CHAPTER11

EXPANSION PLANNING ALGORITHM FOR INTERCONNECTED INFORMATION SYSTEMS

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ABSTRACT:

The tools and techniques for gathering and using information are at the core of civilization. The information technologies of today and tomorrow help us see and visualize the world around us and communicate that information to a wide variety of computing devices that then helps us analyze and understand information. With this information and understanding, nations can begin to create solutions to their problems and control their lives, environment, jobs and even entire societies.

This paper presents a new efficient algorithm for the expansion planning of interconnected information system. The algorithm includes two phases; namely: the short term planning phase and the long term planning phase. In the first phase the generalized optimization network technique can be used to solve the linear topological information network. The decision tree technique can also be used for long term planning of the interconnected information system. The overall algorithm satisfies the system objectives and most of both short term and long term planning constraints.

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3.2. Space-Division Switching Technique:-

Refer to Fig. (2) In this case, the available number of paths between the input and output switches equals to (15), and the (PR) can be calculated as before.

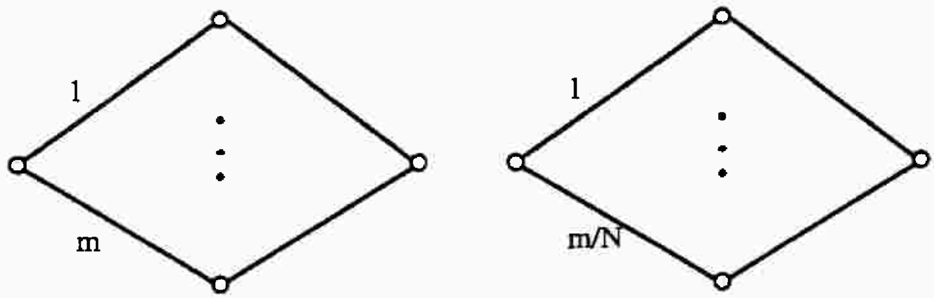
4. RESULTS & CONCLUSIONS:-

From Fig. (3), it is clear that:-

* In the time interval :$0 \leq t \leq 0.4$, the reward (PR) for the SD-channel is superior to the high-speed TD-channel, i.e, the traffic efficiency (1-PR), for the SD-channel is lower than the high-speed TD-channel.

* In the time interval: $t \geq 0.4$, it is clear that the traffic efficiency for the SD-channel is superior to the high-speed TD-channel.

-Also, the reward for the low-speed TD-channel is lower than that of the previous two channels, (and it is not drawn in the figure since the values of (PR) are very small).



a- Low-speed channel graph. **b- Hi gh-speed channel graph.**

Fig. (1). A time division communication channel.

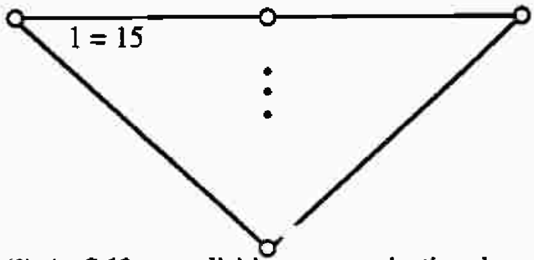


Fig. (2). An S-12 space division communication channel.

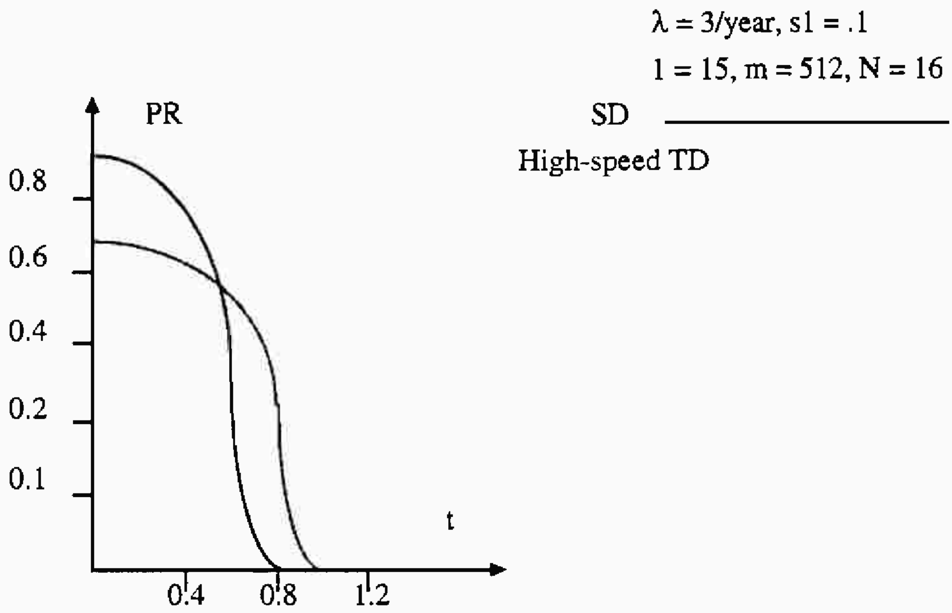


Fig. (3). The Joint Performance Measure (PR) for the SD & TD channel graphs ver. the time, t.

t [year]	PR	t [year]	PR
.2	5.8E-3	1.4	6.3 E-4
.4	5.8E-3	1.6	2. E-4
.6	5.8E-3	1.8	6. E-5
.8	5.7E-3	2.0	1.8 E-5
1.0	4.2E-3	2.2	5.7 E-6
1.2	1.85E-3	2.8	0.0

Table (2). The (PR) ver. the time, t, for the TD channel graph, for the low-speed traffic, where $\lambda = 3$ / year, m = 512, and s1 = 0.1.

t [year]	PR	t [year]	PR
0.2	0.72	1.6	1.6 E-3
0.4	0.69	1.8	4.7 E-4
0.6	0.43	2.0	1.4 E-4
0.8	0.17	2.2	4.4 E-5
1.0	15.5 E-2	2.4	1.4 E-5
1.2	1.7 E-2	2.6	4.1 E-6
1.4	5.2 E-3	2.8	0.0

Table (3). The (PR) ver. the time, t, for the TD high-speed channel graph, where $\lambda = 3$ / year, m = 512, N = 16, and s1 = 0.1.

blocking probability for the given channel graph. Equation (1) can be rewritten as follows:-

$$PR = B.\ (1 - L.\ B), \quad \text{.......... (2)}$$

using Theorem (2), (6). For low-speed calls, we have that:-

$$R = 1 - [1 - r^2]^m, \quad \text{.......... (3)}$$

$$\text{and } B = [1 - s^2]^m, \quad \text{.......... (4)}$$

Multiplying both the two equations (3) & (4), one can evaluate the (PR).

3.1.2. High-Speed Traffic:- Similarly, one can evaluate the Joint Performance Measure (PR) in this case, where the available number of paths equals to (m/N).

t [year]	PR	t [year]	PR
0	.86	.70	.17
.1	.86	.80	.10
.2	.856	.90	5.6E-2
.3	.8	1.0	3.1E-2
.4	.65	2.0	8.0E-5
.5	.46	3.0	0.0
.6	.29		

Table (1). The (PR) ver. the time, t, for the SD channel graph, for λ = 3/year, sl = .1, and l = 15.

average reward.[4] In,[5] the relability analysis for S-12 BISDN was studied using the new relation between the blocking probability and the terminal-pair connectivity.[6] In this paper, a qualitative comparison will be presented between the high-speed TD and the SD switching techniques, using this new relation.

2- MATHEMATICAL MODEL:

Refer to Fig. (1) & Fig. (2), and assume that:

* m = the available number of time slots per frame.
* N = the speed ratio between the high-speed and the low-speed traffics.
* A2 (A1) = the high-(low-) speed offered traffic.
* b = the time period between any two successive gate openings.
* α (β) = the voice (data) traffic rate.
* λ = the link's failure rate.
* : = exp ($-\lambda t$) = the element's reliability.
* l = the number of available paths between the input and the output switches for the SDM channel graph.

3- RELIABILITY ANALYSIS:

3.1. Time Division Switching Technique:-

3.1.1. Low-Speed Traffic:- Refer to Fig. (1). The available number of paths between the input and output switches equals to (m), in this case. Using Theorem (2), (6), we can evaluate the Joint Performance Measure (PR) as follows:-

PR = R. B, ... (1)

where R & B are the terminal-pair connectivity and the end-to end

RELIABILITY ANALYSIS FOR HIGH-SPEED TDS NETWORKS FOR BISDN USING THE NEW RELATION BETWEEN THE END-TO-END BLOCKING PROBABILITY AND THE TERMINAL-PAIR CONNECTIVITY

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KEY WORDS: TDS, SDM, End-to-End blocking, Terminal-pair connectivity.

ABSTRACT: The objective of this paper is to study the reliability of a large-scale high-speed time division switching network for BISDN. The method of attack is to use a new linear relation that was used in the literature giving the terminal-pair connectivity as a function of the probability of internal blocking of the switching network and its architectural parameters. A qualitative comparison - between the SDM and the TDM - will be presented using this new relation. Useful results are obtained.

1- INTRODUCTION:

The performance evaluation of a high-speed time divsion switching network for BISDN was carried out by Koso M. et al, 1987,[1] using the blocking probability as a performance measure.

Two switching techniques were studied, namely, 1) the single-slot, and 2) the multi-slot. It was concluded that the former switching technique is superior to the latter one. In,[2] a new efficient method for the internal blocking evaluation was presented and the same conclusion was obtained. The space-division switching networks can also be used in BISDN. In,[3] the System 12 BISDN with SDM was studied and its performance was evaluated using the

APPENDIX

```
5   REM FAR.
10 INPUT N : L = N - 1
15 DIM F (N, L)
20 P = 0.8
15 GOSUB 100
27 R = 1
30 For I = 2 TO L
35 R (I) = P I
40 r = r * (1 - r (I)) F (N, I)
45 NEXT I
50 PST = 1 - r
55 PRINT PST
60 END
100 Rem GOSUB Routine # 1
105 For I = 4 to n
110 F (I, I - 1) = 2 : Next I
115 For II = 5 to N
120 F (II, 2) = F (II - 1.2) + 1 : NEXT II
125 For J = 5 To N
127 For I = 3 To L
130 F (J, I) = F (J - 1, I) + 2
135 Next I, J
140 Return.
```

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5- F. F. Farahat and Abd El-Samei M. H., "A New Relation Between End-to - End Blocking Probability and the t-p Connectivity for CCN", a presented paper to the second IASTED Int'l conference, Alex., May, 1992.

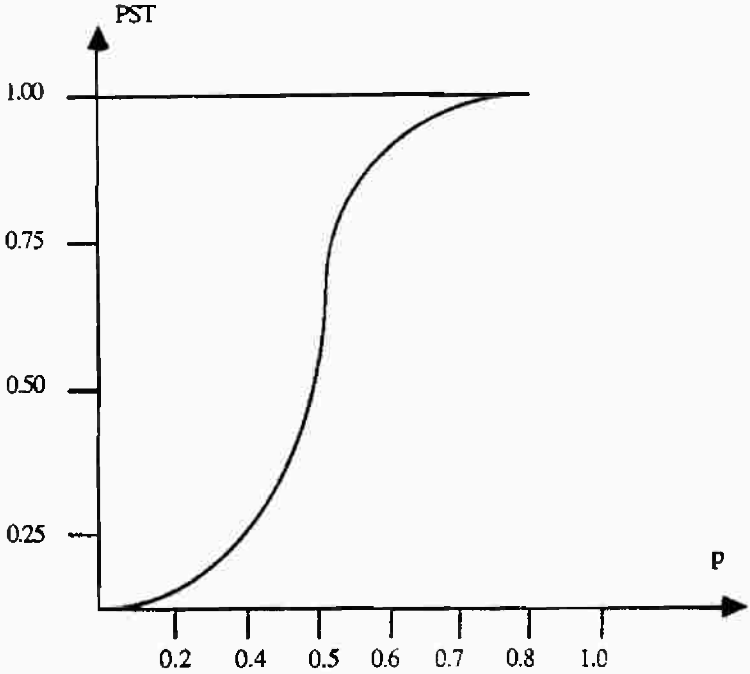


Fig. (7): PST - P, for N = 8.

5- Conclusions:

a- From Fig. (3), it is clear that the t-p connectvity, PST, increases as the number of nodes (N) is enhanced keeping the link reliability 'p' equals to a constant value (= 0.8 in our case).

b- From: Fig. (4), it is clear that the t-p connecrtivity, PST, is a monotonic decreasing function with the failure rate (λ) for an 8-node multibridge ISDN.

c- From Fig. (5), it is clear that the t-p onnectivity is, also a monotonic decreasing function with the time, t, for an 8-node multibridge having a failure rate (λ) equals to (0.5) per year.

d- From Fig. (6), it is clear that the t-p connectivity, PST, is a monotonic non-decreasing function with respect to the link reliability, P.

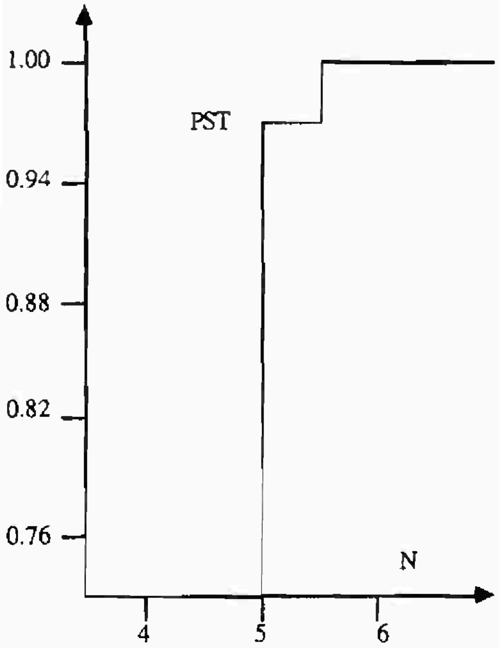


Fig. (4): PST ~ N for P = 0.8

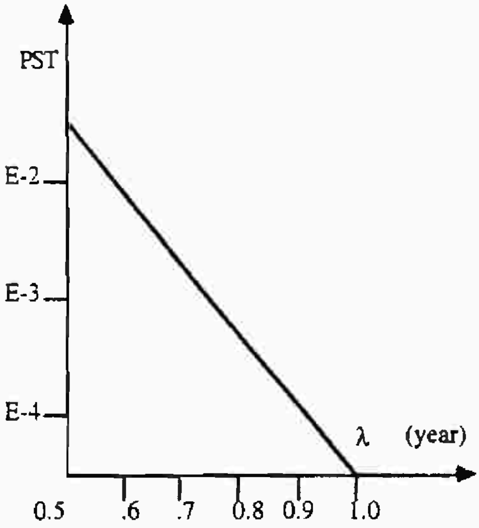


Fig. (5): PST ~λ for an 8-node Multi-bridge ISDN.

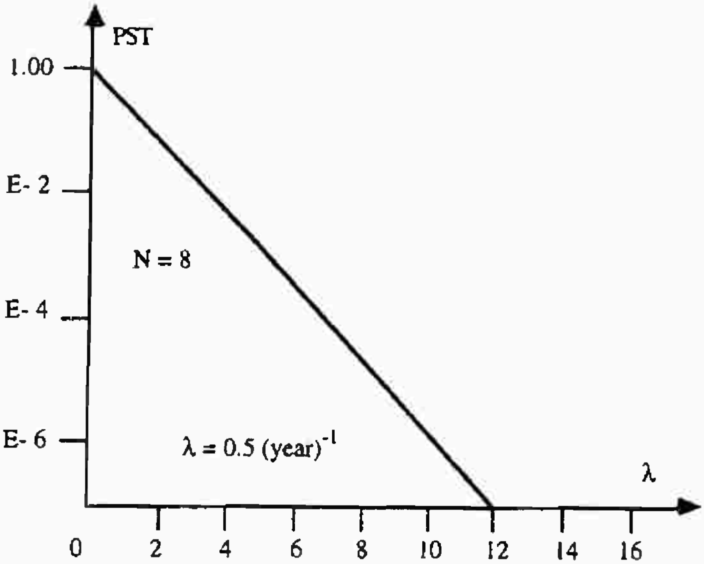


Fig. (6): PST ~λ for an 8-node Multibridge ISDN.

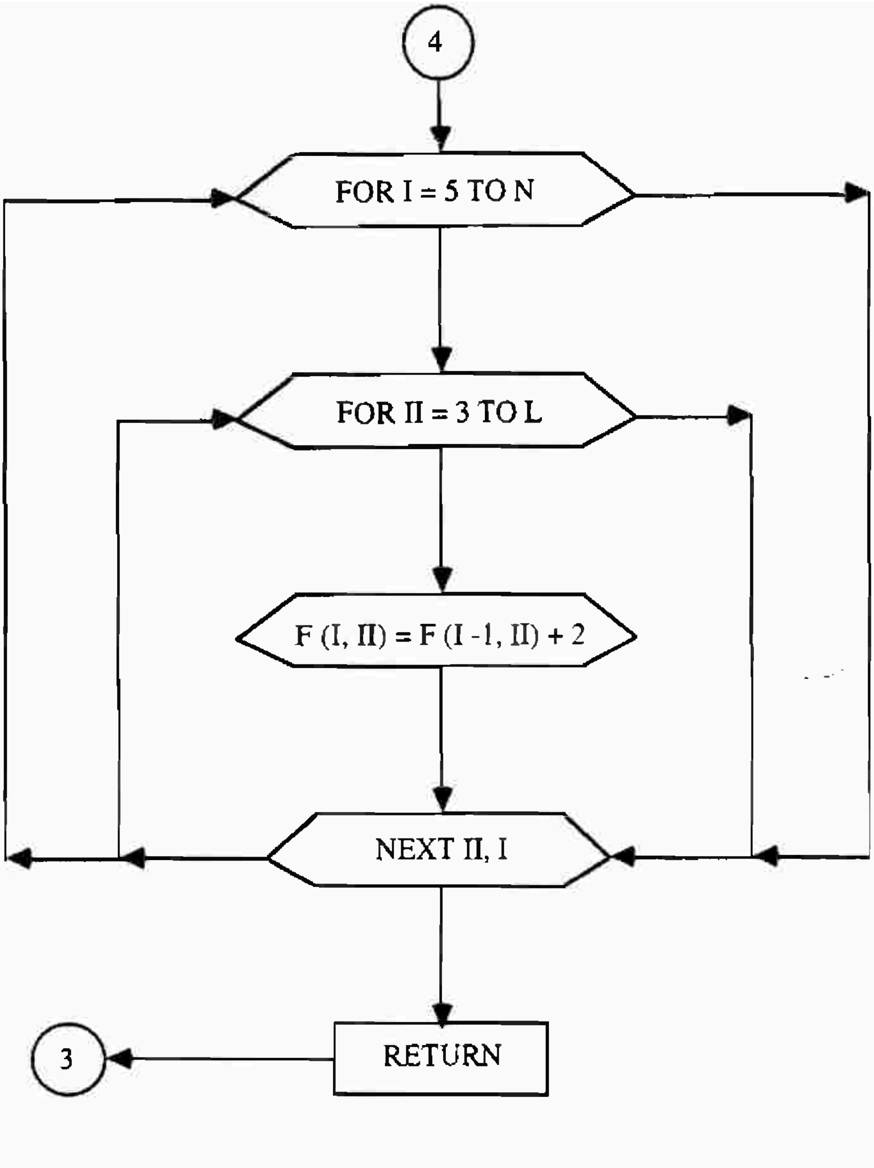
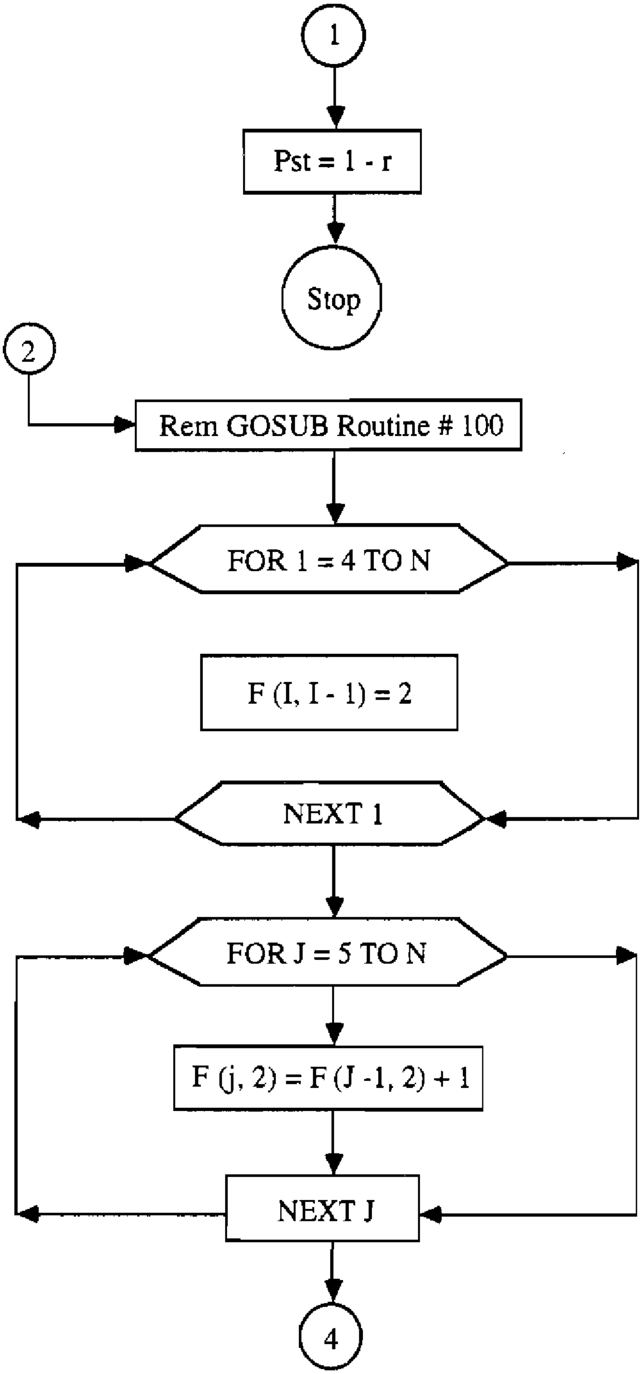
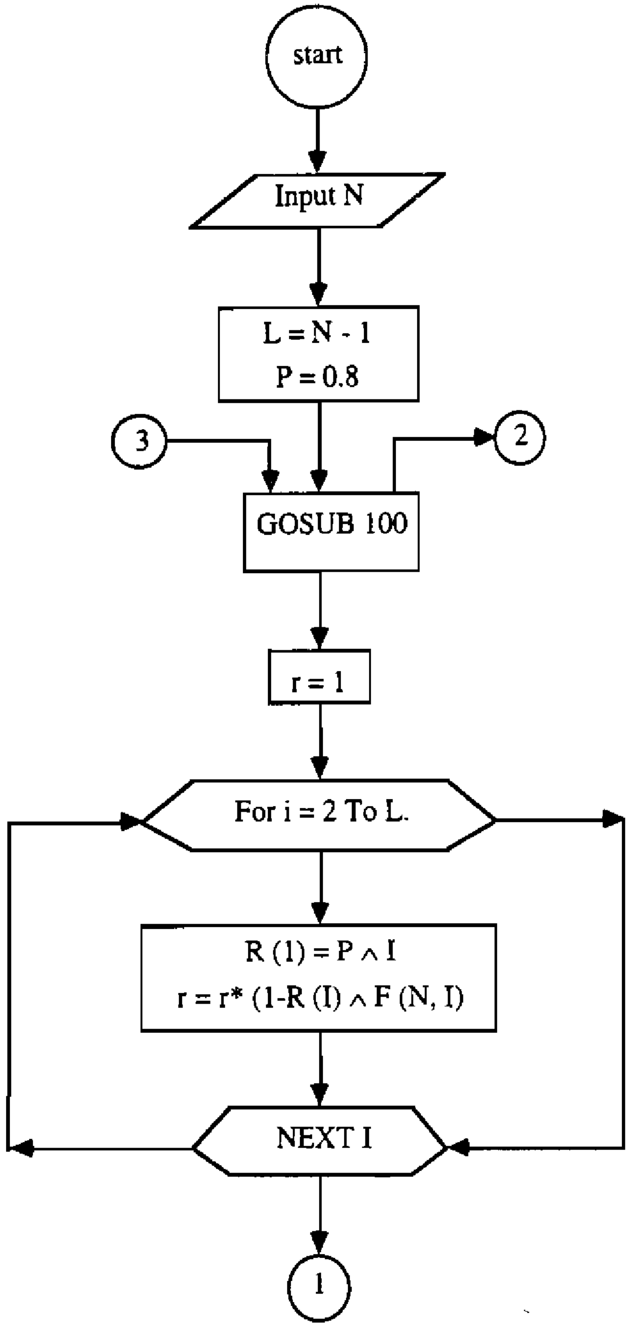


Fig. (3): Flowcharts for the equation (29)


1
Pst = 1 - r
Stop
2
Rem GOSUB Routine # 100
FOR 1 = 4 TO N
F (I, I - 1) = 2
NEXT 1
FOR J = 5 TO N
F (j, 2) = F (J -1, 2) + 1
NEXT J
4


start
Input N
L = N - 1
P = 0.8
3
2
GOSUB 100
r = 1
For i = 2 To L.
R (1) = P ∧ I
r = r* (1-R (I) ∧ F (N, I)
NEXT I
1

I (Year)	PST
0	1.0
1	1.0
2	.73
3	0.29
4	9.7 E-2
5	3.2 E-2
6	1.1 E.2
7	4.0 E-3
8	1.4 E-3
9	5.1 -4
10	1.9 E-4
15	1.4 E-6
16	4.8 E-7
17	0

Table (6): The T-P Connectivity, Pst, Versus the time, t, for an 8-node multibridge ISDN which is operating with failure rate $\lambda = 0.5\ (\text{Year})^{-1}$.

d- Table (6) give the t-p connectivity, PST, as a function of time, t, for an 8-node multi-bridge ISDN which is operating with failure rate λ = 0.5 $(year)^{-1}$.

N	4	5	6	7	8
PST	7618	9926	1.0	1.00	1.00

Table (3): The t-p connectivity, PST, versus the number of nodes (N) for link reliability, P = 0.8

P	.2	0.3	0.4	0.5	0.6	0.7
PST	0.2286	0.5243	0.8163	0.9681	0.9986	1.0

Table (4): The t-p connectivity, PST, versus the link reliablility, P, for an 8-node multi-bridge ISDN.

l $(year)^{-1}$	PST
.025	1.0000
0.033	1.0000
0.50	3.24 E-2
0.60	1.12 E-2
0.70	3.92 E-3
0.80	1.40 E-3
0.90	5.08 E-4
1.00	1.85 E-4
2.00	0.000

Table (5): The t-p connectivity, PST, versus the failure rate (λ); for an 8-node multi-bridge ISDN and operating time 't' equals to '5' years.

$F(x, x-2) = 4,$ .... (25)

for all $x \geq 5$;

$F(x, x-3) = 6;$ .... (26)

for all $x \geq 6$;

$F(x, x-4) = 8,$ .... (27)

for all $x \geq 7$;

Thus, one can get that the number of all available paths, y, between 'S' and 'T' for certain multi-bridge ISDN will be given by:

$$y = \sum_{i=1}^{x-1} F(x, i), \qquad (28)$$

where $x > 1$

Finally, one can evaluate the t-p connectivity, PST, as follows:

$$PST = 1 - \prod_{j=1}^{x-1} (1 - rj)^{Nj}, \qquad (29)$$

where (rj) is the reliability of the path having j-links, j = 1, 2, ..., x-1, or in otherwords

$rj = P^{j}$ and $P = \exp(-\lambda t)$

The flowchart of the equation (29) is depicted in Fig. (3).

4- Simulation Results:

a- Table (3) gives the t-p connectivity (PST) versus the number of nodes (x), for constant value of link reliability, ($P = 0.8$).

b- Table (4) gives the t-p connectivity, PST, versus the link reliability, P, for an 8-node multi-bridge ISDN.

c- Table (5) gives the t-p connectivity PST, versus the failure rate (λ) for an 8-node multi-bridge ISDN, which is operating for a period of time equals to 5-years.

X	N1	N2	N3	N4	N5	N6	N7	N8	....	Y
2	1	0	0	0	0	0	0	0	-	1
3	1	1	-	-	-	-	-	-	-	2
4	0	2	2	-	-	-	-	-	-	4
5	0	3	4	2	-	-	-	-	-	9
6	0	4	6	4	2	-	-	-	-	16
7	0	5	8	6	4+2	2	-	-	-	25
8	0	6	10	8	6	4	2	-	-	36
9	0	7	12	10	8	6	4	2	-	49
10	0	8	14	12	10	8	6	4	2	64

Table (2): Total number of available paths (y) ver. the nomber of nodes "X".

and so on. Generally, we can find that:

F (4, 3) = F (5, 4) = F (6, 5)

.... = F (x, x-1) = 2, ... (20)

for all $x \geq 4$

F (x, x) = 0, for all $x \geq 1$, ... (21)

F (x, 2) = F (x -1, 2) + 1; .... (22)

for all $x \geq 5$;

F (x, i) = F (x-1, i) + 2; .... (23)

for all $x \geq 5$; and $i > 2$,

F (x, x-1) = 1; .... (24)

for all $x < 4$

Also one can get that:

F (5,2) = F (4,2) + 1, ... (12)

F (5,3) = F (4,3) + 2, ... (13)

F (5,2) = F (4,2) + 1, ... (14)

F (5,3) = F (4,3) + 2, ... (15)

F (5,4) = 2, ... (16)

F (6,2) = (5,2) + 1, ... (17)

F (6,3) = F (5,3) 2, ... (18)

F (6,4) = F (5,4) + 2, ... (19)

F (6,5) = 2

X	N1	N2	N3	N4	N5	N6	N7	N8
2	1	-	-	-	-	-	-	-
3	1	1	-	-	-	-	-	-
4	0	1+1	2	-	-	-	-	-
5	0	2+1	2+2	2	-	-	-	-
6	0	3+1	4+2	2+2	2	-	-	-
7	0	4+1	6+2	4+2	2+2	2	-	-
8	0	5+1	8+2	6+2	4+2	2+2	2	-
9	0	6+1	10+2	8+2	6+2	4+2	2+2	2
10	0	7+1	12+2	10+2	8+2	6+2	4+2	2+2

Table (1): Available number of paths N2, N3, ... , N8 versus the number of nodees (X).

Note that the trivial case $x = 1$ gives no connection paths. i.e. $y=0$. which is called a cutset.

Fitting a curve for the previous results obtained in Table (2), one can proceed as follows:

1- Assume that:

$$y = A x^2 + Bx + c, \quad \text{.......... (2)}$$

2- From Table (2), choose the following set of numbers for the couple (x,y) as follows: (4,4), (6,16) and (8,36) and then substituting the last equation (2), one can get the following set of linear equations in A,B & C.

$$16A + 4.B + C = 4; \quad \text{.......... (3)}$$

$$36A + 6.B + C = 16; \quad \text{.......... (4)}$$

$$64A + 8.B + C = 36; \quad \text{.......... (5)}$$

3- Solving the last three equations in:

A, B and C, one can get that:

$A = 1$, $B = -4$ and $C = 4$.

4- Thus, the equation (2) can be rewritten as follows:

$$y = x^2 - 4x + 4, \quad \text{.......... (6)}$$

If we denote by F (x,i) as the number of available paths having i-links for an x-nodes bridge network, then referring to Table (2), one can get that:

$$F(2,1) = 1; \quad \text{.......... (7)}$$

$$F(3,1) = 1; \quad \text{.......... (8)}$$

$$F(3,2) = 1; \quad \text{.......... (9)}$$

$$F(4,2) = 2; \quad \text{.......... (10)}$$

$$F(4,3) = 2; \quad \text{.......... (11)}$$

2- Mathematical model:

Refer to Fig. (1), and assume that:

a) X = the number of reliable nodes,

b) L = the maximum number of links in a path between S & T.

c) y = the total number of available paths for the network graph (G) between S & T.

d) p = the element's reliability, and it is given by: p= expt ($-\lambda t$), where (λ) is the failure's rate.

e) rj = the reliability of the path having j-links, j= 1,2,3,..., X-1.

f) N_i = the number of available paths and each path comprises i-links betwen the two terminals S & T.

g) PST = the t-p connectivity between the two terminals: S & T.

3- Reliability Analysis:

Refer to Fig. (1). Each vertex represents a node and each branch represents a communication link. From Fig. (2.a), it is evident that the number of nodes (x) equals to (4) and the number of available paths (y) equals to (4) also. In Fig. (2.b), the number of nodes equals to (5) and the number of available paths (y) in this case is equal to (9), and so on. Table (1) gives the number of available paths between "S" and "T" having two, three, four, ... at certain value of (X), where (X) takes the values: 4,5,6,..., 10.

Table (2) gives the total number of paths (y) versus the number of nodes (x) for different nulti-bridge structures.

Denote by (Ni) as the number of paths having i-links, then

$$y = \sum_{i=1}^{x-1} Ni, \qquad (1)$$

where $x \geq 2$.

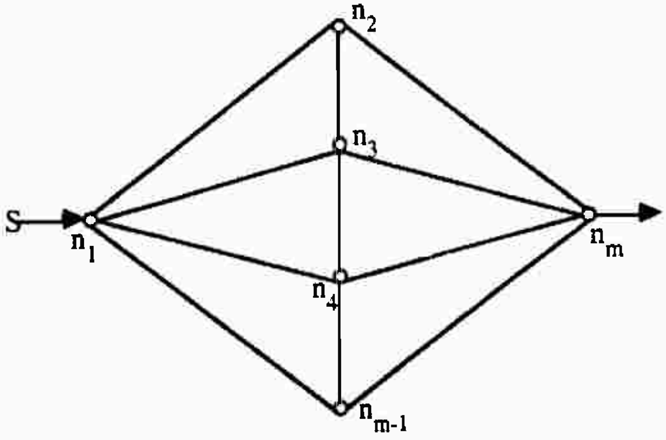


Fig. (1) A multibridge ISDN.

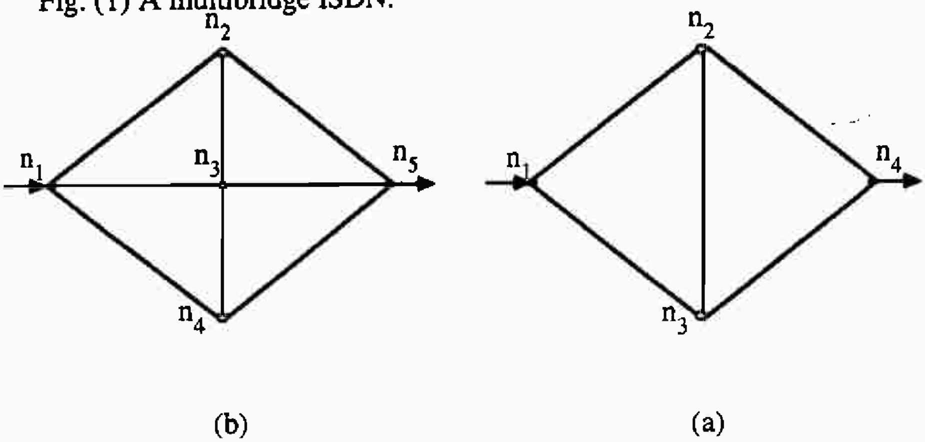


Fig (2). (a) A 4-node and

(b) A 5-node multi-bridge ISDN.

Paths having different number of links. The t-p connectivity, PST, is then evaluated using an analytical formula. The flowchart of the used formula is presented and a computer package is then prepared using the AT IBM-compatible digital computers. Useful results are obtained for those who are working in this area.

AN EFFICIENT METHOD FOR RELIABILITY EVALUATION FOR LARGE - SCALE MULTI - BRIDGE ISDN

Dr. F.F. FARAHAT

KEY WORDS: Multi-bridge ISDN, t-p connectivity.

ABSTRACT : The objective of this paper is to present an efficient and new analytical formula in order to evaluate the t-p connectivity, Pst, for a large scale multi-bridge ISDN in the general case, where the number of nodes grows very high. The set of available number of paths betwen the two terminals S & T are divided into subsets each subset comprises different number of available paths, where each path in one subgroup has a certain number of links for given multi-bridge ISDN, i.e having a certain value of number of nodes. Iterative formulae are deduced giving a recurrence relation between the number of available paths in each subset and the number of available paths in other subsets. Useful results are obtained.

1- Introduction:

The multi-bridge ISDN is a network consisting of multi-bridge structure, refer to Fig. (1). We use the t-p connectivity as a measure of service quality. as the number of nodes increases, then the available number of paths is enhanced, and consequently the computation complexity increases.

In this paper, an efficient and a new method is presented to evaluate the t-p connectivity for large scale multi-bridge ISDN. The method of attacks to find an iterative formulae between the number of available.

6- Conclusions:

From the previous analysis, it is clear that the graph (G1) is more reliable than the graph (G2). The performance measure that is applied here, is the MTFF, MTTR and MTBF.

The calculation of these measures was depending on the MGF. The same results can be obtained using other reliability measures such as the tree connectivity, and/or the multi - terminal connectivity.

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Finally, one can get the values of: MTTRI & MTTR2 for the two graphs: G1 & G2, respectively as follows:

$$\text{MTTR 1} = \frac{77b^2 \cdot c\,(1 + b.c)}{60\,\lambda\, a\,[1 + ab\,(1 + b.c)]}, \qquad (19)$$

$$\text{MTTR 2} = \frac{7b^2 \cdot c\,(1 + a)}{6\,\lambda\, a\,(1 + a.b)}, \qquad (20)$$

Where $a = \mu/(\lambda + \mu)$, $b = 1 - a$, and $c = 1 + a$

We can get the same result obtained before if we use the foowing relation:

$\text{MTTR} = \text{MTFF}\,(1/SA)\text{-}1)$, ... (21)

From the last two equations (19) & (20) it is clear that the MTTRI is greater than the MTTR 2.

5- Calculation of the MTBF:

Applying the definition of the MTBF, one can evaluate the value of MTBF1, and MTBF2 for the two graphs: G1 & G2, respectively, as follows:

MTBFI = MTFF1 + MTTR1.

$$= \frac{77}{60\lambda}\,[1 + \{b^2.c\,(1 + b.c)\,/\,a\,(1 + ab\,(1 + b.c))\}], \qquad (22)$$

and $\text{MTBF2} = (7/6\lambda)\,[1 + \{b^2.\, c/a\,)1{+}ab)\}]$, ... (23)

Also, one can show that the MTBFI is greater than the MTBF2, i.e, the graph G1 is superior to the second graph G2.

From the above two equations (11) & (14) it is clear that for any value of (λ) the MTFF1 is greater than MTFF2, in other words the first bridge network is more reliable than the second one. Thus, we have used the MTFF as a new criterion for comparing between the service quality for the two graphs: G 1 & G 2.

3.2. When the Nodes are Unreliable:

In this case, the reliability functions R1 (t) & R2 (t) for the two graphs: G1 & G2, respectively, will be equal and have the following value:

$R(t) = p^2$, ... (15)

and $p = \exp(-\lambda t)$. The calculation of MTFF in this case will yield that.

$MTFF = 1/2\lambda$, ... (16)

4- Calculation of the MTTR:

When the links are unreliable:

Using the MGF, one can be able to evaluate also the MTTR for the bridge network or any other communication network if we know the availability function, A (t).

The availability, function, A (t), can be found for any network if we substitute the value of the element's availability, a (t) instead of the element's reliability, p. Thus, one can find the value o the availability functions A1(t) & A2 (t) for the two graphs: G1 & G2, respectively as Follows:

$A_1(t) = a(t)\,[1+2.a(t) - 2\ \{a(t)\}^2 + \{a(t)\}^3 + \{a(t)\}^4]$, ... (17)

$A_2(t) = a(t) + \{a(t)\}^2\ \{1-a(t)\}$, ... (18)

The density functions $g_1(t)$ & g2 (t) can be evaluated in a similar way to the density functions $f_1(t)$ & $f_2(t)$, but here the reliability function R (t) is replaced by the availability function, A (t).

$$M_x(t) = \int_0^{\infty} f(t)\, e^{tx}\, d\lambda^{x}, \qquad (7)$$

d- Finally, one can calculate the MTFF using the equation (4).

Calculating the reliability function, R(t), for the graph, G, one can find that:-

$$R(t) = p\,(1 + 2p - 2p^2 - p^3 + p^4), \qquad (8)$$

The density function, f (t), will be given as follows:

$$f(t) = -\frac{d\,R(t)}{d\,t}$$

$$= \lambda P (1 + p^2)(1 + P(2\ \ q - p)) - 2q^2 (1 + P)\, p), \qquad (9)$$

Thus

$$M_{x(t)} = \int_0^{\infty} f(x)\, e^{tx}\, dx, \qquad (10)$$

using equation (4), one can get that

$$MTFFI = 77/60\lambda, \qquad (11)$$

To find the MTFF for the second bridge network (graph G2), then we proceed as above and one can obtain the density function f2 (t) as follows:

$$f2\,(t)\ \lambda p\,(1{=}p - 3\,p^2), \qquad (12)$$

and the MG will be given by:

$$M2 = (1-t/\lambda)^{-1} + (1-t/2\lambda)^{-1} - (1-t/3\lambda)^{-1}, \qquad (13)$$

Finally, one can get that the vaue of MTFF2 will be given by:

$$MTFF2 = 7/6\lambda, \qquad (14)$$

To find the MTFF for a link of the bridge, then we use the equation (1) as follows:

$$M_x(t) = \lambda \int_0^{\infty} e^{tx} \; e^{-\lambda^{x}} \; d^{x} \quad \text{.......... (2)}$$

The integration of the last equation (2) yields the valve as follows:-

$$M_x(t) = [1 - (t l \lambda)]^{1}, \quad \text{.......... (3)}$$

To evaluate the value of MTFF, we use the following equation:

$$MTFF = \left. \frac{\partial M_x}{\partial t} \right| t = 0, \quad \text{.......... (4)}$$

Finally, one can obtain the:

$$MTFF = 1/\lambda, \quad \text{.......... (5)}$$

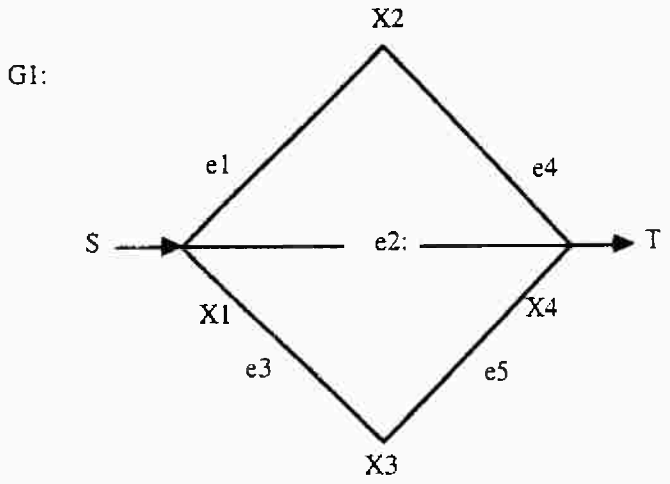
Now, in order to get the MTFF for the given bridge network, we proceed as follow:

a- Find the reliability function, R (t), for the graph, G,

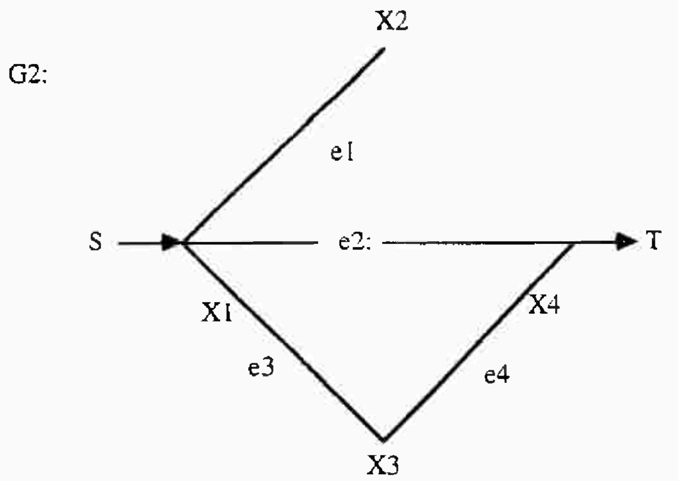
b- Find the density function, f (t), for the given graph, G, using the following equation:-

$$f(t) = -\frac{d\,R(t)}{d\,t}, \quad \text{.......... (6)}$$

c- Then the MGF, $M_{x(t)}$, can be calculated using the following equation



(a) An 5-edge 4- node bridge network



b- An 4- edge 4- node bridge network.

Fig (1). Two bridge networks.

the integration of the availability function, A (t). In this paper, we'll use another criterion for the calculation of both the MTFF & MTTR. This new criterion is the MGF. A comparative analysis is presented between the two graphs and it is concluded that the graph 'G1' is superior to the second one 'G2'.

2- MATHEMATICAL MODEL:

Refer to Fig. (1). The set (x_i) represents the family of nodes, i= 1,2,3,4 and the (ei) represents the set of edges, for the shown network for i= 1,2,3,4,5 for the graph G1, while i= 1,2,3,4 for the graph G2. Assume that:

a- The failure rate (λ) and the repair rate (μ) are s-independent, and negative exponentially distributed.

b- The element's reliability, P, is given by: $P = \exp(-\lambda t)$, and define $q = 1-P$.

c- The element's availability, a, is defined as follows:

$$a = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} \exp(-t(\lambda + \mu))$$

d- f (t) & g (t) are the density functions for the reliability and the availability functions, respectively.

e- SA = the statistical availability, i.e., the value of the availability as the time, t, goes to an infinite value.

3- MTFF Calculation:

3-1 When the links are unreliable:

The moment generating function, $M_x(t)$, can be defined for a random variable, x 9t), as follwos:

$$M_x(t) = E(e^{tx}), \quad \text{..........} \quad (1)$$