CHAPTER 12
Global Positioning System (GPS)

12.1 Global Surveying:

Since ancient time man has sought to find reference points for navigation. He used stars to know his way by night and the sun to know his way in the day. He, eventually discovered the compass. The bottom line has been to find one's position with respect to the earth. He used celestial as well as terrestrial objects as control points, to know position and seasons, to determine land boundaries, provide maps of his environment, control the construction of public works (such as the pyramid) and to know the exact time to plant the crops.

Satellite - based navigation started in the early 1970s and the term. Global Positioning System (GPS) was coined to answer the questions: what time. what position and What velocity, accurately and inexpensively anywhere on the globe at anytime. To provide a continuous global positioning capability, it is required to orbit a sufficient number of satellites (21) placed in circular 12 hour orbits inclined 55° to the equatorial plane. It was found that a constellation of a minimum of 4 satellites are available in good geometric position 24 hours a day anywhere on the earth.

The GPS satellites are configured to provide the user with the capability of determining his position, expressed by latitude, longitude and elevation.

12:2 GPS Equations:

The position of a certain point in space can be found from distances measured from this point to some known positions in space. In order to determine the user position, three satellites and three distances are geometrically required. The trace of a point with constant distance to a fixed point in a plane is a circle. Two satellites and two distances give two possible solutions because two circles intersect at two points. The third circle is needed to uniquely determine the user position in the $2D$ case. The equal distance trace to a fixed point is a sphere in the $3D$ case. Two spheres interest in a circle. This circle intersects another sphere to produce two points. In order to determine the point uniquely, another satellite is needed (Fig. 12.1). Thus, in the $3D$ case, we need 4 satellites and 4 distances. In GPS, the position of the satellite is known from the ephemeris data transmitted by the satellite. However, the distance measured between the receiver and the satellite has a constant unknown bias, because the user clock usually is different from the GPS clock. In order to resolve this error, one more satellite is required. Therefore, in order to find the user position 5 satellites are needed.

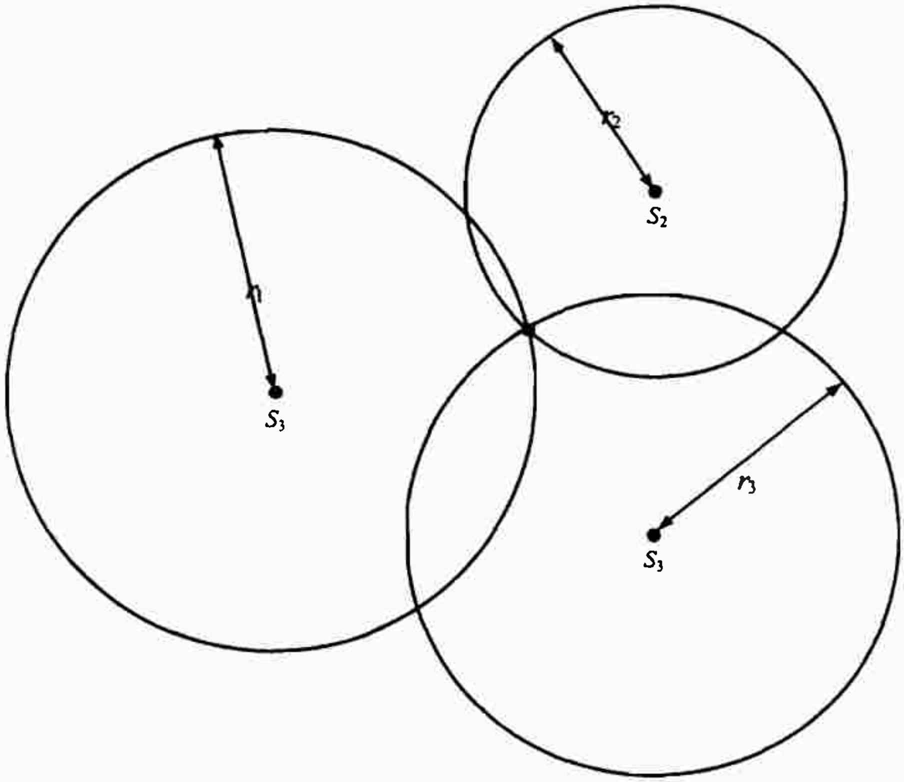


Fig. (12.1) User position in 2D

But we can save one satellite, since with 4 satellites - including the one for bias and 3 for position - we end up with two solution points: one close to the earth and the other in space far from the earth's surface which must be rejected. Therefore, we still need 4 satellites including the bias correction. Forgetting about the bias correction for now, we have three equations as follows: Assume that the distance measured is accurate, and under this condition there are three satellites at known points of locations $r_1(x_1\ y_1\ z_1)$, $r_2(x_2\ y_2\ z_2)$ and $r_3(x_3\ y_3\ z_3)$ and an unknown point at $r_u(x_u\ y_u\ z_u)$. If the distances between the three known points to the unknown point can be measured as ℓ_1, ℓ_2, ℓ_3 these distances can be written as

$$\ell_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2}$$
$$\ell_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2}$$
$$\ell_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} \quad (12-1)$$

There are three unknowns and three equations. Thus, the values of x_u, y_u and

z_u can be determined theoretically. There should be two sets of solutions for second order equations. Since these equations are nonlinear, they are difficult to solve directly. But they may be solved by linearization and iteration approach. In GPS operation, positions of the satellites are given from the data transmitted from the satellites. Thus, the distances from the user to the satellites must be measured simultaneously at a certain instant. Each satellite transmits a signal with a time reference associated with it. By measuring the time of the signal traveling from the satellite to the user the distance between the user and the satellite can be found.

If we assume that every satellite sends a signal at a certain time t_{si}, the receiver will receive the signal at a later time t_u. The distance between the user and the satellite i is

$$\ell_{io} = c\left(t_u - t_{si}\right) \qquad (12-2)$$

where ℓ_{io} is the true value of pseudo range without error from the user to satellite i, t_{si} is the true time of transmission from satellite i, t_u is the true time of reception. Practically, it is impossible to obtain the correct time from the satellite or the user. The actual satellite clock time t'_{si} and the actual user clock time t'_u are related as

$$\begin{aligned} t'_{si} &= t_{si} + \Delta t_{si} \\ t'_u &= t_u + \tau_u \end{aligned} \qquad (12-3)$$

where Δt_{si} is the satellite clock error and Δt_u is the user clock bias error. Additionally, there are other factors affecting the pseudo range measurement.

$$\ell'_i = \ell_0 + \Delta r_i - c\left(\Delta t_{si} - t_u\right) + c\left(\Delta t_{trop} + \Delta t_{ion} + \Delta t_{rel} + \Delta n_i\right) \qquad (12-4)$$

where Δr_i is the satellite position error effect on range, Δt_{trop} is the tropospheric delay error, Δt_{ion} is the ionospheric delay error, Δt_{rel} is the relativistic time correction and Δn_i is the receiver measurement noise error. Even if all errors are corrected or ignored the user clock error in particular will remain unknown. Therefore, we need an extra satellite, and hence, distance measurement. Thus, we have to rewrite eqn. (12 - 1) as

$$\left.\begin{aligned} \ell_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + c\,\tau_u \\ \ell_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + c\,\tau_u \\ \ell_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + c\,\tau_u \\ \ell_4 &= \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + c\,\tau_u \end{aligned}\right\} \qquad (12-5)$$

12.3 Solution of User Position:

We thus have 4 unknowns x_u , y_u , z_u and τ_u in 4 nonlinear simultaneous equations. One common way to solve the problem is to linearize them. We may rewrite eqn. (13 - 1) as

$$\ell_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + c\,\tau_u \qquad (12-6)$$

where $i = 1, 2, 3$ and 4 and x_u, y_u, z_u and $\Delta\tau_u$ are the unknowns. The pseudo range ℓ_i and the positions of the satellites x_i , y_i , z_i are known. Differentiating eqn. (13 - 6),

$$\delta\ell_i = \frac{(x_i - x_u)\,\delta x_u + (y_i - y_u)\,\delta y_u + (z_i - z_u)\,\delta z_u}{\sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2}} + c\,\delta\tau_u$$

$$= \frac{(x_i - x_u)\,\delta x_u + (y_i - y_u)\,\delta y_u + (z_i - z_u)\,\delta z_u}{\ell_i - c\,\tau_u} + c\,\delta\tau_u \qquad (12-7)$$

In this equation, δx_u , δy_u , δz_u and $\delta\tau_u$ are considered the only unknowns referred to some reference points on earth. The quantities x_u , y_u , z_u and τ_u are treated as known values (reference points) since they reveal some initial knowledge of the whereabouts of the user. We can obtain δx_u , δy_u , δz_u and $\delta\tau_u$ by solving the 4 equations (12 - 7). Thus, we use the new value as initial values and keep on reiteration until the added accuracy may be neglected. This is called the interation method. With δx_u , δy_u , δz_u and $\delta\tau_u$ as unknowns we have a set of linear equations. This process is referred to as linearization. The above equations can be written in matrix form as

$$\begin{bmatrix} \delta\ell_1 \\ \delta\ell_2 \\ \delta\ell_3 \\ \delta\ell_4 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{112} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ c\,\delta\tau_u \end{bmatrix} \qquad (12-8)$$

$$\alpha_{i1} = \frac{x_i - x_u}{\ell_i - c\,\tau_u} \qquad (12-9)$$

$$\alpha_{i2} = \frac{y_i - y_u}{\ell_i - c\,\tau_u}$$

$$\alpha_{i3} = \frac{z_i - z_u}{\ell_i - c\,\tau_u}$$

The solution of (12 - 9) is

$$\begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ c\,\delta \tau_u \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{112} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \delta \ell_1 \\ \delta \ell_2 \\ \delta \ell_3 \\ \delta \ell_4 \end{bmatrix} \qquad (12-10)$$

The desired solution may be obtained by iteration. A quantity is used to determine whether the desired solution is reached

$$\sqrt{\delta\, x_u^2 + \delta\, y_u^2 + \delta\, z_u^2 + c\, \delta\tau_u^2} < \in \qquad (12-11)$$

where $\in$ is a certain predetermined threshold

12.4 Code Signals:

In order to perform the user position calculation, the positions of the satellites and pseudoranges to the satellite must be measured. Information needed for the calculation of the positions of the satellites must be transmitted in the satellite signals. There are basically two types of signals: the coarse (or clear) acquisition (C/A) and the precision P(Y) codes. The P(Y) code is classified and used only by the military.

The GPS signal contains two frequency components: link 1 (L_1) and link 2 (L_2). These frequencies are coherent and obtained from a $10.23\ MHz$ clock .

$$L_1 = 1575.42\ MHz = 154 \times 10.23\ MHz$$

$$L_2 = 1227.6\ MHz = 120 \times 10.23\ MHz$$

These frequencies are very accurate as their reference is an atomic frequency standard. The Doppler frequency shift produced by the satellite motion at L_1 frequency is approximately ± 5 kHz .

The signal structure at L_1 frequency S_{L_1} given by

$$S_{L_1} = A_p\ P(t)\ D(t)\cos(2\pi f_1 t + \varphi) + A_c\ C(t)\ D(t)\sin(2\pi f_1 t + \varphi) \qquad (12-12)$$

where A_p is the amplitude of the P code, $P(t) = \pm 1$ represents the phase of the P code, $D(t) = \pm 1$ is the data code, f_1 is the L_1 frequency, ϕ is the initial phase, A_c is the amplitude of the C/A code, $C(t) = \pm 1$ is the phase of the C/A code. The codes and the carrier frequencies are all phase locked together. Thus, the GPS signal is BPSK spread spectrum. The phase change rate is the chip rate and the spectrum shape is a sinc function $\sin x / x$ with the spectrum width proportional to the chip rate. If the chip rate is $1\ MHz$, the main lobe of the spectrum has a null to null width of $2\ MHz$.

The P code is biphase modulated at $10.23\ MHz$. Therefore, the main lobe of the spectrum is $20.46\ MHz$ wide from null to null. The P code is 38 week long, but it is reset every week, which means that this 38 week long code can be divided into 37 different codes and each satellite can use a different portion of the code. There are a total of 32 satellite identification numbers although only 24 of them are in orbit. In order to perform acquisition. The time of the week is found from the C/A code signal and the navigation data rate is carried by the P code through phase modulation at $50\ Hz$ rate.

The C/A code is also a biphase modulated signal with a chip rate of $1.023\ MHz$. Therefore, the null to null bandwidth of the spectrum is 2.046 MHz. The transmitting bandwidth of the GPS satellite in the L_1 frequency is approximately $20\ MHz$ to accommodate the P code signal. Therefore the C/A code transmitted contains the main lobe and several side lobes. The total code period contains $1023\ chips\ (1ms)$. This code repeats itself every millisecond. Fig. (12.2) shows the GPS data format. The first row shows a C/A code with 1023 chips (length $1ms$). The second row shows a navigation data that has a data rate of $50\ Hz$. Thus, a data bit is $20\ ms$ long and contains $20\ C/A$ codes. Thirty data bits make a $600\ ms$ long word. Ten words make a subframe. A page is 30 second long and contains 5 subframes, while 25 pages (called super frame) make a complete data set (12.5 minutes). The information in the first three subframes of a page from 4 satellites determine the user location. Thus, it takes 18 seconds of data from 4 satellites to calculate the user position.

12.5 Generation of C/A code:

The GPS C/A signals belong to the family of pseudo random noise (PRN) spread spectrum codes known as the Gold Codes. The signals are generated from the product of two 1023 bit PRN sequences G_1 and G_2. Both G_1 and G_2 are generated by a maximum length linear shift register of 10 stages and are driven by a $1.023\ MHz$ clock (Fig. 13.3).

The maximum length sequence (MLS) generator can be made from a shift register with proper feedback. If the shift register has n bits, the length of the sequence generated is $2^n - 1$.

Both shift generators in G_1 and G_2 have 10 bits. Thus, the sequence length is 1023 which is $10^{10} - 1$. The feedback circuit is accomplished through modulo 2 adders. The positions of the feedback circuit determine the output pattern of the sequence.

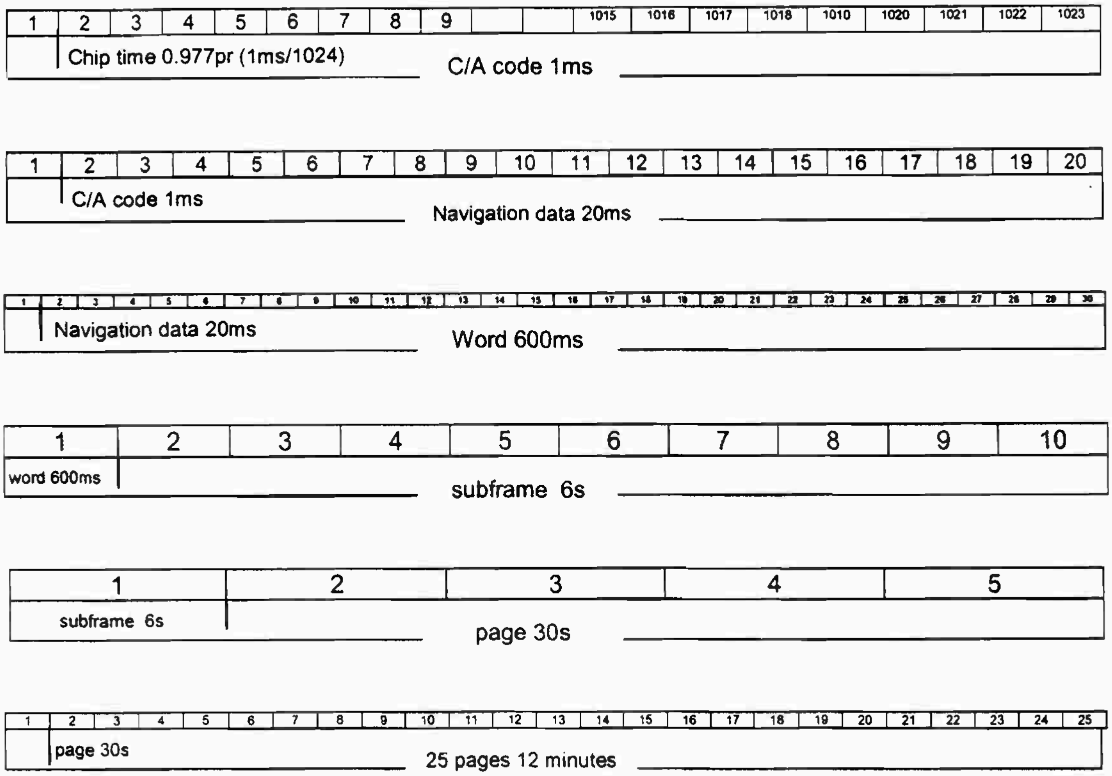
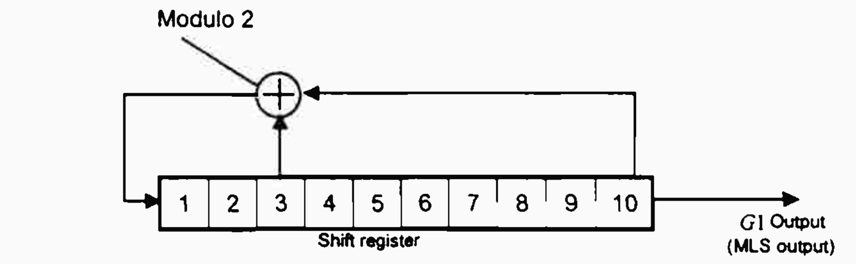


Fig. (12.2) GPS data format

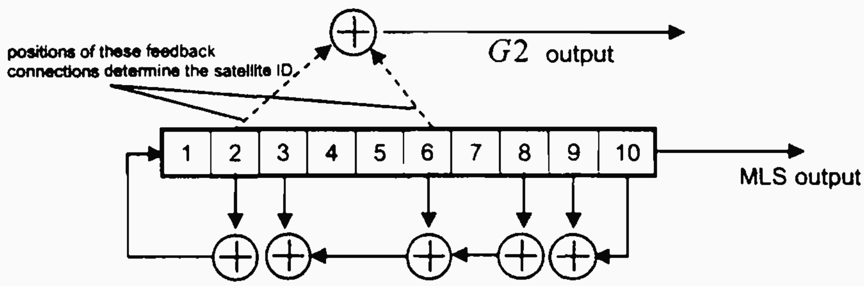
The feedback of G_1 is from bits 3 and 10 (Fig.12.3a). The feedback of G_2 is from bits 2, 3, 6, 8, 9, 10 (Fig. 12.3b).

In general, the output from the last bit of the shift register is the output of the sequence (MLS). However, the G_2 generator does not use the MLS output as the output. The output is generated from two bits which are referred to as the code phase selections though another modulo 2 adder (Fig. 13.3b, c).

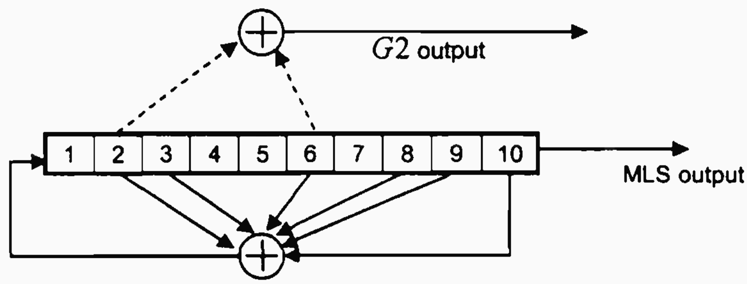
This G_2 output is a delayed version of the MLS output. The delay time is determined by the positions of the two output points selected. Fig. (12.4) shows the C/A code generator. Another modulo 2 adder is used to generate the C/A code, which uses the output from G_1 and G_2 as inputs.



(a) $G1$ generator



(b) $G2$ generator



(c) Simplified notation of $G2$

Fig. (12.3) G_1 and G_2 maximum length sequence generators

The initial values of the two shift registers G_1 and G_2 are all 1's and must be loaded in the registers first. The satellite identification is determined by the two output positions of the G_2 generator. There are 37 unique output positions. Among these 37 outputs, 32 are utilized for the C/A codes of 32 satellite. However there are only 24 C/A code satellites in orbit. The other five outputs are received for other applications such as ground transmission.

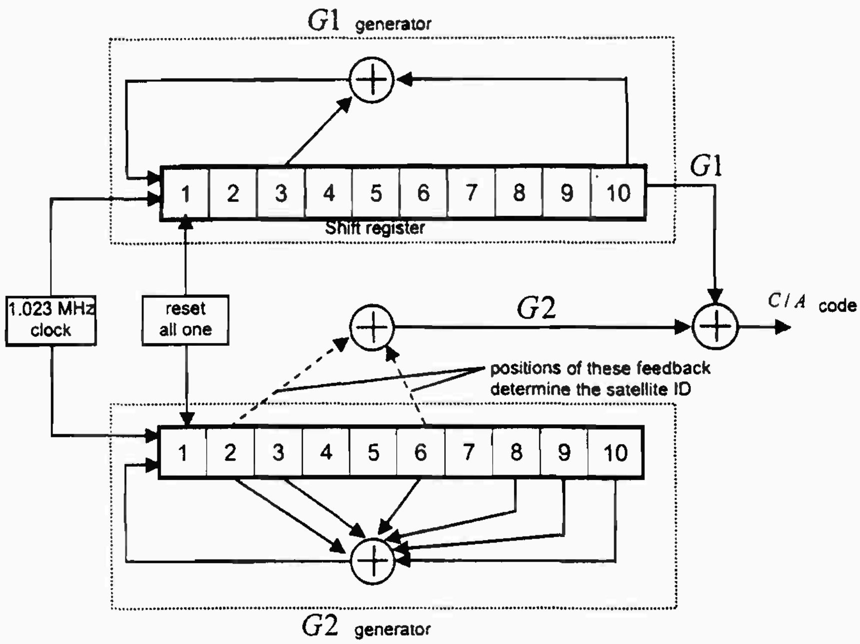


Fig. (12.4) C/A **code generator**

12.6 Acquisition and Tracking:

In order to track and decode the information in the GPS signal an acquisition method must be used to detect the presence of the signal. Once the signal is detected, the necessary parameters must be obtained and passed to a tracking program. From the tracking program information, the navigation data can be obtained. The acquisition method must search over a frequency range of $\pm$ 10 kHz to cover all of the expected Doppler frequency range for high speed aircraft. In order to accomplish the search in a short time, the bandwidth of the searching program cannot be very narrow. Searching through with a wide bandwidth filter will provide relatively poor sensitivity. On the other hand, the tracking method has a very narrow bandwidth, thus, high sensitivity can be achieved. Once the signal is found, the information will immediately pass to the tracking hardware. If the acquisition is slow, the time elapse is long and the information passed to the tracking program obtained from old date might be out of date. It is important to build a real time receiver with high speed acquisition.

The basic idea of acquisition is to despread the input signal and find the carrier frequency by multiplying with the code, at the receiver, hence obtaining the continuous wave of the carrier. The beginning of the C/A code and the carrier frequency are the parameters passed to the tracking program. The tracking process will follow the signal and obtain the information of the navigation data.

Problems

1. Estimate the time needed to determine the user position from 4 different satellites.

2. Using the linearization procedure show the detailed, steps to determine the user position from information from 4 different satellites.

3. Venify the operation of G_1 and G_2 generators (Fig. 12.3).

4. Derive the output of the C/A generator(Fig. 12.4).

5. Referring to Fig. (12.4), show that there are 37 codes as we change the feedback points.

References

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