# CHAPTER 5
# Angle Modulation

## 5.1 Concept of Instantaneous Frequency:

The phasor representation of a sinusoid with a constant magnitude $A$ and a phase angle $\theta(t)$ is given by

$$\upsilon(t) = A\cos\theta(t) \qquad (5-1)$$

If $\theta(t)$ increases linearly with time, i.e, $\theta(t) = \omega_0 t$, we say that the phasor has a constant angular frequency $\omega_o$ rad/s.

If the angular rate is not constant, we can express $\theta(t)$ as

$$\theta(t) = \int_o^t \omega_i(\tau)\,d\tau + \theta_o, \qquad (5-2)$$

where $\omega_i(t)$ is the instantaneous angular rate $\omega_i(t)$.

Taking the derivative of both sides of eqn. (5-2)

$$\omega_i(t) = \frac{d\theta(t)}{dt} \qquad (5-3)$$

If the phase angle $\theta(t)$ is varied linearly with the input signal $\upsilon_m(t)$, we have

$$\theta(t) = \omega_c t + k_p \upsilon_m(t) + \theta_0 \qquad (5-4)$$

where $\omega_c, k_p, \theta_0$ are constants.

Because the phase is linearly related to $\upsilon_m(t)$, this type of angle modulation is called phase modulation (PM). The instantaneous frequency of this phase modulated signal is

$$\omega_i = \frac{d\theta}{dt} = \omega_c + k_p \frac{d\upsilon_m(t)}{dt} \qquad (5-5)$$

Another possibility is to make the instantaneous frequency proportional to the modulating signal

$$\omega_i = \omega_c + k_f\, \upsilon_m(t), \qquad (5-6)$$

where $\omega_c, k_f$, are constants. Because the frequency is linearly related to $\upsilon_m(t)$, this type of angle modulation is called frequency modulation (FM). Thus,

$$\theta(t) = \int_o^t \omega_i(\tau)\,d\tau = \omega_c t + k_f \int_o^t \upsilon_m(\tau)\,d\tau + \theta_0 \qquad (5-7)$$

We notice that if we integrate the modulating signal first, and then use it to phase modulate a carrier, we obtain FM, i.e,

$$k_f \int_0^t \upsilon_m(\tau)\,d\tau \text{ (FM) corresponds to } k_p \upsilon_m(t) \text{ (PM)} \qquad (5-8)$$

In the case of AM, there is a linear relationship between the modulating signal and the modulated signal. In the case of FM or PM, this is not the case. We may express the PM modulated signal $V_{PM}(t)$ by the phasor

$$V_{PM}(t) = Ae^{j\theta(t)} = Ae^{j(\omega_c t+\theta_0)}e^{jk_p\upsilon_m(t)} \qquad (5-9)$$

$$= Ae^{j(\omega_c t+\theta_0)}\left[1 + jk_p\upsilon_m(t) - \frac{1}{2}k_p^2\upsilon_m^2(t) - j\frac{1}{6}k_p^3\upsilon_m^3(t) + \ldots\right] \qquad (5-10)$$

In the series expansion of the exponential term, only when $|k_p \upsilon_m(t)| \ll 1$, we have linearity.

## 5.2 Narrowband FM:

If we let $\upsilon_m(t) = a\cos\omega_m t$, then from eqn. (5-6),

$$\omega_i = \omega_c + k_f\upsilon_m(t) = \omega_c + ak_f\cos\omega_m t \qquad (5-11)$$

We call $k_f$; the frequency modulation constant ( rad / s-volt).

We define the peak angular frequency deviation $\Delta\omega$

$$\hat{\Delta\omega} = ak_f \qquad (5-12)$$

$$\omega_i = \omega_c + \hat{\Delta\omega}\cos\omega_m t \qquad (5-13)$$

Then, from eqn. ( 5 7 ),

$$\theta(t) = \omega_c t + \frac{\hat{\Delta\omega}}{\omega_m}\sin\omega_m t = \omega_c t + \beta\sin\omega_m t, \qquad (5-14)$$

where

$$\beta = \frac{\hat{\Delta\omega}}{\omega_m}, \qquad (5-15)$$

$\beta$ is a dimensionless ratio of the peak frequency deviation to the modulating frequency. The resulting FM signal becomes

$$V_{FM}(t) = Ae^{j(\omega_c t+\beta\sin\omega_m t)} \qquad (5-16)$$

$$V_{FM}(t) = \Re_e V_{FM}(t) = A\cos(\omega_c t + \beta\sin\omega_m t) \qquad (5-17)$$

Expanding eqn. (5 16),

$$V_{FM}(t) = Ae^{j\omega_c t}\left[1 + j\beta\sin\omega_m t - \frac{1}{2}\beta^2\sin^2\omega_m t - j\frac{1}{6}\beta^3\sin^3\omega_m t + \ldots\right] \qquad (5-18)$$

Hence, the bandwidth is dependent on the value of $\beta$ . For small values of $\beta$ ( < 0.3 ), only the constant and the first term are significant. This is called narrowband FM ( NBFM)

$$V_{NBFM}(t) = Ae^{j\omega_c t}\left[1 + j\beta \sin \omega_m t\right] \qquad (5-19)$$

$$= Ae^{j\omega_c t}\left[1 + \frac{1}{2}\beta e^{j\omega_m t} - \frac{1}{2}\beta e^{-j\omega_m t}\right] \qquad (5-20)$$

For AM, we have

$$V_{AM}(t) = Ae^{j\omega_c t}\left[1 + m_a \cos \omega_m t\right] \qquad (5-21)$$

$$= Ae^{j\omega_c t}\left[1 + \frac{1}{2}m_a e^{j\omega_m t} + \frac{1}{2}m_a e^{-j\omega_m t}\right] \qquad (5-22)$$

By comparing eqns. (5-19) and (5-20), we deduce that $\beta$ is the modulation index of FM, and that the bandwidth for narrowband FM is $2\omega_m$ (just like AM).

We can represent eqns. (5-20) and (5-22) by the phasor representation (Fig 5.1), by taking $Ae^{j\omega_c t}$ as a reference.

Similarly, for phase modulation (PM), we have from eqn. (5-10), taking $\theta_o = O$ and $v_m(t) = a\cos\omega_m t$, $\Delta\theta = \hat{\Delta\theta}\cos\omega_m t$, and $\hat{\Delta\theta} = k_p a$.

$$V_{NBPM}(t) = Ae^{j\omega_c t}\left[1 + jk_p a\cos\omega_m t\right] \qquad (5-23)$$

$$= Ae^{j\omega_c t}\left[1 + \frac{1}{2}j\beta_\phi e^{j\omega_m t} + \frac{1}{2}j\beta_\phi e^{-j\omega_m t}\right] \qquad (5-24)$$

where

$$\beta_\phi = \hat{\Delta\theta} = ak_p \qquad (5-25)$$

We see from Fig. 5.1 that in AM the resultant modulation is added in phase with the carrier, whereas in NBFM the resultant modulation is added in quadrature with the carrier. In the NBFM case, we have phase variations with very little amplitude change (in the resultant) as time goes on, whereas in the AM case, we have large amplitude variations with no phase change.

The instantaneous frequency deviation from the carrier frequency is given by $d\,\gamma(t)/dt$, where

$$\gamma(t) = \tan^{-1}(\beta \sin \omega_m t) \qquad (5-26)$$

$$\frac{d\gamma(t)}{dt} = \frac{\beta\omega_m \cos\omega_m t}{1 + \beta^2 \sin^2 \omega_m t} \qquad (5-27)$$

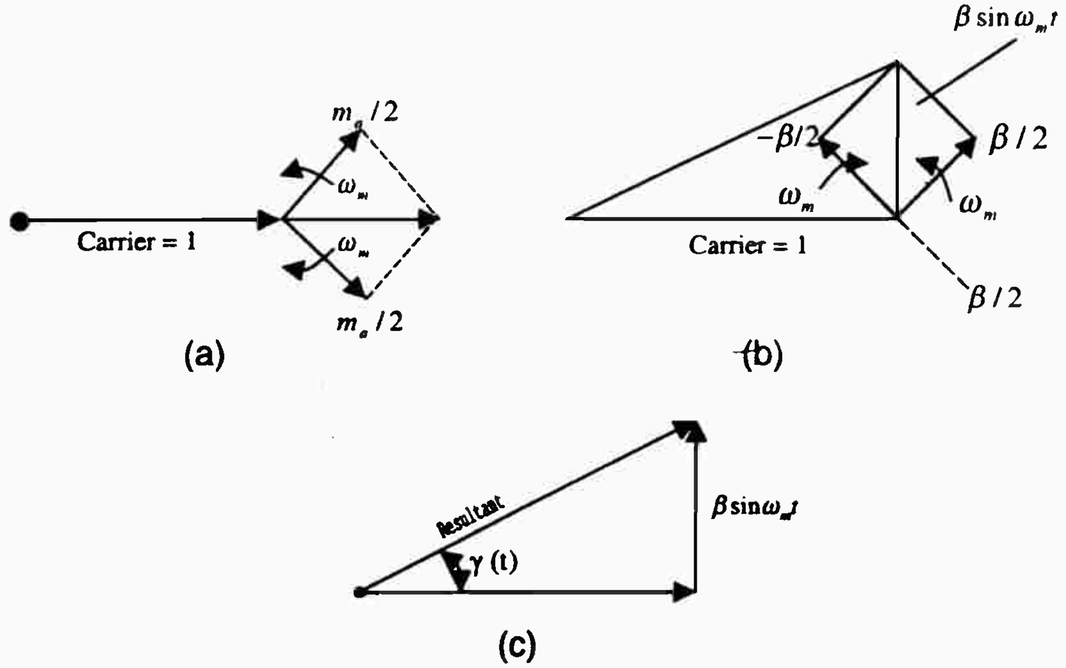


**Fig. 5.1 Phasor diagram for NBFM**
a) AM phasor b) NBFM phasor c) NBFM Resultant

This should equate $\hat{\Delta\omega}\cos\omega_m t$ and $\beta\omega_m\cos\omega_m t$
Thus,

$$\frac{\beta\omega_m\cos\omega_m t}{1+\beta^2\sin^2\omega_m t} = \beta\omega_m\cos\omega_m t \qquad (5-28)$$

This is true if $\beta^2 << 1$, in which case the resultant from Fig 5.1b is

$$A\sqrt{1+\beta^2\sin^2\omega_m t} = A \quad \text{for } \beta^2 << 1 \qquad (5-29)$$

Hence, the condition $\beta^2 << 1$ ensures NBFM. Usually, we take $\beta^2 << 0.1$ (or $\beta^2 << 0.316$).

For NBFM,

$$BW = 2f_m \qquad (5-30)$$

For large $\beta$, Carson found out

$$BW = 2\left(\hat{\Delta f} + f_m\right)$$

$$= 2f_m(1+\beta) \qquad (5-31)$$

**Ex. 5.1:**

Suggest ways for generating AM, NBPM, NBFM, using balanced modulators. From eqn. (5-20), taking the real part,

**Solution:**

$$\Re e\ V_{NBFM}(t) = A\left[\cos\omega_c t + \frac{\beta}{2}\cos(\omega_c + \omega_m)t - \frac{\beta}{2}\cos(\omega_c - \omega_m)t\right]$$

$$= A[\cos\omega_c t - \beta\sin\omega_m t\ \sin\omega_c t] \qquad (5-32)$$

From eqn. (5 – 23), taking the real part,

$$\Re e\ V_{NBFM}(t) = A\left[\cos\omega_c t - \frac{1}{2}k_p\, a\sin(\omega_c + \omega_m)t - \frac{1}{2}k_p\, a\sin(\omega_c - \omega_m)t\right]$$

$$= A\left[\cos\omega_c t - \frac{1}{2}k_p\, a\cos\,\omega_m t\ \sin\omega_c t\right] \qquad (5-33)$$

Similarly, from eqn. ( 5-22),

$$\Re e\ V_{AM}(t) = A[\cos\omega_c t + m_a\ \cos\omega_m t\ \cos\,\omega_c t] \qquad (5-34)$$

From eqn. (5 34), with $\upsilon_m(t) = a\cos\omega_m t = mA\cos\omega_m t$, Fig (5-2a) follows for AM. From eqn. (5 32), Fig (5.2b) follows for NBFM. Using eqn. (5-33), Fig (5-2c) follows for NBPM.

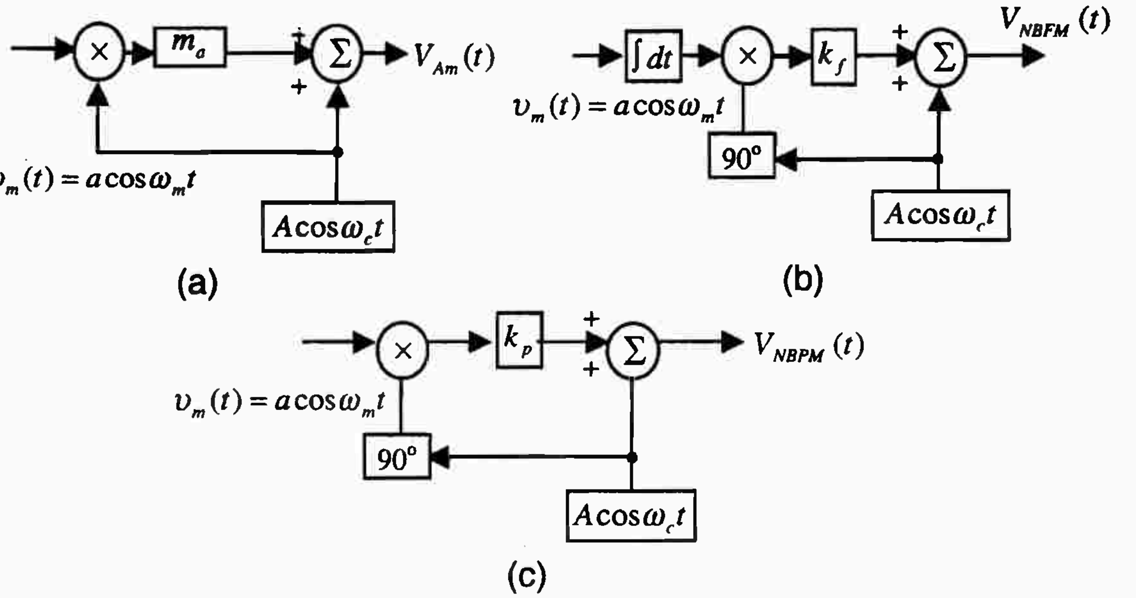


**Fig. 5.2 Generation of signals using balanced modulators**

a) AM b) NBFM $(k_f = \hat{\Delta\omega}/a = \beta\omega_m/a)$

c) NBPM $(k_p = \hat{\Delta\theta}/a) = (\beta_\varphi/a)$

**FM Transmission:**

An FM receiver is similar to an AM receiver except for signal and oscillator frequencies and the type of demodulator (Fig. 5.3).

It is common practice to use a high IF frequency, e.g., 10.7 MHz. The FM band extends from 88 - 108MHz. Thus, the tuning range of the local oscillator is from 98.7 to 118.7 MHz.

The IF stage ahead of the demodulator is often a limiter stage. Its function is to remove or clip off any amplitude variations caused by noise and interference. The carrier frequencies are usually spaced at 200kHz intervals, and the peak frequency deviation is set at 75kHz.

The 200kHz available to each station - in comparison with 10kHz for AM broadcasting - allows for the transmission of high fidelity (Hi Fi ) programs. If the modulating frequency is 15kHz (maximum audio frequency in FM transmission), we use Carson's rule $BW = 2(\hat{\Delta f} + f_m) = 180\ kHz$

## 5.4 Stereophonic FM Broadcasting:

Stereo systems reproduce the multidirectionality of sound. In basic stereo reproduction systems, microphones are placed on the left and right sides of the pickup area. The left channel signal will contain primarily the sound picked up by the left microphone - in addition to some sounds from the right microphone - and vice versa.

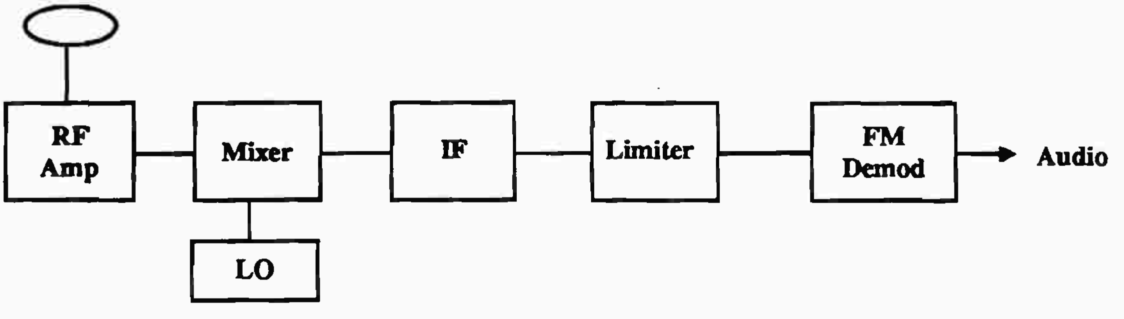


**Fig. 5.3 Block diagram of FM receiver**

The left ear and the right ear in a real situation will compose the stereo effect we all experience in real sounds. In stereophonic broadcasting, we try to simulate the same situation, as if we were sitting in front of an orchestra. The left signal and the right signal are processed separately (Fig. 5.4). However, in order not to undermine the right of a user of the broadcasting service who has a monophonic receiver (or monaural transmission), the left and right signals must somehow be combined to make the service available for all, for the sake of compatibility.

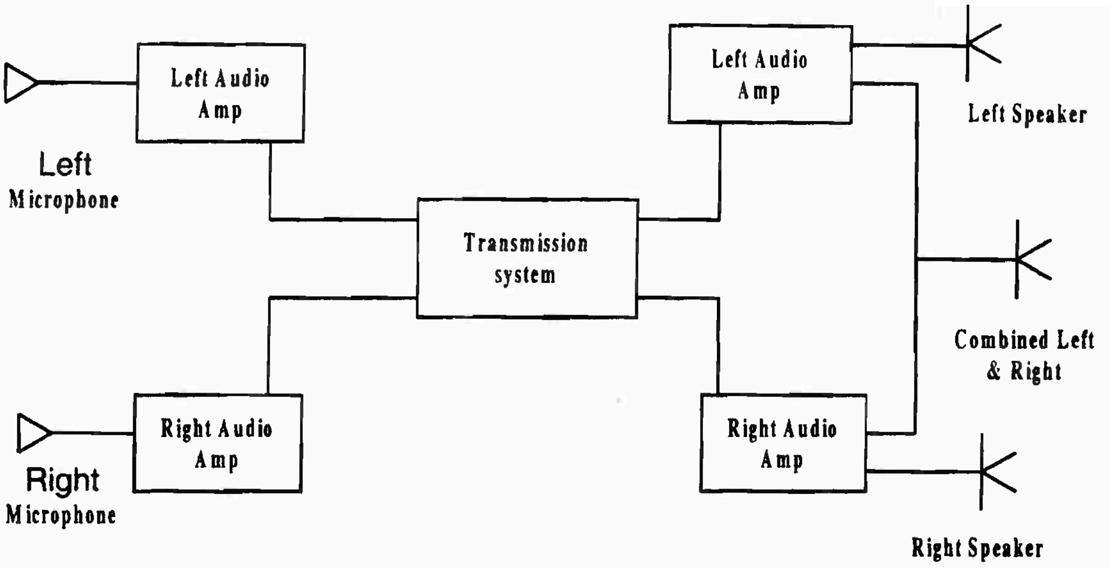


**Fig. 5.4 Compatibility between stereophonic and monophonic receivers**

Stereo signals from two microphones are processed in two separate channels, and are finally delivered to two separate speakers, or combined in one speaker.

The stereo FM broadcast system solves the problem of reproducing a good monaural sound in a conventional FM receiver by a technique known as FM multiplex. By this system, more than one signal can be combined and used to modulate the same carrier.

The transmitted signal must produce a normal output on a nonstereo FM receiver. This signal is obtained by combining both left and right signals, so as not to deprive the listener with a monaural receiver any substantive content of the real world audio signal. This signal is called L+R signal. In order to be able to separate the left and right signals for the listener with a stereo receiver, however, a signal L-R is also generated by subtracting right from left signals.

At the receiver side, if we combine both L+R and L-R signals, we obtain the left signal.

$$(L+R)+(L-R)=2L \qquad (5-35)$$

and similarly, if we subtract L   R from L + R, we obtain the right signal.

$$(L+R)-(L-R)=2R \qquad (5-36)$$

The two signals L + R and L   R are transmitted on the same carrier. The frequency locations are shown (Fig. 5.5). A subcarrier at 67kHz for background music is used for NBFM signaling. We should note that the entire signal extends to 75kHz, which is the peak frequency deviation, while the 15kHz is the maximum modulating audio frequency.

The AM carrier is removed in a balanced modulator, and a pilot carrier at one half the subcarrier frequency is transmitted. The (L–R) is modulated as DSBSC. The pilot - separated out at half its frequency - makes the recovery a bit easier (it has only 10 % of the total power). The block diagrams for the transmitter and receiver in stereophonic systems are shown in Figs. 5.6 and 5.7, respectively.

We should note that the transmission of one audio channel (e.g. 15kHz) requires a peak frequency deviation of 75kHz and a total BW of 180kHz (or 200kHz). This leaves room for additional program material within the bandwidth allocated. Stereo multiplexing and auxiliary transmission - such as the one at 67kHz called SCA ( Subsidiary Communication Authorization ) signal - occupies the higher unused frequency portions of the modulating spectrum.

The concept of multiplexing information through the use of a subcarrier and a built-in subsidiary modulation is an important tool in communication to make utmost use of the bandwidth, without impairing the requirements for the transmission of the original signal. This type of multiplexing is hidden within an - otherwise - unused spectrum, reserved for the modulating signal on account of the peak frequency deviation. In the receiver, this multiplex information is picked up from the received transmitted signal.

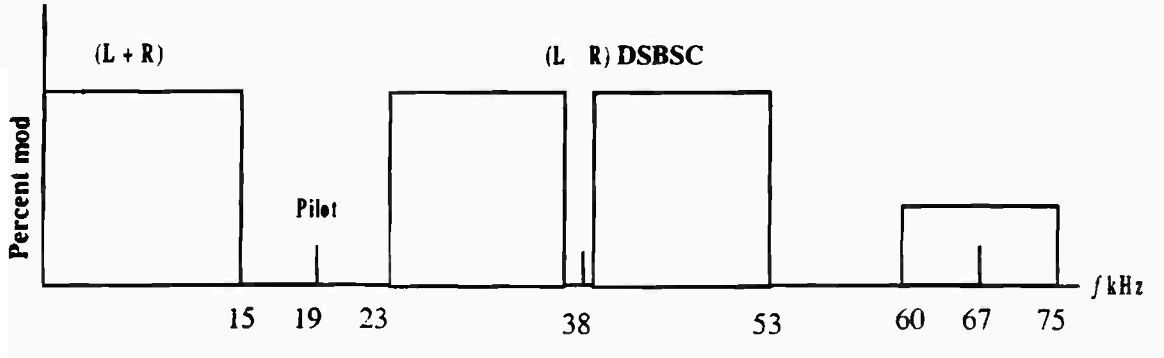


**Fig. 5.5 Distribution of signals in a stereo broadcast channel**

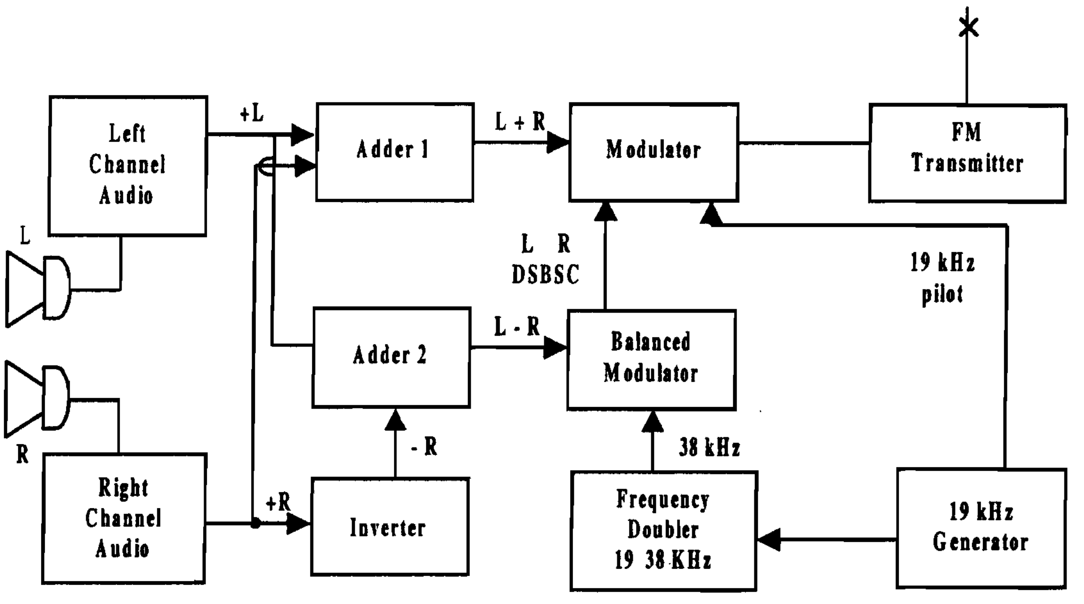


**Fig. 5.6 Block diagram of an FM stereo transmitter**

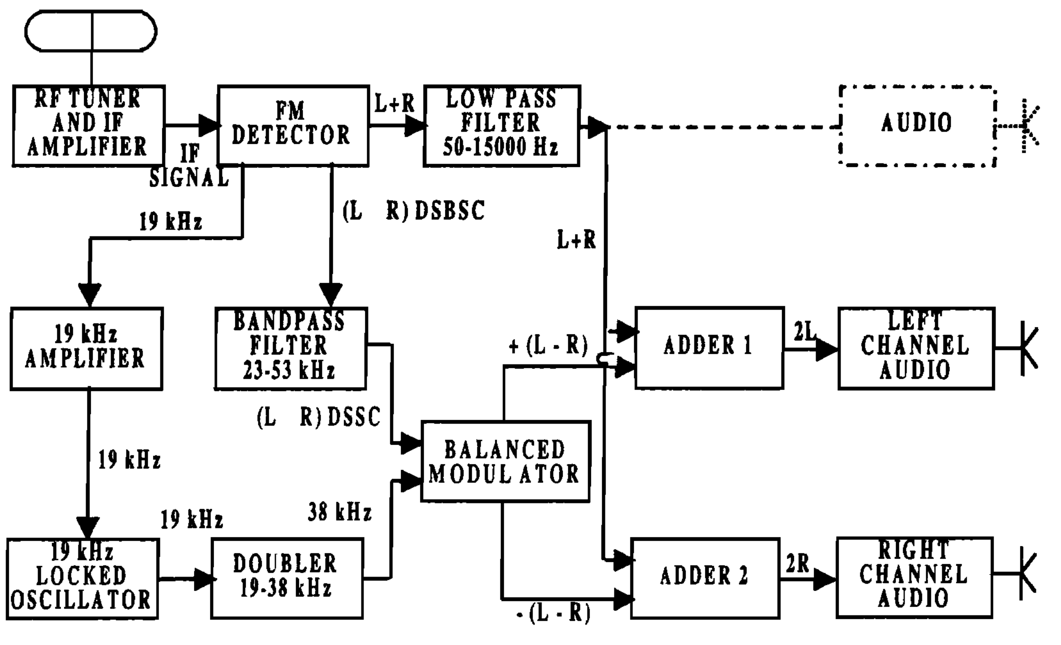


**Fig. 5.7 Block diagram of an AM stereo receiver**

**5.5 Phase Modulation:**

The phase in the modulated waveform is proportional to the input signal amplitude in PM, and to the integral of the input signal in FM. For an FM signal , the instantaneous frequency is

$$\omega_i(t) = \omega_c + a\,k_f \cos\omega_m t \qquad (5-37)$$

$$= \omega_c + \Delta\hat{\omega}_{FM} \cos\omega_m t \qquad (5-38)$$

$$\Delta\hat{\omega}_{FM} = a\,k_f\,, \qquad (5-39)$$

where $\Delta\omega_{FM}$ FM is the peak angular frequency deviation ( rad/s ) and $k_f$ is the frequency modulator constant ( rad/s-v).

For PM,

$$\theta(t) = \omega_c t + a\,k_p \cos\omega_m t + \theta_0 \qquad (5-40)$$

$$= \omega_c t + \Delta\hat{\theta} \cos\omega_m t + \theta_0\,, \qquad (5-41)$$

where $\Delta\hat{\theta} = a k_p$ is the peak phase deviation ( rad ), and $k_p$ is the phase modulator constant ( rad/V ).

The instantaneous frequency is

$$\omega_i(t) = \frac{d\theta(t)}{dt} \qquad (5-41)$$

$$= \omega_c - a\,k_p\,\omega_m \sin\omega_m t \qquad (5-42)$$

$$= \omega_c - \Delta\hat{\omega}_{PM} \sin\omega_m t\,, \qquad (5-43)$$

where

$$\Delta\hat{\omega}_{PM} = a\,k_p\omega_m \qquad (5-44)$$

$$= \Delta\hat{\theta}\,\omega_m = \beta_\varphi\omega_m \qquad (5-45)$$

Comparing eqns. (5-39) and (5-45), we see that the peak angular frequency deviation in PM is proportional not only to the amplitude of the modulating waveform but also to its frequency.

$$\beta_{FM} = \frac{\Delta\hat{\omega}_{FM}}{\omega_m} = \frac{a\,k_f}{\omega_m} \qquad (5-46)$$

$$\beta_{PM} = \frac{\Delta\hat{\omega}_{FM}}{\omega_m} = a\,k_p = \beta_\varphi = \Delta\hat{\theta} \qquad (5-47)$$

The bandwidth for PM is given by

$$BW_{PM} = 2(\hat{\Delta f} + f_m) \qquad (5-48)$$

$$= 2f_m(1+\hat{\Delta\theta}) = 2f_m(1+\beta_\varphi) \qquad (5-49)$$

**5.6 FM Modulator:**

First, we explore the indirect method. We have seen in Ex. 5.1 that the NBFM can be generated by first integrating the modulating signal, and then using NBPM for values of $\beta < 0.2$. To generate wideband FM, we use a frequency multiplier, which is a nonlinear device having a characteristic of the form

$$v_0(t) = a\, v_i^2(t) \qquad (5-50)$$

If the input is FM signal,

$$v_i(t) = A\cos(\omega_c t + \beta \sin\omega_m t) \qquad (5-51)$$

Then

$$v_0(t) = a A^2 \cos^2(\omega_c t + \beta \sin\omega_m t)$$

$$= \frac{1}{2} a A^2 \left[1 + \cos(2\omega_c t + 2\beta \sin\omega_m t)\right] \qquad (5-52)$$

The first term is a constant which can be removed by a filter. We note that the carrier frequency and the modulation index have been doubled. If we use an $n^{th}$ law device, the carrier frequency and the modulation index $\beta$ will be multiplied by a factor of $n$. A frequency converter may be used to control the increase in carrier frequency due to multiplication, since frequency conversion entails frequency translation without affecting the spectral content. The method of obtaining wideband FM from narrowband waveform using frequency multiplication is called Armstrong transmitter (Fig. 5.8).

A direct method for generating FM uses an oscillator with a tank circuit whose capacitance is a varactor. This is called voltage controlled oscillator (VCO).

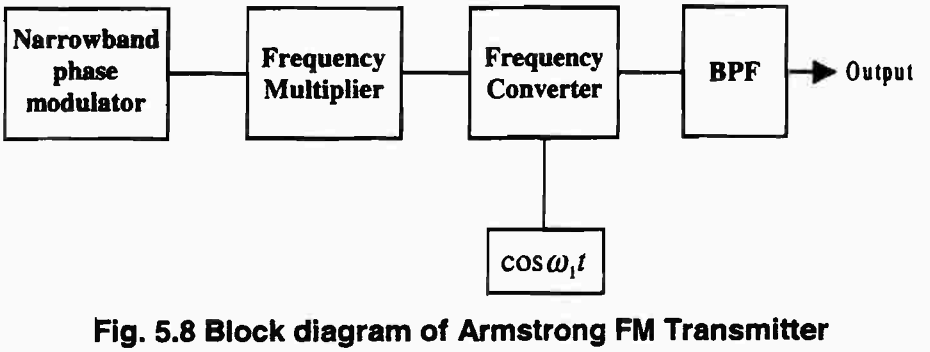


**Fig. 5.8 Block diagram of Armstrong FM Transmitter**

**Ex. 5.2:**

A varactor has a characteristic $C = Co/\sqrt{1+V}$. Such a diode is used in a tank circuit tuned to a center frequency of 5MHz when the reverse bias is 4V. Determine the modulation constant $k_f$ and the peak frequency deviation for a maximum error of 1% of linear frequency voltage characteristic.

**Solution:**

We can write

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{(1+V)^{1/4}}{2\pi\sqrt{LC_0}} \qquad (5-53)$$

At the operating point $V = V_o$ , $f = f_o$

$$f_0 = \frac{(1+V_0)^{1/4}}{2\pi\sqrt{LC_0}} \qquad (5-54)$$

Thus,

$$f = f_0 \frac{(1+V)^{1/4}}{(1+V_0)^{1/4}} \qquad (5-55)$$

Let $\Delta V$ be an incremental voltage about the operating point. So, $V = V_0 + \Delta V$ , $f = f_0 + \Delta f$

$$f_0 + \Delta f = \frac{f_o}{(1+V_0)^{1/4}}(1+V_0+\Delta V)^{1/4} = f_0\left[1+\frac{\Delta V}{1+V_0}\right]^{1/4} \qquad (5-56)$$

Let $K = 1 + V_0$

$$f_0 + \Delta f = f_0\left[1+\frac{\Delta V}{K}\right]^{1/4} \qquad (5-57)$$

$$= f_0\left[1+\frac{1}{4}\frac{\Delta V}{K} - \frac{3}{32}\left(\frac{\Delta V}{K}\right)^2 .....\right] \qquad (5-58)$$

The modulation constant $k_f$ is the slope of the linear frequency voltage characteristic, and is given by

$$k_f = \frac{1}{4}\left(\frac{\Delta V}{K}\right)\frac{f_0}{\Delta V} = \frac{f_0}{4K} = \frac{f_0}{4(1+V_0)} \qquad (5-59)$$

For the given operating point, $f_o$ = 5MHz, and $V_o$ = 4V, $k_f$ = 0.25MHz/V. Most of the error will arise from the second order term

$$\frac{\frac{3}{32}\left(\frac{\Delta V}{K}\right)^2}{\frac{1}{4}\left(\frac{\Delta V}{K}\right)} \leq 0.01$$

This gives $\Delta V \leq \frac{8K}{300}$

$$\hat{\Delta f} = k_f V_{max} = \left(\frac{f_0}{4K}\right)\left(\frac{8K}{300}\right) = \frac{f_0}{150} = 33.3kHz$$

**5.7 FM Demodulator:**

One method to demodulate FM is to use a system with a linear frequency to voltage transfer characteristic. Such a system is called frequency discriminator. If we use a simple differentiator,

$$V_{FM}(t) = A\cos\left[\omega_c t + k_f \int_o^t \upsilon_m(\tau)d\tau\right], \tag{5 - 60}$$

where $A$ is a constant ( a limiter is inserted prior to the differentiator )

$$\frac{dV_{FM}(t)}{dt} = -A\left[\omega_c + k_f\ \upsilon_m(t)\right]\sin\left[\omega_c t + k_f \int_o^t \upsilon_m(\tau)d\tau\right] \tag{5 - 61}$$

If $k_f\ \upsilon_m(t) << \omega_c$, this is an AM signal whose envelope is

$$A\omega_c\left[1 + \frac{k_f}{\omega_c}\upsilon_m(t)\right] \tag{5 - 62}$$

and whose carrier frequency is

$$\omega_c + k_f\ \upsilon_m(t) \tag{5 - 63}$$

The differentiator has, thus, changed FM into AM with some shift in carrier frequency. The resulting AM signal can be detected by an envelope detector. The action of the ideal differentiator can be approximated by any device whose magnitude transfer function is reasonably linear within the range of frequencies of interest. An R-L circuit approximates a differentiator, and is followed by an envelope detector. A band pass version can also function as a linear detector. Such discriminators are called slope detectors. But such circuits have very limited linearity range. A more common circuit uses the differences between two band pass amplitude responses. This discriminator uses triple tuned circuits which ensure better sensitivity and linearity. Such circuits also provide carrier frequency cancellation (Fig. 5.9). Conventional circuits are Foster-Seeley discriminator and ratio detector (Fig. 5.10, probs. 5-3 and 5-4). Advanced circuits use PLL (chapters 9, 10, 11).

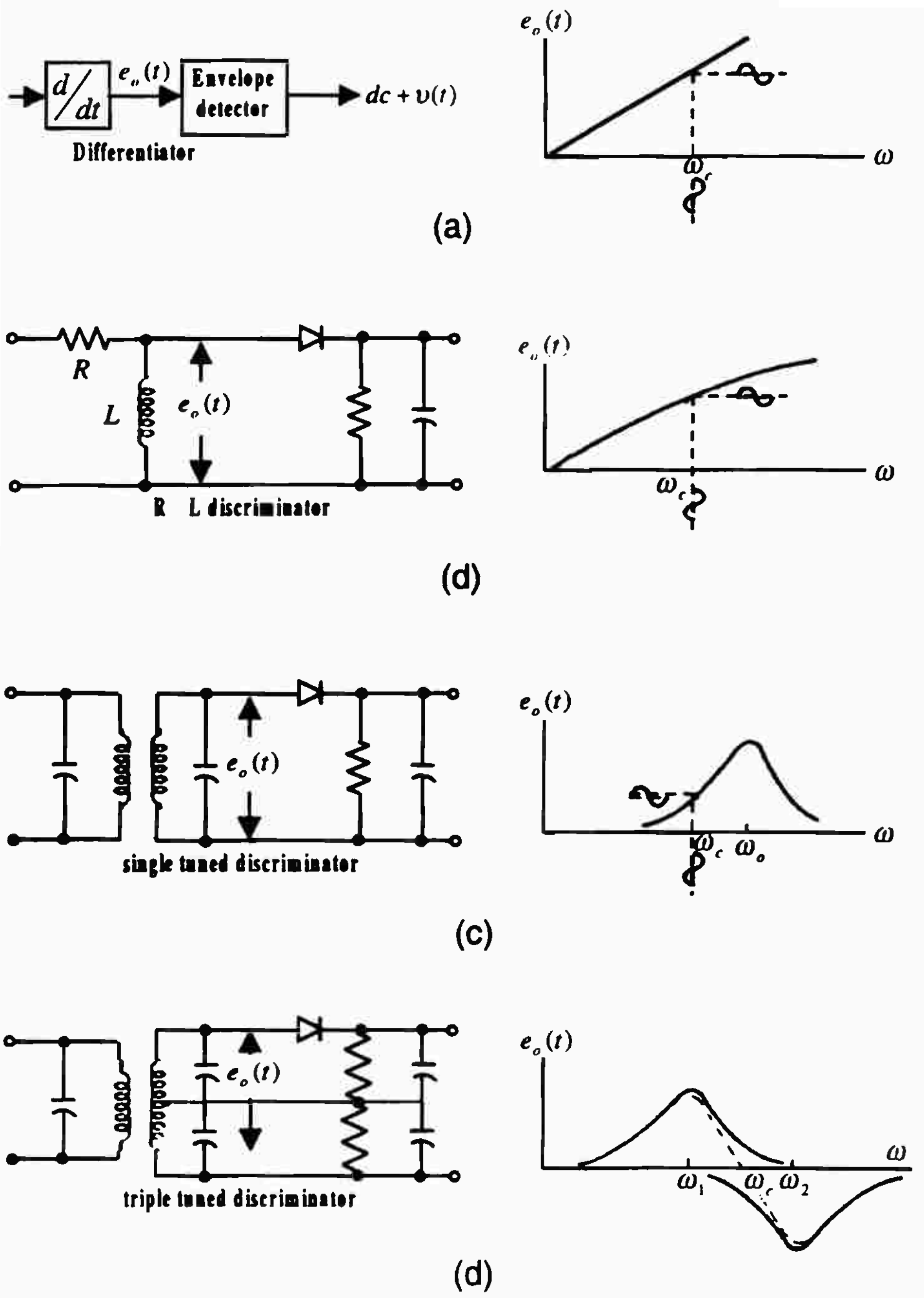


**Fig. 5.9 FM demodulation using slope discriminators**

a) differentiator b) R – L discriminator

c) single tuned discriminator d) triple tuned discriminator

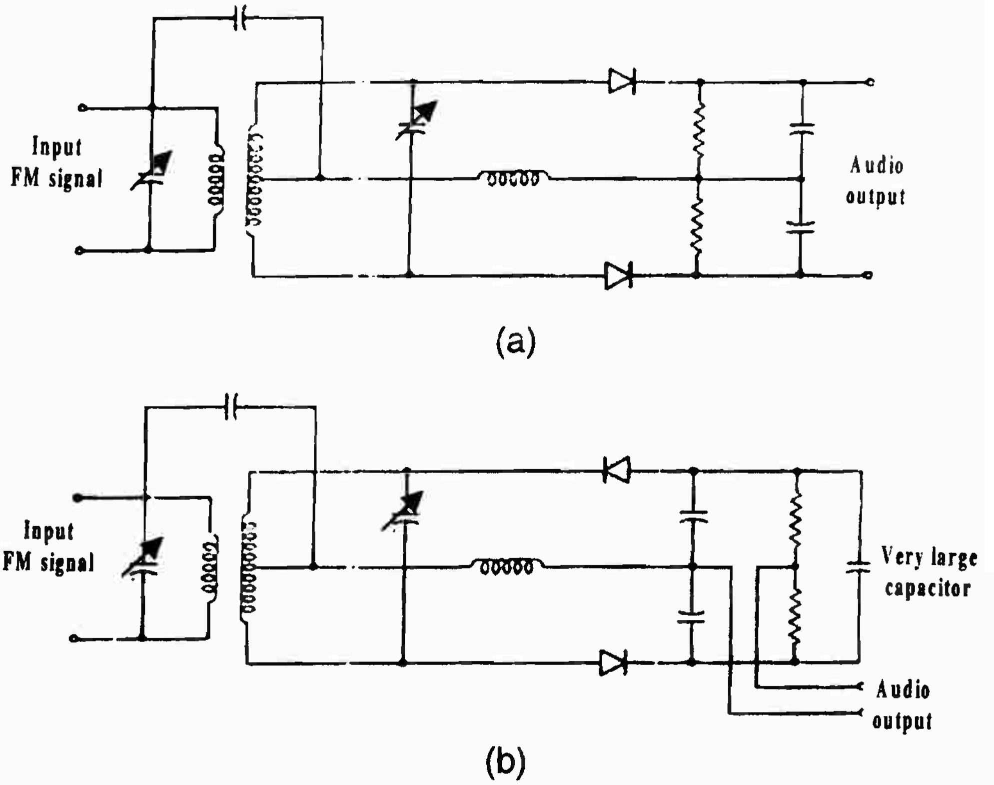


**Fig. 5.10 FM demodulators**

a) Foster Seeley discriminator b) Ratio detector

**Problems:**

1- A 1 GHz carrier is frequency modulated by a 20kHz sinusoid so that the peak frequency deviation is 5kHz. Determine the $BW$ of the FM signal.

2- Repeat if the bandwidth of the modulating signal amplitude is doubled. What happens if the bandwidth of the modulating signal frequency is doubled?

3- Analyze the Foster-Seeley discriminator circuit shown in (Fig. 5.10a).

4- Analyze the ratio detector circuit shown (Fig. 5-10b).

5- Consider $v_m(t) = 0.1\cos\omega_m t$, $f_m = 1kHz$ and $v_c(t) = 10\cos\omega_c t$, $f_c = 1MHz$. For FM, $k_f = 10^{-2}$ rad/Vs, write down $\omega_i(t)$, $\theta(t)$ and find the $BW$.

6- Repeat the problem above for PM, where $k_p = 10^{-2}$ rad.

7- A given angle modulated signal has a peak frequency deviation of 20Hz for an input sinusoid of unit amplitude and a frequency of 50Hz. Determine the frequency multiplication factor $n$ to produce a peak frequency deviation of 20kHz.

8- A 10MHz carrier is frequency modulated by a sinusoidal signal such that the peak frequency deviation is 50kHz. Determine the bandwidth of the FM signal for 500kHz input signal if the input frequency is a) 500Hz b) 10kHz.

9- $V(t) = 10\cos(10^6\pi\, t + 6\sin 10^3\pi\, t)$. Determine the type of modulation, the modulation index, the peak frequency deviation and the bandwidth.

10- A 1MHz carrier is phase modulated by a sinusoidal signal $v_m(t)$. The peak phase deviation is 2 rad when the peak input amplitude is 1Volt. Express $V_{PM}(t)$, and obtain the bandwidth if $v_m(t) = 10\cos 200\pi t$.

11- Show how PM can be used to generate FM, and how FM can be used to generate PM. Express steps both in block diagrams and in mathematical relations. Compare the bandwidth in each case.

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