CHAPTER 9
The Phase Detector and Voltage Controlled Oscillator

9.1 Emitter Coupled Differential Amplifier:

The circuit shown (Fig. 9.1a) is that of an emitter coupled pair. We assume that the emitter current source resistance R_{EE} is infinite, and so is the output resistance of each transistor.

We first note that:

$$V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0 \qquad (9-1)$$

Assuming $V_{be1}, V_{be2} >> V_T$, we have

$$V_{be1} = V_T \, \ell n \frac{I_{c1}}{I_{s1}} \qquad (9-2)$$

$$V_{be2} = V_T \, \ell n \frac{I_{c2}}{I_{s2}} \qquad (9-3)$$

Assuming $I_{s_1} = I_{s_2}$, we combine eqns. (9-1), (9-2) and (9-3). We find

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{i_1} - V_{i_2}}{V_T}\right) = \exp\frac{V_{id}}{V_T}, \qquad (9-4)$$

where V_{id} is the difference $V_{i_1} - V_{i_2}$.

At the emitters of the transistors, $\left(I_{e_1} + I_{e_2}\right) = I_{EE} \cong I_{c_1} + I_{c_2}$ (9 - 5)

From eqn. (9 - 5), we get:

$$I_{c_1} = \frac{I_{EE}}{1 + \exp\left(-Vid/V_T\right)} \qquad (9-6)$$

$$I_{c_2} = \frac{I_{EE}}{1 + \exp\left(Vid\,/\,V_T\right)} \qquad (9-7)$$

The two currents are shown as functions of V_{id} (Fig. 9.1b). Note that for input voltage differences in excess of several hundred millivolts, the collector current becomes independent of V_{id}, i.e., current flows in one transistor at a time.

For difference voltages < 50 mV, the circuit behaves approximately in a linear relation.

Now, we calculate the output voltages:

$$V_{01} = V_{CC} - I_{c_1} R_c \qquad (9-8)$$

$$V_{02} = V_{CC} - I_{c_2} R_c \qquad (9-9)$$

The output signal is often $V_{od} = V_{01} - V_{02}$, and is given by the difference output current

$$V_{od} = V_{01} - V_{02} = I_{EE}\, R_c \tanh\left(-Vid / 2V_T\right) \qquad (9-10)$$

ΔI_c is given by:

$$\Delta I_c = I_{EE} R_c \tanh\left(-Vid / 2V_T\right) \qquad (9-11)$$

This differential output voltage is shown (Fig. 9.1c). It is clear that when V_{id} is zero, V_{od} is zero

From eqn. (9-11),

$$\tanh\left(\frac{V_{id}}{2V_T}\right) \approx \frac{V_{id}}{2V_T}, \qquad \text{for } \frac{V_{id}}{2V_T} << 1 \qquad (9-12)$$

Hence,

$$\Delta I_c = I_{EE} \frac{V_{id}}{2V_T} \qquad (9-13)$$

9.2 Analog Multiplier:

We can develop the emitter circuit as an analog multiplier by considering the following circuit (Fig. 9.2a)

From the current mirror circuit (Fig. 9.2b), we sum the currents at the collector of Q_1, assuming identical transistors, and noting that $I_{b1} = I_{b2}$

$$I_{ref} - I_{c1} = 2I_b = 2I_c / \beta \qquad (9-14)$$

$$I_{c1} = \frac{I_{ref}}{1 + 2/\beta} = I_{c2} \qquad (9-15)$$

If $\beta >> 1$,

$$I_{c1} = I_{ref} = I_{c2} = \frac{V_{cc} - V_{BE_{(ON)}}}{R} \qquad (9-16)$$

Hence, in the circuit of (Fig. 9.2a),

$$I_{EE} = \frac{V_{i2} - V_{BE_{(ON)}}}{R} \qquad (9-17)$$

$$= K_o\left(V_{i2} - V_{BE_{(ON)}}\right) \qquad (9-19)$$

Hence, eqn. (9-13) becomes

$$\Delta I_c = K_o V_{id} \left[\frac{V_{i2} - V_{BE_{(ON)}}}{2V_T}\right] \qquad (9-19)$$

Thus, we have a circuit that works as a multiplier for $V_{id} << V_T$ and $V_{i2} > V_{BE_{(ON)}}$. This means that the multiplier functions in two quadrants of $\frac{V_{id}}{V_{i2}}$ plane. We wish to remove this restriction to have a four quadrant multiplier.

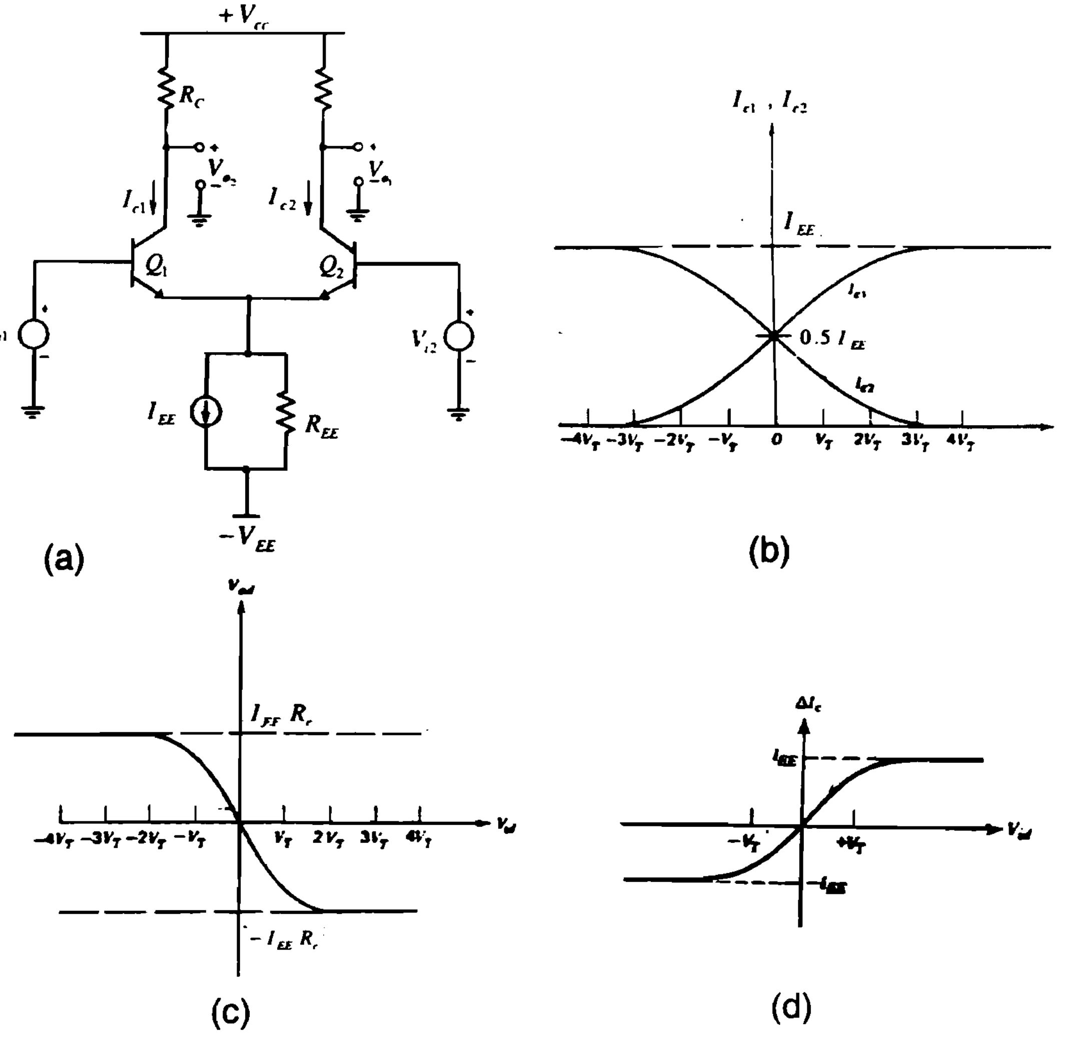

Fig. 9.1 Emitter coupled pair

a) circuit b) collector currents

c) differential output d) differential output current

The circuit shown is called the Gilbert multiplier cell. We assume that all transistors are identical, the output resistance of the current source and of all transistors are infinite, and the base currents are neglected for transistors Q_3 and Q_4. Using eqns. (9-6) and (9-7),

$$I_{c3} = \frac{I_{c1}}{1+\exp\left(\frac{-V_1}{V_T}\right)} \tag{9-20}$$

$$I_{c4} = \frac{I_{c1}}{1+\exp\left(\frac{V_1}{V_T}\right)} \tag{9-21}$$

$$I_{c5} = \frac{I_{c2}}{1+\exp\left(\frac{V_1}{V_T}\right)} \tag{9-22}$$

$$I_{c6} = \frac{I_{c2}}{1+\exp\left(\frac{-V_1}{V_T}\right)} \tag{9-23}$$

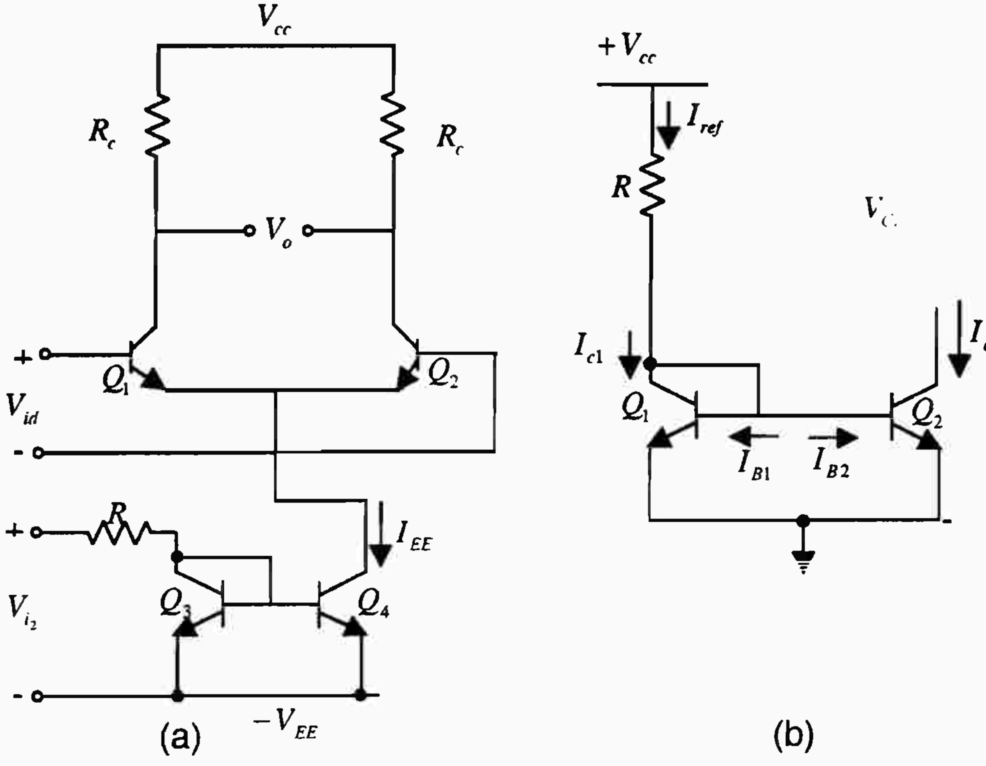

Fig. 9.2 Two quadrant analog multiplier
a) circuit b) current mirror

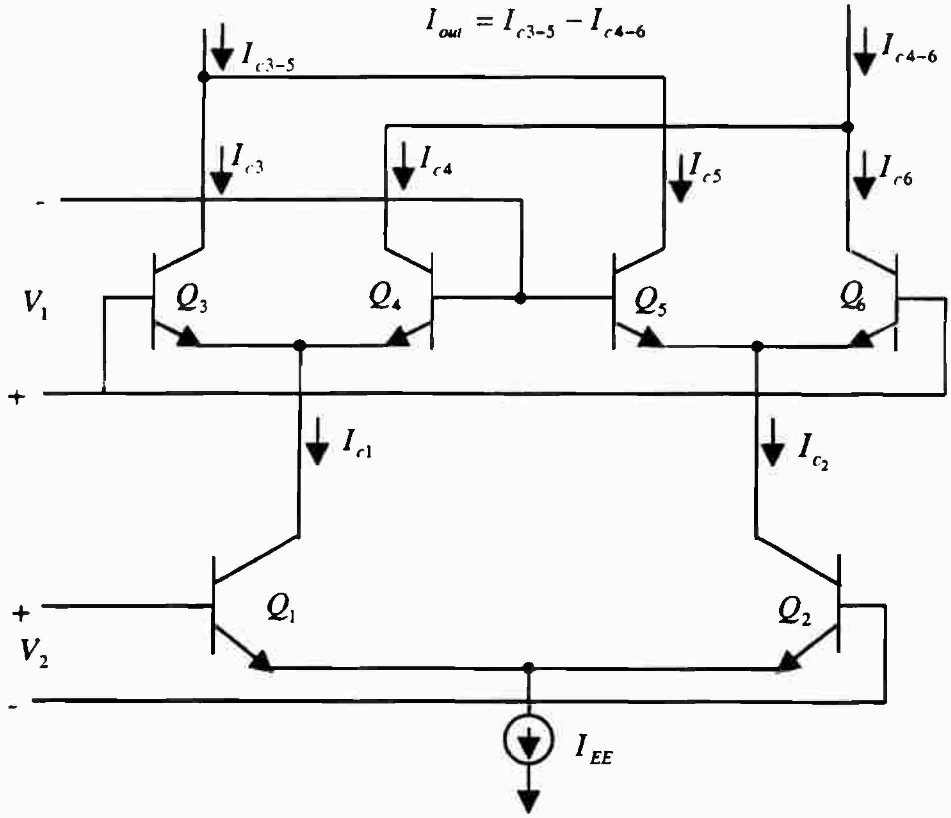

Fig. 9.3 The Gilbert multiplier cell

The two currents I_{c1} and I_{c2} are related to V_2,

$$I_{c1} = \frac{I_{EE}}{1+\exp\left(\frac{-V_2}{V_T}\right)} \qquad (9-24)$$

$$I_{c2} = \frac{I_{EE}}{1+\exp\left(\frac{V_2}{V_T}\right)} \qquad (9-25)$$

Thus,

$$I_{c3} = \frac{I_{EE}}{\left[1+\exp\left(\frac{-V_1}{V_T}\right)\right]\left[1+\exp\left(\frac{-V_2}{V_T}\right)\right]} \qquad (9-26)$$

$$I_{c4} = \frac{I_{EE}}{\left[1+\exp\left(\frac{-V_2}{V_T}\right)\right]\left[1+\exp\left(\frac{V_1}{V_T}\right)\right]} \qquad (9-27)$$

$$I_{c5} = \frac{I_{EE}}{\left[1+\exp\left(\frac{V_1}{V_T}\right)\right]\left[1+\exp\left(\frac{V_2}{V_T}\right)\right]} \qquad (9-28)$$

$$I_{c6} = \frac{I_{EE}}{\left[1+\exp\left(\frac{V_2}{V_T}\right)\right]\left[1+\exp\left(\frac{-V_1}{V_T}\right)\right]} \qquad (9-29)$$

Then

$$\begin{aligned}\Delta I &= I_{c3-5} - I_{c4-6} \\ &= I_{c3} + I_{c5} - (I_{c4} + I_{c6}) \\ &= (I_{c3} - I_{c6}) - (I_{c4} - I_{c5}) \qquad (9-30)\end{aligned}$$

Hence, we find that:

$$\Delta I = I_{EE}\left[\tanh\left(\frac{V_1}{2V_T}\right)\right]\left[\tanh\left(\frac{V_2}{2V_T}\right)\right] \qquad (9-31)$$

For small values of V_1 and $V_2 << V_T$,

$$\Delta I = I_{EE}\left(\frac{V_1}{2V_T}\right)\left(\frac{V_2}{2V_T}\right) \qquad V_1, V_2 < V_T \qquad (9-32)$$

Thus, for small amplitude signals, the circuit performs as an analog multiplier.

9.3 The Gilbert Cell Modulator:

In this application, the Gilbert cell multiplies a continuously varying function with a square wave, instead of two continuously varying functions. This can be done by applying a sufficiently large signal >> $2V_T$ to the cross coupled pair, so that two of the four transistors alternately turn off, and the other two conduct all the current. A sinusoid is applied to the small signal input and a square wave is applied to the large signal input. For the large signal input, tanh $(V_1/2V_T)$ amounts to +1 or -1. Thus, effectively, the output is the small signal input V_2 multiplied by +1 or -1, and is actually independent of the magnitude of the square wave V_1.

We note that the Gilbert cell here performs modulation coming from the switching action (Fig. 9.4). The Gilbert cell in this case is called a balanced modulator.

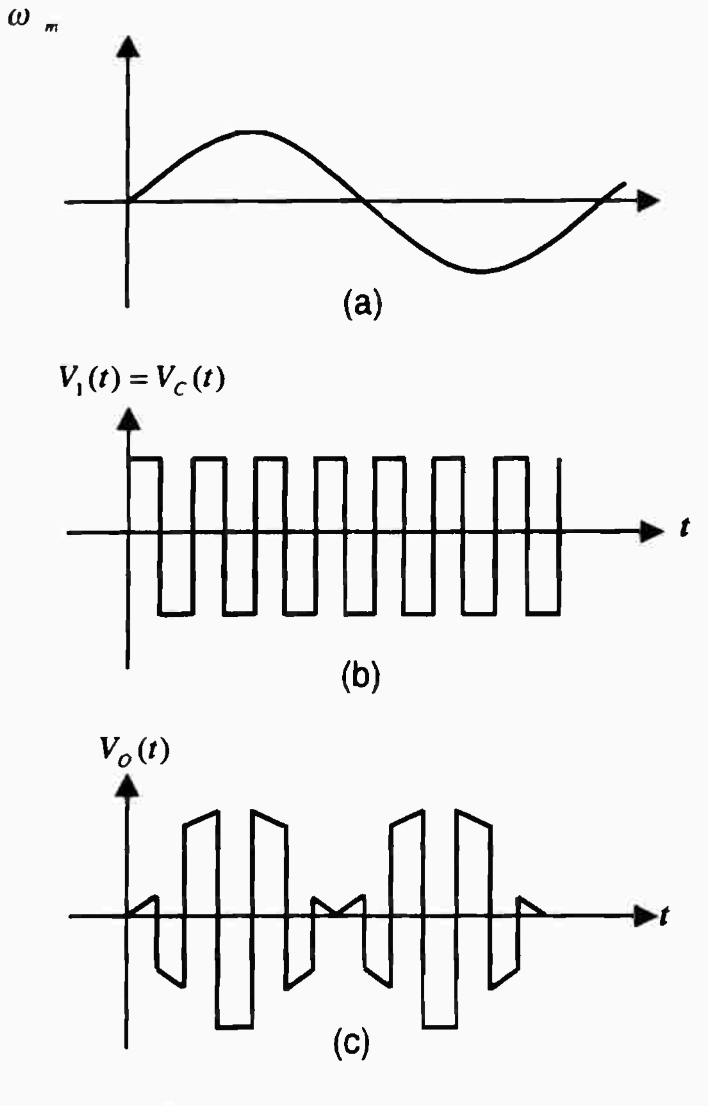

Fig. 9.4 Gilbert cell as a balanced modulator
a) small signal input b) large signal modulating input c) output

Ex. 9.1:

Find the Fourier spectrum of the output of Gilbert cell balanced modulator for a low frequency modulating signal $V_m(t) = V_m \cos\omega_m t$, and for high frequency square wave whose amplitude is ± 1 and whose frequency is ω_c. Show how a balanced modulator can also be used for the demodulation of such a waveform.

Solution:
We have,

$$v_c(t) = \sum_{n=1}^{\infty} A_n \cos n\omega_c t \qquad (9-33)$$

$$A_n = \frac{\sin n\frac{\pi}{2}}{n\frac{\pi}{4}} \qquad (9-34)$$

The output signal is:

$$v_o(t) = k[v_c(t)v_m(t)]$$

$$= k\sum_{n=1}^{\infty} A_n V_m \cos\omega_m t \cos n\omega_c t \qquad (9-35)$$

$$= k\sum_{n=1}^{\infty} \frac{A_n V_m}{2}[\cos(n\omega_c + \omega_m)t + \cos(n\omega_c - \omega_m)t] \qquad (9-36)$$

The spectrum has components located at frequencies ω_m above and below each of the harmonics of ω_c, but no component at ω_c or its harmonics, which is a property of balanced modulators (Fig. 9 - 5)

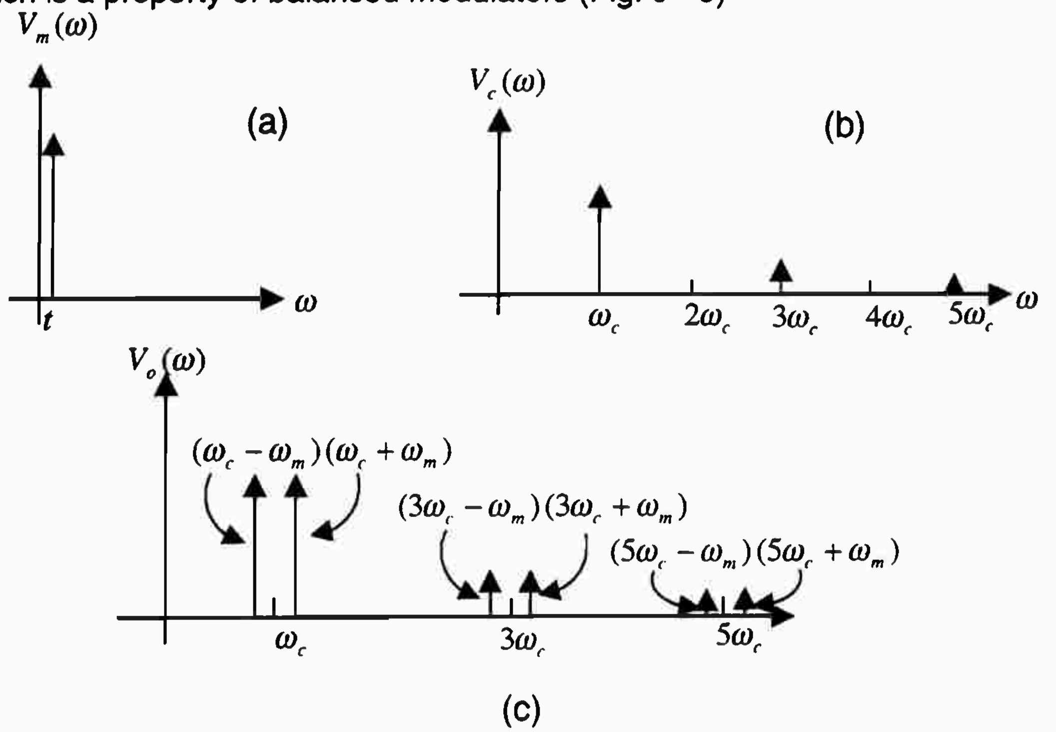

Fig. 9.5 Spectrum for a balanced modulator
a) modulating signal b) square wave carrier c) modulated carrier

We notice that the balanced modulator has translated the information from ω_m to the spectral components near ω_c and its harmonics. Thus, in frequency translation, the sum and difference components are obtained at the output. The balanced modulator may also be used for demodulation. Multiplying the output [eqn. (9-35)] by a square wave [eqn. (9-33)], the result is $\upsilon_m(t)$, since squaring the square wave whose amplitude is ± 1 is always +1.

9.4 The Gilbert Cell Phase Detector:

If unmodulated signals of identical frequency ω_o are applied to the two inputs of the Gilbert cell, it behaves as a phase detector whose output has a dc component proportional to the phase difference between the two inputs (Fig. 9.6). We assume - for simplicity - that both inputs are square waves of large amplitude so that the transistors behave as switches.

The output waveform (Fig. 9.6a) consists of a dc component and a component at twice the incoming frequency. The dc component is given

$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} \upsilon_o(t)d\theta \qquad (9-37)$$

$$= -\frac{1}{\pi}(A_1 - A_2) \qquad (9-38)$$

Note that the minus sign relates to the polarity assigned for the output (Fig. 9.6a) and A_1 and A_2 are magnitude areas. Thus,

$$V_{av} = -\left[I_{EE}R_c\frac{\pi-\varphi}{\pi} - \frac{I_{EE}R_c\varphi}{\pi}\right] = I_{EE}R_c\left(\frac{2\varphi}{\pi} - 1\right) \qquad (9-39)$$

The phase relation of eqn. (9.39) is plotted in Fig (9.7)

We should note that if the input signal amplitude is large the actual waveform shape is unimportant, since the transistor behaves simply as a switch. If the amplitude of the applied signal at V_{in2} is small compared to V_T, the circuit behaves as a balanced modulator. The output waveform is then a sinusoid multiplied by a synchronous square wave (Fig. 9.8). The dc component in the output becomes:

$$V_{av} = \frac{1}{\pi}g_m R_c V_i\left[\int_o^{\phi}\sin\omega t\, d(\omega t) - \int_{\varphi}^{\pi}\sin\omega t\, d(\omega t)\right] \qquad (9-40)$$

$$= \frac{-2g_m R_c V_i\cos\varphi}{\pi} \qquad (9-41)$$

We note from eqn. (9-41) that the dc output is proportional to $\cos\varphi$. It also depends on the magnitude V_i of the input of the small signal $V_{in2}(t)$ as long as it is small compared to V_T.

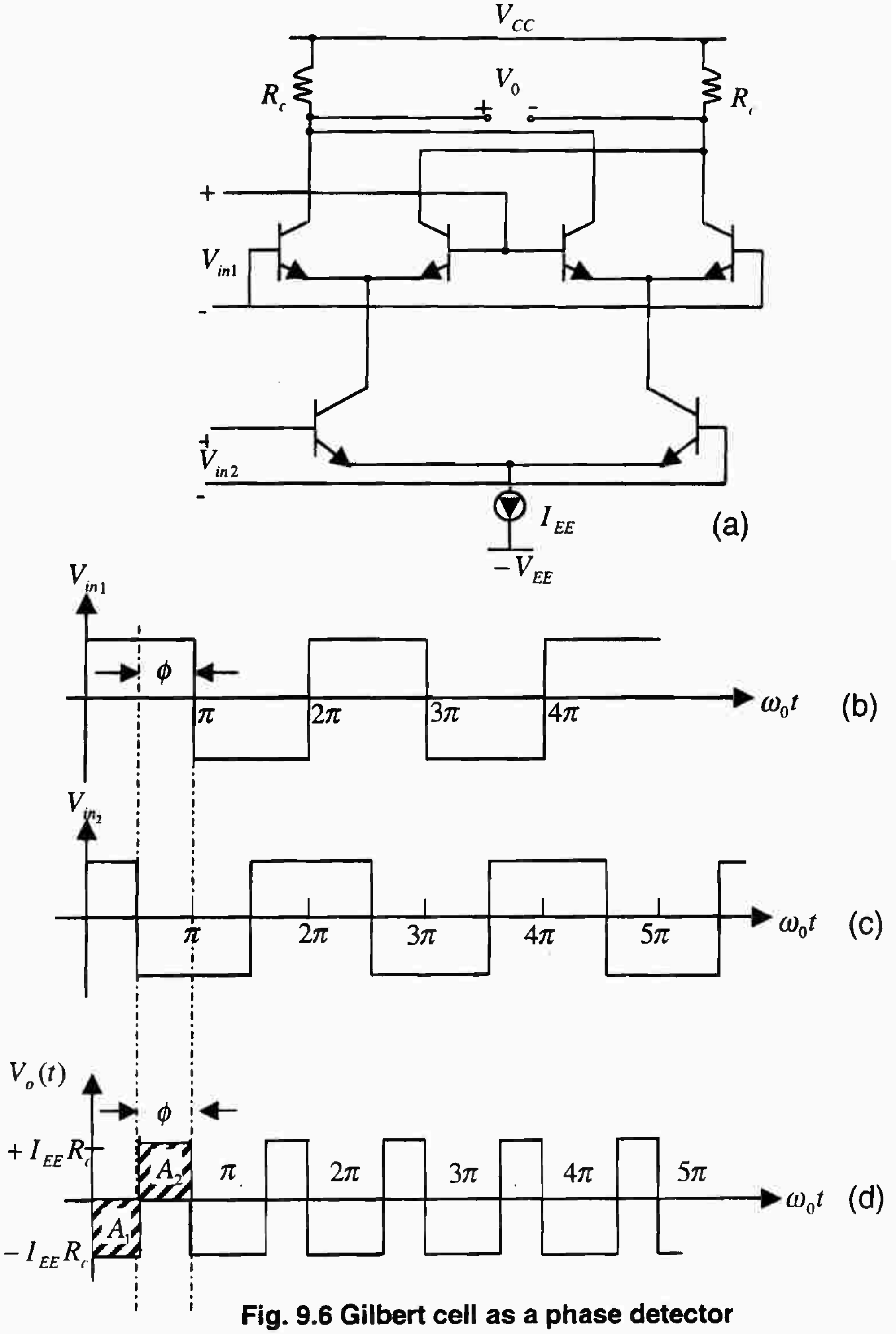

Fig. 9.6 Gilbert cell as a phase detector

a) circuit b) $V_{in1}(t)$ c) $V_{in2}(t)$ d) $V_o(t)$

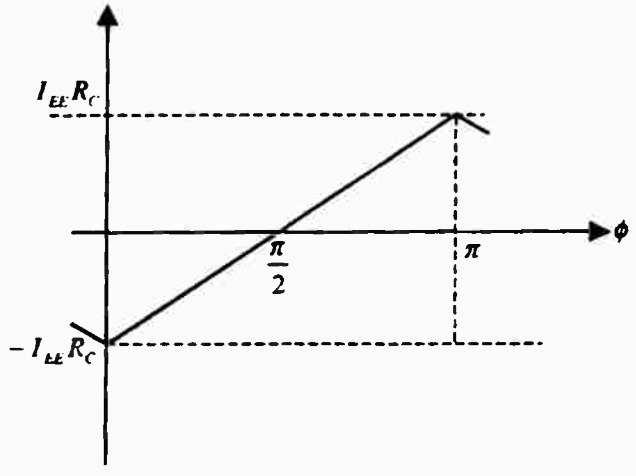

Fig 9.7 Phase detector output versus phase difference

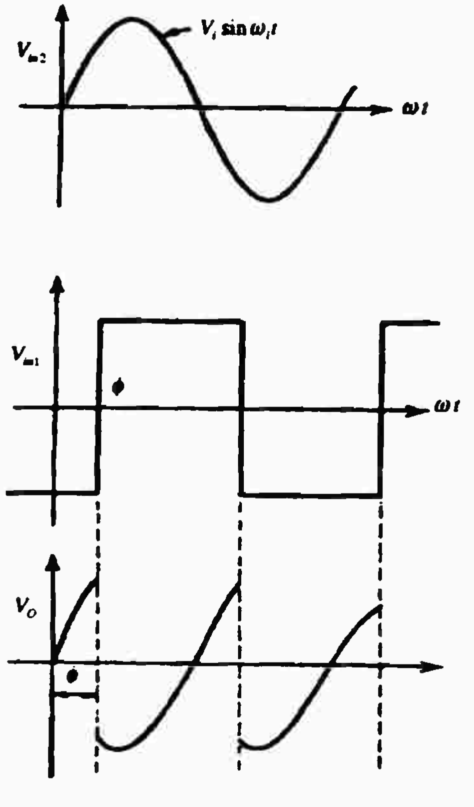

Fig. 9.8 Phase detector output when a sinusoid is multiplied by a synchronous square wave

a) $V_{in2}(t)$ b) $V_{in1}(t)$ c) $V_O(t)$

9.5 Phase Detector Characteristics:

We thus have to distinguish three cases in phase detector characteristics.

1) Case 1: when both inputs are large signal square waves. In this case, the phase detector output is a linear function of φ.
2) Case 2: when one signal is a small signal sinusoid and the other is a square wave. In this case, the dc output is proportional to $\cos\varphi$. We can also think of the square wave as a Fourier series. We can then show the output of the phase detector to be proportional to $\cos\varphi$ (Prob. 9.2)
3) Case 3: when both inputs are sinusoidal. In this case, the dc output is also proportional to $\cos\varphi$ (Prob. 9.3). It is often required to place a limiter before the phase detector to force the linear operation, and to make the dc output independent of the amplitude of the incoming signals.

9.6 The Voltage Controlled Oscillator:

Usually the design of oscillators seeks high stability, which means that the oscillator frequency be constant. In many communication problems, it is required to design an oscillator whose frequency can be varied by changing a dc voltage applied to it. Such an oscillator is called voltage controlled oscillator (VCO). Fig.9.9 shows an emitter coupled RC multivibrator VCO. The key is to show that the charging current in the capacitor is proportional to the control input voltage V_{in}. Assume that Q_1 is turned off and Q_2 is on (Fig. 9.10). The voltage drop IR is assumed to be large enough to turn on diode Q_6.

Thus, the voltage on Q_6 is clamped to nearly the forward cut-in voltage of a diode (V_γ). Thus, the base of Q_4 is $V_{CC}-V_\gamma$. The emitter of Q_4 is at $V_{CC}-2V_\gamma$ which is above the voltage at the base of Q_1. Because Q_1 is off, the base of Q_3 is at V_{CC} and its emitter is at $V_{CC}-V_\gamma$. Thus, the emitter of Q_2 is at $V_{CC}-2V_\gamma$. Because Q_1 is off, the current I_1 is charging the capacitor, so that the emitter of Q_1 is becoming more negative.

Q_1 will turn on when the voltage at its emitter becomes equal to $V_{CC}-3V_\gamma$. The resulting collector current in Q_1 will turn on Q_5. The base of Q_3 then drops by V_γ, causing the base of Q_2 to rise by V_γ because Q_6 is off and the emitter base junction of Q_2 is reverse braised by V_γ.

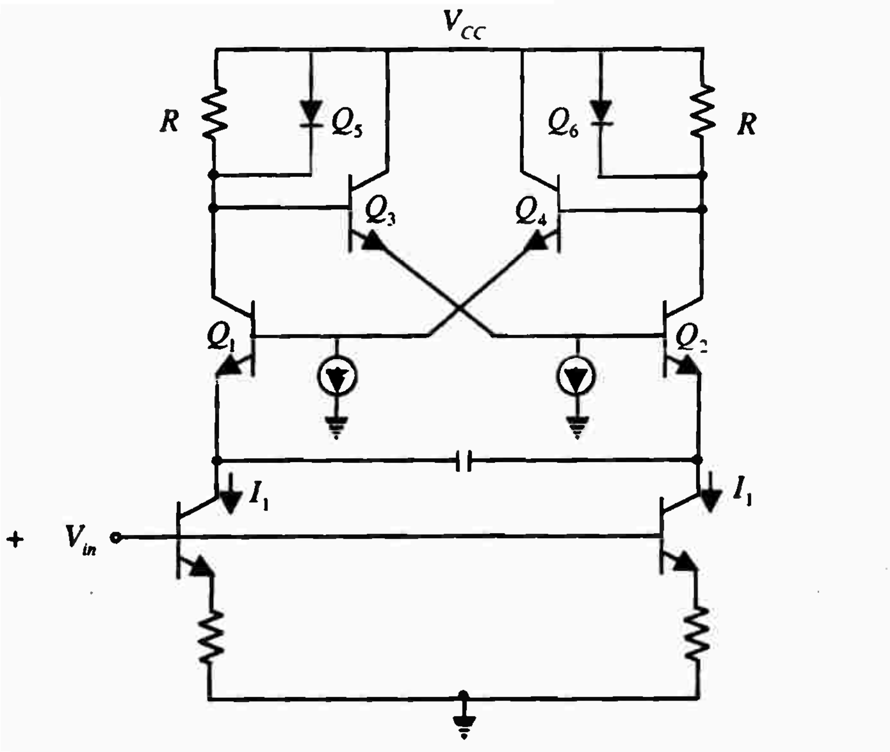

Fig. 9.9 Emitter coupled multivibrator VCO

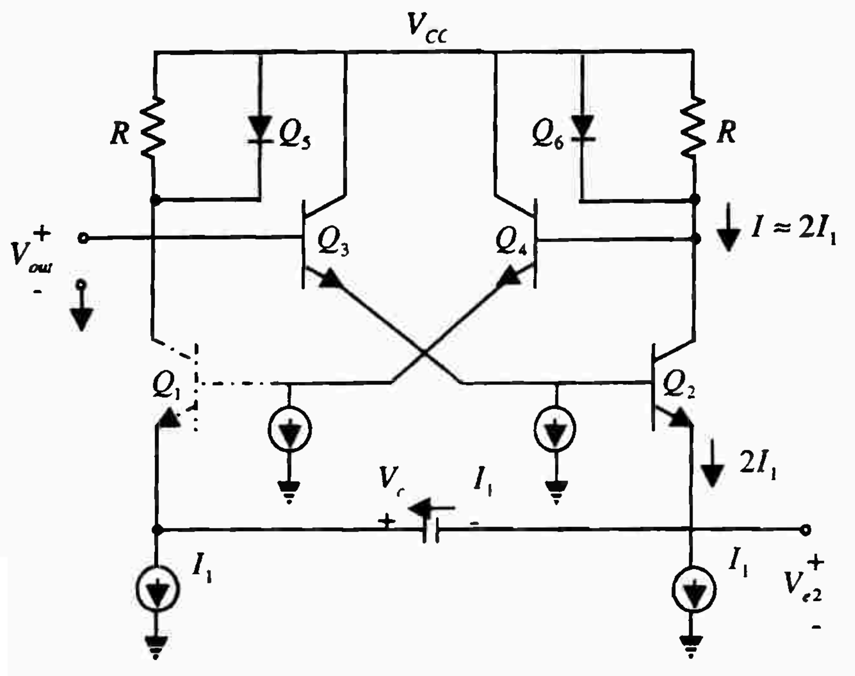

Fig. 9.10 Circuit operation during half cycle

But the voltage on C cannot change instantly. Current I_1 must now charge the capacitor in the negative direction by an amount = $2V_\gamma$ before the circuit switches back again. Because of the symmetry, the half period is given by the time required to charge the capacitor

$$\frac{T}{2} = \frac{Q}{I_1}, \qquad (9-42)$$

where $Q = C\Delta V = 2CV_\gamma$

The frequency of the oscillator becomes

$$f = \frac{I}{T} = \frac{I_1}{4CV_\gamma} \qquad (9-43)$$

The waveforms are shown in Fig. 9-11

We assume that the transistors used are not driven into saturation, because the voltage swings are small and that $V_{in} > V_\gamma$, so that $I_1 = \frac{V_{in}}{R_E}$. By controlling V_{in}, we can vary I_1 so that eqn. (9-43) becomes

$$f = \frac{V_{in}}{4CR_E V_\gamma} \qquad (9-44)$$

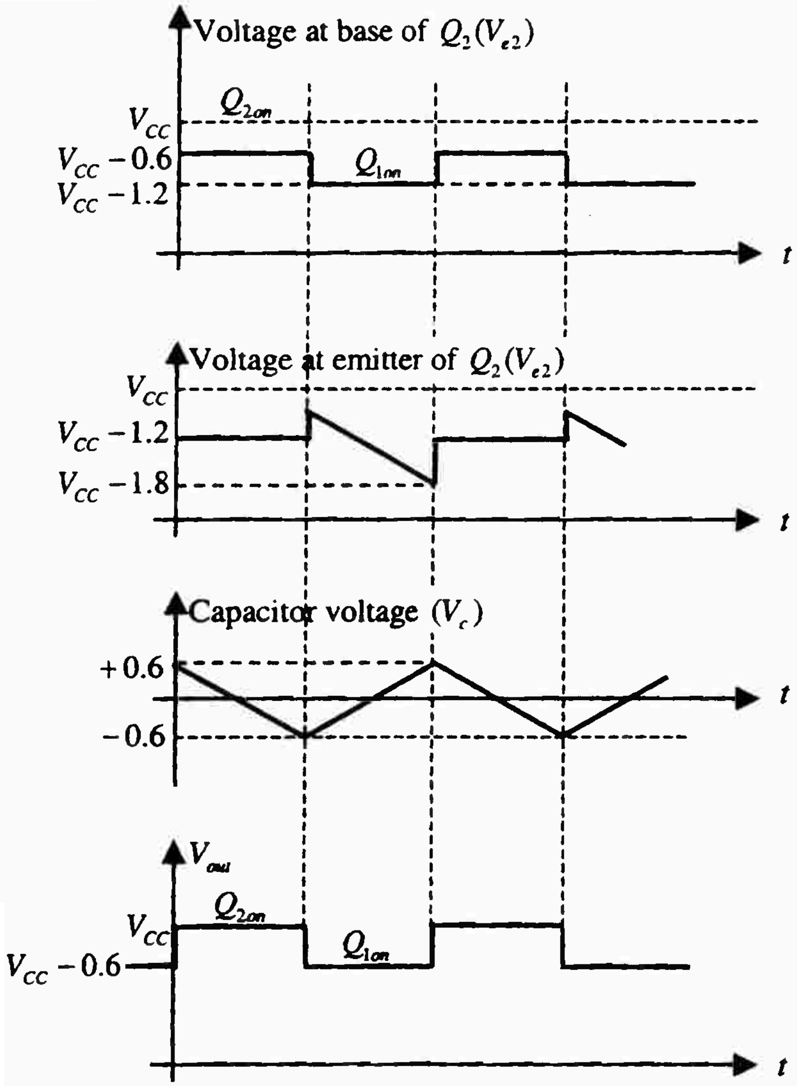

Fig. 9.11 waveforms of emitter coupled multivibrator VCO
a) $V_{b2}(t)$ b) $V_{e2}(t)$ c) $V_c(t)$ d) $V_O(t)$

Problems:

1- If a dc component is added to the modulating input in Ex 9.1, find the spectrum? What do you conclude?

2- If one input of a phase detector is a sinusoid and the other is a square wave, show that the dc output of the phase detector is proportional to $\cos\varphi$.

3- Repeat the above problem if both inputs are sinusoidal. Show that the dc output is also proportional to $\cos\varphi$.

4- Obtain the dc transfer curve I_{out} versus V_2 for the Gilbert multiplier for $V_1 = 0.1\,V_T$, $0.5V_T$, V_T and $10\,V_T$.

5- Show that an XOR circuit can be used as a phase detector, and obtain its characteristic.

6- Discuss the temperature sensitivity of the VCO frequency in eqn. (9-43).

7- Analyze the phase detector shown in Fig. (9-12).

8- Analyze the VCO shown in Fig. (9-13).

9- A reverse biased semiconductor diode has a characteristic given by: $C = C_o / \sqrt{1+2|V_r|}$, where $|V_r|$ is the reverse voltage. This diode is used in a parallel resonant LC circuit with a center frequency of 10 MHz at $|V_r| = 4\,V$. Show how this circuit may be used as a VCO.

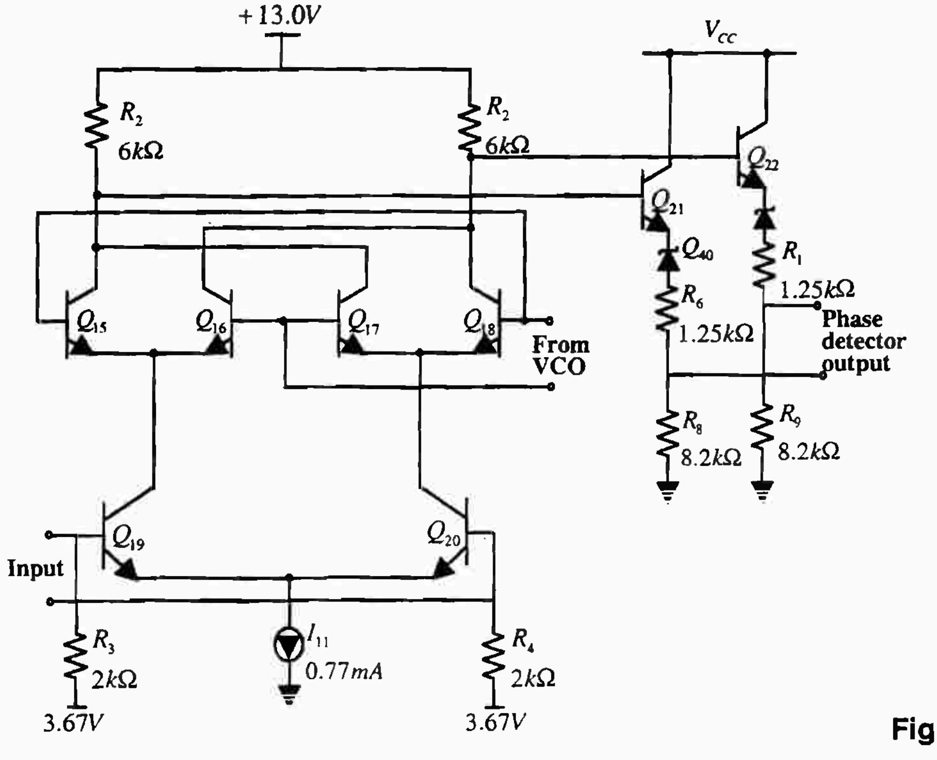

Fig 9.12

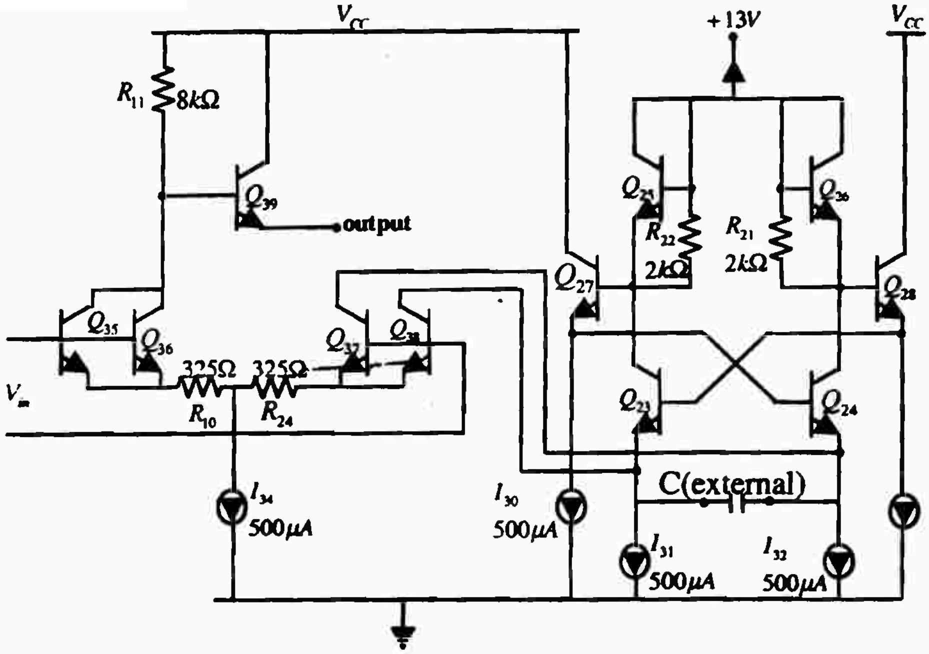

Fig. 9.13

10-Analyze the circuit shown in Fig. 9.14, and obtain an expression for the VCO output frequency.

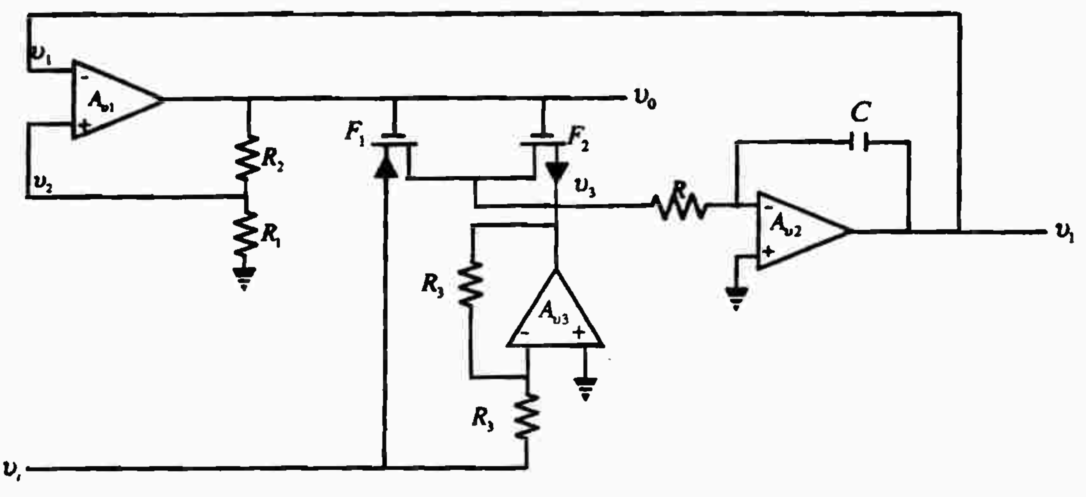

Fig. 9.14

Hint: F_1 and F_2 are n channel and p channel FETs acting as an electronic switch. When v_o is negative, F_1 conducts and F_2 is cut off and $v_3 = v_i$. When V_o is positive, F_1 is cut off and F_2 conducts, thus $v_3 = -v_i$. A_{v3} circuit multiples v_i by -1.

References:

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