.

## III CONCLUSION

The transmitted spectral light current distribution from an optical resonator depends upon the radiating solid angle of the point light source. The transmitted light current is of Airy - distribution type, only when the deformed point light source illuminates the optical resonator with a solid angle (w) such that

$$w \leq A/\left[a + 2\,(N-1)\ D\right]^2$$

Two different formula for the average transmitted light current are also found for the two cases when :

$$w > \text{ or } \leq A/\left[a + 2\ (N-1)\ D\right]^2$$

There-fore it is important in the experimental measurements to take into account the radiating solid angle of the point light source which illuminates the optical resonator.

## IV - REFERENCES

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## ملخص بحث

### التوزيع الطيفى لهدب التداخل المتكونة من رنان ضوئى

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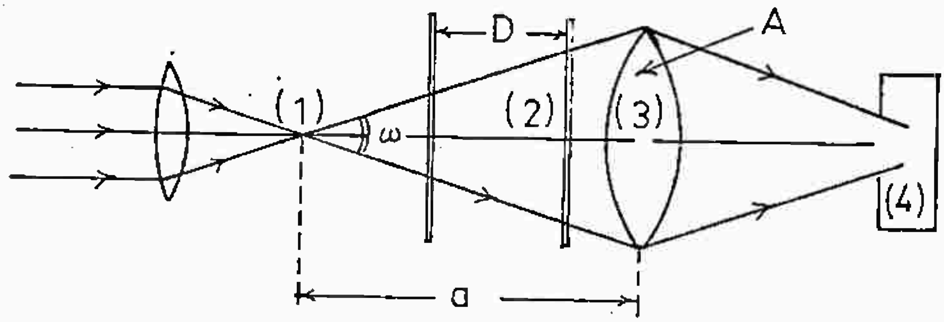
تعتمد الدراسات النظرية للتوزيع الطيفى للشدة الضوئية لهدب التداخل على اعتبار ان الاضاءة الساقطة على الرنان الضوئى تساوى كميا الكثافة الضوئية للمصدر الضوئى نفسه أى أن المصدر الضوئى يضىء الرنان فى زاوية مجسمة قيمتها الوحدة •

ويعالج هذا البحث نظريا هذه الدراسات باعتبار ان الاضاءة الساقطة تساوى تيارا ضوئيا اى ان الرنان يضاء بزاوية مجسمه لا تساوى بالضرورة الوحدة •

وقد وجد ان التوزيع الطيفى لهدب التداخل يأخذ صورة معادلة ايرى المعروفة اذا كانت الزاوية المجسمة اقل من قيمة معينة • •

كذلك يتضمن البحث معادلات رياضية مستنتجة للتوزيع الطيفى عندما تكون الزاوية المجسمة اكبر او أقل عن هذه القيمة المعينة •




1_Point Light Source.
2_Resonotor.
3_Collimating.lens.
4_Detector.

Fig (1)

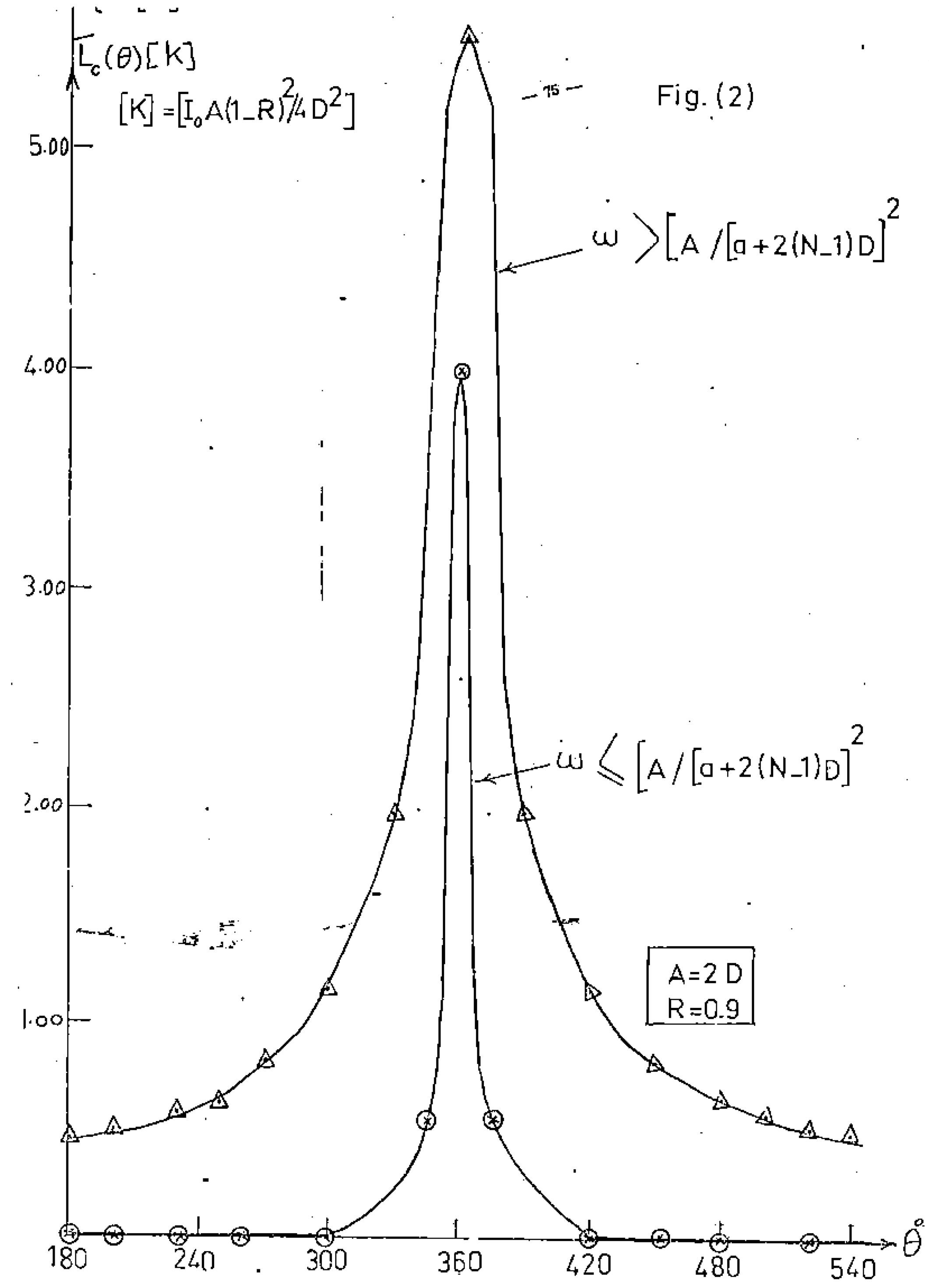
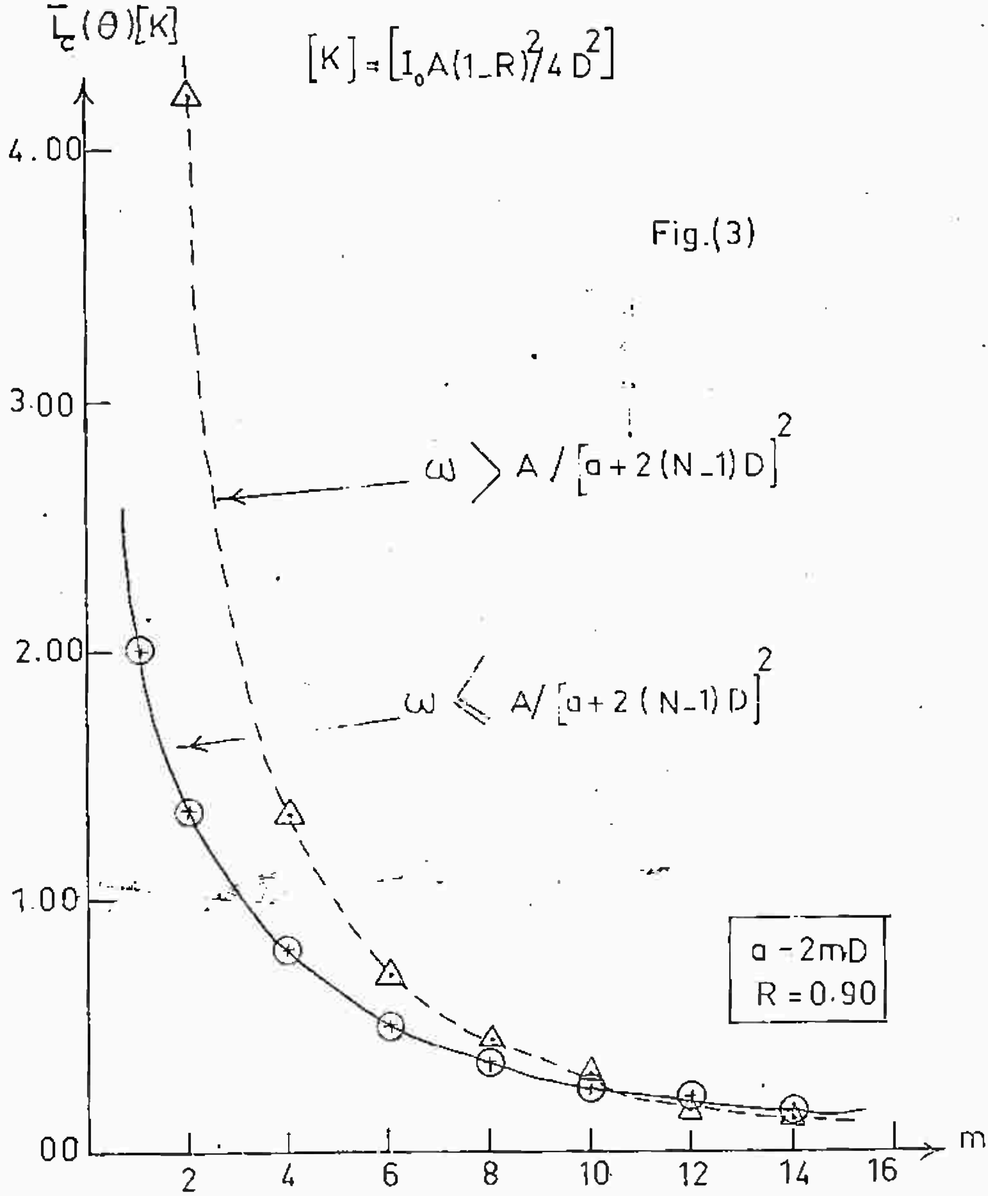

$L_c(\theta)[K]$
$[K] = [I_0 A(1-R)^2/4D^2]$
5.00
4.00
3.00
2.00
1.00
$\omega > [A/[a+2(N-1)D]^2$
$\omega \leqslant [A/[a+2(N-1)D]^2$
A=2 D
R=0.9
180
240
300
360
420
480
540
$\theta^\circ$

Fig. (2)



$\bar{I}_c(\theta)[K]$
$[K] = [I_0 A(1-R)^2/4D^2]$
4.00
3.00
2.00
1.00
00
$\omega > A/[a+2(N-1)D]^2$
$\omega < A/[a+2(N-1)D]^2$
a - 2mD
R = 0.90
2
4
6
8
10
12
14
16
m


Fig.(3)