ON THE PLANE TEMPERATURE WAVES METHOD FOR THE DETERMINATION OF THERMAL PROPERTIES OF SOLIDS

BY

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Abstract:

Theory of plane temperature waves was analysed for a flat heater generating periodic heat flux and sandwitched between two identical solid specimens, taking into consideration the role of both heater and radiation.

Theoretical expressions for the determination of different theraml parameters from data of temperature oscillations in different ways are obtained.

The experimental results for olivine confirm the proposed scheme of measuring the thermal diffusivity, conductivity and heat capacity coefficients in one experiment using a multiproperty apparatus.

Introduction:

Measurement of the theraml properties of solids may be done using a great variety of steady and nonsteady state method. The best of these are the so-called periodic temperature methods, since they enable us to measure the heat capacity, thermal diffusivity and conductivity coefficients in one experiment[1,2]. When such an experiment is properly designed, it is possible to have extra information about the distribution of the amplitude and phases of the temperature oscillations in the investigated sample. So an extra control for the results treated in various ways is provided. Moreover, the random errors at suchan experiment are reduced.

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In this work we deduced the essential formulae for the determination of thermal diffusivity, conductivity, capacity coefficients in different ways by processing the experimental diagrams of temperature oscillations corresponding to emf variations of two thermocouples. One of them is sandwitched with the heater generating periodic heat flux between two identical specimens, while the other is fixed on the outer surface of the specimen this set-up is explained elsewhere[3,4].

Theoretical approach:

The plane temperature waves is an infinite slab are described by the following differential equation[5,6].

$$\partial^2\theta/\partial x^2 - (i\omega/a)\,\theta = 0 \qquad (1)$$

where θ represents the temperature oscillations.

ω the angular frequency and

a the thermal diffusivity coefficient

solution of this equation:

$$\theta = A \exp\left(\frac{i\omega}{a}\right)^{1/2} x + B \exp\left(-\frac{i\omega}{a}\right)^{1/2} x \qquad (2)$$

The boundary conditions to find A and B for the case under discussion (Fig. 1), considering radiation from the unheated surface (x = 0), are of the following from:

$$-\lambda\, \partial\theta/\partial x\Big|_{x=-L} = \tilde{q} \quad \text{and} \quad -\lambda\, \partial\theta/\partial x\Big|_{x=0} = \alpha\,\theta \qquad (3)$$

Where λ is the thermal conductivity coefficient, $\tilde{q}$ - heat flux,

α - heat transfer coefficient.

From the above an expression for the temperature oscillations at both heated (θ_h) and unheated surfaces (θ_{uh}) is obtained:

$$\theta_h = \tilde{q}/\lambda\left(\frac{i\omega}{a}\right)^{1/2}\cdot\left[\exp.(-\varkappa\sqrt{i}) + \exp.(\varkappa\sqrt{i})\right] + \frac{Bi}{\varkappa\sqrt{i}}\left[\exp.(\varkappa\sqrt{i}) - \exp(\varkappa\sqrt{i})\right] \quad (4)$$

$$/\left[\exp.(\varkappa\sqrt{i} - \exp(-\varkappa\sqrt{i})\right] - \frac{Bi}{\varkappa\sqrt{i}}\left[\exp.(-\varkappa\sqrt{i}) + \exp.(\varkappa\sqrt{i})\right]$$

$$\theta_{uh} = 2\,\tilde{q}/\lambda\left(\frac{i\omega}{a}\right)^{1/2}/\left[\exp(\varkappa\sqrt{i}) - \exp(-\varkappa\sqrt{i})\right] - \frac{Bi}{\varkappa\sqrt{i}}\left[\exp.(-\varkappa\sqrt{i}) + \exp.(\varkappa\sqrt{i})\right] \quad (5)$$

The reduced amplitude and the phase of temperature oscillations at both heated and unheated surfaces could be expressed.

$$F_h = |\theta_h/\theta_o| = \left[\left((s_1 + s_2) + \frac{Bi\sqrt{2}}{\varkappa}(s_4^2 + s_6^2 + s_1 s_2)\right)^2 + \left((s_2 - s_1) - \frac{Bi\sqrt{2}}{\varkappa}(s_5^2 + s_3^2)\right)^2\right]^{1/2} /$$

$$s_3^2 + s_2^2 + \frac{Bi\sqrt{2}}{\varkappa}(s_1 - s_2)$$

$$\phi_h = \arctan\left[(s_2 - s_1) - \frac{Bi\sqrt{2}}{\varkappa}(s_5^2 - s_3^2)\right] / (s_1 + s_2) + \frac{Bi\sqrt{2}}{\varkappa}(s_4^2 + s_6^2 + s_1 s_2) \quad (6)$$

$$F_{uh} = |\theta_{uh}/\theta_o| = \varkappa / \left[(s_3^2 + s_5^2) + \frac{Bi}{\varkappa\sqrt{2}}(s_2 - s_1)\right]^{1/2}$$

(7)

$$\phi_{uh} = \arctan\left[(s_3 + s_5) - \frac{Bi\sqrt{2}}{\varkappa}s_4\right] / (s_5 - s_3) + \frac{Bi\sqrt{2}}{\varkappa}s_6$$

where $\varkappa = \left(\frac{\omega}{a}\right)^{1/2} L$, $Bi = \alpha L/\lambda$ = Biot number (8)

$\theta_o = \tilde{q}/MWC_p$, $\alpha = 4\sigma\epsilon T^3$

σ - stefan - Boltzman constant, ϵ - emissivity (9)

$S_1 = Ch\,\varkappa/\sqrt{2}\;Sh\,\varkappa/\sqrt{2}$, $S_2 = \sin\varkappa/\sqrt{2}\cos\varkappa/\sqrt{2}$

$S_3 = Ch\,\varkappa/\sqrt{2}\,\sin\varkappa/\sqrt{2}$, $S_4 = SH\,\varkappa/\sqrt{2}\,\sin\varkappa/\sqrt{2}$

$S_5 = SH\,\varkappa/\sqrt{2}\,\cos\varkappa/\sqrt{2}$, $S_6 = CH\,\varkappa/\sqrt{2}\cos\varkappa/\sqrt{2}$

The dependence of $|\theta_h/\theta_o|$, $\left|\frac{\theta_{uh}}{\theta_o}\right|$, ϕ_h & ϕ_{uh} on $\varkappa$ and Bi[5] is plotted in Figures 2,3,4 and 5.

The ratio of the complex temperature oscillations at both heated and unheated surfaces could be expressed as:

$$\psi_1 = \theta_h / \theta_{uh} = \frac{1}{2}\left[\exp(\varkappa\sqrt{i}) + \exp(-\varkappa\sqrt{i})\right] + \frac{Bi}{\varkappa\sqrt{i}}\left[\exp(\varkappa\sqrt{i}) - \exp(-\varkappa\sqrt{i})\right] \quad (10)$$

Modulus of this expression is:

$$\theta_h/\theta_{uh} = \left[\left(s_6 + Bi/\varkappa\sqrt{2}(s_5 + s_3)\right)^2 + \left(s_4 + Bi/\varkappa\sqrt{2}(s_3 - s_6)\right)^2\right]^{1/2} \quad (11)$$

The difference of phases is:

$$\phi_h - \phi_{uh} = \Delta\phi = \arctan\left[s_4 + Bi/\varkappa\sqrt{2}(s_3 - s_5)\right] / \left[s_6 + \frac{Bi}{\varkappa\sqrt{2}}(s_5 + s_3)\right] \quad (12)$$

The dependence of θ_h/θ_{uh} and $\Delta\phi$ upon factor $\varkappa$ and Bi is shown in Figures 6 and 7.

The difference between θ_h and θ_{uh} will be:

$$(\theta_h - \theta_{uh}) = \tilde{q}/\lambda\left(\frac{\omega}{a}\right)^{1/2}\left[\varkappa\left(\exp(\varkappa\sqrt{i}) + \exp(-\varkappa\sqrt{i})\right) + \frac{Bi}{\varkappa\sqrt{i}}\left(\exp(\varkappa\sqrt{i}) - \exp(-\varkappa\sqrt{i})\right) - 2\right]$$

$$\Big/ \; 2\sqrt{i}\left(\exp(\varkappa\sqrt{i}) - \exp(-\varkappa\sqrt{i})\right) - \frac{Bi}{\varkappa\sqrt{i}}\left(\exp(-\varkappa\sqrt{i}) + \exp(\varkappa\sqrt{i})\right) \quad (13)$$

Modulus of this expression is given by

$$|\theta_h - \theta_{uh}| = |\Delta\theta| = \tilde{q}L/\lambda\varkappa\left[(s_1 - s_2 + 2(s_3 - s_5) + Bi\sqrt{2}/\varkappa(2s_6 - s_6^2 - s_4^2)\right] \Big/$$

$$s_5^2 + s_3^2 + Bi\sqrt{2}/\varkappa(s_2 - s_1) \quad (14)$$

In Figure 8, $|\Delta\theta|$ as a function of $\varkappa$ and Bi is shown.

Using the previous formulae and graphs, it is possible to determine factor $\varkappa$ by processing thhe experimental curves to get ϕ_h, ϕ_{uh} and $\Delta\phi$. It is also easy to determine $\varkappa$ by measuring the ratio θ_h / θ_{uh}.

Factor Bi is found from the measurement of ϕ_h or ϕ_{uh} at two frequencies ω and 2ω (6).

Thermal diffusivity coefficient is then obtained from

$$a = \frac{\omega}{\varkappa^2} L^2 \tag{15}$$

It could be also obtained using the following relation (quasi-steady state method).

$$a = \frac{\pi^2 L^2 \theta_0}{\tau \; \Delta T} \tag{16}$$

where

τ = period, ΔT = steady-state temperature difference across the specimen, or by measuring time lag between the power and the temperature oscillations at unheated surface

So,

$$a = \frac{L^2}{6 \Delta t}$$

where Δt = time lag.

The heat capacity can be determined using either.

$$C_p = \frac{\tilde{q}}{M \omega \theta_h} F_h \tag{17}$$

or

$$C_p = \frac{\tilde{q}}{M \omega \theta_{uh}} F_{uh} \tag{18}$$

M-mass of the specimen.

The thermal conductivity coefficient is determined from the steady-state temperature gradient across the specimen according to

$$\lambda = \frac{\tilde{q} L}{f \Delta T} \tag{19}$$

where f = area of the specimen.

It can also be determined using the relation:

$$\lambda = \frac{\tilde{q} L}{f \, |\Delta\theta|} \; F \tag{20}$$

Finally λ can be determined from

$$\lambda = \rho \, C a \tag{21}$$

Thus using the information about the temperature oscillations obtained from the two thermocopuples, the thermal properties may be determined according to the previous scheme.

It is worthy to mention, that in case of rectangular modulation of the heat flux (switching on and off of the heating current), it is possible to use a much simpler way for processing the experimental curves, to deduce the thermal properties.

This adds extra informations necessary for internal control of the obtained results.

Role of heater:

For the given arrangement in Figure (1) we should take into consideration the distribution of amplitudes and phases of temperature oscillations due to the presence of the heater.

It is worthy to mention that the presence of the heater does not affect the amplitude ratio method.

The influence of the heater upon the amplitude difference is expressed as;

$$\frac{\delta(\Delta\theta)}{(\Delta\theta)} = \frac{M_1 C_1}{MC} \cdot (s_1^2 + s_2^2)^{1/2} / (s_5^2 + s_3^2)$$

When $\frac{M_1 \; C_1'}{MC}$ is small (less than 1%) this correction may be neglected, otherwise it must be taken into consideration.

All the illustrated functions have been computed using computer ICL-600.

Experimental:

This theory was used for processing the experimental curves obtained during the course of calibration of the mentioned apparatus, while varying widely the experimental condtions.

In Table I the obtained results of measuring the thermal diffusivity for olivine using phase method, time lag method, amplitude ratio method and quasi-steady state method are given.

From the table, one can see that the obtained experimental values differ by about 2% randomly each other.

The experimental error in the phase method and amplitude-ratio method was 4-7%, in time lag method (for rectangular heat flux modulation) the error was 3.5% and in the quasi-steady state it was 6-8%.

Table (I): The thermal diffusivity coefficient of olivine ($a \times 10^{-3} \frac{cm^2}{sec}$)

Temp. °K	Biot number	Phase method at heat, surface		Phase method at unheated surface		Phase difference method		Time lag method		Amplitude method		Quasi steady state method	
		2ω	ω	2ω	ω	2ω	ω	2ω	ω	2ω	ω	2ω	ω
		$\tau=30.5$ sec	$\tau=61.5$ sec	$\tau=30.5$ sec	$\tau=61.5$ sec	$\tau=30.5$ sec	$\tau=61.5$ sec	$\tau=30.5$ sec	$\tau=61.5$ sec	$\tau=30.5$ sec	$\tau=61.5$ sec	$\tau=30.5$ sec	$\tau=61.5$ sec
435	0.10	8.9	8.2	8.2	8.1	8.1	8.1	8.3	8.2	8.2	8.4	8.1	8.1
545	0.15	6.6	6.6	7.1	7.1	7.0	6.6	6.8	7.2	7.4	7.0	6.9	6.7

The heat capacities of olivine calculated by different methods is tabulated in Table II.

Table (II): Heat capacity of olivine

Temp.	Biot number	Heat capacity at heared surface Cal/gm. °K		Heat capacity at unheated surface	
		τ =30.5sec	τ =61.5sec	τ=30.5 sec	τ =61.6 sec
435	0.10	0.176	0.179	0.177	0.178
545	0.15	0.214	0.213	0.213	0.215

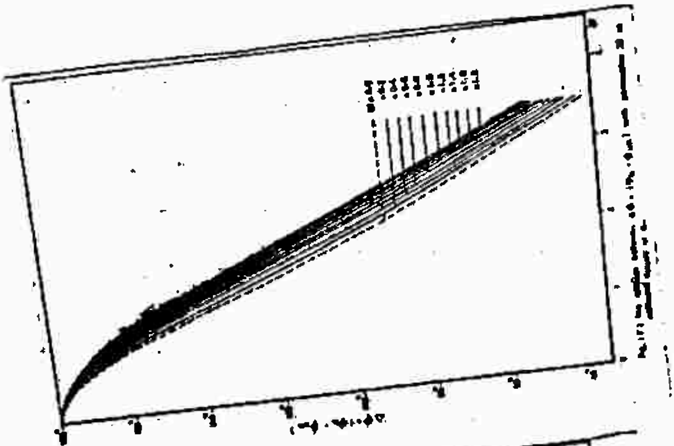
The obtained experimental results differe by 2-2.5% for the heat capacity of olivine. This seems to be quite satisfactory agreement with the experimental results.The experimental error of measuring heat capacity was, 5%.

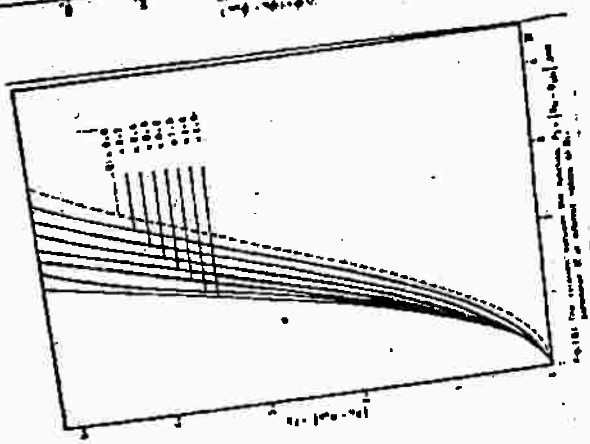
The thermal conductivity of olivine calculated by different methods is tabulated in Table III.

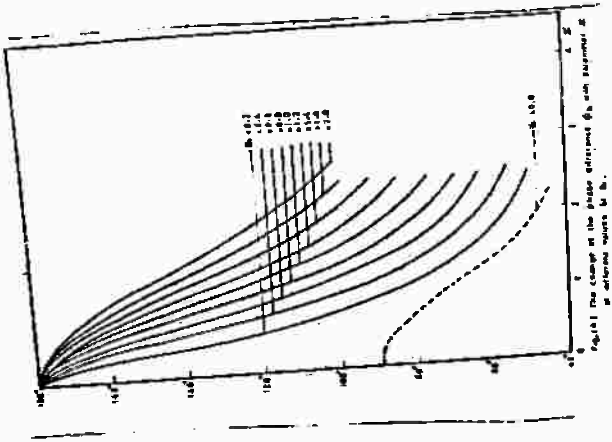
Table (III): The thermal conductivity coefficient of olivine

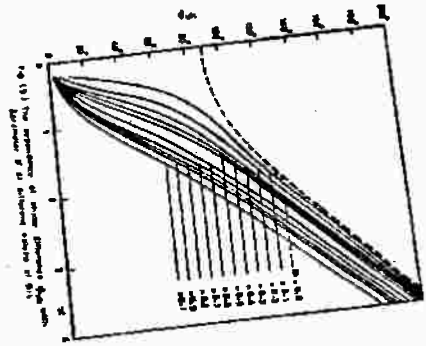
Temp	Biot number	Using equation (λ= Ca) Cal/cm sec. °K				Difference method
		heat. surface		unheat. surface		
		τ =30.5	τ =61.5	τ =30.5	τ =61.5	
435	0.10	0.0041	0.0040	0.0042	0.0043	0.0039
545	0.15	0.039	0.0039	0.0040	0.0040	0.0034

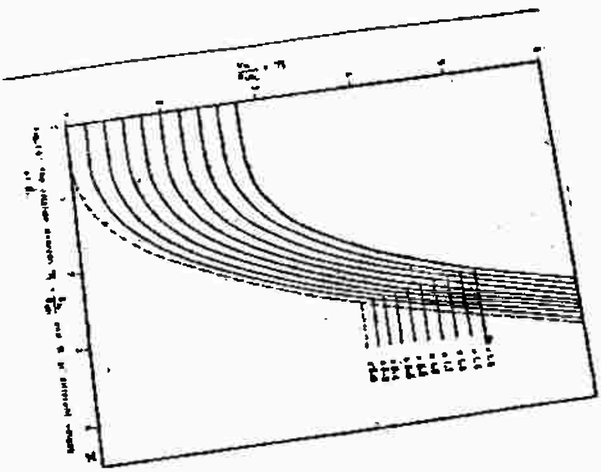
The agreement among the obtained values from different methods lies within 2-4%, which is less than the general experimental error of 2-5% for the mentioned methods except for formula (21) which gives 3-7%.

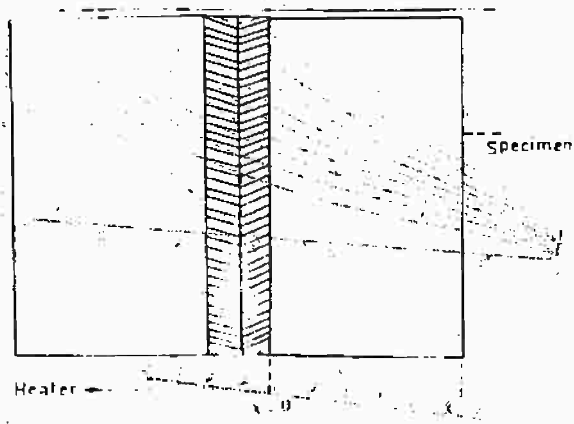


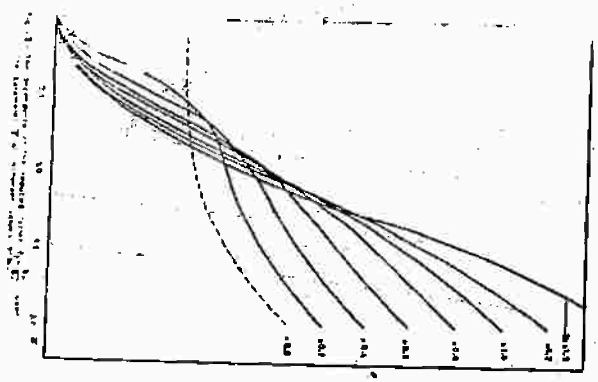












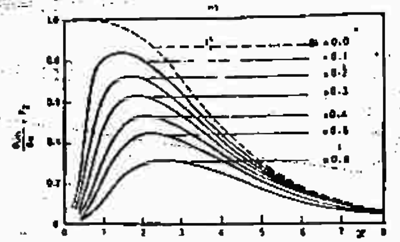


Fig. (3) The dependence of the dimensionless $f_3 = \frac{\theta_{sh}}{\theta_m}$ on parameter $\mathcal{X}$ at different values of Bi.