CHAPTER 8
Evaluation of Analog Modulation Schemes

8.1 Coherent Detection:

We have seen before that the AM signal is given by

$$\upsilon_{AM}(t) = A_c\left(1 + m_a \cos\omega_m t\right)\cos\omega_c t$$

This type of AM produces a carrier and two sidebands. The carrier represents a power loss. If we suppress the carrier, we have

$$\upsilon_{AM}(t) = m_a A_c \cos\omega_m t \cos\omega_c t \qquad (8-1)$$

In general, we have

$$\upsilon_{AM}(t) = f(t)\cos\omega_c t \qquad (8-2)$$

Applying the modulation property of the Fourier transform,

$$V_{AM}(\omega) = \frac{1}{2}F(\omega+\omega_c) + \frac{1}{2}F(\omega-\omega_c) \qquad (8-3)$$

Thus, AM translates the frequency spectrum of a signal by $\pm\,\omega_c$, while the spectral shape remains unchanged (Fig. 8.1).

For demodulation, we may multiply the modulated signal - given by eqn. (8-3) - by $\cos\omega_c t$

$$\upsilon_{AM}(t)\cos\omega_c t = f(t)\cos^2\omega_c t$$

$$= \frac{1}{2}f(t) + \frac{1}{2}f(t)\cos 2\omega_c t \qquad (8-4)$$

If we take Fourier transform of both sides (Fig. 8.2),

$$\mathcal{F}\left[\upsilon_{AM}(t)\cos\omega_c t\right] = \frac{1}{2}F(\omega) + \frac{1}{4}F(\omega+2\omega_c) + \frac{1}{4}F(\omega-2\omega_c)$$

A LPF is required to separate out the useful signal from the double frequency terms, for $\omega_c > W$. This type of demodulation is called synchronous (or coherent) detection. It comprises an important premise in communication. We notice, however, that the correct phase and frequency must be known to correctly demodulate the DSBSC waveforms. This consideration must be kept in mind in regenerating a carrier at the receiver with the correct frequency and phase as in the transmitter, i.e., synchronization.

8.2 Quadrature Multiplexing:

It is possible to transmit different signals on the same carrier using the orthogonality property of sines and cosines (see Appendix A). This is called quadrature multiplexing. A simple scheme for achieving this is shown (Fig 8.3).

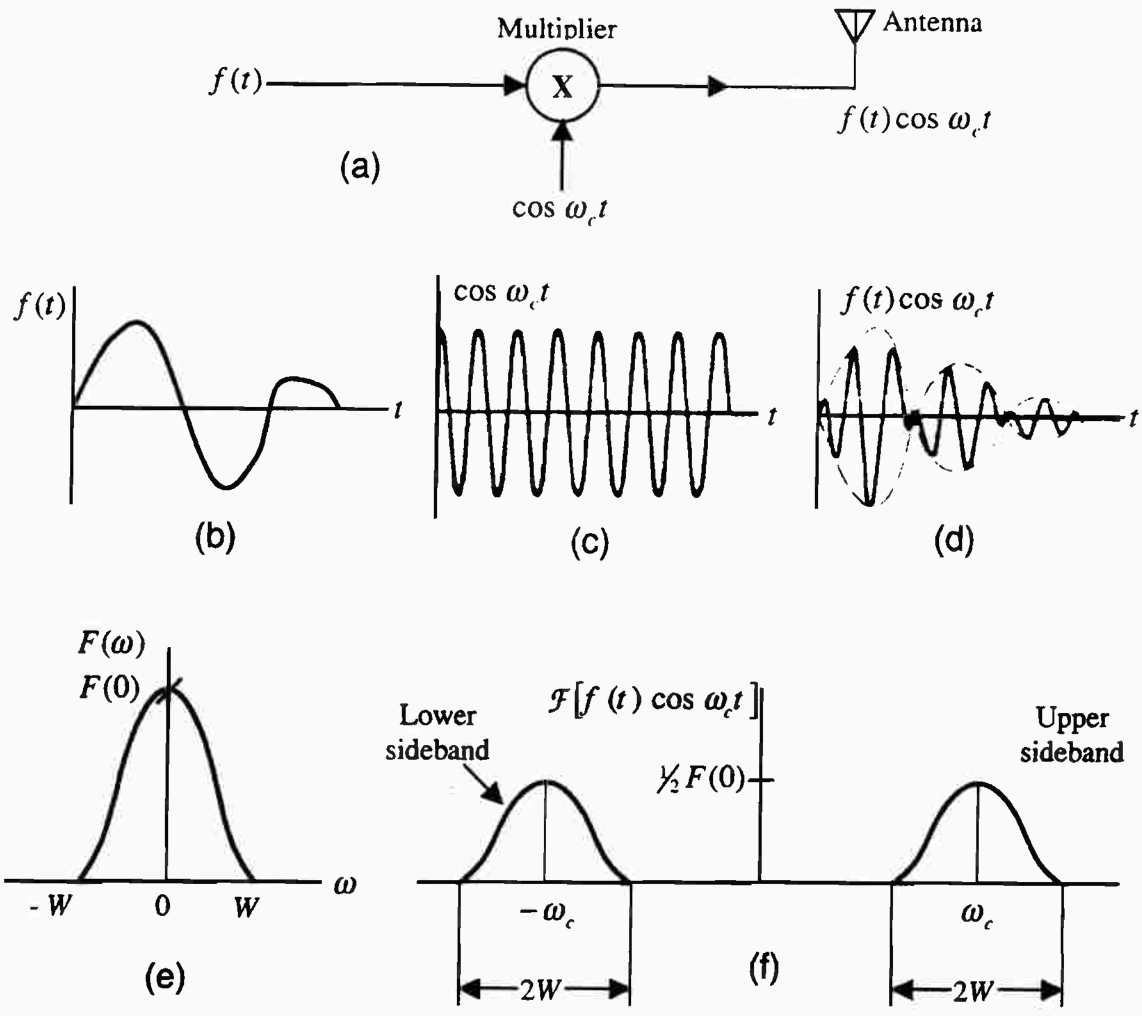

Fig. 8.1 Spectral response of DSBSC – AM signal

a) transmilter block diagram b) modulating signal c) carrier
d) modulated signal e) spectral response of modulating signal
f) spectral response of modulated signal

A single carrier frequency ω_c is used. But the modulating signals use this carrier in different phases, $\cos\omega_c t$ and $\sin\omega_c t$.

Thus, the transmitted signal $v_{QM}(t)$ is given by

$$v_{QM}(t) = f_1(t)\cos\omega_c t + f_2(t)\sin\omega_c t \qquad (8-6)$$

In the receiver, synchronous detection is used, in which $\cos\omega_c t$ and $\sin\omega_c t$ are separately used to detect the original signals:

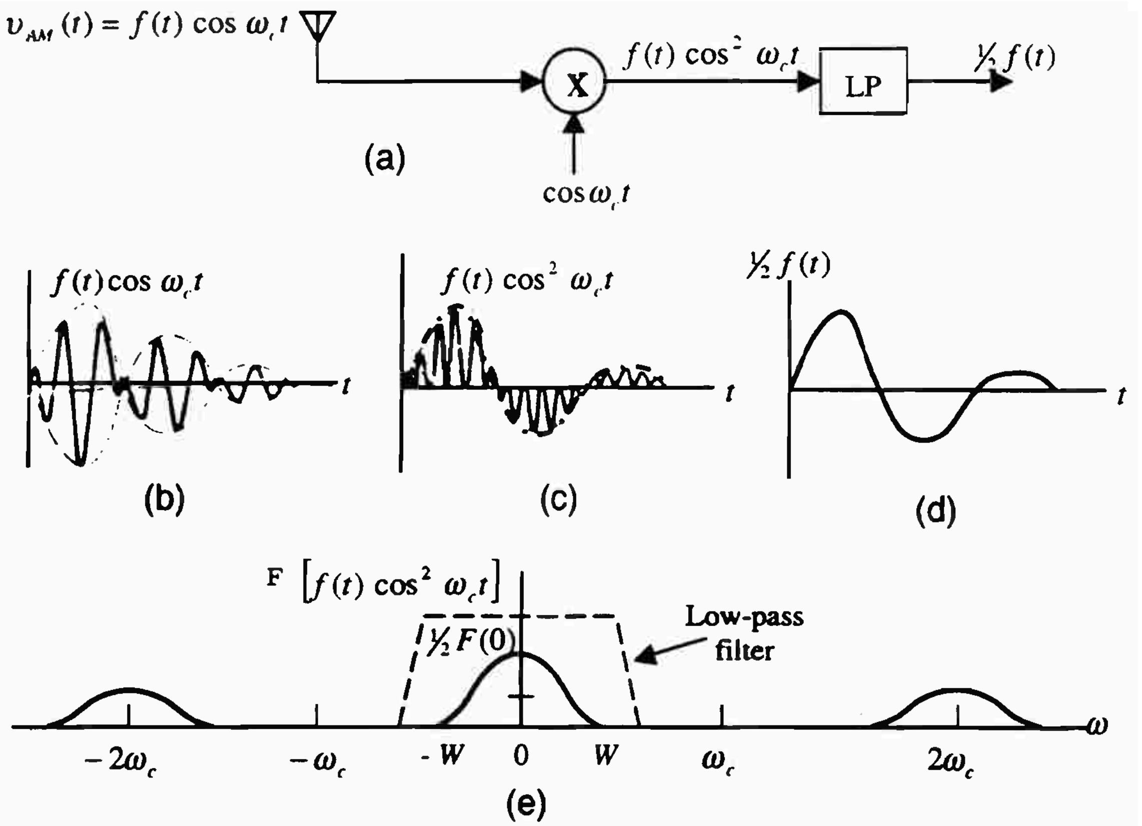

Fig. 8.2 Demodulation of DSBSC – AM Signal

a) demolulator block diagram b) received signal

c) signal after coherent multiplexing

d) signal at LPF output (recovered modulating signal)

e) spectral response

$$v_{QM}(t)\cos\omega_c t = f_1(t)\cos^2\omega_c t + f_2(t)\sin\omega_c t\cos\omega_c t$$

$$= \frac{1}{2}f_1(t) + \frac{1}{2}f_1(t)\cos 2\omega_c t + \frac{1}{2}f_2(t)\sin 2\omega_c t \qquad (8-7)$$

Also,

$$v_{QM}(t)\sin\omega_c t = f_1(t)\cos\omega_c t\sin\omega_c t + f_2(t)\sin^2\omega_c t$$

$$= \frac{1}{2}f_1(t)\sin 2\omega_c t + \frac{1}{2}f_2(t) - \frac{1}{2}f_2(t)\cos 2\omega_c t \qquad (8-8)$$

From eqns. (8-7) and (8-8), and with two LPFs installed, we have

$$e_1(t) = \frac{1}{2}f_1(t) \qquad (8-9)$$

$$e_2(t) = \frac{1}{2}f_2(t) \qquad (8-10)$$

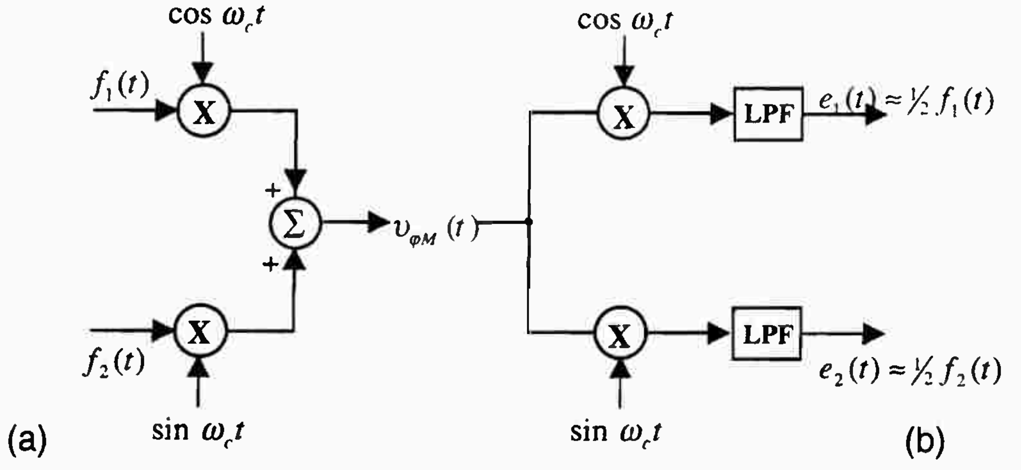

Fig. 8.3 Quadrature multiplexing scheme
a) transmitter b) receiver

Thus, we have recovered the original signals which had been modulated, using the same carrier for the same bandwidth. This procedure requires, however, precise phase synchronization between the transmitter and the receiver. Quadrature multiplexing has many applications, among them is color TV transmission.

8.3 Time Representation of Bandpass Noise:

Although noise is random, yet for bandpass systems - which limit the bandwidth of noise - some of the noise fluctuations are restricted, and it is possible to write time relations using phasor representation.

As the bandwidth of the noise is small compared to the center frequency of the bandpass circuit, we may represent the noise phasor as

$$[n_{ip}(t) + j\, n_{qp}(t)]e^{j\omega_0 t}, \qquad (8-11)$$

where $n_{ip}(t)$ is the in - phase component and $n_{qp}(t)$ is the quadrature phase component (Fig 8.4), and ω_0 is the center frequency. Both $n_{ip}(t)$ and $n_{qp}(t)$ are slowly varying compared to $e^{j\omega_0 t}$ for narrow bandpass condition.

Taking the real part of eqn. (8-11)

$$n(t) = \Re e\left\{\left[n_{ip}(t) + jn_{qp}(t)\right]e^{j\omega_0 t}\right\} \qquad (8-12)$$

$$= n_{ip}(t)\cos\omega_0 t - n_{qp}(t)\sin\omega_0 t \qquad (8-13)$$

This is known as the bandpass representation of noise, where $n_{ip}(t)$ and $n_{qp}(t)$ are fluctuations limited by the bandwidth of the bandpass noise.

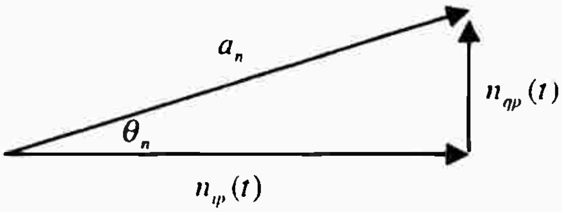

Fig. 8.4 Phasor representation of narrowband random noise

To obtain properties of $n_{ip}(t)$, let us multiply eqn. (8-13) by $\cos\omega_0 t$,

$$n(t)\cos\omega_0 t = n_{ip}(t)\cos^2\omega_0 t - n_{qp}(t)\sin\omega_0 t\cos\omega_0 t \qquad (8-14)$$

Taking the output of the LPF,

$$[n(t)\cos\omega_0 t]_{LPF} = \frac{1}{2}n_{ip}(t) \qquad (8-15)$$

Since

$$\mathcal{F}[f(t)\cos\omega_0 t] = \frac{1}{2}[F(\omega+\omega_0)+F(\omega-\omega_0)] \qquad (8-16)$$

and

$$S(\omega) = Lim_{T\to\infty}\frac{|F_T(\omega)|^2}{T} \qquad (8-17)$$

Thus,

$$S_{n_{ip}}(\omega) = \left\{Lim_{T\to\infty}\frac{|N_T(\omega-\omega_0)+N_T(\omega+\omega_0)|^2}{T}\right\}_{LPF} \qquad (8-18)$$

where $N(\omega)$ is the F.T. of $n(t)$

For random noise, the average of the cross products in eqn. (8-18) is zero. Thus,

$$S_{n_{ip}}(\omega) = [S_n(\omega-\omega_0)+S_n(\omega+\omega_0)]_{LPF} \qquad (8-19)$$

For $n_{qp}(t)$, we multiply eqn. (8 13) by $\sin\omega_0 t$

$$S_{n_{qp}}(\omega) = [S_n(\omega-\omega_0)+S_n(\omega+\omega_0)]_{LPF} \qquad (8-20)$$

Thus

$$S_{n_{ip}}(\omega) = S_{n_{qp}}(\omega) = [S_n(\omega-\omega_0)+S_n(\omega+\omega_0)]_{LPF} \qquad (8-21)$$

It follows, using eqns.(8-13) and (8-21), that

$$\overline{n^2(t)} = \frac{1}{2}\overline{n_{ip}^2(t)} + \frac{1}{2}\overline{n_{qp}^2(t)}$$

$$\overline{n^2(t)} = \overline{n_{ip}^2(t)} = \overline{n_{qp}^2(t)} \qquad (8-22)$$

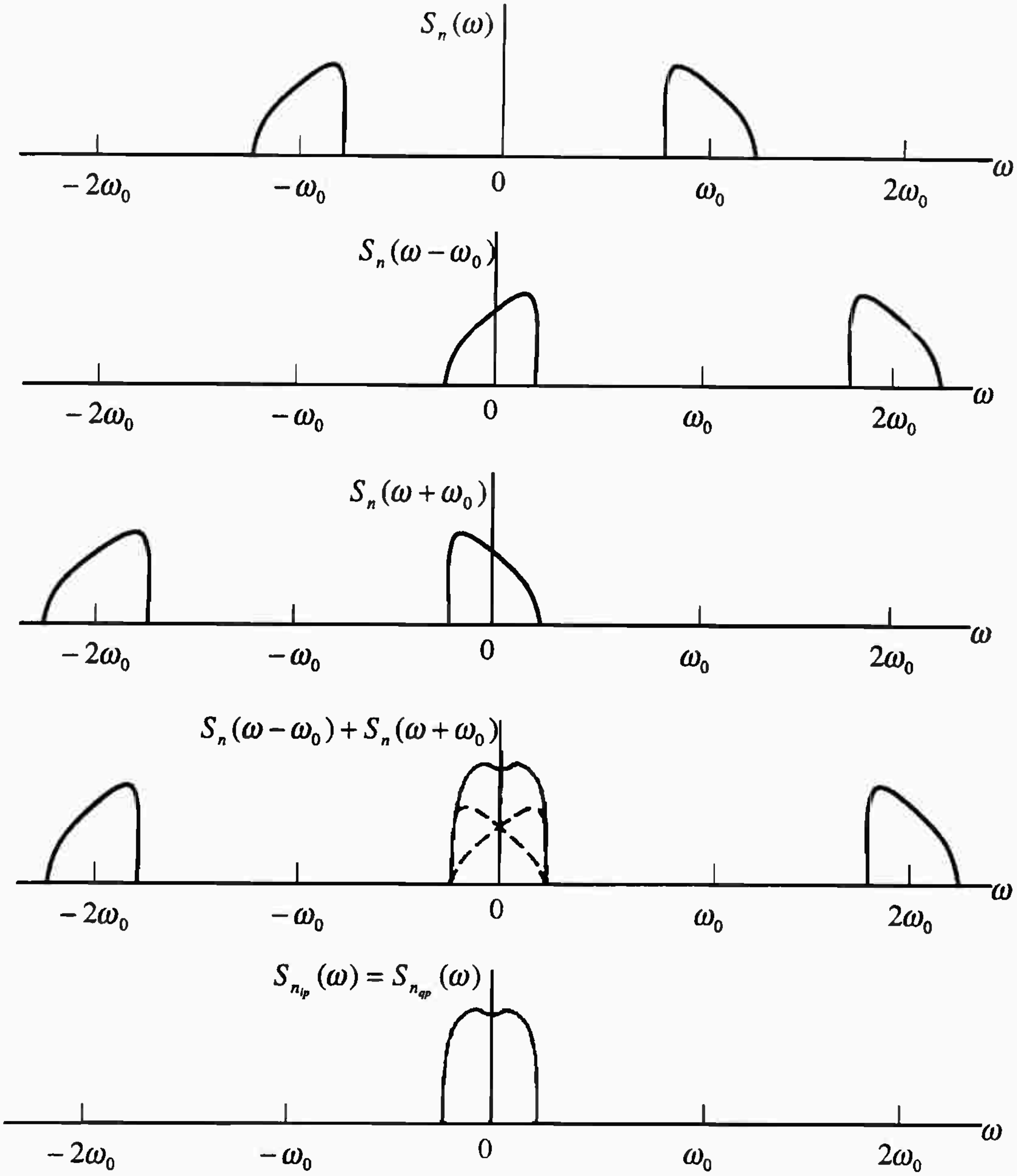

Fig. 8.5 In-phase and quadrature noise spectral densities for bandpass noise

Thus, the mean noise power is divided equally between the cosine and sine terms. It is the random nature of the noise which tends to distribute the noise components equally over both the cosine and sine terms. This is in direct contrast to a deterministic signal in which we can control the phase to give only cosine or sine terms.

A sketch of a given bandpass noise spectral density and the resulting low pass $S_{n_{ip}}(\omega)$ and $S_{n_{qp}}(\omega)$ are shown (Fig 8.5).

On this basis, we may compare different modulation systems in terms of their signal to noise ratio assuming bandpass noise.

8.4 The DSBSC Case: Synchronous Detector

In the case when the detector is synchronous (Fig 8.2)

$$S_i = \overline{[f(t)\cos\omega_c t]^2} = \frac{1}{2}\overline{f^2(t)} \qquad (8-23)$$

The useful output signal is given by $\frac{1}{2}f(t)$, according to eqn. (8-4).

$$S_o = \overline{\left[\frac{1}{2}f(t)\right]^2} = \frac{1}{4}\overline{f^2(t)} = \frac{1}{2}S_i \qquad (8-24)$$

The noise output of the detector n_d is $n_i(t)\cos\omega_c t$ filtered by LPF. The noise output of the detector $n_d(t)$ is given - using eqn.(8-13) - as

$$n_d(t) = \frac{1}{2}n_{ip}(t) + \frac{1}{2}n_{ip}(t)\cos 2\omega_c t - \frac{1}{2}n_{qp}(t)\sin 2\omega_c t \qquad (8-25)$$

The noise output of the LPF $n_0(t)$ is given by

$$n_0(t) = \frac{1}{2}n_{ip}(t) \qquad (8-26)$$

Defining

$$\overline{n_i^2(t)} = N_i \qquad (8-27)$$

Using eqn. (8-22),

$$N_0 = \overline{n_0^2(t)} = \frac{1}{4}\overline{n_{ip}^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \frac{1}{4}N_i \qquad (8-28)$$

Noting, from eqn. (8-22), that

$$\overline{n_i^2} = \overline{n_{iq}^2} = \overline{n_{qp}^2} \qquad (8-29)$$

Thus ,

$$\frac{S_0}{N_0} = 2\frac{S_i}{N_i}, \text{ for DSBSC} \qquad (8-30)$$

Hence, there is an improvement in S/N by a factor of 2. The reason for this improvement is that the synchronous detector accepts only the in - phase noise component, and rejects the quadrature phase component, thus, reducing the noise to half.

8.5 The DSBLC Case: Synchronous Detector

In the case of DSBLC, synchronous detection can be analyzed in similar steps as for DSBSC, except now we replace $f(t)$ by $A+f(t)$. Hence, for $\overline{f(t)}=O$

$$S_i=\frac{1}{2}A^2+\frac{1}{2}\overline{f^2(t)} \qquad (8-31)$$

$$S_0=\frac{1}{4}\overline{f^2(t)} \qquad (8-32)$$

$$N_0=\frac{1}{4}N_i \qquad (8-33)$$

Thus,

$$\frac{S_0}{N_0}=\frac{\frac{1}{4}\overline{f^2(t)}}{\frac{1}{4}N_i} \qquad (8-34)$$

$$\frac{S_i}{N_i}=\frac{\frac{1}{2}A^2+\frac{1}{2}\overline{f^2(t)}}{N_i} \qquad (8-35)$$

Dividing eqn. (8-34) by eqn. (8-35),

$$\frac{S_0}{N_0}=\frac{2\overline{f^2(t)}}{A^2+\overline{f^2(t)}}\,\frac{S_i}{N_i} \qquad (8-36)$$

The phasor diagram for DSBLC signals is shown (Fig 8.6) for large and small S/N . Since $|f(t)|<A$, the signal to noise ratio is poorer for a large carrier system than for a suppressed carrier system.

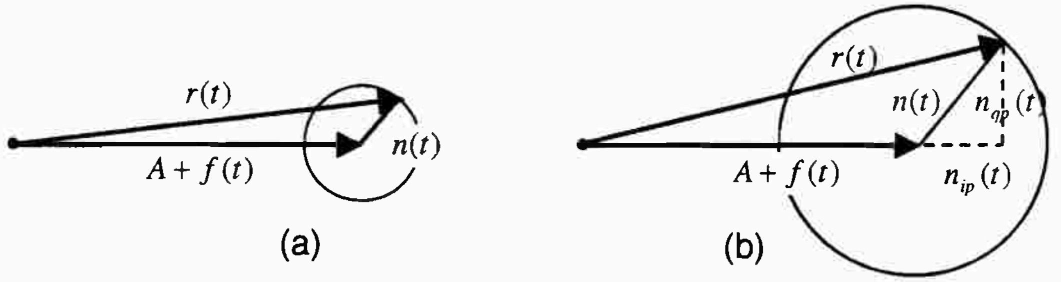

Fig. 8.6 Phasor diagram of DSBLC signals
a) large S/N b) small S/N

8.6 The DSBLC Case: Envelope Detector

We may now consider the envelope detector. The input signal and noise can be written - using noise bandpass representation - as:

$$s_i(t)+n_i(t)=[A+f(t)]\cos\omega_c t+n_{ip}(t)\cos\omega_c t-n_{qp}(t)\sin\omega_c t \qquad (8-37)$$

From eqn. (8-37), the envelope is

$$r(t)=\sqrt{[A+f(t)+n_{ip}(t)]^2+[n_{qp}(t)]^2} \qquad (8-38)$$

For high S/N,

$$r(t)\approx A+f(t)+n_{ip}(t) \qquad (8-39)$$

The useful signal is $f(t)$, while the noise is $n_{ip}(t)$.

Thus,

$$S_0=\overline{f^2(t)} \qquad (8-40)$$

$$N_0=\overline{n_i^2(t)}=\overline{n_{ip}^2(t)}=N_i \qquad (8-41)$$

$$S_i=\overline{\{[A+f(t)]\cos\omega_c t\}^2}=\frac{1}{2}A^2+\frac{1}{2}\overline{f^2(t)} \qquad (8-42)$$

Thus,

$$\frac{S_0}{N_0}=\frac{2\overline{f^2(t)}}{A^2+\overline{f^2(t)}}\frac{S_i}{N_i} \qquad (8-43)$$

We see that this result is identical to that for synchronous detection, as in eqn. (8-36), for large S/N.

For sinusoidal modulation, $f(t)=m_a A\cos\omega_m t$, eqn. (8-43) reduces to

$$\frac{S_0}{N_0}=\frac{2m_a^2}{2+m_a^2}\frac{S_i}{N_i} \qquad (8-44)$$

The maximum S/N occurs for $m_a=100\%$

$$\frac{S_0}{N_0}=\frac{2}{3}\frac{S_i}{N_i} \qquad (8-45)$$

When S/N is low - or when the noise level is high - the signal becomes heavily mutilated at the detector output. The threshold at which this occurs is when the modulated carrier power is on the same level as the average noise power. The threshold effect is a property of the envelope detector.

We observe no such effect in synchronous detector. Thus, in a synchronous detector the output signal can always be separated from noise, and S/N improvement is true for all noise conditions. Thus, the performance of the envelope detector at best may approach that of the synchronous detector for large S/N. But for high noise levels, the envelope detector suffers from the threshold effect, and becomes inferior to the synchronous detector.

The real advantage of synchronous detection lies in its ability to lock on the weak signal and extract it from a noisy background.

In the case of white input noise, the power spectral density is $\frac{\eta}{2}\ W/Hz$ and $P_n = \eta B$ watts for bandwidth f_m, $N_i = 2\eta f_m$ for DSB and $N_i = \eta f_m$ for SSB. Thus, from eqn. (8.30) $\frac{S_0}{N_0} = \frac{S_i}{\eta f_m}$ (8 - 46)

Fig 8.7 shows S_0/N_0 as a function of $S_i/\eta f_m$. We see that as S_0/N_0 increases, the performance of the envelope detector approaches that of the synchronous detector, while at low S_0/N_0, the envelope detector deteriorates in performance.

8.7 Spectral Power in FM:

We must draw a distinguishing line between the concept of instantaneous frequency and the frequency content. The instantaneous frequency is related to the amplitude of the modulating signal. The peak frequency deviation is, hence, related to the peak of the amplitude of the modulating signal.

The frequency content is related to the Fourier spectrum of the FM signal. Fourier transform for a general case of FM is not possible to obtain. But the analysis may be restricted to the sinusoidal case.

Let

$$f(t) = a \cos \omega_m t \qquad (8-47)$$

$$\omega_i(t) = \omega_c + a k_f \cos \omega_m t = \omega_c + \Delta\hat{\omega}_f \cos \omega_m t \qquad (8-48)$$

Where $\Delta\hat{\omega}_f$ is the maximum angular frequency deviation in the case of FM.

$$\theta(t) = \int_0^t \omega_i(\tau)d\tau = \omega_c t + \frac{\Delta\hat{\omega}_f}{\omega_m}\sin \omega_m t = \omega_c t + \beta_f \sin \omega_m t \quad ,(8-49)$$

where β_f denotes β for the FM case.

Using the complex notation,

$$v_{FM}(t) = \Re e\left[A\, e^{j\theta(t)}\right]$$
$$= \Re e\left[A e^{j\omega_c t}\ e^{j\beta_f \sin\omega_m t}\right] = A\cos(\omega_c t + \beta_f \sin \omega_m t) \quad (8-50)$$

we note that $e^{j\beta_f \sin \omega_m t}$ is a periodic function of time with a fundamental frequency ω_m rad/s. It can be expressed in terms of Fourier series:

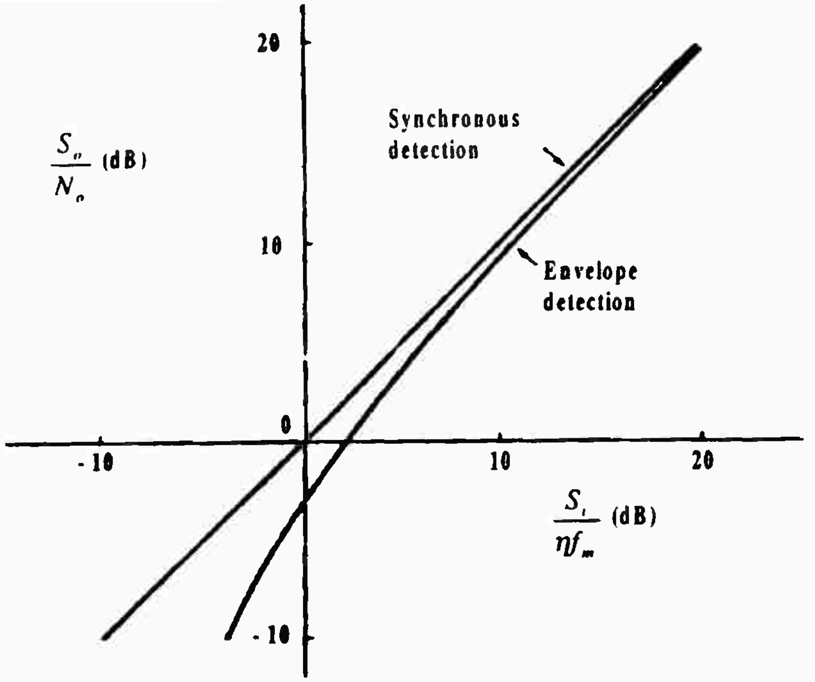

Fig. 8.7 Performance of S/N for AM detectors

$$e^{j\beta_f \sin \omega_m t} = \sum_{-\infty}^{\infty} F_n \, e^{jn\omega_m t} \qquad (8-51)$$

$$F_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{j\beta_f \sin \omega_m t} \, e^{-jn\omega_m t} \, dt \qquad (8-52)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta_f \sin x - nx)} \, dx \qquad (8-53)$$

where

$$x = \omega_m t = \frac{2\pi t}{T} \qquad (8-54)$$

The integral in eqn. (8-53) is called Bessel function of the first kind $J_n(\beta)$ of order n and argument β, where n is an integer (positive or negative), and β has continuous positive values. $J_n(\beta)$ can be evaluated numerically. Bessel functions have the following properties:

$$J_n = J_{-n}(\beta) \quad \text{for even } n \qquad (8-55a)$$

$$J_n = -J_{-n}(\beta) \quad \text{for odd } n \qquad (8-55b)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \qquad (8-55c)$$

Thus, eqn. (8-51) can be expressed as

$$e^{j\beta_f \sin\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta_f)\, e^{jn\omega_m t} \qquad (8-56)$$

Hence, eqn (8-50) becomes

$$\upsilon_{FM}(t) = \Re e\left[A e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta_f) e^{jn\omega_m t}\right]$$

$$= A \sum_{n=-\infty}^{\infty} J_n(\beta_f) \cos(\omega_c + n\omega_m)t \qquad (8-57)$$

From the properties of $J_n(\beta)$, eqn. (8-55),

$$\upsilon_{FM}(t) = A\lfloor J_0(\beta_f)\cos\omega_c t\rfloor$$
$$+ J_1(\beta_f)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t]$$
$$+ J_2(\beta_f)[\cos(\omega_c + 2\omega_m)t + \cos(\omega_c - 2\omega_m)t]$$
$$+ J_3(\beta_f)[\cos(\omega_c + 3\omega_m)t - \cos(\omega_c - 3\omega_m)t] + \ldots\ldots \qquad (8-58)$$

This means that FM has an infinite number of sidebands. The extent of bandwidth is determined by a rule of thumb: a sideband is significant if its magnitude is equal to or exceeds 1% of the unmodulated carrier, or $|J_n(\beta)| \geq 0.01$. It is seen that n/β approaches one as β becomes very large (Fig 8.9).

There are two ways to change β_f: by varying ω_m or $\Delta\hat{\omega}_f$ (Fig. 8.10). The bandwidth for very large β_f can then be approximated by taking the last significant sideband at $n = \beta_f$

$$W = 2nf_m = 2\beta_f f_m = 2\Delta\hat{f}_f \qquad \text{(for large } \beta_f\text{)} \qquad (8-59)$$

Whereas for very small values of β_f, the only significant Bessel functions are $J_0(\beta_f)$ and $J_1(\beta_f)$. Hence,

$$BW = 2f_m \quad \text{(for very small } \beta_f\text{)} \qquad (8-60)$$

Hence, a general rule proposed by Carson takes the intermediate case

$$BW = 2(\Delta\hat{f}_f + f_m) \qquad (8-61)$$

$$= 2f_m(1+\beta_f) \qquad (8-62)$$

As an approximation, it holds well for general band - limited modulating signals. From eqn. (8-50), We have

$$\overline{\upsilon_{FM}^2(t)} = \frac{A^2}{2}, \qquad (8-63)$$

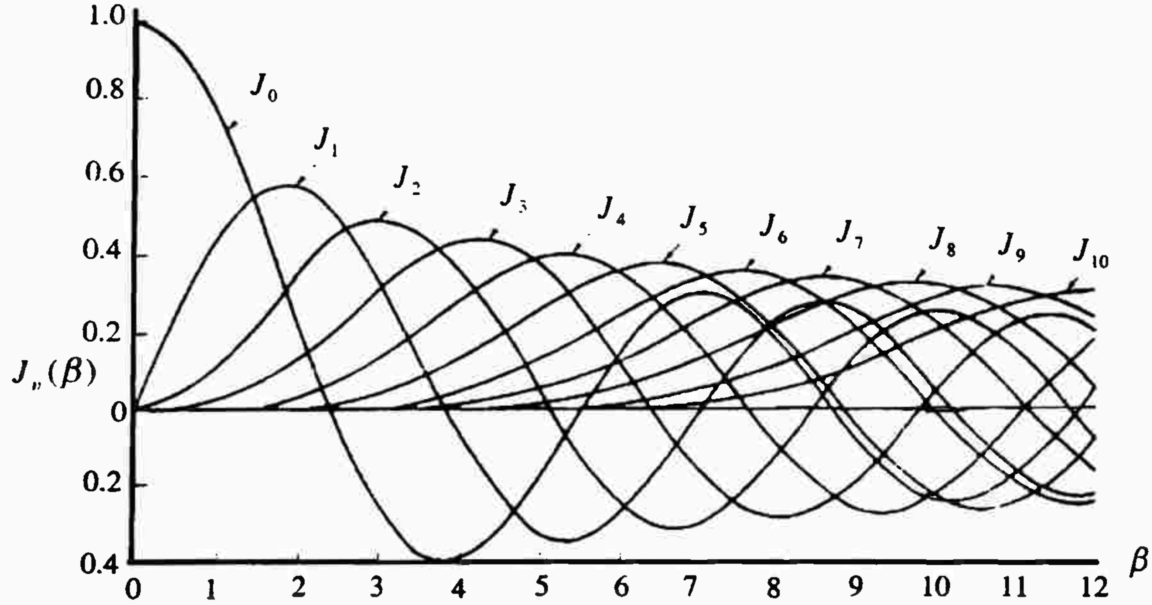

Fig. 8.8 Plot of Bessel function of the first kind $J_n(\beta)$

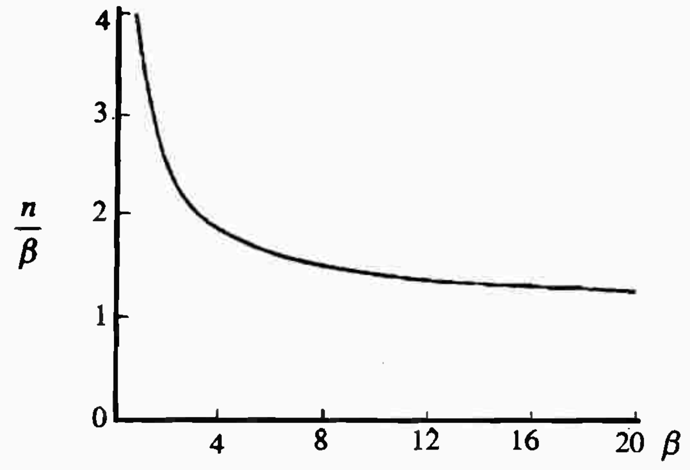

Fig. 8.9 Number of FM sidebands for $|J_n(\beta)| \geq 0.01$

This shows that the total average power in an FM waveform is constant, regardless of β_f. In other words, the average power of FM signal is the same as that of the unmodulated carrier. We can show (prob 8.5) that

$$\overline{v_{FM}^2(t)} = \frac{1}{2}A^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta_f) = \frac{1}{2}A^2 \qquad (8-64)$$

Thus, the mean square value of each sideband

$$\overline{v_{SB_n}^2} = \frac{1}{2}A^2 J_n^2(\beta_f) \qquad (8-65)$$

8.8 Spectral Power in PM:

The only difference between PM and FM is that in PM the phase in the modulated waveform is proportional to the input signal amplitude, whereas it is propotional to the integral of the input signal in FM.

For an FM signal with sinusoidal modulation, $f(t) = a\cos\omega_m t$,

$$\omega_i(t) = \omega_c + a k_f \cos\omega_m t = \omega_c + \Delta\hat{\omega}_f \cos\omega_m t \qquad (8-66)$$

$$\theta(t) = \int\omega_i(\tau)d\tau = \int(\omega_c + \Delta\hat{\omega}_f \cos\omega_m \tau)d\tau \qquad (8-67)$$

$$= \omega_c t + \frac{\Delta\hat{\omega}_f}{\omega_m}\sin\omega_m t \qquad (8-68)$$

$$\upsilon_{FM}(t) = A\cos(\omega_c t + \beta_f \sin\omega_m t) \qquad (8-69)$$

For PM,

$$\theta(t) = \omega_c t + a k_p \cos\omega_m t + \theta_0 \qquad (8-70)$$

$$= \omega_c t + \Delta\hat{\theta}\cos\omega_m t + \theta_0, \qquad (8-71)$$

Where $\Delta\hat{\theta}$ is the peak phase deviation (rad) and k_p is the phase modulator constant (rad / V). The instantaneous frequency is given by

$$\omega_i(t) = \frac{d\theta}{dt} \qquad (8-72)$$

$$= \omega_c - a k_p \omega_m \sin\omega_m t \qquad (8-73)$$

$$= \omega_c - \Delta\hat{\omega}_p \sin\omega_m t \qquad (8-74)$$

$$\upsilon_{PM}(t) = A\cos\left(\omega_c t + \Delta\hat{\theta}\cos\omega_m t + \theta_0\right), \qquad (8-75)$$

where $\Delta\hat{\omega}_p$ is the maximum angular frequency deviation in the case of PM

Thus, we see that the peak frequency deviation in PM is proportional not only to the amplitude of the modulating waveform, but also its frequency, or

$$\Delta\hat{\omega}_f = a k_f \qquad FM \qquad (8-76)$$

$$\Delta\hat{\omega}_p = a k_p \omega_m = \Delta\hat{\theta}\,\omega_m \qquad PM \qquad (8-77)$$

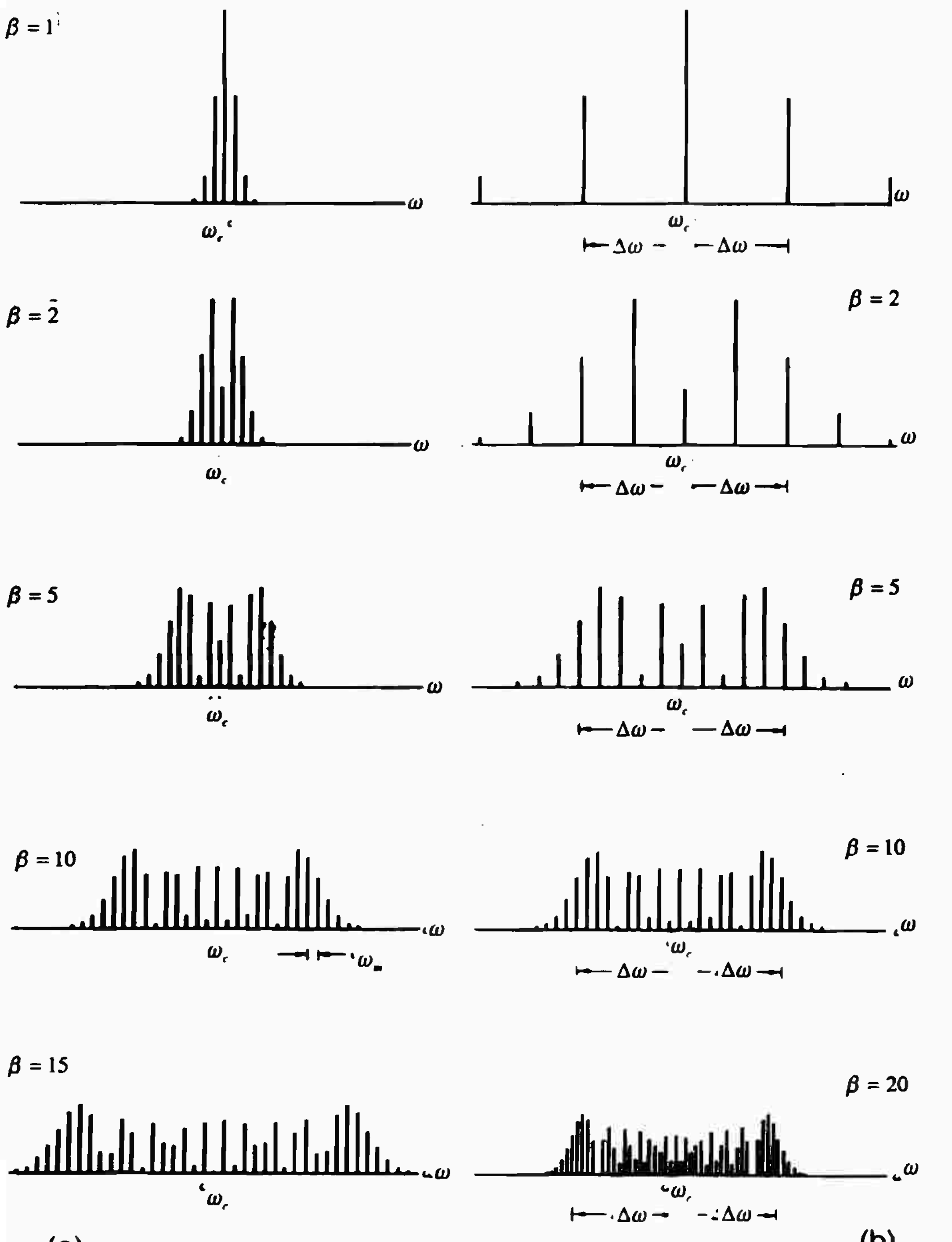

Fig. 8.10 Line spectra for FM waveforms

a) for constant ω_m b) for constant $\hat{\Delta\omega}_f$

This makes PM less popular to transmit when $\Delta\hat{\omega}_p$ is fixed. In PM analysis, we can compute $\Delta\hat{\omega}_p = a k_p \omega_m = \Delta\hat{\theta}\,\omega_m$, and then proceed as if the modulation were FM, with β_p representing numerically $\Delta\hat{\theta}$. Then, we proceed as before in FM analysis using Bessel functions.

Ex. 8.1:

A phase modulation scheme uses a sinusoidal signal of 5kHz unit amplitude and peak phase deviation of 1 radian. Calculate the bandwidth of the PM signal. If the PM transmitter for no modulation is 50 watts. The peak phase deviation is increased from zero until the first sideband amplitude in the output is zero. Determine the spectral power in the significant components

Solution:

$\Delta f_p = \Delta\hat{\theta}\, f_m = 5kHz$

From Carson's rule,

$$BW_p = 2(\Delta\hat{f}_p + f_m) = 2(5+5) = 20kHz$$

From Bessel function expansion, $\beta_p = \Delta\hat{\theta} = 1$ and $n = 3$, we have only three significant Bessel function components (Fig. 8.8)

$$BW = 2nf_m = 2\times3\times5 = 30kHz$$

We note from Fig. 8.8, $J_1(\beta) = 0$ at $\beta = 3.8$, $J_0(3.8) = 0.40$, $J_2(3.8) = 0.41$, $J_3(3.8) = 0.41$. The average carrier power is

$$P_0 = J_0^2(3.8)\times 50 = 8W$$

$$P_1 = 0$$

$$P_2 = 2J_2^2(3.8)50 = 17W$$

$$P_3 = 2J_3^2(3.8)50 = 17W$$

Power in the remaining sidebands $= 50 - 42 = 8W$

8.9 S/N in FM:

The block diagram of an idealized FM receiver is shown (Fig. 8.11)

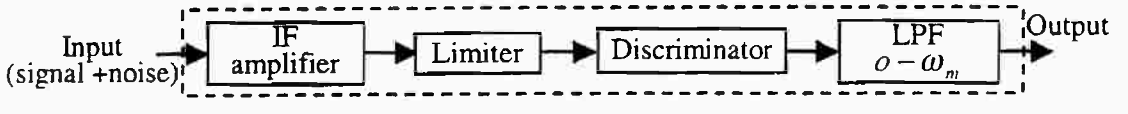

Fig. 8.11 Block diagram of an idealized FM receiver

At the discrimiantor input, we have the FM signal $s_i(t)$ given by

$$s_i(t) = A\cos\theta(t) = A\cos\left[\omega_c t + k_f \int_0^t f(\tau)d\tau\right] \quad (8-78)$$

The discriminator output is proportional to the difference between the instantaneous frequency of $s_i(t)$ and the carrier frequency (take the constant of proportionality = 1 for simplicity).

$$s_o(t) = \frac{d\theta}{dt} - \omega_c = k_f f(t) \quad (8-79)$$

$$S_o = \overline{s_o^2(t)} = k_f^2 \overline{f^2(t)} \quad (8-80)$$

We now calculate the mean output noise power in the presence of an unmodulated carrier using the bandpass representation of band limited noise.

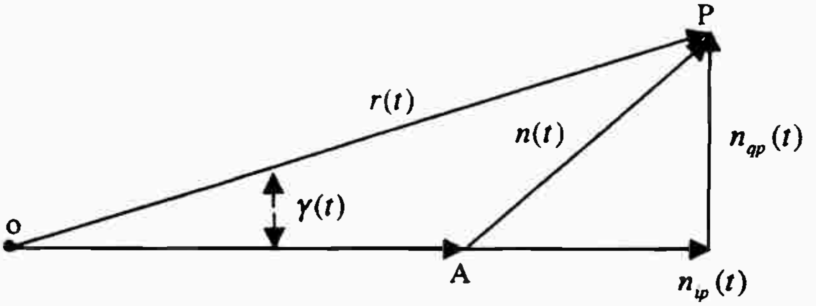

Fig. 8.12 Phasor diagram for FM (large S/N)

From Fig. 8.12,

$$A\cos\omega_c t + n_i(t) = A\cos\omega_c t + n_{ip}(t)\cos\omega_c t - n_{qp}(t)\sin\omega_c t$$

$$= r(t)\cos\left[\omega_c t - \gamma(t)\right] \quad (8 \quad 81)$$

Thus, the addition of noise produces noise in the amplitude $r(t)$ and noise phase in $\gamma(t)$. In AM, we accounted for noise in $r(t)$ not $\gamma(t)$. Here, we are interested in the noise of $\gamma(t)$, assuming that the limiter is ideal so that the noise in $r(t)$ is ironed out.

Now,

$$\gamma(t) = \tan^{-1}\left[\frac{n_{qp}(t)}{A + n_{ip}(t)}\right] \quad (8-82)$$

Assuming that $n_{ip}(t)$ and $n_{qp}(t)$ are both << A,

$$\gamma(t) = \tan^{-1}\left[\frac{n_{qp}(t)}{A}\right] = \frac{n_{qp}(t)}{A} \quad (8-83)$$

The discriminator output is proportional to the difference between the instantaneous frequency and the carrier frequency

$$n_0(t) = \frac{d\gamma}{dt} = \frac{1}{A}\,\frac{d}{dt}\left[n_{qp}(t)\right] \qquad (8-84)$$

The Fourier transform equivalent of time differentiation is multiplication by $j\omega$. Thus,

$$S_{n_o}(\omega) = \frac{\omega^2 S_{qp}(\omega)}{A^2} \qquad (8-85)$$

Thus, the spectral components at higher frequencies are emphasized as a result of differentiation. The bandwidth of the discriminator output is limited by a LPF with a cutoff frequency ω_m rad/s.

Using eqn. (8-20),

$$S_{qp}(\omega) = \left[S_n(\omega - \omega_c) + S_n(\omega + \omega_c)\right]_{LP} \qquad (8-86)$$

For white noise at the discriminator input,

$$S_{n_i}(\omega) = \frac{\eta}{2} \qquad (8-87)$$

From eqn. (6-86),

$$S_{qp}(\omega) = \eta \qquad (8-88)$$

Then, eqn. (6-85) becomes

$$S_{n_O}(\omega) = \eta\frac{\omega^2}{A^2} \qquad (8-89)$$

Thus, for white noise at the discriminator input, the output noise spectral density is parabolic (Fig 8.13)

From eqn (8-89), the mean square value of the output noise may be obtained from

$$N_0 = \overline{n_0^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{n_o}(\omega)\, d\omega \qquad (8-90)$$

$$= \frac{\eta}{\pi A^2}\int_0^{\omega_m} \omega^2\, d\omega \qquad (8-91)$$

$$= \frac{\eta\,\omega_m^3}{3\pi A^2} \qquad (8-92)$$

The mean carrier power is

$$S_c = \frac{A^2}{2} \qquad (8-93)$$

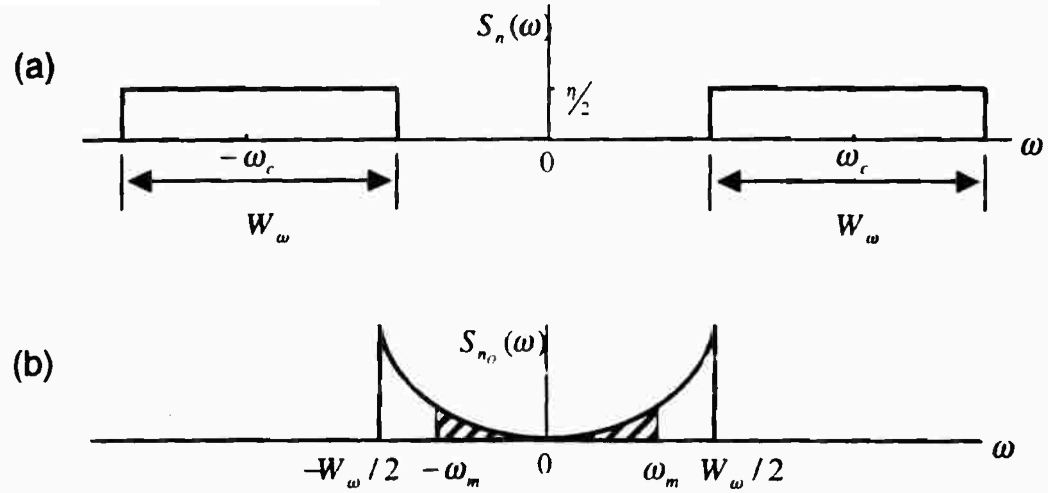

Fig. 8.13 Noise in FM demodulator for high carrier to noise ratio ($W_\omega = 2\pi W$)
a) at discriminator input. b) at output.

From eqns. (8-92), and (8-93), we see that the output noise power is inversely proportional to the mean carrier power in FM. This effect of reduction in output noise with an increase in the carrier power is called noise quieting.
From eqns. (8-80) and (8-92),

$$\frac{S_0}{N_0} = 3\pi A^2 \frac{k_f^2\, \overline{f^2(t)}}{\eta\, \omega_m^3} \qquad (8-94)$$

We see that the signal to noise ratio at the output increases as the square of the bandwidth for wideband FM, since $\Delta\hat{\omega}_f$ is proportional to k_f ($\Delta\hat{\omega}_f = a\, k_f$ for the sinusoidal case $f(t) = a\cos\omega_m t$)
Thus,

$$\frac{S_0}{N_0} = \frac{3\pi\, A^2 (\Delta\hat{\omega}_f)^2}{2\,\eta\,\omega_m^3} \qquad (8-95)$$

Now, we wish to compare FM with AM. Assume that the modulation is sinusoidal in each case. The mean square noise at the IF for the AM case is given by

$$N_c = \frac{2}{2\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} \frac{1}{2}\eta\, d\omega = \frac{\eta\,\omega_m}{\pi} \qquad (8-96)$$

From eqns. (8-93) and (8-96), eqn. (8-95) becomes

$$\frac{S_0}{N_0} = 3\left(\frac{\Delta\hat{\omega}_f}{\omega_m}\right)^2 \frac{S_c}{N_c} = 3\beta_f^2\,\frac{S_c}{N_c} \qquad (8-97)$$

For AM at the output of the IF amplifier,

$$S_0 = \overline{f^2(t)}$$

$$N_0 = N_i$$

$$S_c = \frac{A^2}{2}$$

$$N_c = N_i$$

Then,

$$\frac{S_0}{N_0} = \frac{\overline{f^2(t)}}{N_i} \tag{8-98}$$

$$\frac{S_c}{N_c} = \frac{A^2/2}{N_i} \tag{8-99}$$

Thus,

$$\frac{S_0}{N_0} = \frac{2\overline{f^2(t)}}{A^2}\frac{S_c}{N_c} \tag{8-100}$$

When $f(t) = m_a A \cos\omega_m t$, eqn. (8-100) can be rewritten as

$$\frac{S_0}{N_0} = m_a^2\frac{S_c}{N_c} \tag{8-101}$$

Thus, the maximum signal to noise ratio of an AM system is equal to the carrier to noise ratio (CNR) at the output of the IF amplifier (when $m_a = 1$).In this case,

$$\left(\frac{S_0}{N_0}\right)_{AM} = \frac{S_c}{N_c} = \frac{A^2/2}{\eta\omega_m/\pi} = \frac{\pi A^2}{2\eta\omega_m} \tag{8-102}$$

Incorporating eqn. (8-102) into eqn. (8-97),

$$\left(\frac{S_0}{N_0}\right)_{FM} = 3\beta_f^2\left(\frac{S_0}{N_0}\right)_{AM} \tag{8-103}$$

We conclude that that signal to noise ratio in FM can be made to surpass that in AM by increasing β_f. The factor $3\beta_f^2$ is called noise quieting. As β_f is increased, the bandwidth is increased, yet noise is quieter. This is an interesting feature of FM. Of course, an increase in bandwidth has other problems in limiting the number of channels to be used, yet it provides enough bandwidth for high fidelity ($Hi\ Fi$) operation.*

* We should note that increasing β_f increases the bandwidth by a factor β_f for which we expect S/N to decrease. But we find that S/N increases by a factor β_f^2 due to noise quieting.

We note that noise quieting occurs as long as $\beta_f > \frac{1}{\sqrt{3}} = 0.577$

This condition is approximately the transition point between narrowband and wideband FM. Thus, we conclude that there is no signal to noise ratio improvement in narrowband FM over AM. We must remember that the previous analysis was based on the assumption that noise was small compared to carrier power. Therefore, as we exchange bandwidth for S/N, the improvement cannot go on without limit. In fact, as the noise power increases with continued increase in bandwidth, the noise becomes comparable to the signal power, giving rise to a threshold effect.

Ex. 8.2:

Show why PM surpasses AM in S/N.

Solution:

For PM,

$$s_i(t) = A\cos\left[\omega_c t + k_p\ f(t)\right] \qquad (8-104)$$

The phase detector output is proportional to the difference between the instantaneous phase of $s_i(t)$ and the carrier phase

$$s_o(t) = \left[\theta(t) - \omega_c t\right] = k_p\ f(t) \qquad (8-105)$$

From eqn. (8-83),

$$n_0(t) = \frac{n_{qp}(t)}{A} \qquad (8-106)$$

$$S_{n_o}(\omega) = \frac{1}{A^2}\,S_{qp}(\omega) \qquad (8-107)$$

For white input noise equal to $\eta/2\ \ W/Hz$,

$$\overline{n_0^2(t)} = \frac{1}{\pi}\int_0^{\omega_m} \frac{1}{A^2}\,\eta d\omega = \frac{\eta\,\omega_m}{\pi A^2} \qquad (8-108)$$

Then,

$$\frac{S_0}{N_0} = \frac{\overline{s_o^2(t)}}{\overline{n_o^2(t)}} = \pi A^2\,\frac{k_p^2\,\overline{f^2(t)}}{\eta\,\omega_m} \qquad (8-109)$$

For the sinusoidal case,

$$\overline{f^2(t)} = \frac{a^2}{2} \quad , \quad \hat{\Delta\theta} = a\,k_p$$

$$\frac{S_0}{N_0} = \frac{\pi A^2}{2\eta\,\omega_m}\,(\hat{\Delta\theta})^2 \qquad (8-110)$$

From eqn. (8-102), by comparing to the AM case with 100% modulation, eqn. (8-110) becomes

$$\left(\frac{S_0}{N_0}\right)_{PM} = \left(\hat{\Delta\theta}\right)^2 \left(\frac{S_0}{N_0}\right)_{AM} \qquad (8-111)$$

8.10 Threshold Effects in FM:

The case when the noise becomes comparable to signal is important because the main challenge in communication arises when signals are weak rather than when signals are strong

In Fig (8.14), phasor A represents the signal and phasor B represents the noise. The mean square value of the phase angle θ_c represents the average noise in the angle. We have

$$\theta_c = \tan^{-1}\frac{B\sin\theta_b}{A + B\cos\theta_b} \qquad (8-112)$$

The mean square value is:

$$\overline{\theta_c^2} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\tan^{-1}\left(\frac{B\sin\theta_b}{A + B\cos\theta_b}\right)\right]^2 d\theta_b \qquad (8-113)$$

This can be solved numerically as shown (Fig 8.14 b). In this model, A/B represents a ratio of signal to noise voltages. The noise increases sharply for $A/B < 3$. The magnitude of the phasor $n(t)$ may exceed that of the carrier A. Thus, the phase angle $\gamma(t)$ may change by $\pm 2\pi$ within a very short interval as P sweeps around O (Fig 8.15). Since the output of the FM discriminator is proportional to $d\gamma/dt$, impulse-like noise spikes result in the output (Fig 8.15c) giving rise to click noise.

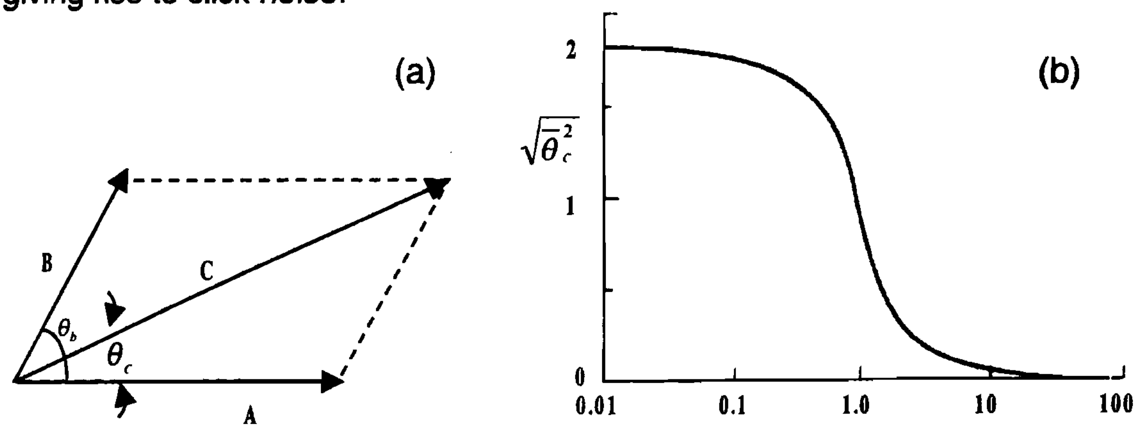

Fig. 8.14 Phasor diagram for phase noise

a) phasors A, B b)the mean square value $\sqrt{\overline{\theta_c^2}}$ as a function of A/B

This is a threshold effect that deteriorates the performance of the FM discriminator. Fig. 8.16 shows the S/N performance characteristics of a wideband FM system.
Note that for small input S/N, the FM system may be inferior to an AM system.

8.11 Pre-emphasis and De-emphasis:
In signals related to music and speech, most of the power lies at the low frequency end, whereas the signal power is weak at the high frequency end. However, it is at this high end that noise power increases parabolically. Therefore, it is important to reduce the noise power, where the signal power is weak, to counteract the noise effect. In other words, we want to emphasize the high frequency components of the signal at the input, and de-emphasize the noise components at the high frequency end in the output. Because the noise power spectral density at the FM demodulator output rises parabolically with frequency, we must use a matching pre-emphasis for operation the signal, i.e., $|H(\omega)|^2 = \omega^2$ or $H(\omega) = j\omega$, (differentiation).

We need a filter whose transfer function is constant for low frequencies, and behaves as a differentiator at the higher frequencies (Fig 8.17a), and a corresponding de-emphasis network for the receiver (Fig 8.17b).
From eqn. (7-89),

$$S_{n_o}(\omega) = \frac{\eta \omega^2}{A^2} \qquad (8-114)$$

The transfer function of the de-emphasis filter can be written as

$$H(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_1}} \qquad (8-115)$$

The mean square value of the noise after the de-emphasis filter is

$$N_0' = \frac{1}{\pi}\int_0^{\omega_m} S_{n_o}(\omega)\left|H(\omega)\right|^2 d\omega \qquad (8-116)$$

$$= \frac{\eta}{\pi A^2}\int_0^{\omega_m} \frac{\omega^2}{1+\left(\dfrac{\omega}{\omega_1}\right)^2} d\omega \qquad (8-117)$$

Without the de-emphasis filter, the noise would be.

$$N_0 = \frac{\eta}{\pi A^2}\int_0^{\omega_m} \omega^2\, d\omega \qquad (8-118)$$

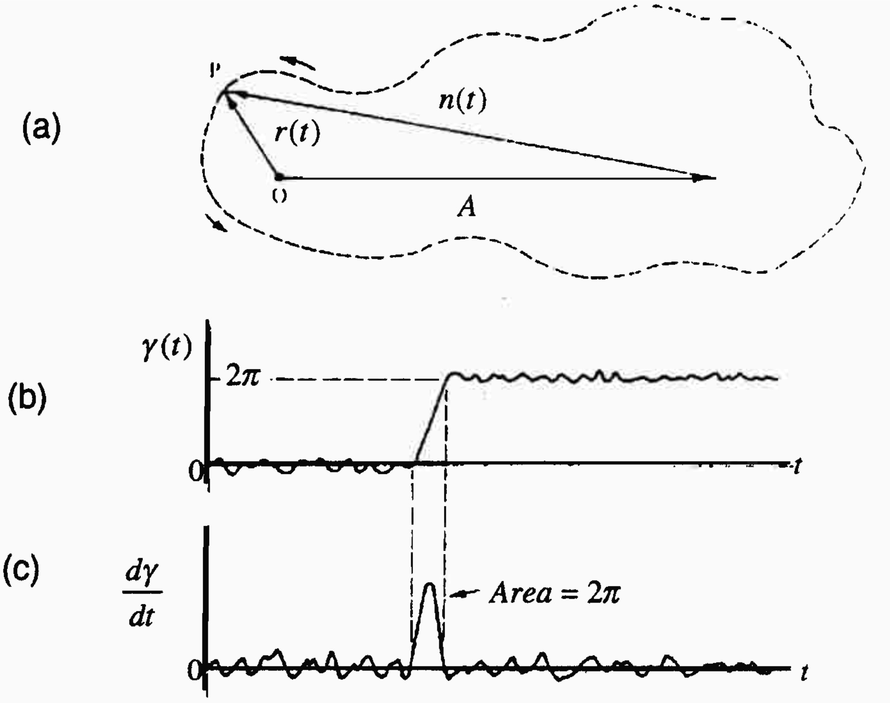

Fig. 8.15 FM threshold effect

a) phasor diagram of the sum b) $\gamma(t)$ c) $\frac{d\gamma(t)}{dt}$

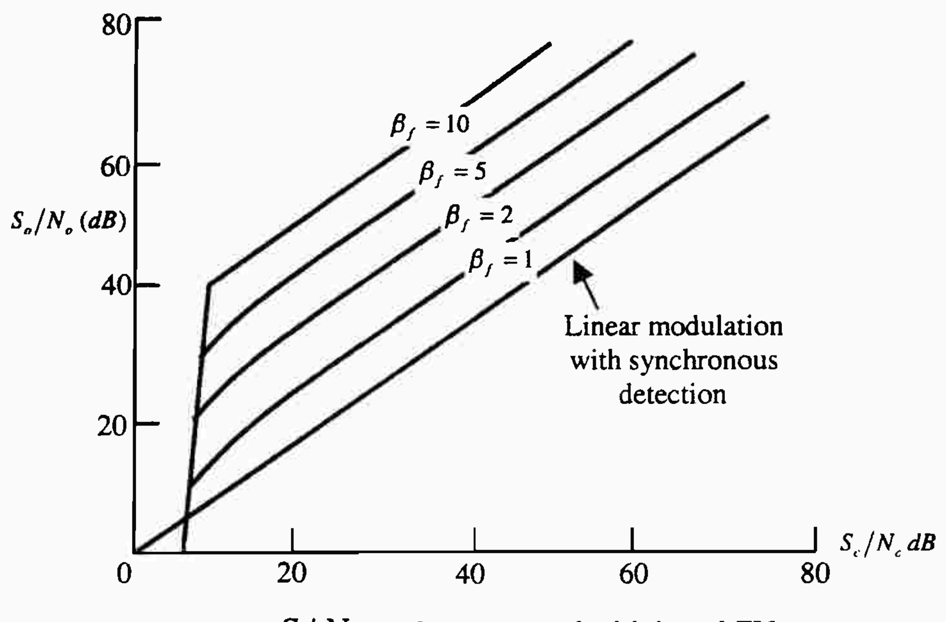

Fig. 8.16 S/N performance of wideband FM

The noise improvement factor Γ is given by

$$\Gamma = \frac{N_0}{N_0'} \tag{8-119}$$

From eqns. (8.117) and (8.118).

$$\Gamma = \frac{1}{3}\frac{(\omega_m/\omega_1)^3}{\left(\frac{\omega_m}{\omega_1}\right) - \tan^{-1}\left(\frac{\omega_m}{\omega_1}\right)} \tag{8-120}$$

The S/N improvement using de-emphasis is shown Fig. 8.18 and Fig. 8.19.

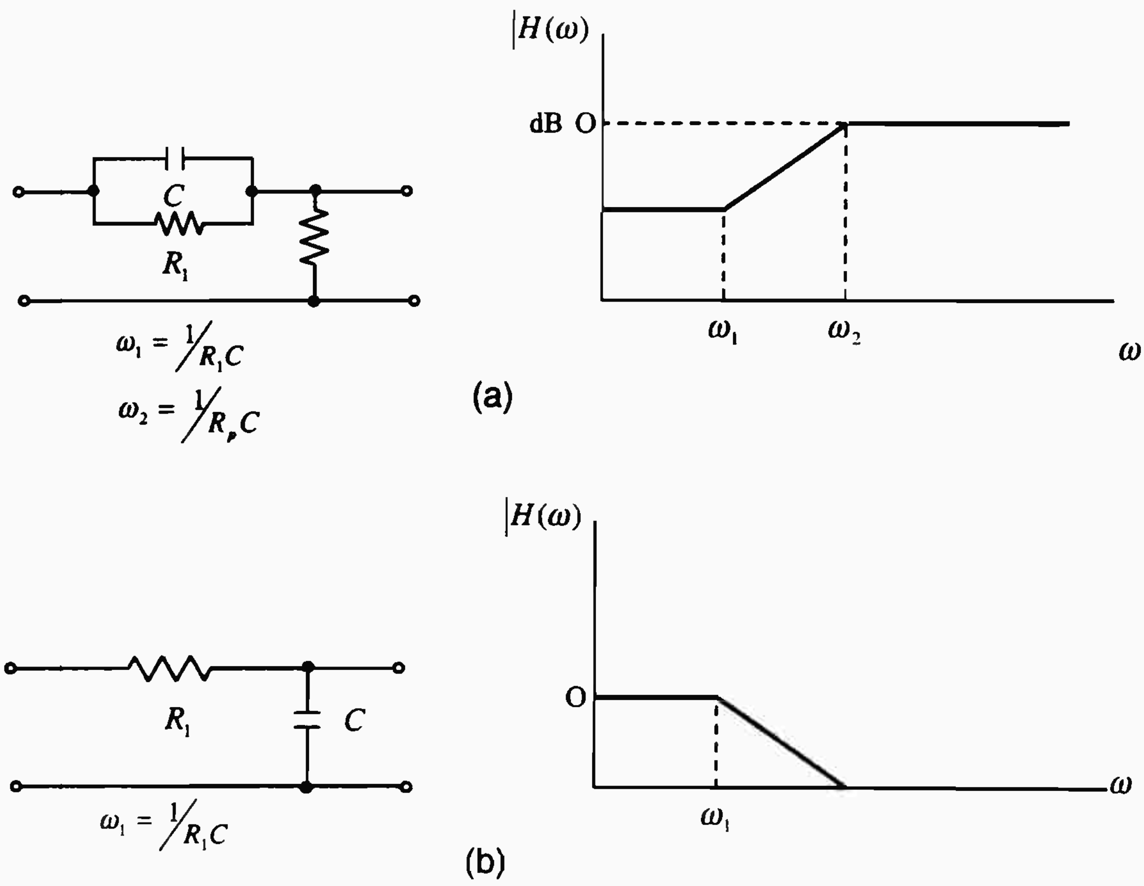

Fig. 8.17 Pre-emphasis and de-emphasis
a) pre-emphasis b) de-emphasis

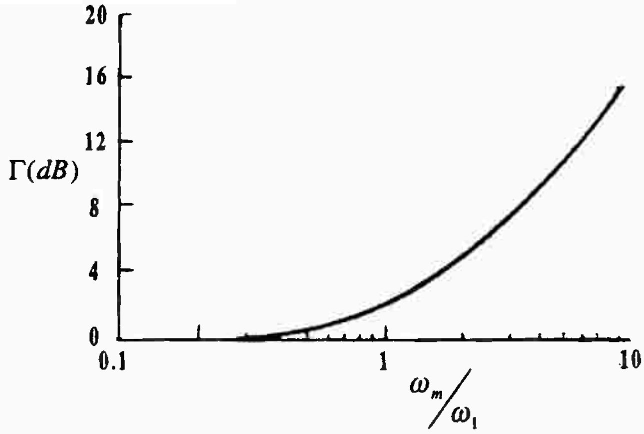

Fig. 8.18 S/N improvement using de-emphasis

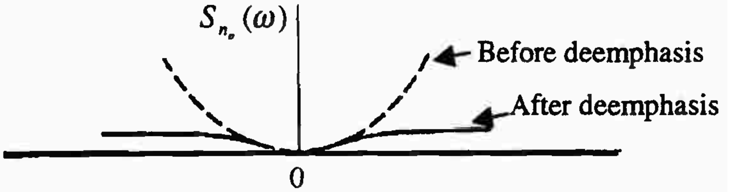

Fig. 8.19 Noise spectral density after de-emphasis

Problems:

1- Consider coherent modulation with a generated carrier whose frequency is in error $\delta\omega$ and phase in error by $\delta\theta$, find the output. What happens if the phase error approaches $\pm$ 90° ?

2- Obtain the outputs of quadrature multiplexing scheme when we have phase error $\Delta\theta$

   a. when $\Delta\theta$ is small.

   b. when $\Delta\theta$ is large.

3- Consider an FM system with two possible values of frequency. The modulating signal is a periodic wave of unit amplitude. Obtain the FM spectrum.

4- A 10 MHz carrier with $\hat{\Delta f_f} = 50$ MHz. Determine the bandwidth first by studying the line spectrum, and secondly from Carson's rule. Compare the two results. Take the modulating frequency to be.

   a) 500 kHz   b) 500 Hz   c) 50 Hz

   What do you conclude?

5- From eqn. (8-58), verify eqns. (8-63),(8-64).

6- Find the value of β_f such that the power in the carrier of an FM signal is zero and all the power lies in the sidebands.

7- For a given FM transmitter, the output in 500 ohm resistor is 100 Watt. The peak frequency deviation is gradually increased until the first sideband amplitude is zero. Determine the average power in the carrier and the first three sidebands.

8- A phase modulator uses a periodic symmetric square wave of unit amplitude. Determine $\hat{\Delta\theta}$ for which $P_o = O$.

9- A receiver has noise temperature of 210^oK. Estimate the minimum required signal strength at the test generator terminals to obtain full quieting. The input resistance is 300Ω. The signal generator is at 290^oK and matched to the receiver. The IF bandwidth is 180kHz and the threshold is 10 dB.

10-If the power density of the signal is $C/[1+(\omega/\omega_1)^2]$, determine the required change in modulation level with and without pre-emphasis for a fixed modulation power. Then, find the net S/N improvement.

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