

CONTINUOUS BEAMS ON SOIL

2

2.1) INTRODUCTION

The design of footings (strip or isolated) and beams resting on the soil, based on the assumption that " the reaction of the soil on the foundation is uniform or varying linearly ", is only valid if the foundations are very rigid.

In practice, the flexural rigidity of the soil-foundation interaction must be taken into consideration and hence, any assumption concerning the shape of the reaction of the soil might lead to very big errors.

In this chapter the classical Winkler model is used. The soil is considered uniform with constant modulus of subgrade reaction in every direction under the foundation. The term [L_0] or [$1/\alpha$] which is equal to :

$$\left(4 E_b I_b / K_s B \right)^{1/4}$$

is the main parameter governing the phenomena of the soil-foundation interaction, where :

- E_b is the Youngs modulus of the foundation materials .
- I_b is the moment of itnertia of the foundation cross-section .
- K_s is the modulus of subgrade reaction .

Table No. (2-1) gives the values of K_s proposed by Terzaghi .

- B is the breadth of the foundation .

Table (2.1) : Values of K_s (kg/cm^3) for different soil formations.

Kind of soil	K_s (kg/cm^3)
Dense sand	9.61
Medium sand	4.96
Loose sand	1.04
Silty sand	3.60
Clayey sand	5.60
Stiff clay	4.80
Medium clay	3.60
Soft clay	1.79

The equations giving the solution of the problem developed by Hetenyi are plotted in curves to facilitate their use . The principal of superposition can be used to deal with more than one load.

The values of soil reaction/m', deflection, bending moment and shearing forces for a beam resting on elastic soil subjected to concentrated load P Figure (2.1) may be obtained from the following simple expressions :

$$\text{SOIL REACTION} \quad p = P \alpha C_p / B \quad (2.1)$$

$$\text{DEFLECTION} \quad Y = p_s / K_s \quad (2.2)$$

$$\text{BENDING MOMENT} \quad M = P \times C_M / (2 \times \alpha) \quad (2.3)$$

$$\text{SHEARING FORCE} \quad Q = P \times C_Q \quad (2.4)$$

where :

P = Concentrated Load (t)

B = Foundation breadth (m)

α = Relative rigidity (m^{-1}) = $(K_s B / 4 E_b I_b)^{1/4}$

K_s = Coefficient of subgrade reaction (tm^{-3})

C_p = Soil reaction coefficient (Dimensionless)

C_M = Bending moment coefficient (Dimensionless)

C_Q = Shearing force coefficient (Dimensionless)

$E_b I_b$ = Foundation rigidity (tm^2)

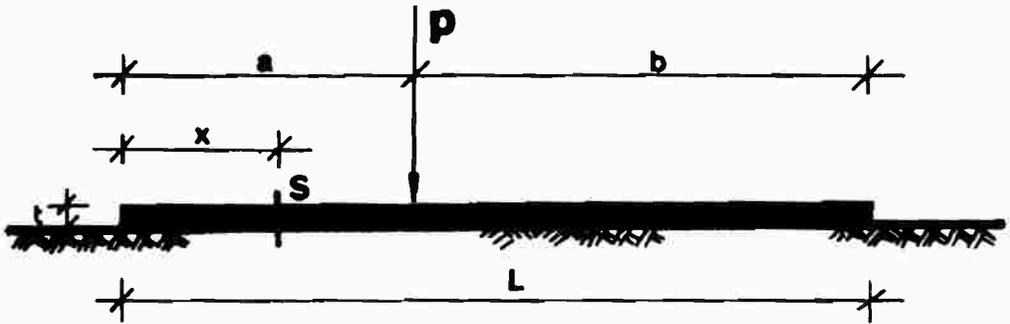


Fig. 2.1

2.2) Hetenyi equations

$$C_p = \frac{1}{\sinh^2 \alpha L - \sin^2 \alpha L} \{ 2 (\cosh \alpha x \cos \alpha x) (\sinh \alpha L \cos \alpha a \cosh \alpha b - \sin \alpha L \cosh \alpha a \cos \alpha b) + (\cosh \alpha x \sin \alpha x + \sinh \alpha x \cos \alpha x) [\sinh \alpha L (\sin \alpha a \cosh \alpha b - \cos \alpha a \sinh \alpha b) + \sin \alpha L (\sinh \alpha a \cos \alpha b - \cosh \alpha a \sin \alpha b)] \} \quad (2.5)$$

$$C_M = \frac{1}{\sinh^2 \alpha L - \sin^2 \alpha L} \{ 2 (\sinh \alpha x \sin \alpha x) (\sinh \alpha L \cos \alpha a \cosh \alpha b - \sin \alpha L \cosh \alpha a \cos \alpha b) + (\cosh \alpha x \sin \alpha x - \sinh \alpha x \cos \alpha x) [\sin \alpha L (\sin \alpha a \cosh \alpha b - \cos \alpha a \sinh \alpha b) + \sin \alpha L (\sinh \alpha a \cos \alpha b - \cosh \alpha a \sin \alpha b)] \} \quad (2.6)$$

$$C_Q = \frac{1}{\sinh^2 \alpha L - \sin^2 \alpha L} \{ (\cosh \alpha x \sin \alpha x + \sinh \alpha x \cos \alpha x) (\sinh \alpha L \cos \alpha a \cosh \alpha b - \sin \alpha L \cosh \alpha a \cos \alpha b) + (\sinh \alpha x \sin \alpha x) [\sinh \alpha L (\sin \alpha a \cosh \alpha b - \cos \alpha a \sinh \alpha b) + \sin \alpha L (\sinh \alpha a \cos \alpha b - \cosh \alpha a \sin \alpha b)] \} \quad (2.7)$$

Where :

a , b , and L are distances shown in Figure. (2.1)

The values of coefficients C_p , C_M , C_Q are calculated and plotted hereinafter .

2.3) HOW TO USE THE CURVES TO GET C_p , C_M & C_Q

- (1) According to the kind of soil get K_s from table (2.1)
- (2) From (K_s) and depth (d) get (α) from the expression $(K_s B / 4E_b I_b)^{1/4}$ or from chart (2.1)
- (3) According to (αL) and load position (a / L) get C_p , C_M , C_Q from the corresponding curves .

The curves are numbered in the form ($C_p - \alpha L$), ($C_M - \alpha L$) and ($C_Q - \alpha L$) .

For example the notation ($C_Q - 5$) gives values of C_Q for $\alpha L = 5.00$.

For intermediate values of (αL), (a) and (b) the solution can be obtained by linear interpolation .

The values of C_p , C_M and C_Q are also tabulated in appendix A .

EXAMPLE :

Find the soil reaction, the bending moment and shearing force at section (S) due to the shown loads Figure (2.2).

The beam is 10 ms long and 1.0 m in breadth and 1.0 m in depth. The soil is loose sand.

From table (2.1) $K_s = 1.04 \text{ kg/cm}^3$

From chart (2.1) $\alpha = 0.002 \text{ 1/cm}$

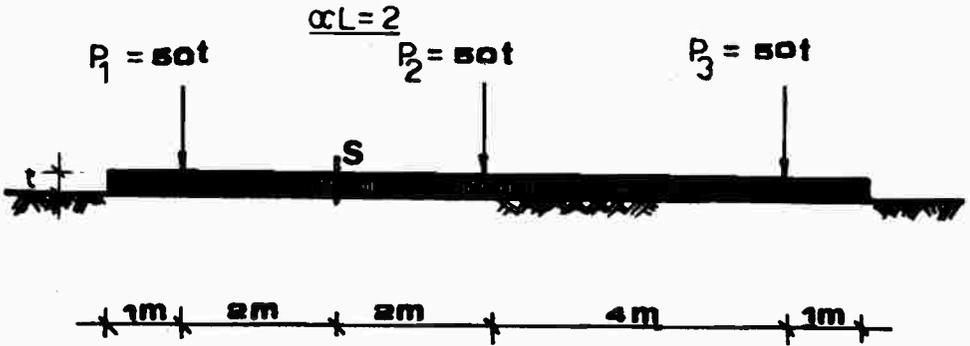


Fig. 2.2

From Fig. (C_p , 2), Fig. (C_M , 2) and Fig. (C_Q , 2)

we have :

For P_1 : $C_p = 0.94$, $C_M = -0.25$ and $C_Q = -0.17$

For P_2 : $C_p = 0.53$, $C_M = 0.15$ and $C_Q = 0.27$

For P_3 : $C_p = 0.00$, $C_M = -0.14$ and $C_Q = 0.18$

Soil reaction (kg/cm^2)
$P_1 : 0.94 \times 50 \times 10^3 \times 0.002 / 100 = 0.94$
$P_2 : 0.53 \times 50 \times 10^3 \times 0.002 / 100 = 0.53$
$P_3 : 0.00 \times 50 \times 10^3 \times 0.002 / 100 = 0.00$
$p = 1.47 \text{ k/cm}^2$

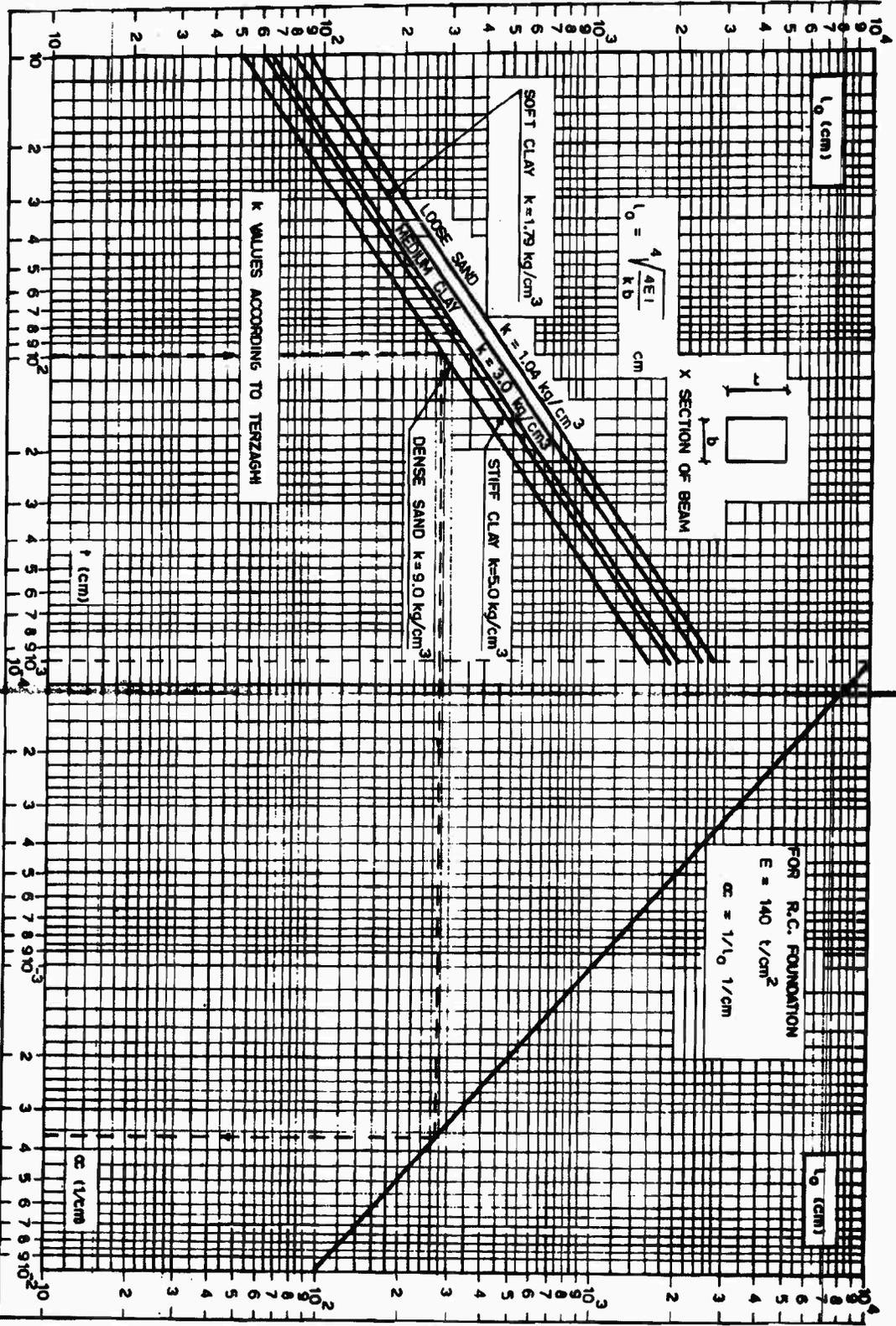


CHART 2.1

Deflection (cm)	B.M. (m.t)	S.F. (t)
P1 : 0.94 / 1.04 = 0.9	-0.25 x 50 / 2 x 1 = -6.25	-0.17 x 50 = -8.5
P2 : 0.5 / 1.04 = 0.48	0.15 x 50 / 2 x 1 = 3.83	0.27 x 50 = 13.5
P3 : 0.0	-0.14 x 50 / 2 x 1 = -3.5	0.18 x 50 = 9.0
y = 1.4 cm	M = -5.9 mt	Q = 14.0 t

If the foundation is very flexible, the problem can be considerably simplified. If both αa and αb are $> \pi$, the foundation may be considered of infinite length.

If $a = \text{Zero}$ and $\alpha b > \pi$ the foundation may be considered of semi infinite length. The solution of the problem in these cases are obtained from the following equations :

2.4) FOUNDATIONS WITH INFINITE LENGTH

$$(\alpha a, \alpha b) > \pi$$

2.4.1) *Infinite long beam subjected to single concentrated load (P)*

$$p = P \alpha / 2 \times F_2 \quad (2.8)$$

$$M = P / 4 \alpha \times F_4 \quad (2.9)$$

$$Q = -P / 2 \times F_1 \quad (2.10)$$

2.4.2) *Infinite long beam subjected to single concentrated couple (M₀)*

$$p = M_0 \alpha^2 \times F_3 \quad (2.11)$$

$$M = M_0 / 2 \times F_1 \quad (2.12)$$

$$Q = M_0 \alpha / 2 \times F_2 \quad (2.13)$$

2.5) FOUNDATIONS WITH SEMI-INFINITE LENGTH [$a = 0$, $(\alpha b > \pi)$]

2.5.1) *Semi-infinite long beam subjected to a concentrated load (P) at its end .*

$$p = 2P \alpha \times F_1 \quad (2.14)$$

$$M = -P/\alpha \times F_3 \quad (2.15)$$

$$Q = -P \times F_4 \quad (2.16)$$

2.5.2) *Semi-infinite long beam subjected to a concentrated couple (M_o) at its end.*

$$p = 2 M_o \alpha^2 \times F_4 \quad (2.17)$$

$$M = -M_o \times F_2 \quad (2.18)$$

$$Q = 2 M_o \alpha \times F_4 \quad (2.19)$$

The functions F_1 , F_2 , F_3 , and F_4 are plotted in figure (2.3) and can be obtained from the following analytical expressions :

$$F_1 = e^{-\alpha x} \cos(\alpha x)$$

$$F_2 = e^{-\alpha x} [\cos(\alpha x) + \sin(\alpha x)]$$

$$F_3 = e^{-\alpha x} \sin(\alpha x)$$

$$F_4 = e^{-\alpha x} [\cos(\alpha x) - \sin(\alpha x)]$$

For problems with more than one load acting on the foundation, we may use the principle of superposition.

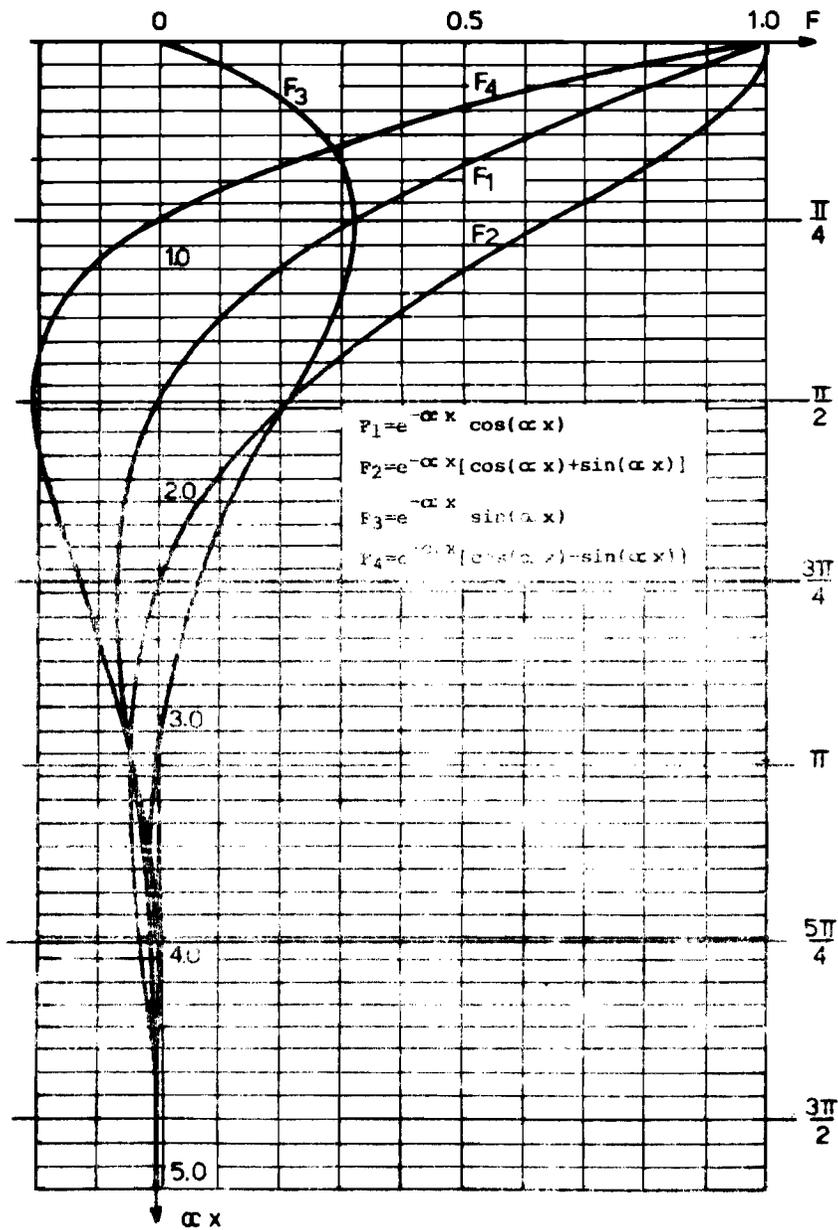


Figure 2.3

APPENDIX A

DIMENSIONLESS COEFFICIENTS For $\alpha L = 0.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	8.0048	0.0000	0.0000
0.0	8.0048	0.0000	-1.0000
0.1	6.8022	-0.0810	-0.6298
0.2	5.6001	-0.1280	-0.3198
0.3	4.3985	-0.1469	-0.0698
0.4	3.1977	-0.1439	0.1201
0.5	1.9977	-0.1249	0.2500
0.6	0.7982	-0.0959	0.3199
0.7	-0.4008	-0.0630	0.3298
0.8	-1.5995	-0.0320	0.2798
0.9	-2.7980	-0.0090	0.1699
1.0	-3.9964	-0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	6.8022	0.0000	0.0000
0.1	5.8412	0.0162	0.3161
0.1	5.8412	0.0162	-0.6839
0.2	4.8802	-0.0384	-0.4159
0.3	3.9193	-0.0686	-0.1959
0.4	2.9588	-0.0792	-0.0239
0.5	1.9987	-0.0750	0.1000
0.6	1.0390	-0.0608	0.1759
0.7	0.0795	-0.0414	0.2039
0.8	-0.8797	-0.0216	0.1839
0.9	-1.8389	-0.0062	0.1159
1.0	-2.7980	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	5.6001	0.0000	0.0000
0.1	4.8802	0.0134	0.2620
0.2	4.1602	0.0512	0.4880
0.2	4.1602	0.0512	-0.5120
0.3	3.4401	0.0098	-0.3220
0.4	2.7198	-0.0144	-0.1680
0.5	1.9997	-0.0250	-0.0500
0.6	1.2797	-0.0256	0.0320
0.7	0.5598	-0.0198	0.0780
0.8	-0.1600	-0.0112	0.0880
0.9	-0.8797	-0.0034	0.0620
1.0	-1.5995	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	4.3985	0.0000	0.0000
0.1	3.9193	0.0106	0.2079
0.2	3.4400	0.0408	0.3919
0.3	2.9606	0.0882	0.5519
0.3	2.9606	0.0882	-0.4481
0.4	2.4807	0.0504	-0.3120
0.5	2.0006	0.0250	-0.2000
0.6	1.5204	0.0096	-0.1120
0.7	1.0401	0.0018	-0.0479
0.8	0.5598	-0.0008	-0.0080
0.9	0.0795	-0.0006	0.0080
0.1	-0.4008	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	3.1977	0.0000	0.0000
0.1	2.9588	0.0078	0.1539
0.2	2.7198	0.0304	0.2959
0.3	2.4807	0.0666	0.4259
0.4	2.2413	0.1152	0.5439
0.4	2.2413	0.1152	-0.4561
0.5	2.0013	0.0750	-0.3500
0.6	1.7609	0.0448	-0.2559
0.7	1.5204	0.0234	-0.1739
0.8	1.2797	0.0096	-0.1039
0.9	1.0389	0.0022	-0.0459
1.0	0.7982	-0.0000	0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	1.9977	0.0000	0.0000
0.1	1.9987	0.0050	0.0999
0.2	1.9997	0.0200	0.1999
0.3	2.0006	0.0450	0.2999
0.4	2.0013	0.0800	0.3999
0.5	2.0016	0.1250	0.5000
0.5	2.0016	0.1250	-0.5000
0.6	2.0013	0.0800	-0.3999
0.7	2.0006	0.0450	-0.2999
0.8	1.9997	0.0200	-0.1999
0.9	1.9987	0.0050	-0.0999
1.0	1.9977	-0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 1.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	4.0378	0.0000	0.0000
0.0	4.0378	0.0000	-1.0000
0.1	0.4176	-0.1617	-0.6272
0.2	2.8005	-0.2550	-0.3164
0.3	2.1884	-0.2923	-0.0670
0.4	1.5821	-0.2859	0.1215
0.5	0.9814	-0.2478	0.2496
0.6	0.3857	-0.1900	0.3179
0.7	-0.2063	-0.1245	0.3269
0.8	-0.7957	-0.0632	0.2768
0.9	-1.3839	-0.0178	0.1678
1.0	-1.9717	-0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	3.4176	0.0000	0.0000
0.1	2.9295	0.0325	0.3174
0.1	2.9295	0.0325	-0.6826
0.2	2.4414	-0.0763	-0.4141
0.3	1.9547	-0.1363	-0.1943
0.4	1.4706	-0.1573	-0.0231
0.5	0.9896	-0.1488	0.0999
0.6	0.5116	-0.1205	0.1749
0.7	0.0360	-0.0820	0.2023
0.8	-0.4380	-0.0427	0.1822
0.9	-0.9111	-0.0123	0.1148
1.0	-1.3839	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	2.8005	0.0000	0.0000
0.1	2.4414	0.0268	0.2621
0.2	2.0816	0.1024	0.4883
0.2	2.0816	0.1024	-0.5117
0.3	1.7204	0.0197	-0.3216
0.4	1.3588	-0.0286	-0.1677
0.5	0.9977	-0.0498	-0.0499
0.6	0.6375	-0.0510	0.0319
0.7	0.2783	-0.0394	0.0777
0.8	-0.0800	-0.0223	0.0876
0.9	-0.4380	-0.0068	0.0617
1.0	-0.7957	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	2.1884	0.0000	0.0000
0.1	1.9547	0.0211	0.2072
0.2	1.7204	0.0813	0.3909
0.3	1.4845	0.1759	0.5512
0.3	1.4845	0.1759	-0.4488
0.4	1.2457	0.1002	-0.3123
0.5	1.0049	0.0494	-0.1997
0.6	0.7630	0.0187	-0.1113
0.7	0.5207	0.0032	-0.0472
0.8	0.2783	-0.0018	-0.0072
0.9	0.0360	-0.0013	0.0085
1.0	-0.2063	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	1.5821	0.0000	0.0000
0.1	1.4706	0.0154	0.1526
0.2	1.3588	0.0603	0.2941
0.3	1.2457	0.1323	0.4244
0.4	1.1300	0.2293	0.5432
0.4	1.1300	0.2293	-0.4568
0.5	1.0102	0.1488	-0.3498
0.6	0.8875	0.0886	-0.2549
0.7	0.7630	0.0460	-0.1724
0.8	0.6375	0.0188	-0.1023
0.9	0.5116	0.0043	-0.0449
1.0	0.3857	0.0000	0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	0.9814	0.0000	0.0000
0.1	0.9896	0.0098	0.0986
0.2	0.9977	0.0395	0.1979
0.3	1.0049	0.0891	0.2981
0.4	1.0102	0.1587	0.3988
0.5	1.0124	0.2486	0.5000
0.5	1.0124	0.2486	-0.5000
0.6	1.0102	0.1587	-0.3988
0.7	1.0049	0.0891	-0.2981
0.8	0.9977	0.0395	-0.1979
0.9	0.9896	0.0098	-0.0986
1.0	0.9814	0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 1.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.7910	0.0000	0.0000
0.0	2.7910	0.0000	-1.0000
0.1	2.3243	-0.2407	-0.6164
0.2	1.8680	-0.3768	-0.3022
0.3	1.4285	-0.4288	-0.0552
0.4	1.0079	-0.4164	0.1273
0.5	0.6059	-0.3585	0.2481
0.6	0.2200	-0.2734	0.3099
0.7	-0.1537	-0.1783	0.3147
0.8	-0.5193	-0.0901	0.2642
0.9	-0.8808	-0.0252	0.1591
1.0	-1.2410	0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	2.3243	0.0000	0.0000
0.1	1.9779	0.0497	0.3227
0.1	1.9779	0.0497	-0.6773
0.2	1.6311	-0.1116	-0.4067
0.3	1.2891	-0.1995	-0.1877
0.4	0.9559	-0.2293	-0.0195
0.5	0.6328	-0.2161	0.0995
0.6	0.3194	-0.1744	0.1708
0.7	0.0137	-0.1182	0.1957
0.8	-0.2866	-0.0615	0.1752
0.9	-0.5842	-0.0176	0.1099
1.0	-0.8808	0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	1.8680	0.0000	0.0000
0.1	1.6311	0.0403	0.2624
0.2	1.3921	0.1539	0.4892
0.2	1.3921	0.1539	-0.5108
0.3	1.1482	0.0302	-0.3202
0.4	0.9027	-0.0419	-0.1664
0.5	0.6290	-0.0733	-0.0493
0.6	0.4184	-0.0751	0.0315
0.7	0.1812	-0.0580	0.0764
0.8	-0.0535	-0.0328	0.0860
0.9	-0.2866	-0.0099	0.0604
1.0	-0.5193	0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	1.4285	0.0000	0.0000
0.1	1.2891	0.0311	0.2038
0.2	1.1482	0.1202	0.3867
0.3	1.0017	0.2609	0.5480
0.3	1.0017	0.2609	-0.4520
0.4	0.8455	0.1467	-0.3133
0.5	0.6825	0.0705	-0.1987
0.6	0.5163	0.0250	-0.1088
0.7	0.3488	0.0028	-0.0439
0.8	0.1812	-0.0038	-0.0041
0.9	0.0137	-0.0022	0.0105
1.0	-0.1537	0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	1.0079	0.0000	0.0000
0.1	0.9559	0.0223	0.1473
0.2	0.9027	0.0876	0.2867
0.3	0.8455	0.1935	0.4179
0.4	0.7793	0.3374	0.5399
0.4	0.7793	0.3374	-0.4601
0.5	0.7002	0.2164	-0.3490
0.6	0.6111	0.1268	-0.2505
0.7	0.5163	0.0647	-0.1659
0.8	0.4184	0.0258	-0.0958
0.9	0.3194	0.0057	-0.0405
1.0	0.2200	0.0000	0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	0.6059	0.0000	0.0000
0.1	0.6328	0.0138	0.0929
0.2	0.6590	0.0561	0.1898
0.3	0.6825	0.1281	0.2905
0.4	0.7002	0.2307	0.3943
0.5	0.7073	0.3649	0.5000
0.5	0.7073	0.3649	-0.5000
0.6	0.7002	0.2307	-0.3943
0.7	0.6825	0.1281	-0.2905
0.8	0.6590	0.0561	-0.1898
0.9	0.6328	0.0138	-0.0929
1.0	0.6059	0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 2.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.2752	0.0000	0.0000
0.0	2.2752	0.0000	-1.0000
0.1	1.8263	-0.3150	-0.5901
0.2	1.4016	-0.4838	-0.2679
0.3	1.0149	-0.5402	-0.0269
0.4	0.6708	-0.5150	0.1410
0.5	0.3676	-0.4360	0.2442
0.6	0.0991	-0.3273	0.2903
0.7	-0.1433	-0.2105	0.2855
0.8	-0.3688	-0.1051	0.2341
0.9	-0.5857	-0.0291	0.1386
1.0	-0.7999	-0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.8263	0.0000	0.0000
0.1	1.5284	0.0691	0.3355
0.1	1.5284	0.0691	-0.6645
0.2	1.2296	-0.1396	-0.3888
0.3	0.9413	-0.2498	-0.1719
0.4	0.6724	-0.2846	-0.0109
0.5	0.4260	-0.2654	0.0985
0.6	0.2006	-0.2120	0.1609
0.7	-0.0080	-0.1425	0.1799
0.8	-0.2052	-0.0735	0.1584
0.9	-0.3964	-0.0209	0.0982
1.0	-0.5857	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	1.4016	0.0000	0.0000
0.1	1.2296	0.0538	0.2631
0.2	1.0527	0.2059	0.4915
0.2	1.0527	0.2059	-0.5085
0.3	0.8640	0.0421	-0.3166
0.4	0.6716	-0.0525	-0.1631
0.5	0.4829	-0.0935	-0.0478
0.6	0.3015	-0.0957	0.0306
0.7	0.1276	-0.0737	0.0734
0.8	-0.0405	-0.0416	0.0820
0.9	-0.2052	-0.0126	0.0574
1.0	-0.3688	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	1.0149	0.0000	0.0000
0.1	0.9413	0.0396	0.1956
0.2	0.8640	0.1545	0.3763
0.3	0.7740	0.3384	0.5404
0.3	0.7740	0.3384	-0.4596
0.4	0.6617	0.1841	-0.3157
0.5	0.5344	0.0827	-0.1959
0.6	0.4002	0.0239	-0.1024
0.7	0.2639	-0.0029	-0.0360
0.8	0.1276	-0.0085	0.0032
0.9	-0.0080	-0.0039	0.0151
1.0	-0.1433	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	0.6708	0.0000	0.0000
0.1	0.6724	0.0269	0.1343
0.2	0.6716	0.1075	0.2688
0.3	0.6617	0.2418	0.4024
0.4	0.6322	0.4289	0.5322
0.4	0.6322	0.4289	-0.4678
0.5	0.5734	0.4664	-0.3468
0.6	0.4929	0.1496	-0.2399
0.7	0.4002	0.0721	-0.1504
0.8	0.3015	0.0267	-0.0802
0.9	0.2006	0.0053	0.0300
1.0	0.0991	0.0000	0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	0.3676	0.0000	0.0000
0.1	0.4260	0.0155	0.0794
0.2	0.4829	0.0650	0.1703
0.3	0.5344	0.1532	0.2722
0.4	0.5734	0.2840	0.3832
0.5	0.5893	0.4606	0.5000
0.5	0.5893	0.4606	-0.5000
0.6	0.5734	0.2840	-0.3832
0.7	0.5344	0.1532	-0.2722
0.8	0.4829	0.0650	-0.1703
0.9	0.4260	0.0155	-0.0794
1.0	0.3676	0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 2.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0736	0.0000	0.0000
0.0	2.0736	0.0000	-1.0000
0.1	1.5729	-0.3810	-0.5447
0.2	1.1177	-0.5648	-0.2097
0.3	0.7317	-0.6083	0.0199
0.4	0.4207	-0.5594	0.1624
0.5	0.1791	-0.4573	0.2361
0.6	-0.0055	-0.3322	0.2567
0.7	-0.1487	-0.2073	0.2368
0.8	-0.2657	-0.1008	0.1846
0.9	-0.3698	-0.0273	0.1050
1.0	-0.4699	0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.5729	0.0000	0.0000
0.1	1.2836	0.0923	0.3571
0.1	1.2836	0.0923	-0.6429
0.2	0.9916	-0.1550	-0.3586
0.3	0.7177	-0.2782	-0.1455
0.4	0.4776	-0.3113	0.0031
0.5	0.2758	-0.2843	0.0965
0.6	0.1091	-0.2224	0.1440
0.7	-0.0298	-0.1466	0.1534
0.8	-0.1505	-0.0744	0.1306
0.9	-0.2616	-0.0209	0.0789
1.0	-0.3698	0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	1.1177	0.0000	0.0000
0.1	0.9916	0.0672	0.2637
0.2	0.8559	0.2583	0.4950
0.2	0.8559	0.2583	-0.5050
0.3	0.6971	0.0562	-0.3104
0.4	0.5304	-0.0590	-0.1570
0.5	0.3704	-0.1077	-0.0447
0.6	0.2234	-0.1101	0.0293
0.7	0.0899	-0.0843	0.0682
0.8	-0.0333	-0.0472	0.0751
0.9	-0.1505	-0.0142	0.0520
1.0	-0.2657	0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.7317	0.0000	0.0000
0.1	0.7177	0.0454	0.1812
0.2	0.6971	0.1805	0.3583
0.3	0.6530	0.4025	0.5278
0.3	0.6530	0.4025	-0.4722
0.4	0.5682	0.2057	-0.3187
0.5	0.4570	0.0796	-0.1902
0.6	0.3352	0.0105	-0.0911
0.7	0.2116	-0.0166	-0.0228
0.8	0.0899	-0.0174	0.0149
0.9	-0.0298	-0.0068	0.0223
1.0	-0.1487	0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	0.4207	0.0000	0.0000
0.1	0.4776	0.0275	0.1123
0.2	0.5304	0.1146	0.2385
0.3	0.5682	0.2679	0.3763
0.4	0.5718	0.4919	0.5198
0.4	0.5718	0.4919	-0.4802
0.5	0.5236	0.2868	-0.3423
0.6	0.4388	0.1467	-0.2215
0.7	0.3352	0.0613	-0.1245
0.8	0.2234	0.0177	-0.0545
0.9	0.1091	0.0020	-0.0130
1.0	-0.0055	0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	0.1791	0.0000	0.0000
0.1	0.2758	0.0132	0.0569
0.2	0.3704	0.0609	0.1377
0.3	0.4570	0.1548	0.2414
0.4	0.5236	0.3056	0.3646
0.5	0.5513	0.5214	0.5000
0.5	0.5513	0.5214	-0.5000
0.6	0.5236	0.3056	-0.3646
0.7	0.4570	0.1548	-0.2414
0.8	0.3704	0.0609	-0.1377
0.9	0.2758	0.0132	-0.0569
1.0	0.1791	0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 3.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0131	0.0000	0.0000
0.0	2.0131	0.0000	-1.0000
0.1	1.4283	-0.4367	-0.4849
0.2	0.9183	-0.6151	-0.1354
0.3	0.5164	-0.6266	0.0770
0.4	0.2260	-0.5434	0.1857
0.5	0.0328	-0.4181	0.2223
0.6	-0.0853	-0.2858	0.2129
0.7	-0.1517	-0.1681	0.1763
0.8	-0.1875	-0.0772	0.1249
0.9	-0.2088	-0.0198	0.0652
1.0	-0.2260	-0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.4283	0.0000	0.0000
0.1	1.1343	0.1198	0.3846
0.1	1.1343	0.1198	-0.6154
0.2	0.8337	-0.1565	-0.3204
0.3	0.5590	-0.2822	-0.1125
0.4	0.3335	-0.3066	0.0200
0.5	0.1624	-0.2701	0.0930
0.6	0.0394	-0.2037	0.1222
0.7	-0.0470	-0.1296	0.1203
0.8	-0.1100	-0.0637	0.0964
0.9	-0.1612	-0.0174	0.0555
1.0	-0.2088	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.9183	0.0000	0.0000
0.1	0.8337	0.0801	0.2629
0.2	0.7324	0.3101	0.4986
0.2	0.7324	0.3101	-0.5014
0.3	0.5914	0.0712	-0.3020
0.4	0.4359	-0.0614	-0.1479
0.5	0.2903	-0.1154	-0.0394
0.6	0.1647	-0.1169	0.0283
0.7	0.0597	-0.0884	0.0615
0.8	-0.0296	-0.0489	0.0657
0.9	-0.1100	-0.0146	0.0446
1.0	-0.1875	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.5164	0.0000	0.0000
0.1	0.5590	0.0478	0.1614
0.2	0.5914	0.1960	0.3344
0.3	0.5869	0.4502	0.5126
0.3	0.5869	0.4502	-0.4874
0.4	0.5178	0.2089	-0.3203
0.5	0.4098	0.0603	-0.1806
0.6	0.2897	-0.0147	-0.0756
0.7	0.1716	-0.0376	-0.0065
0.8	0.0597	-0.0295	0.0280
0.9	-0.0470	-0.0105	0.0298
1.0	-0.1517	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	0.2260	0.0000	0.0000
0.1	0.3335	0.0236	0.0840
0.2	0.4359	0.1071	0.1996
0.3	0.5178	0.2688	0.3435
0.4	0.5500	0.5230	0.5054
0.4	0.5500	0.5230	-0.4946
0.5	0.5045	0.2749	-0.3347
0.6	0.4082	0.1169	-0.1969
0.7	0.2897	0.0320	-0.0919
0.8	0.1647	-0.0008	-0.0237
0.9	0.0394	-0.0039	0.0069
1.0	-0.0853	-0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	0.0328	0.0000	0.0000
0.1	0.1624	0.0068	0.0293
0.2	0.2903	0.0429	0.0973
0.3	0.4098	0.1311	0.2026
0.4	0.5045	0.2926	0.3407
0.5	0.5452	0.5442	0.5000
0.5	0.5452	0.5442	-0.5000
0.6	0.5045	0.2926	-0.3407
0.7	0.4098	0.1311	-0.2026
0.8	0.2903	0.0429	-0.0973
0.9	0.1624	0.0068	-0.0293
1.0	0.0328	0.0000	0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 3.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0022	0.0000	0.0000
0.0	2.0022	0.0000	-1.0000
0.1	1.3254	-0.4830	-0.4197
0.2	0.7604	-0.6390	-0.0588
0.3	0.3481	-0.6054	0.1306
0.4	0.0824	-0.4836	0.2020
0.5	-0.0653	-0.3391	0.2021
0.6	-0.1295	-0.2090	0.1661
0.7	-0.1420	-0.1095	0.1174
0.8	-0.1269	-0.0443	0.0699
0.9	-0.1003	-0.0099	0.0300
1.0	-0.0708	-0.0000	-0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.3254	0.0000	0.0000
0.1	1.0291	0.1504	0.4124
0.1	1.0291	0.1504	-0.5876
0.2	0.7194	-0.1476	-0.2818
0.3	0.4422	-0.2685	-0.0801
0.4	0.2286	-0.2799	0.0353
0.5	0.0824	-0.2338	0.0879
0.6	-0.0069	-0.1663	0.0997
0.7	-0.0555	-0.0997	0.0878
0.8	-0.0794	-0.0462	0.0637
0.9	-0.0915	-0.0119	0.0336
1.0	-0.1003	0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.7604	0.0000	0.0000
0.1	0.7194	0.0915	0.2592
0.2	0.6523	0.3588	0.5007
0.2	0.6523	0.3588	-0.4993
0.3	0.5227	0.0845	-0.2922
0.4	0.3697	-0.0622	-0.1360
0.5	0.2302	-0.1181	-0.0317
0.6	0.1184	-0.1169	0.0284
0.7	0.0347	-0.0826	0.0545
0.8	-0.0281	-0.0466	0.0551
0.9	-0.0794	-0.0136	0.0361
1.0	-0.1269	0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.3481	0.0000	0.0000
0.1	0.4422	0.0465	0.1384
0.2	0.5227	0.2011	0.3081
0.3	0.5513	0.4828	0.4984
0.3	0.5513	0.4828	-0.5016
0.4	0.4874	0.1974	-0.3175
0.5	0.3728	0.0304	-0.1662
0.6	0.2488	-0.0454	-0.0576
0.7	0.1347	-0.0600	0.0092
0.8	0.0347	-0.0413	0.0384
0.9	-0.0555	-0.0139	0.0346
1.0	-0.1420	0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	0.0824	0.0000	0.0000
0.1	0.2286	0.0161	0.0545
0.2	0.3697	0.0881	0.1595
0.3	0.4874	0.2502	0.3106
0.4	0.5414	0.5305	0.4934
0.4	0.5414	0.5305	-0.5066
0.5	0.4913	0.2411	-0.3232
0.6	0.3797	0.0708	-0.1698
0.7	0.2488	-0.0068	-0.0596
0.8	0.1184	-0.0234	0.0045
0.9	-0.0069	-0.0109	0.0239
1.0	-0.1295	0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.0653	0.0000	0.0000
0.1	0.0824	-0.0020	0.0030
0.2	0.2302	0.0163	0.0577
0.3	0.3728	0.0908	0.1636
0.4	0.4913	0.2562	0.3160
0.5	0.5446	0.5407	0.5000
0.5	0.5446	0.5407	-0.5000
0.6	0.4913	0.2562	-0.3160
0.7	0.3728	0.0908	-0.1636
0.8	0.2302	0.0163	-0.0577
0.9	0.0824	-0.0020	-0.0030
1.0	-0.0653	0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 4.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0016	0.0000	0.0000
0.0	2.0016	0.0000	-1.0000
0.1	1.2351	-0.5219	-0.3560
0.2	0.6251	-0.6442	0.0095
0.3	0.2159	-0.5610	0.1712
0.4	-0.0158	-0.4040	0.2060
0.5	-0.1180	-0.2486	0.1758
0.6	-0.1397	-0.1288	0.1223
0.7	-0.1190	-0.0526	0.0696
0.8	-0.0805	-0.0141	0.0294
0.9	-0.0368	-0.0012	0.0058
1.0	0.0076	0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.2351	0.0000	0.0000
0.1	0.9451	0.1824	0.4367
0.1	0.9451	0.1824	-0.5633
0.2	0.6314	-0.1338	-0.2481
0.3	0.3550	-0.2468	-0.0530
0.4	0.1546	-0.2442	0.0463
0.5	0.0310	-0.1899	0.0810
0.6	-0.0321	-0.1242	0.0792
0.7	-0.0552	-0.0676	0.0607
0.8	-0.0562	-0.0282	0.0379
0.9	-0.0478	-0.0065	0.0169
1.0	-0.0368	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.6251	0.0000	0.0000
0.1	0.6314	0.1005	0.2516
0.2	0.6001	0.4021	0.5004
0.2	0.6001	0.4021	-0.4996
0.3	0.4778	0.0929	-0.2816
0.4	0.3216	-0.0642	-0.1217
0.5	0.1833	-0.1179	-0.0218
0.6	0.0811	-0.1120	0.0298
0.7	0.0141	-0.0791	0.0478
0.8	-0.0278	-0.0411	0.0445
0.9	-0.0562	-0.0116	0.0274
1.0	-0.0805	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.2159	0.0000	0.0000
0.1	0.3550	0.0420	0.1143
0.2	0.4778	0.1972	0.2820
0.3	0.5333	0.5036	0.4878
0.3	0.5333	0.5036	-0.5122
0.4	0.4658	0.1770	-0.3089
0.5	0.3378	-0.0021	-0.1474
0.6	0.2075	-0.0731	-0.0388
0.7	0.0989	-0.0771	0.0216
0.8	0.0141	-0.0489	0.0435
0.9	-0.0552	-0.0157	0.0349
1.0	-0.1190	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.0158	0.0000	0.0000
0.1	0.1546	0.0066	0.0278
0.2	0.3216	0.0626	0.1233
0.3	0.4658	0.2209	0.2822
0.4	0.5354	0.5264	0.4863
0.4	0.5354	0.5264	-0.5137
0.5	0.4747	0.1993	0.3080
0.6	0.3462	0.0225	-0.1428
0.7	0.2075	-0.0439	-0.0323
0.8	0.0811	-0.0434	0.0250
0.9	-0.0321	-0.0166	0.0344
1.0	-0.1397	-0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.1180	0.0000	0.0000
0.1	0.0310	-0.0109	-0.0174
0.2	0.1833	-0.0119	0.0253
0.3	0.3378	0.0459	0.1296
0.4	0.4747	0.2114	0.2933
0.5	0.5400	0.5269	0.5000
0.5	0.5400	0.5269	-0.5000
0.6	0.4747	0.2114	-0.2933
0.7	0.3378	0.0459	-0.1296
0.8	0.1833	-0.0119	-0.0253
0.9	0.0310	-0.0109	0.0174
1.0	-0.1180	-0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 4.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0012	0.0000	0.0000
0.0	2.0012	0.0000	-1.0000
0.1	1.1487	-0.5545	-0.2964
0.2	0.5049	-0.6364	0.0661
0.3	0.1118	-0.5053	0.1960
0.4	-0.0782	-0.3219	0.1973
0.5	-0.1369	-0.1658	0.1453
0.6	-0.1266	-0.0629	0.0843
0.7	-0.0890	-0.0103	0.0353
0.8	-0.0461	0.0064	0.0049
0.9	-0.0052	0.0043	-0.0065
1.0	0.0341	-0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.1487	0.0000	0.0000
0.1	0.8739	0.2145	0.4561
0.1	0.8739	0.2145	-0.5439
0.2	0.5611	-0.1190	-0.2208
0.3	0.2888	-0.2237	-0.0324
0.4	0.1031	-0.2085	0.0524
0.5	0.0004	-0.1487	0.0729
0.6	-0.0421	-0.0866	0.0618
0.7	-0.0490	-0.0405	0.0403
0.8	-0.0388	-0.0136	0.0202
0.9	-0.0226	-0.0022	0.0063
1.0	-0.0052	-0.0000	-0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.5049	0.0000	0.0000
0.1	0.5611	0.1062	0.2403
0.2	0.5666	0.4381	0.4978
0.2	0.5666	0.4381	-0.5022
0.3	0.4477	0.0946	-0.2702
0.4	0.2845	-0.0689	-0.1055
0.5	0.1452	-0.1163	-0.0104
0.6	0.0510	-0.1034	0.0320
0.7	-0.0021	-0.0684	0.0417
0.8	-0.0274	-0.0333	0.0343
0.9	-0.0388	-0.0089	0.0192
1.0	-0.0461	0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.1118	0.0000	0.0000
0.1	0.2888	0.0346	0.0903
0.2	0.4477	0.1857	0.2574
0.3	0.5254	0.5155	0.4814
0.3	0.5254	0.5155	-0.5186
0.4	0.4479	0.1522	-0.2948
0.5	0.3028	-0.0319	-0.1252
0.6	0.1663	-0.0930	-0.0208
0.7	0.0653	-0.0855	0.0299
0.8	-0.0021	-0.0503	0.0431
0.9	-0.0490	-0.0154	0.0311
1.0	-0.0890	0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.0782	0.0000	0.0000
0.1	0.1031	-0.0036	0.0056
0.2	0.2845	0.0346	0.0929
0.3	0.4479	0.1874	0.2591
0.4	0.5294	0.5190	0.4841
0.4	0.5294	0.5190	-0.5159
0.5	0.4543	0.1591	-0.2897
0.6	0.3086	-0.0190	-0.1172
0.7	0.1663	-0.0720	-0.0112
0.8	0.0510	-0.0567	0.0366
0.9	-0.0421	-0.0200	0.0380
1.0	-0.1266	0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.1369	0.0000	0.0000
0.1	0.0004	-0.0185	-0.0308
0.2	0.1452	-0.0366	0.0015
0.3	0.3028	0.0046	0.1019
0.4	0.4543	0.1683	0.2733
0.5	0.5315	0.5136	0.5000
0.5	0.5315	0.5136	-0.5000
0.6	0.4543	0.1683	-0.2733
0.7	0.3028	0.0046	-0.1019
0.8	0.1452	-0.0366	-0.0015
0.9	0.0004	-0.0185	0.0308
1.0	-0.1369	0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 5.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0006	0.0000	0.0000
0.0	2.0006	0.0000	-1.0000
0.1	1.0648	-0.5814	-0.2413
0.2	0.3974	-0.6187	0.1111
0.3	0.0308	-0.4446	0.2068
0.4	-0.1142	-0.2457	0.1789
0.5	-0.1341	-0.0989	0.1133
0.6	-0.1019	-0.0170	0.0532
0.7	-0.0590	0.0144	0.0131
0.8	-0.0220	0.0160	-0.0068
0.9	0.0074	0.0062	-0.0103
1.0	0.0335	0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	1.0648	0.0000	0.0000
0.1	0.8126	0.2459	0.4709
0.1	0.8126	0.2459	-0.5291
0.2	0.5039	-0.1052	-0.1991
0.3	0.2373	-0.2023	-0.0174
0.4	0.0669	-0.1767	0.0546
0.5	-0.0167	-0.1139	0.0641
0.6	-0.0431	-0.0571	0.0474
0.7	-0.0400	-0.0206	0.0259
0.8	-0.0258	-0.0037	0.0092
0.9	-0.0092	0.0005	0.0005
1.0	0.0074	0.0000	0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.3974	0.0000	0.0000
0.1	0.5039	0.1085	0.2260
0.2	0.5458	0.4666	0.4939
0.2	0.5458	0.4666	-0.5061
0.3	0.4264	0.0897	-0.2578
0.4	0.2534	-0.0760	-0.0881
0.5	0.1130	-0.1135	0.0013
0.6	0.0270	-0.0922	0.0341
0.7	-0.0135	-0.0555	0.0359
0.8	-0.0260	-0.0244	0.0252
0.9	-0.0258	-0.0058	0.0120
1.0	-0.0220	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	0.0308	0.0000	0.0000
0.1	0.2373	0.0250	0.0672
0.2	0.4264	0.1680	0.2348
0.3	0.5227	0.5206	0.4787
0.3	0.5227	0.5206	-0.5213
0.4	0.4312	0.1256	-0.2766
0.5	0.2681	-0.0566	-0.1014
0.6	0.1276	-0.1036	-0.0044
0.7	0.0362	-0.0848	0.0345
0.8	-0.0135	-0.0461	0.0388
0.9	-0.0400	-0.0132	0.0248
1.0	-0.0590	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.1142	0.0000	0.0000
0.1	0.0669	-0.0135	-0.0120
0.2	0.2534	0.0067	0.0679
0.3	0.4312	0.1533	0.2403
0.4	0.5235	0.5122	0.4854
0.4	0.5235	0.5122	-0.5146
0.5	0.4323	0.1241	-0.2695
0.6	0.2701	-0.0506	-0.0935
0.7	0.1276	-0.0893	0.0043
0.8	0.0270	-0.0623	0.0412
0.9	-0.0431	-0.0206	0.0364
1.0	-0.1019	-0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.1341	0.0000	0.0000
0.1	-0.0167	-0.0238	-0.0379
0.2	0.1130	-0.0555	-0.0147
0.3	0.2681	-0.0296	0.0795
0.4	0.4323	0.1309	0.2553
0.5	0.5222	0.5047	0.5000
0.5	0.5222	0.5047	-0.5000
0.6	0.4323	0.1309	-0.2553
0.7	0.2681	-0.0296	-0.0795
0.8	0.1130	-0.0555	0.0147
0.9	-0.0167	-0.0238	0.0379
1.0	-0.1341	-0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 5.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0002	0.0000	0.0000
0.0	2.0002	0.0000	-1.0000
0.1	0.9839	-0.6031	-0.1902
0.2	0.3020	-0.5931	0.1458
0.3	-0.0306	-0.3826	0.2067
0.4	-0.1310	-0.1789	0.1547
0.5	-0.1194	-0.0489	0.0829
0.6	-0.0745	0.0105	0.0292
0.7	-0.0337	0.0245	0.0000
0.8	-0.0066	0.0174	-0.0105
0.9	0.0101	0.0057	-0.0092
1.0	0.0231	-0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	0.9839	0.0000	0.0000
0.1	0.7601	0.2762	0.4820
0.1	0.7601	0.2762	-0.5180
0.2	0.4572	-0.0932	-0.1818
0.3	0.1966	-0.1835	-0.0064
0.4	0.0410	-0.1494	0.0542
0.5	-0.0256	-0.0860	0.0551
0.6	-0.0394	-0.0355	0.0356
0.7	-0.0305	-0.0076	0.0158
0.8	-0.0162	0.0020	0.0030
0.9	-0.0025	0.0018	-0.0021
1.0	0.0101	-0.0000	0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.3020	0.0000	0.0000
0.1	0.4572	0.1074	0.2096
0.2	0.5335	0.4878	0.4894
0.2	0.5335	0.4878	-0.5106
0.3	0.4095	0.0792	-0.2442
0.4	0.2252	-0.0844	-0.0703
0.5	0.0851	-0.1093	0.0122
0.6	0.0085	-0.0794	0.0352
0.7	-0.0205	-0.0420	0.0302
0.8	-0.0234	-0.0156	0.0174
0.9	-0.0162	-0.0030	0.0063
1.0	-0.0066	-0.0000	-0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	-0.0306	0.0000	0.0000
0.1	0.1966	0.0137	0.0457
0.2	0.4095	0.1457	0.2141
0.3	0.5219	0.5209	0.4786
0.3	0.5219	0.5209	-0.5214
0.4	0.4144	0.0989	-0.2561
0.5	0.2345	-0.0757	-0.0779
0.6	0.0931	-0.1062	0.0094
0.7	0.0132	-0.0773	0.0360
0.8	-0.0205	-0.0380	0.0324
0.9	-0.0305	-0.0099	0.0177
1.0	-0.0337	0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.1310	0.0000	0.0000
0.1	0.0410	-0.0224	-0.0250
0.2	0.2252	-0.0194	0.0474
0.3	0.4144	0.1204	0.2242
0.4	0.5181	0.5069	0.4885
0.4	0.5181	0.5069	-0.5115
0.5	0.4105	0.0944	-0.2486
0.6	0.2334	-0.0728	-0.0718
0.7	0.0931	-0.0967	0.0154
0.8	0.0085	-0.0614	0.0411
0.9	-0.0394	-0.0191	0.0315
1.0	-0.0745	0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.1194	0.0000	0.0000
0.1	-0.0256	-0.0268	-0.0401
0.2	0.0851	-0.0682	-0.0251
0.3	0.2345	-0.0561	0.0608
0.4	0.4105	0.0994	0.2382
0.5	0.5141	0.5000	0.5000
0.5	0.5141	0.5000	-0.5000
0.6	0.4105	0.0994	-0.2382
0.7	0.2345	-0.0561	-0.0608
0.8	0.0851	-0.0682	0.0251
0.9	-0.0256	-0.0268	0.0401
1.0	-0.1194	0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 6.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0000	0.0000	0.0000
0.0	2.0000	0.0000	-1.0000
0.1	0.9059	-0.6198	-0.1430
0.2	0.2183	-0.5614	0.1716
0.3	-0.0752	-0.3218	0.1986
0.4	-0.1340	-0.1224	0.1282
0.5	-0.0990	-0.0140	0.0561
0.6	-0.0496	0.0238	0.0119
0.7	-0.0152	0.0250	-0.0066
0.8	0.0019	0.0141	-0.0098
0.9	0.0087	0.0040	-0.0063
1.0	0.0123	0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	0.9059	0.0000	0.0000
0.1	0.7154	0.3051	0.4898
0.1	0.7154	0.3051	-0.5102
0.2	0.4186	-0.0835	-0.1675
0.3	0.1637	-0.1674	0.0020
0.4	0.0221	-0.1264	0.0522
0.5	-0.0296	-0.0641	0.0466
0.6	-0.0338	-0.0203	0.0261
0.7	-0.0221	0.0001	0.0090
0.8	-0.0093	0.0046	-0.0003
0.9	0.0005	0.0022	-0.0028
1.0	0.0087	0.0000	0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.2183	0.0000	0.0000
0.1	0.4186	0.1032	0.1921
0.2	0.5267	0.5029	0.4853
0.2	0.5267	0.5029	-0.5147
0.3	0.3944	0.0647	-0.2294
0.4	0.1985	-0.0928	-0.0528
0.5	0.0609	-0.1035	0.0212
0.6	-0.0051	-0.0659	0.0348
0.7	-0.0234	-0.0292	0.0246
0.8	-0.0197	-0.0080	0.0111
0.9	-0.0093	-0.0007	0.0022
1.0	0.0019	0.0000	0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	-0.0752	0.0000	0.0000
0.1	0.1637	0.0016	0.0265
0.2	0.3944	0.1207	0.1954
0.3	0.5212	0.5183	0.4803
0.3	0.5212	0.5183	-0.5197
0.4	0.3968	0.0733	-0.2349
0.5	0.2026	-0.0895	-0.0561
0.6	0.0640	-0.1029	0.0200
0.7	-0.0034	-0.0658	0.0351
0.8	-0.0234	-0.0282	0.0254
0.9	-0.0221	-0.0063	0.0112
1.0	-0.0152	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.1340	0.0000	0.0000
0.1	0.0221	-0.0297	-0.0339
0.2	0.1985	-0.0432	0.0307
0.3	0.3968	0.0898	0.2094
0.4	0.5134	0.5032	0.4919
0.4	0.5134	0.5032	-0.5081
0.5	0.3899	0.0692	-0.2281
0.6	0.1998	-0.0875	-0.0522
0.7	0.0640	-0.0967	0.0233
0.8	-0.0051	-0.0558	0.0381
0.9	-0.0338	-0.0161	0.0252
1.0	-0.0496	-0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.0990	0.0000	0.0000
0.1	-0.0296	-0.0275	-0.0389
0.2	0.0609	-0.0750	-0.0314
0.3	0.2026	-0.0756	0.0446
0.4	0.3899	0.0728	0.2215
0.5	0.5080	0.4983	0.5000
0.5	0.5080	0.4983	-0.5000
0.6	0.3899	0.0728	-0.2215
0.7	0.2026	-0.0756	-0.0446
0.8	0.0609	-0.0750	0.0314
0.9	-0.0296	-0.0275	0.0389
1.0	-0.0990	-0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 6.50$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0000	0.0000	0.0000
0.0	2.0000	0.0000	-1.0000
0.1	0.8312	-0.6319	-0.0997
0.2	0.1458	-0.5252	0.1897
0.3	-0.1053	-0.2643	0.1848
0.4	-0.1273	-0.0766	0.1019
0.5	-0.0772	0.0084	0.0343
0.6	-0.0295	0.0277	0.0007
0.7	-0.0034	0.0206	-0.0088
0.8	0.0055	0.0094	-0.0075
0.9	0.0061	0.0022	-0.0035
1.0	0.0046	0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	0.8312	0.0000	0.0000
0.1	0.6777	0.3323	0.4950
0.1	0.6777	0.3323	-0.5050
0.2	0.3866	-0.0760	-0.1553
0.3	0.1367	-0.1537	0.0086
0.4	0.0082	-0.1070	0.0494
0.5	-0.0306	-0.0470	0.0387
0.6	-0.0279	-0.0100	0.0185
0.7	-0.0151	0.0040	0.0045
0.8	-0.0047	0.0051	-0.0017
0.9	0.0016	0.0020	-0.0025
1.0	0.0061	-0.0000	0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.1458	0.0000	0.0000
0.1	0.3866	0.0962	0.1742
0.2	0.5233	0.5127	0.4820
0.2	0.5233	0.5127	-0.5180
0.3	0.3795	0.0478	-0.2135
0.4	0.1727	-0.0999	-0.0363
0.5	0.0400	-0.0960	0.0280
0.6	-0.0142	-0.0527	0.0329
0.7	-0.0232	-0.0182	0.0192
0.8	-0.0154	-0.0023	0.0063
0.9	-0.0047	0.0009	-0.0003
1.0	0.0055	-0.0000	0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	-0.1053	0.0000	0.0000
0.1	0.1367	-0.0106	0.0099
0.2	0.3795	0.0947	0.1785
0.3	0.5199	0.5143	0.4830
0.3	0.5199	0.5143	-0.5170
0.4	0.3786	0.0496	-0.2140
0.5	0.1729	-0.0988	-0.0369
0.6	0.0402	-0.0956	0.0275
0.7	-0.0141	-0.0528	0.0326
0.8	-0.0232	-0.0187	0.0189
0.9	-0.0151	-0.0031	0.0060
1.0	-0.0034	-0.0000	-0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.1273	0.0000	0.0000
0.1	0.0082	-0.0350	-0.0393
0.2	0.1727	-0.0611	0.0170
0.3	0.3786	0.0621	0.1951
0.4	0.5095	0.5009	0.4948
0.4	0.5095	0.5009	-0.5052
0.5	0.3705	0.0473	-0.2084
0.6	0.1697	-0.0966	-0.0350
0.7	0.0402	-0.0917	0.0285
0.8	-0.0142	-0.0474	0.0338
0.9	-0.0279	-0.0123	0.0188
1.0	-0.0295	0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.0772	0.0000	0.0000
0.1	-0.0306	-0.0263	-0.0354
0.2	0.0400	-0.0768	-0.0347
0.3	0.1729	-0.0890	0.0303
0.4	0.3705	0.0498	0.2050
0.5	0.5042	0.4982	0.5000
0.5	0.5042	0.4982	-0.5000
0.6	0.3705	0.0498	-0.2050
0.7	0.1729	-0.0890	-0.0303
0.8	0.0400	-0.0768	0.0347
0.9	-0.0306	-0.0263	0.0354
1.0	-0.0772	0.0000	-0.0000

DIMENSIONLESS COEFFICIENTS For $\alpha L = 7.00$

LOAD At 0.0L			
Dist	Defl.	Moment	Shear
0.0	2.0000	0.0000	0.0000
0.0	2.0000	0.0000	-1.0000
0.1	0.7596	-0.6398	-0.0599
0.2	0.0838	-0.4860	0.2011
0.3	-0.1237	-0.2114	0.1675
0.4	-0.1146	-0.0408	0.0777
0.5	-0.0565	0.0211	0.0177
0.6	-0.0146	0.0261	-0.0057
0.7	0.0030	0.0147	-0.0086
0.8	0.0060	0.0050	-0.0049
0.9	0.0037	0.0007	-0.0014
1.0	0.0004	0.0000	0.0000

LOAD At 0.1L			
Dist	Defl.	Moment	Shear
0.0	0.7596	0.0000	0.0000
0.1	0.6461	0.3575	0.4982
0.1	0.6461	0.3575	-0.5018
0.2	0.3598	-0.0709	-0.1444
0.3	0.1140	-0.1420	0.0140
0.4	-0.0022	-0.0904	0.0463
0.5	-0.0298	-0.0336	0.0317
0.6	-0.0222	-0.0033	0.0126
0.7	-0.0096	0.0056	0.0016
0.8	-0.0018	0.0046	-0.0021
0.9	0.0018	0.0015	-0.0019
1.0	0.0037	0.0000	0.0000

LOAD At 0.2L			
Dist	Defl.	Moment	Shear
0.0	0.0838	0.0000	0.0000
0.1	0.3598	0.0869	0.1565
0.2	0.5220	0.5185	0.4798
0.2	0.5220	0.5185	-0.5202
0.3	0.3641	0.0300	-0.1970
0.4	0.1479	-0.1050	-0.0213
0.5	0.0224	-0.0871	0.0323
0.6	-0.0197	-0.0403	0.0297
0.7	-0.0209	-0.0096	0.0142
0.8	-0.0110	0.0015	0.0028
0.9	-0.0018	0.0017	-0.0015
1.0	0.0060	0.0000	0.0000

LOAD At 0.3L			
Dist	Defl.	Moment	Shear
0.0	-0.1237	0.0000	0.0000
0.1	0.1140	-0.0222	-0.0040
0.2	0.3641	0.0688	0.1633
0.3	0.5179	0.5102	0.4860
0.3	0.5179	0.5102	-0.5140
0.4	0.3599	0.0282	-0.1939
0.5	0.1456	-0.1043	-0.0206
0.6	0.0215	-0.0861	0.0320
0.7	-0.0201	-0.0401	0.0289
0.8	-0.0209	-0.0106	0.0133
0.9	-0.0096	-0.0006	0.0023
1.0	0.0030	0.0000	0.0000

LOAD At 0.4L			
Dist	Defl.	Moment	Shear
0.0	-0.1146	0.0000	0.0000
0.1	-0.0022	-0.0383	-0.0416
0.2	0.1479	-0.0756	0.0057
0.3	0.3599	0.0377	0.1810
0.4	0.5064	0.4997	0.4970
0.4	0.5064	0.4997	-0.5030
0.5	0.3523	0.0279	-0.1899
0.6	0.1429	-0.1018	-0.0201
0.7	0.0215	-0.0838	0.0317
0.8	-0.0197	-0.0380	0.0289
0.9	-0.0222	-0.0085	0.0130
1.0	-0.0146	-0.0000	-0.0000

LOAD At 0.5L			
Dist	Defl.	Moment	Shear
0.0	-0.0565	0.0000	0.0000
0.1	-0.0298	-0.0236	-0.0306
0.2	0.0224	-0.0744	-0.0360
0.3	0.1456	-0.0974	0.0175
0.4	0.3523	0.0297	0.1884
0.5	0.5019	0.4987	0.5000
0.5	0.5019	0.4987	-0.5000
0.6	0.3523	0.0297	-0.1884
0.7	0.1456	-0.0974	-0.0175
0.8	0.0224	-0.0744	0.0360
0.9	-0.0298	-0.0236	0.0306
1.0	-0.0565	-0.0000	-0.0000

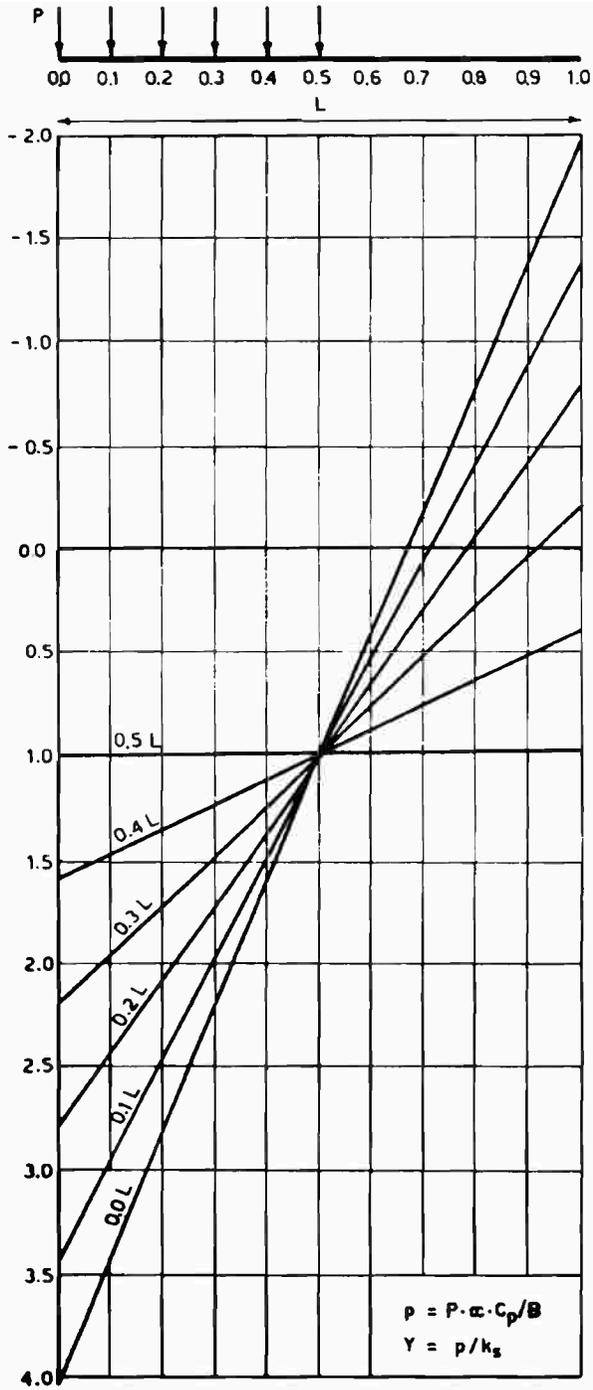


Fig. C_p -1: SOIL REACTION COEFFICIENT (C_p).

$\alpha L = 1.0$

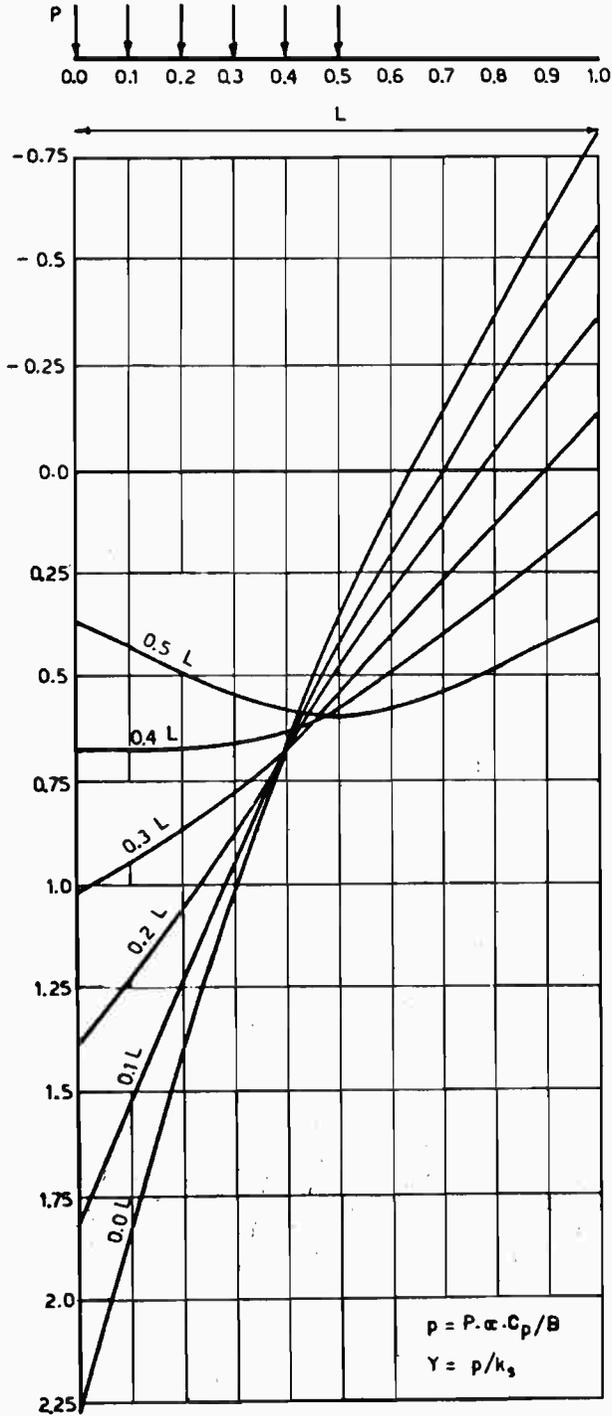


Fig. C_p-2; SOIL REACTION COEFFICIENT (C_p).
 $\alpha L = 2.0$

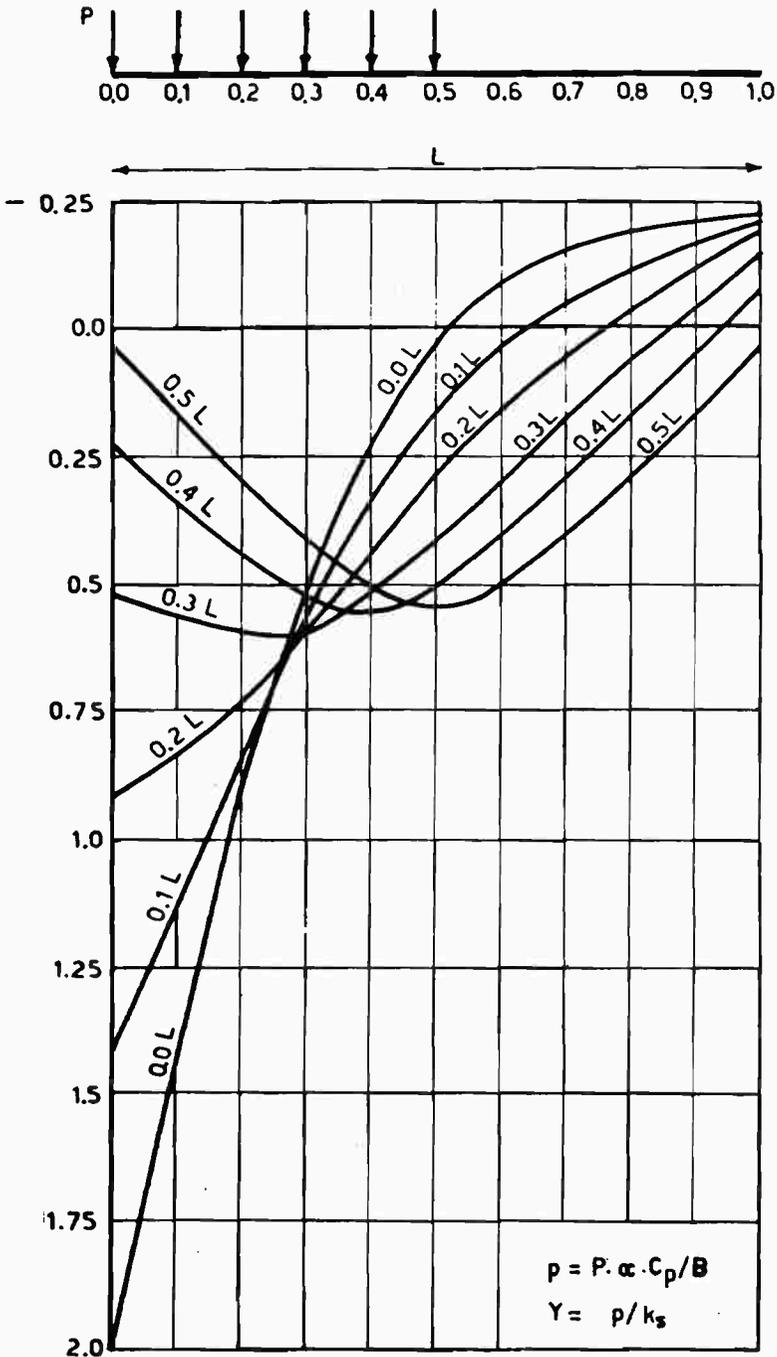


Fig. C_p-3: SOIL REACTION COEFFICIENT (C_p).
 $\alpha L = 3.0$

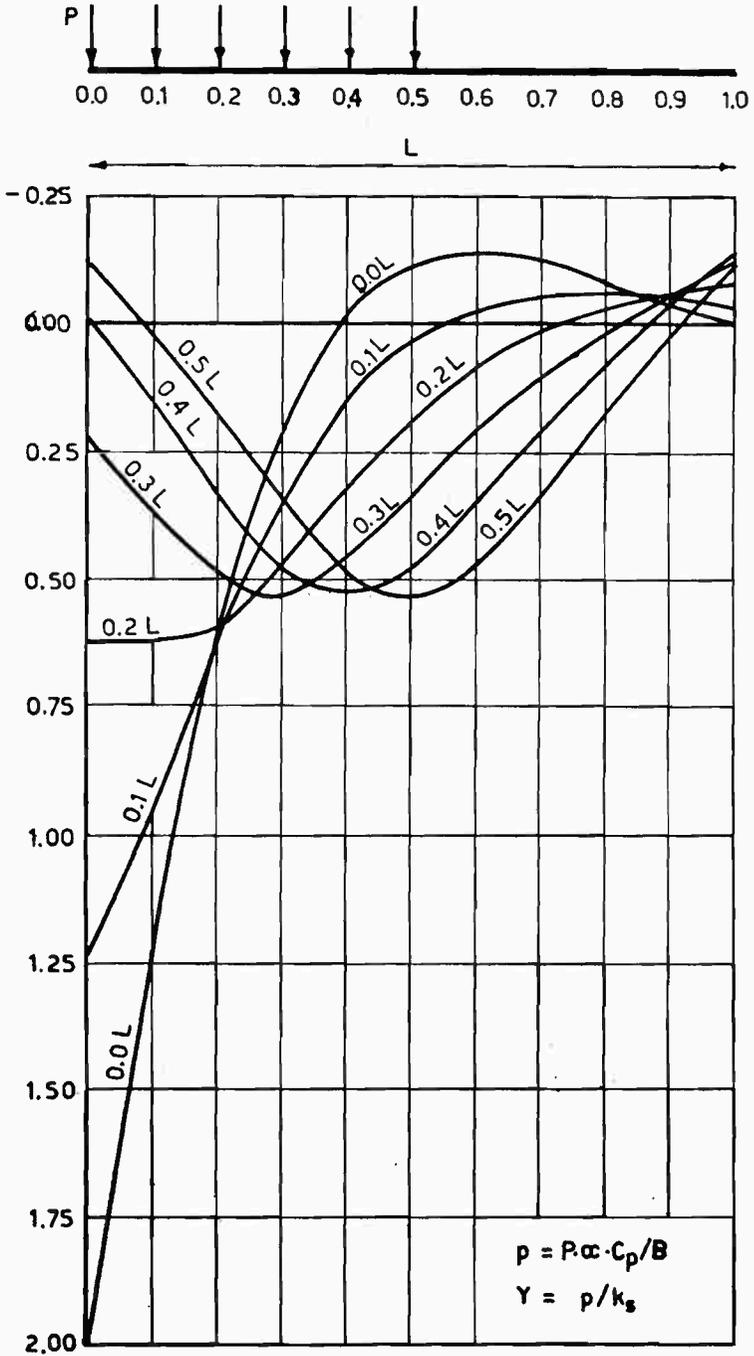


Fig. C_p -4: SOIL REACTION COEFFICIENT (C_p).
 $\alpha L = 4.0$

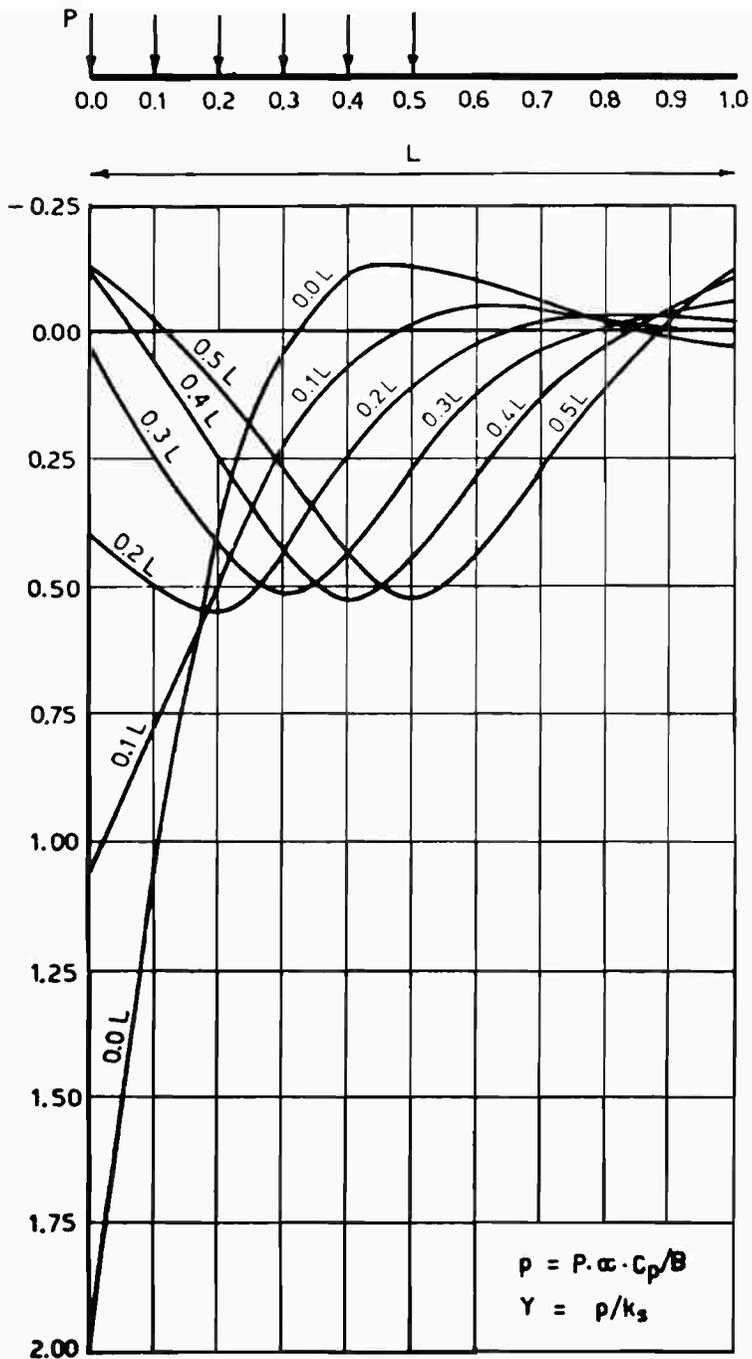


Fig.C_p-5:SOIL REACTION COEFFICIENT (C_p).

$\alpha L = 5.0$

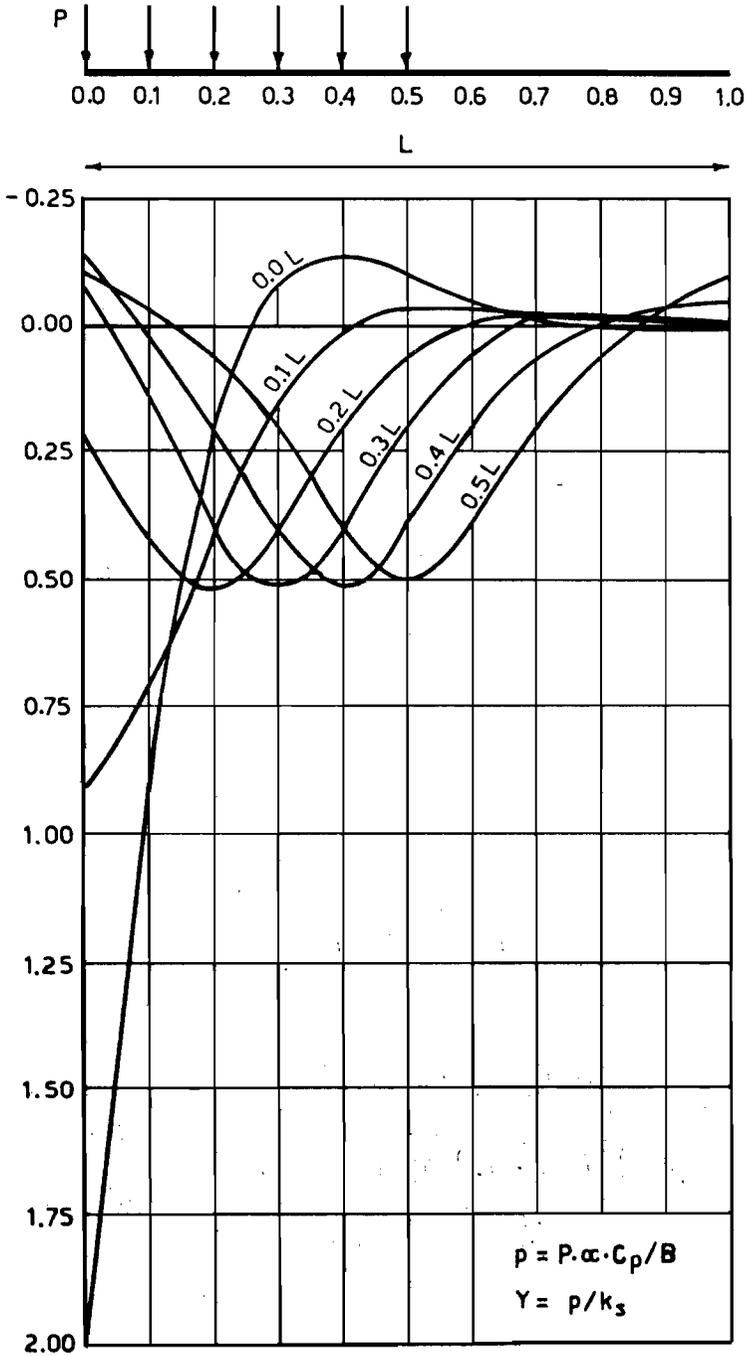


Fig. C_p-6: SOIL REACTION COEFFICIENT (C_p).

$$\alpha L = 6.0$$

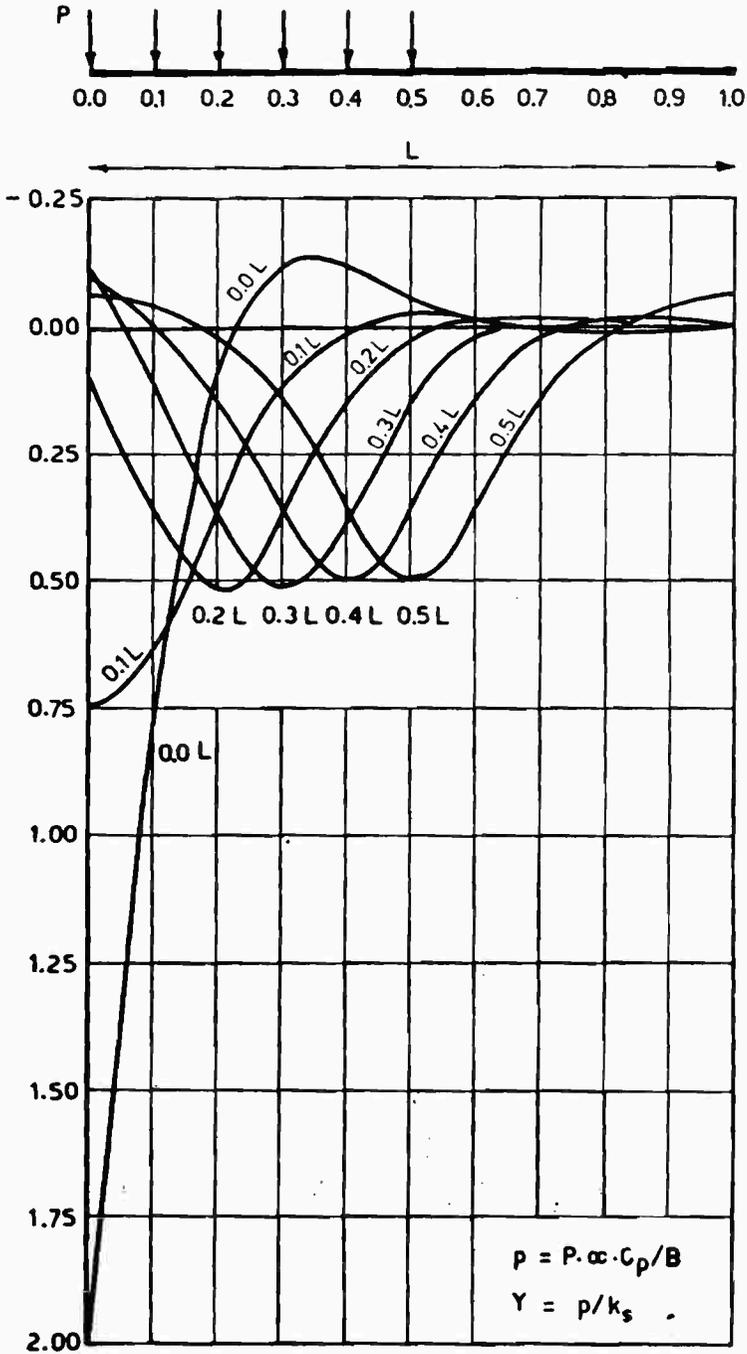


Fig. C_p -7: SOIL REACTION COEFFICIENT (C_p).
 $\alpha L = 7.0$

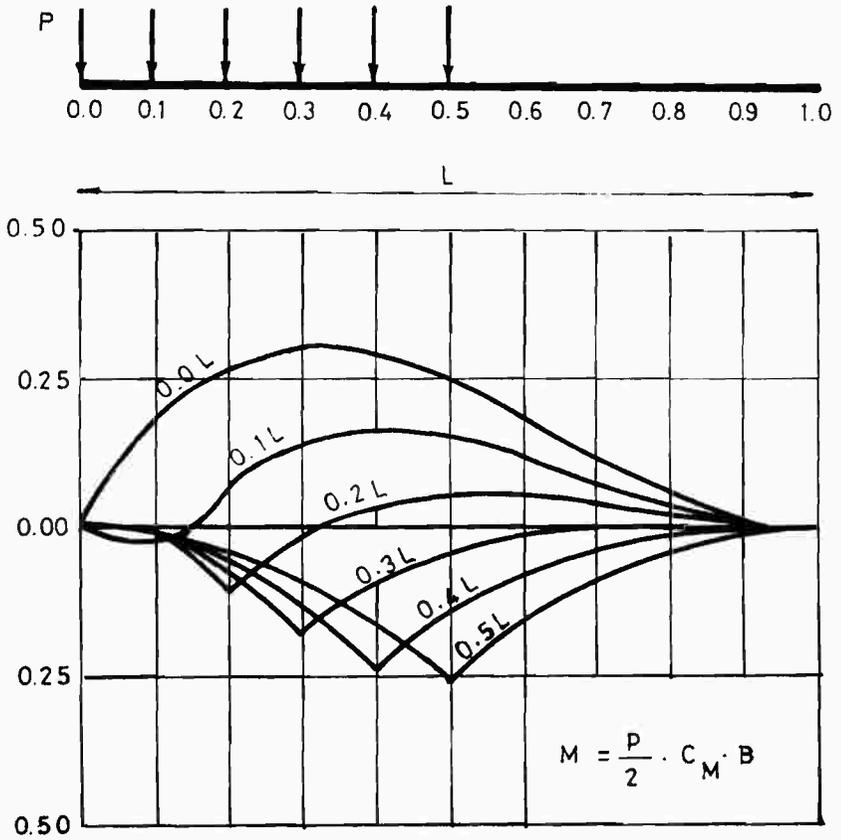


Fig. C_M -1: BENDING MOMENT COEFFICIENT (C_M).

$\propto L = 1.0$

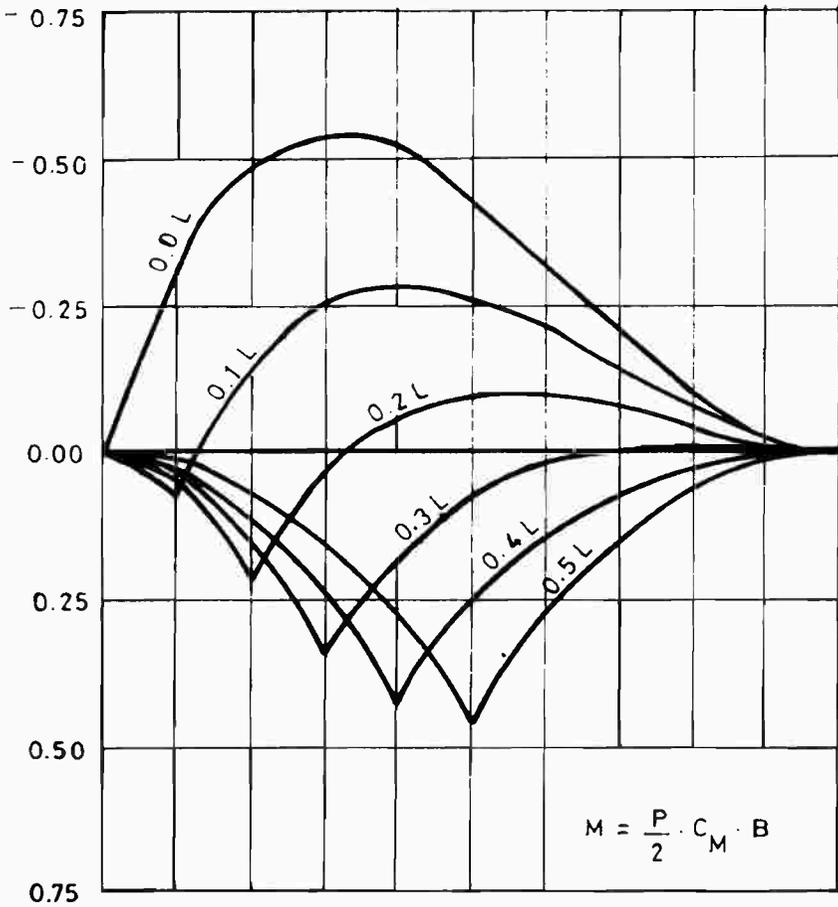
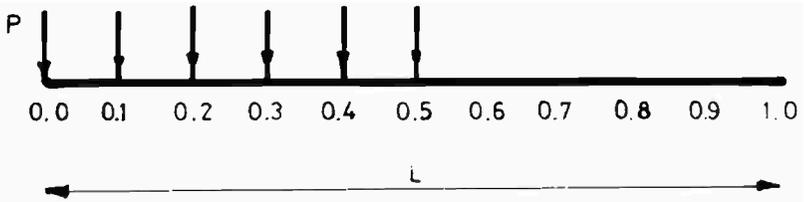


Fig. C_M -2 BENDING MOMENT COEFFICIENT (C_M)
 $\propto L = 2.0$

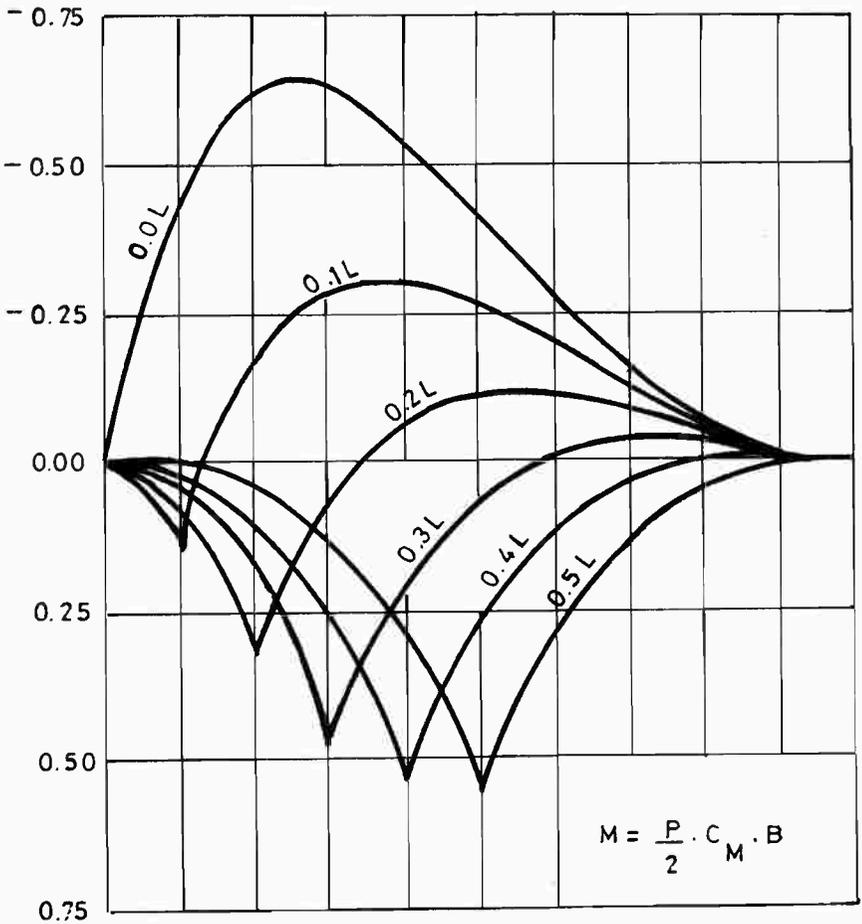
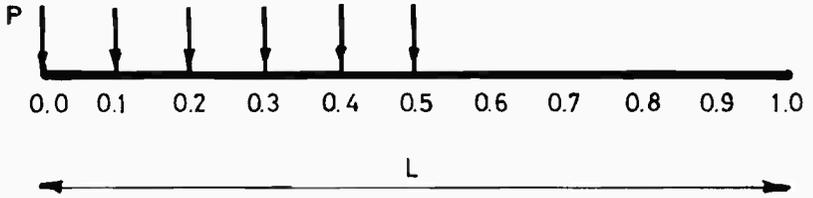


Fig. C_M -3 : BENDING MOMENT COEFFICIENT (C_M).
 $\propto L = 3.0$

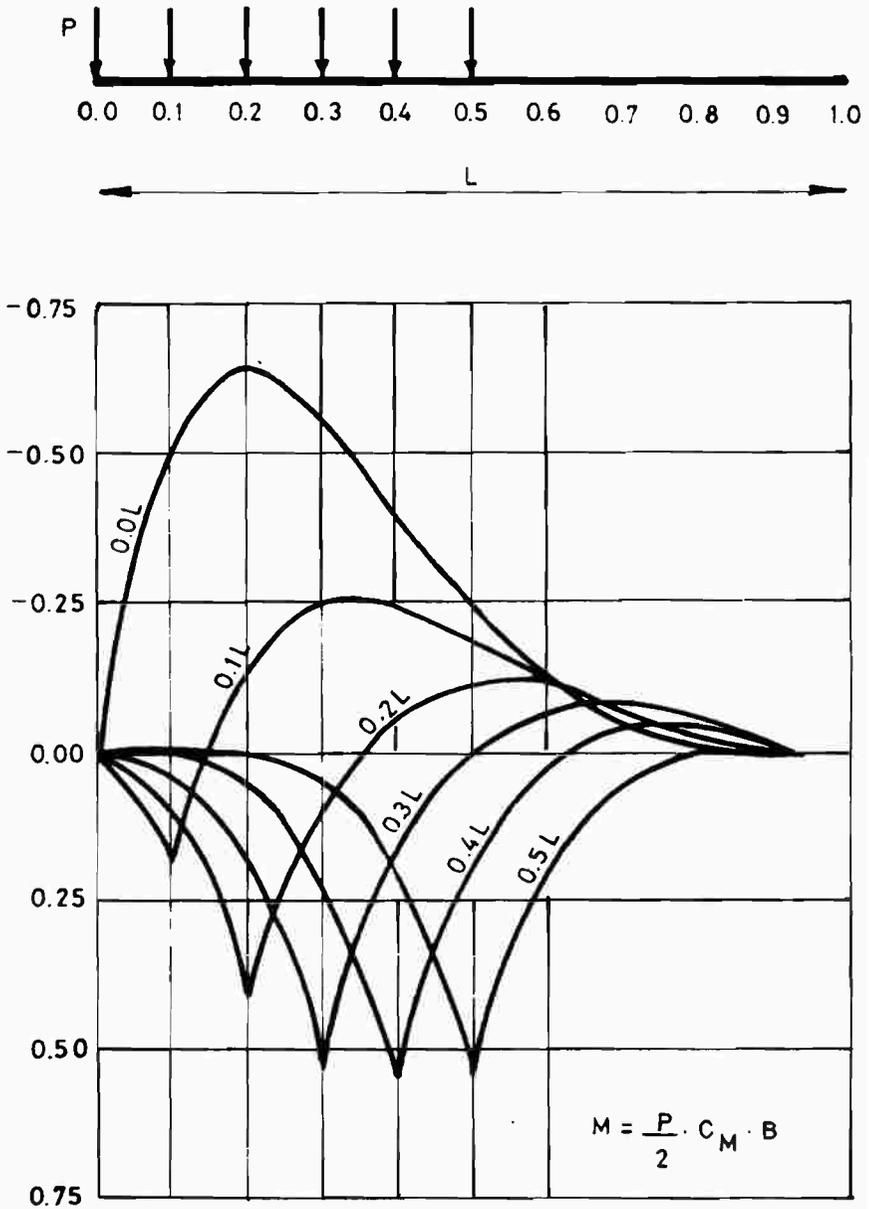


Fig.C_M-4: BENDING MOMENT COEFFICIENT (C_M).

$$\alpha L = 4.0$$

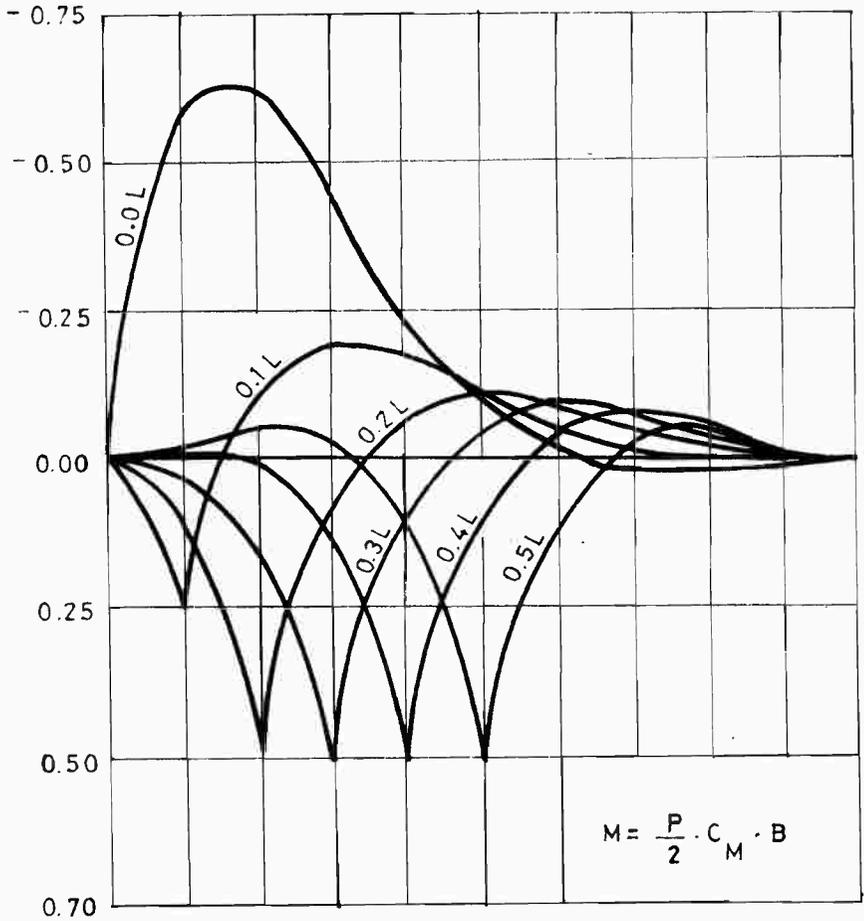
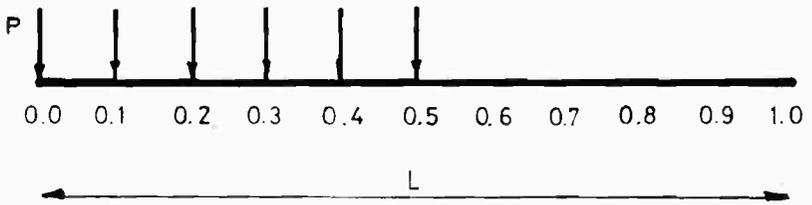


Fig. C_M-5: BENDING MOMENT COEFFICIENT (C_M)
 $\infty L = 5.0$

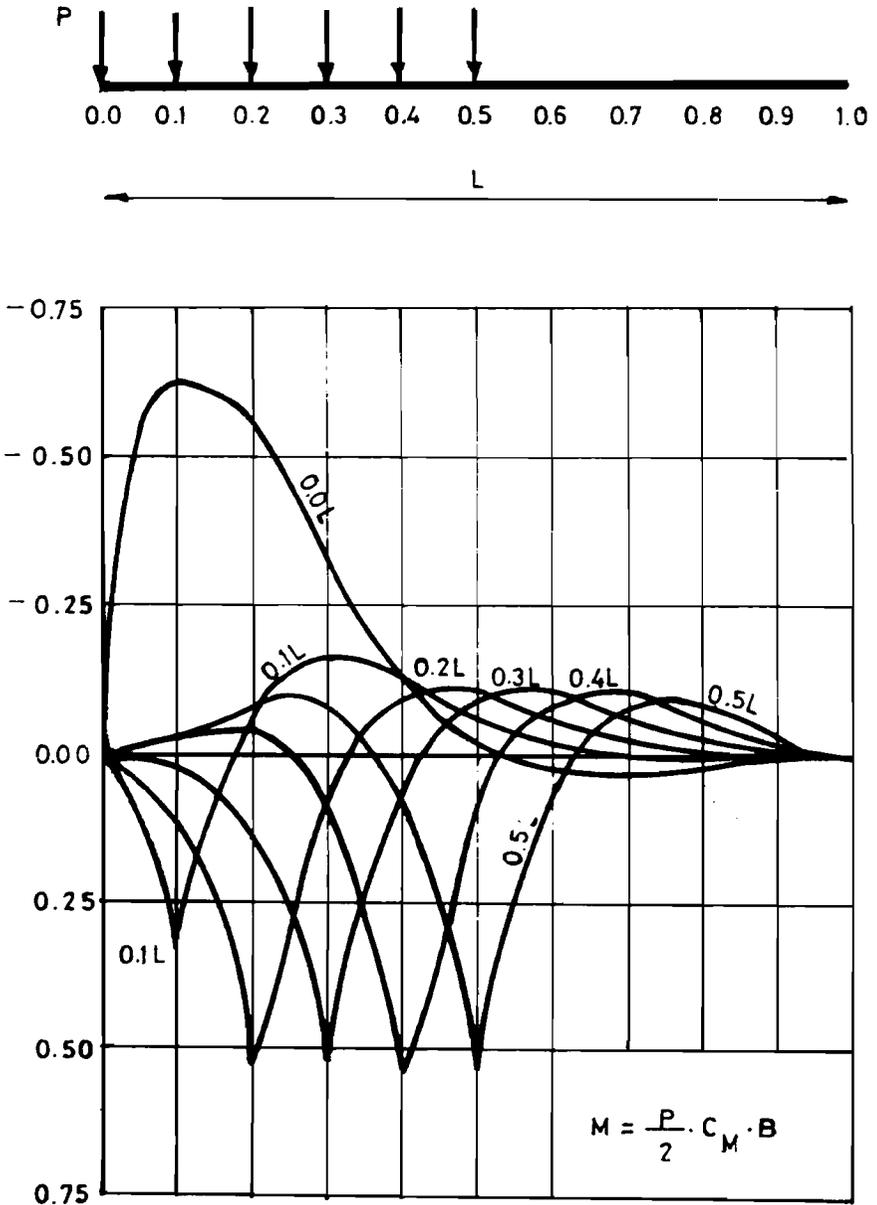


Fig. C_M -6: BENDING MOMENT COEFFICIENT (C_M).
 $\alpha L = 6.0$

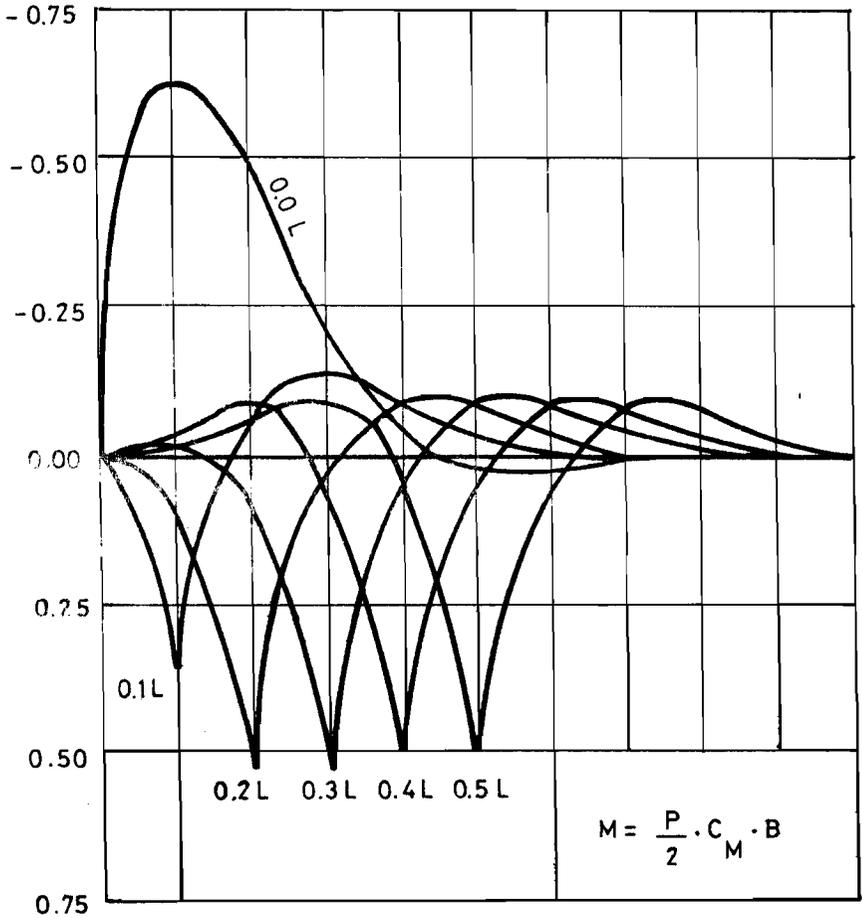
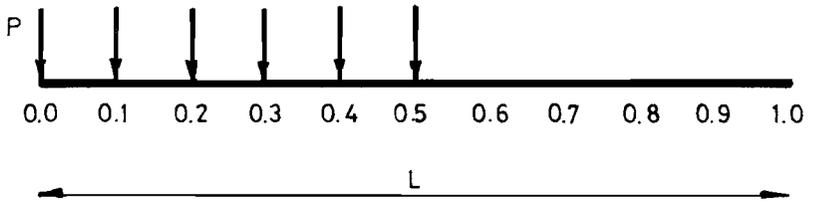


Fig. C_M -7: BENDING MOMENT COEFFICIENT (C_M)
 $\propto L = 7.0$

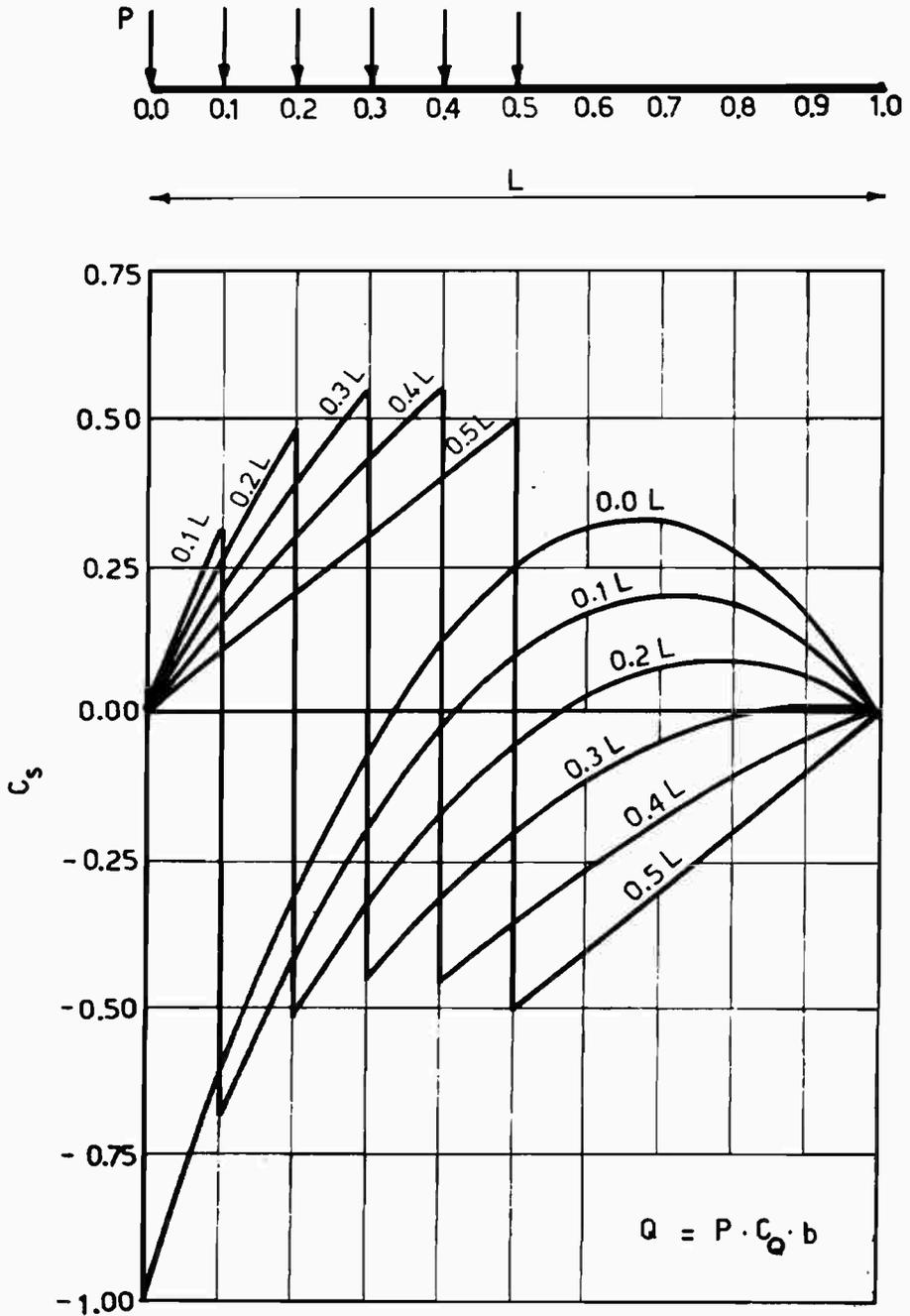


Fig. C_Q-1 : SHEARING FORCE COEFFICIENT (C_Q).
 $\propto L = 1.0$

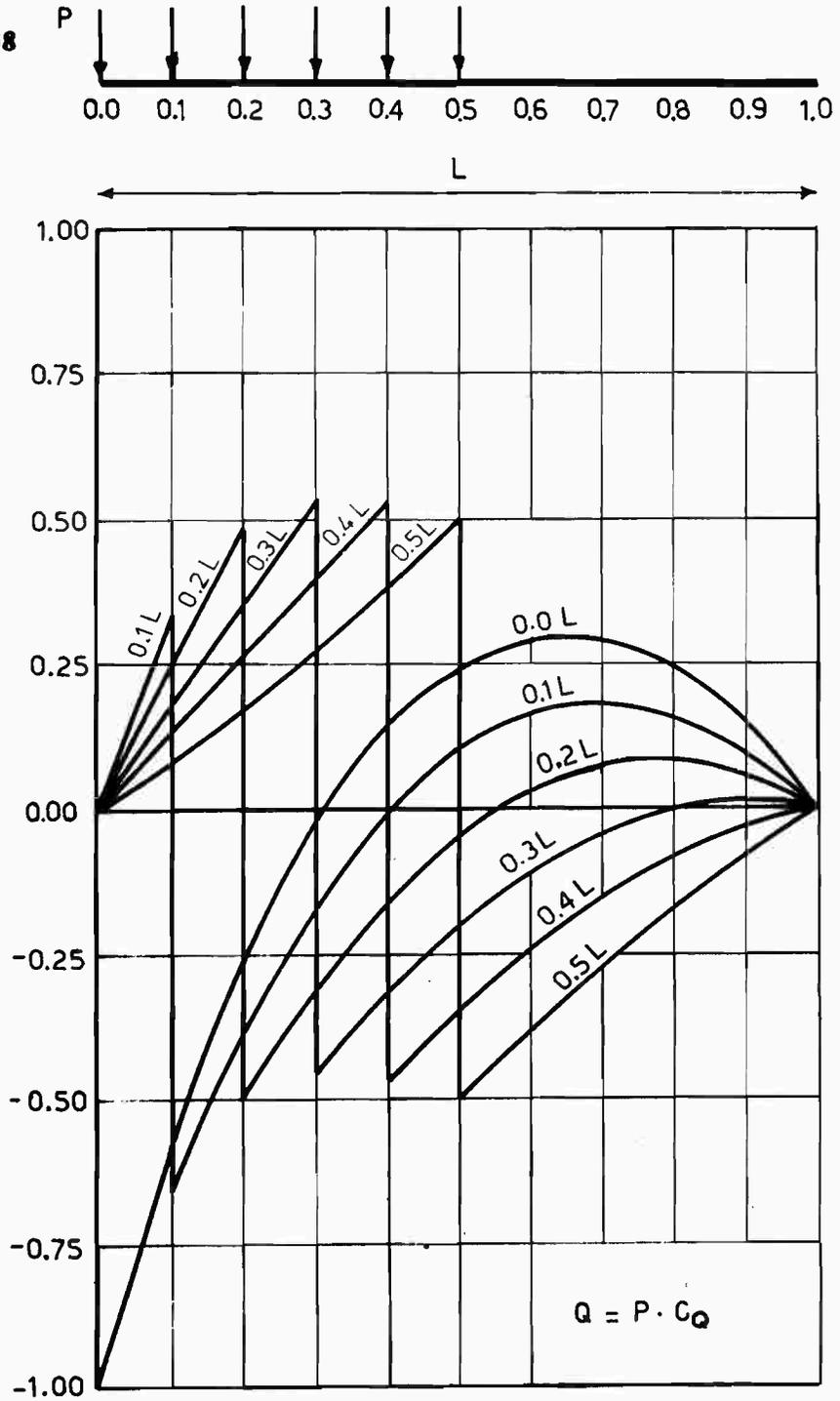


Fig. C_Q -2: SHEARING FORCE COEFFICIENT (C_Q).
 $\propto L = 2.0$

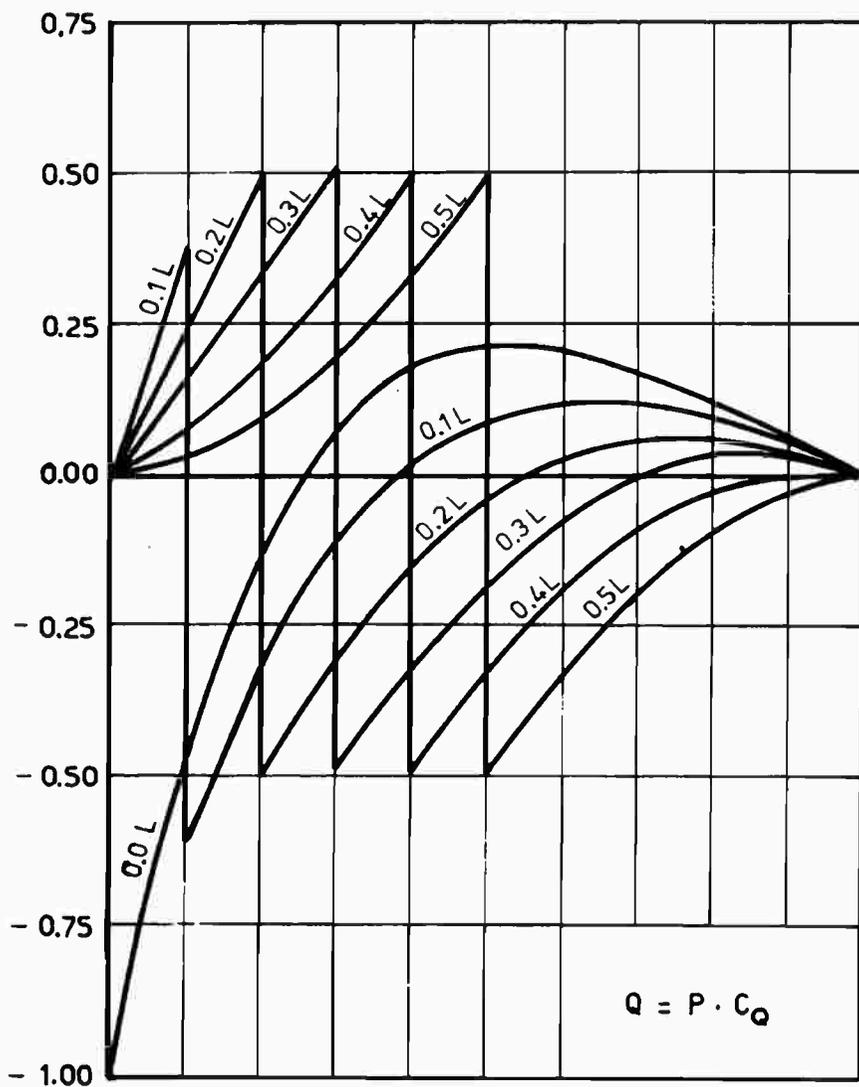
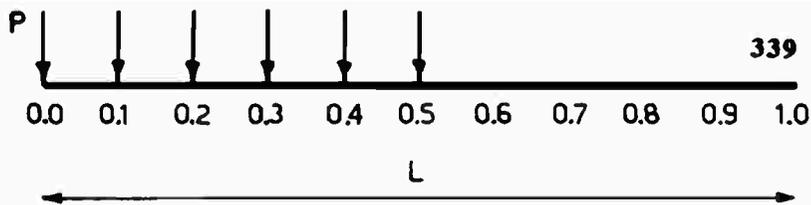


Fig. C_Q -3 : SHEARING FORCE COEFFICIENT (C_Q).
 $\alpha L = 3.0$

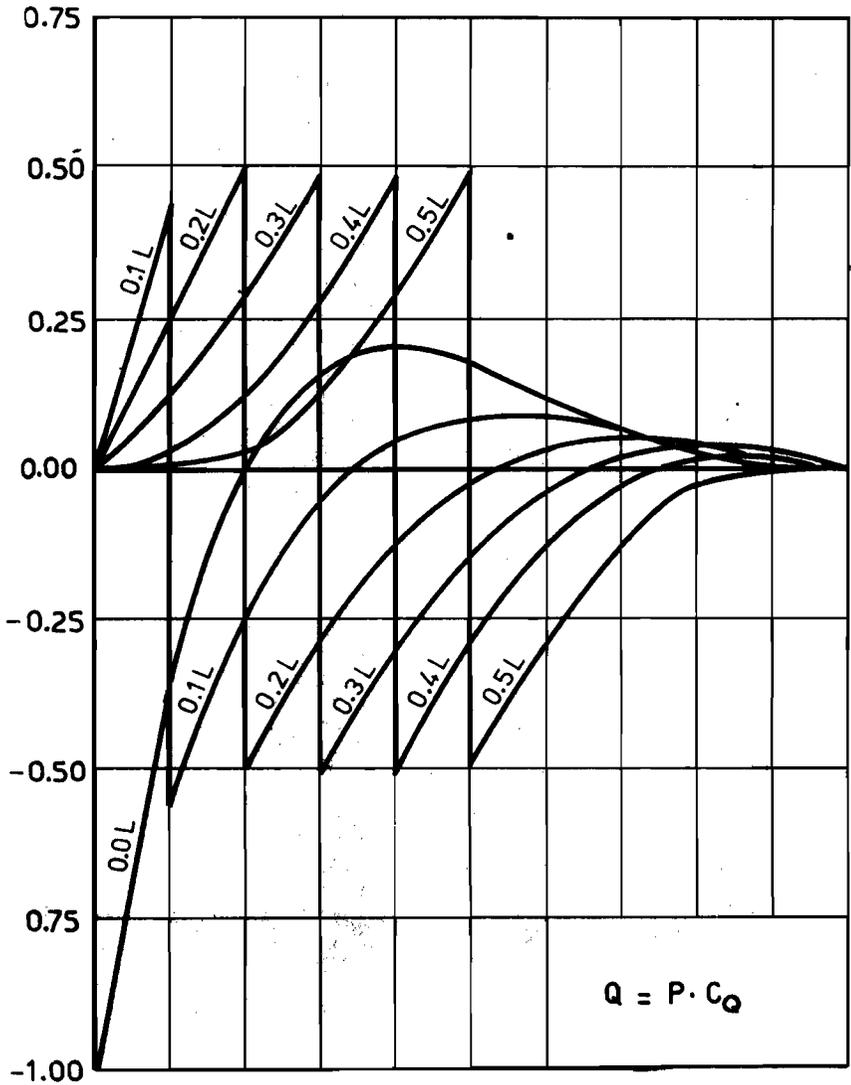
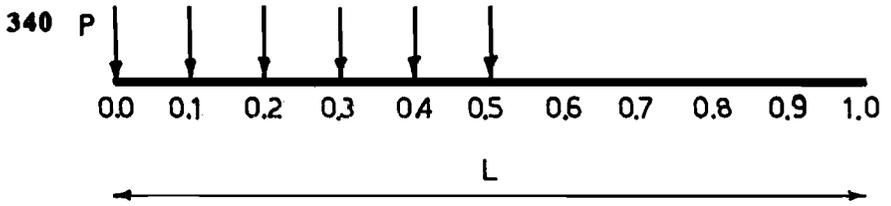


Fig. C-4 SHEARING FORCE COEFFICIENT (C_Q).
 $\alpha L = 4.0$

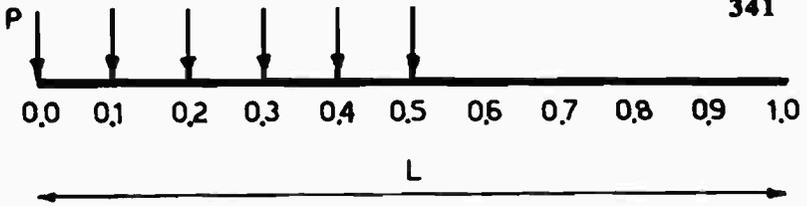


Fig. C_Q-5 SHEARING FORCE COEFFICIENT (C_Q).
 $\propto L = 5.0$

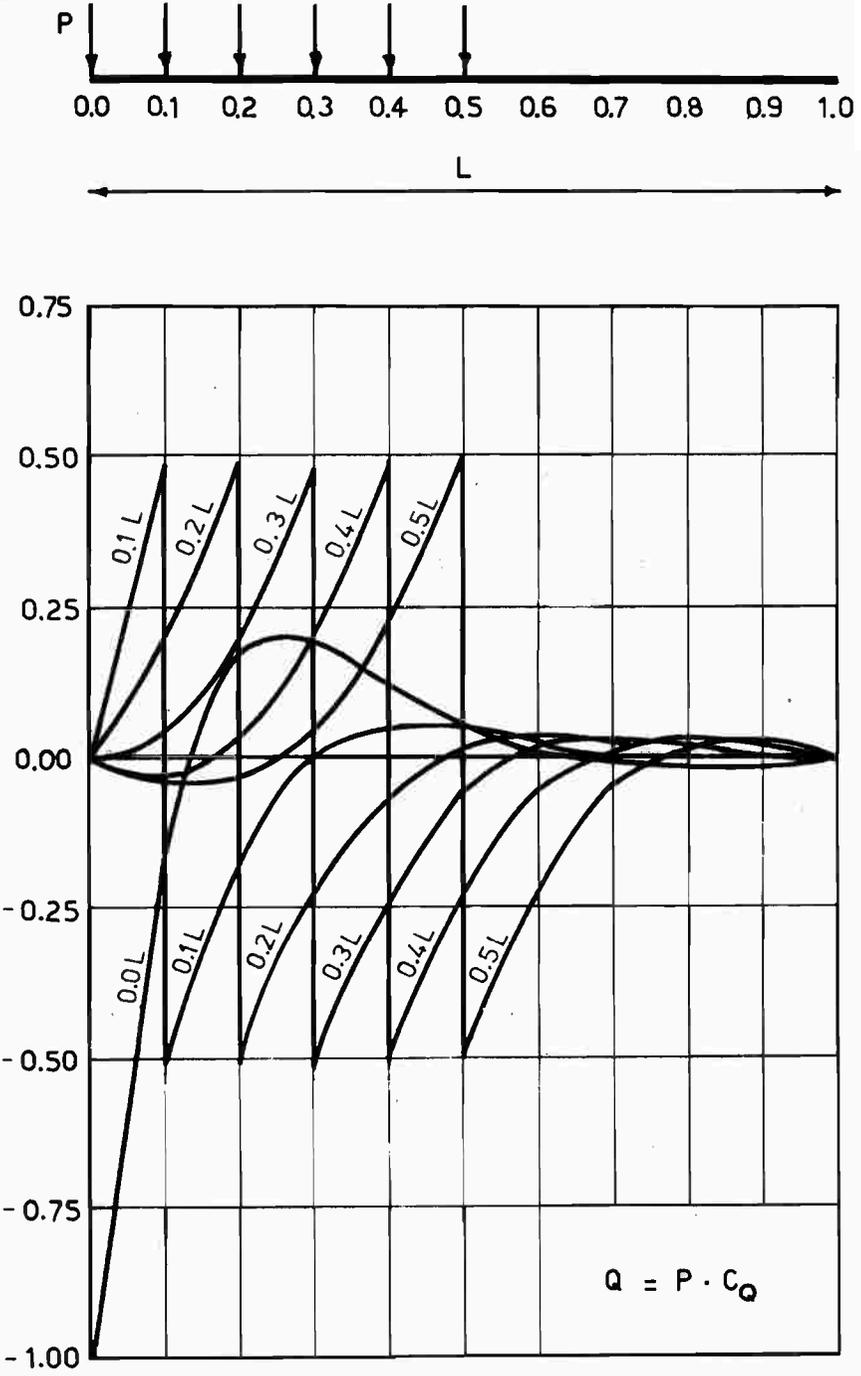


Fig. C_Q -6 : SHEARING FORCE COEFFICIENT (C_Q).
 $\propto L = 6.0$

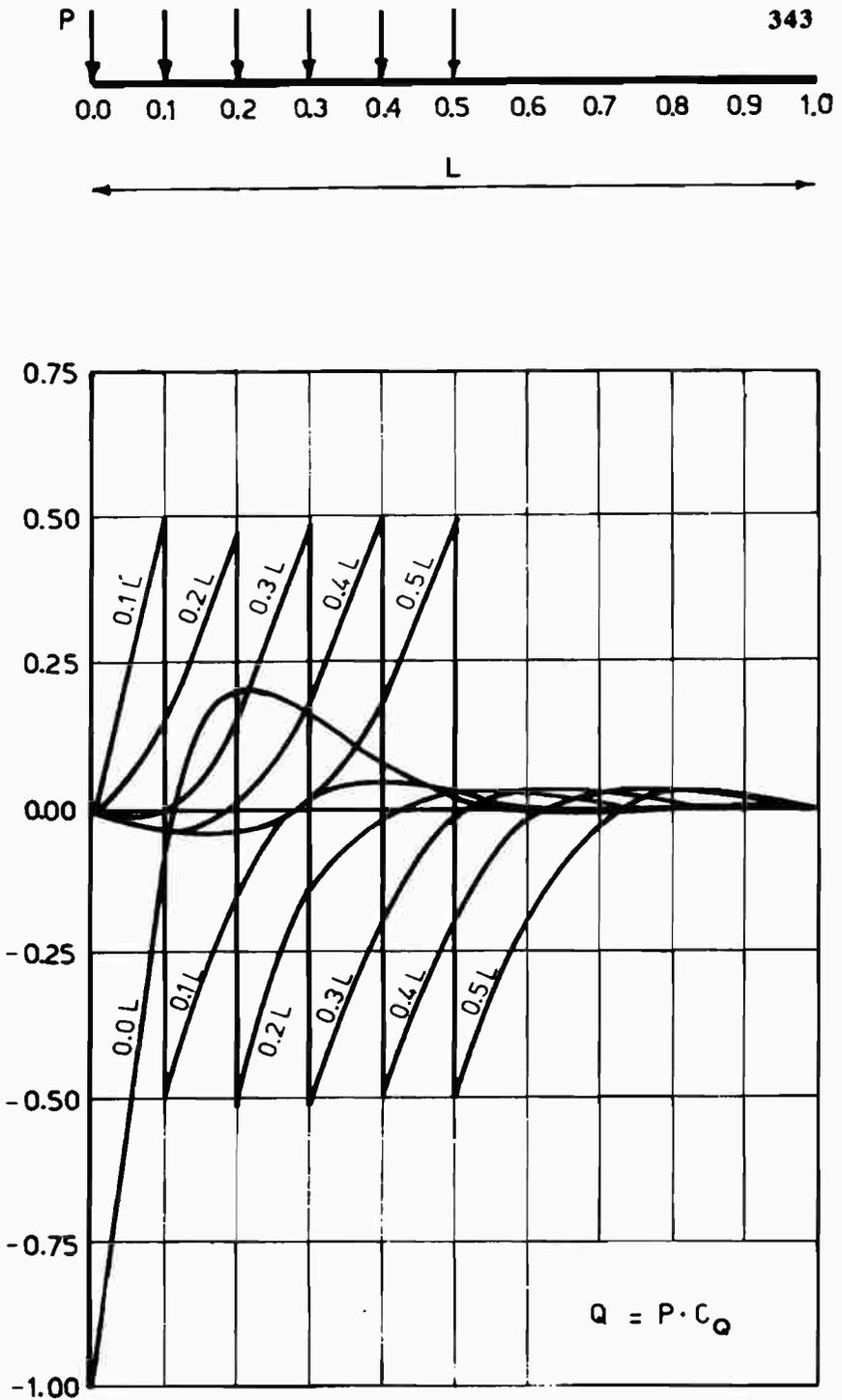


Fig. C_Q -7: SHEARING FORCE COEFFICIENT (C_Q).
 $\alpha L = 7.0$