CHAPTER 8
Bandpass Modulation

8.1 Need for Bandpass Modulation:

Modulation is modification of the characteristics of a signal (amplitude, frequency, phase) by another (called modulating signal). If the signal is sent without modulation (1 being voltage level and 0 no voltage) this is called basesband transmission. If we use pulse modulation where 1 and 0 are represented by two levels while the carrier is a rectangular pulse train and the characteristics are adjusted for pulse amplitudes, we call this pulse modulation. In PCM, the coded waveform may use different forms of coding called line codes. Collectively, all of these forms may still be called baseband transmission even if pulse modulation is used. Baseband modulation is often called formatting to distinguish it from bandpass modulation. Bandpass modulation is reserved for the case when we use high frequency, i.e., radio frequency RF (or IF)pulses. In this case, the carrier is a sinusoid and the characteristics may be adjusted for amplitude, frequency or phase. When do we need bandpass modulation?

1. to be able to use wireless transmission.
2. to use antennas of reasonable length since the antenna's a length is roughly $\lambda/4$, so we need to decrease λ to reasonable values by increasing f since $f = c/\lambda$.
3. to be able to use FDM for multiuse operation.
4. to prevent interference.

The modulated signal is sent down a channel where noise (AWGN) is added to it. When the channel bandwidth is smaller than the signal bandwidth, the channel is band limited. Severe bandwidth limitation causes ISI,.

In bandpass modulation, a sequence of digital symbols are used to alter the parameters of a high frequency sinusoidal signal (carrier). We have three types of binary carrier modulation. They are amplitude shift keying (ASK) or on/off keying (OOK), frequency shift keying (FSK) and phase shift keying (PSK) as shown (Fig. 8.1).

Based on these three basic schemes, a variety of modulation schemes can be derived from their combinations. For example, combining two binary PSK (BPSK) signals with orthogonal carriers a new scheme called quadrature phase shift keying (QPSK) can be generated. By modulating both amplitude and phase of the carrier we can obtain a scheme called quadrature amplitude modulation (QAM). The purpose of digital system design is to efficiently transmit digital bits and recover them from corruption due to noise, distortion and interference.

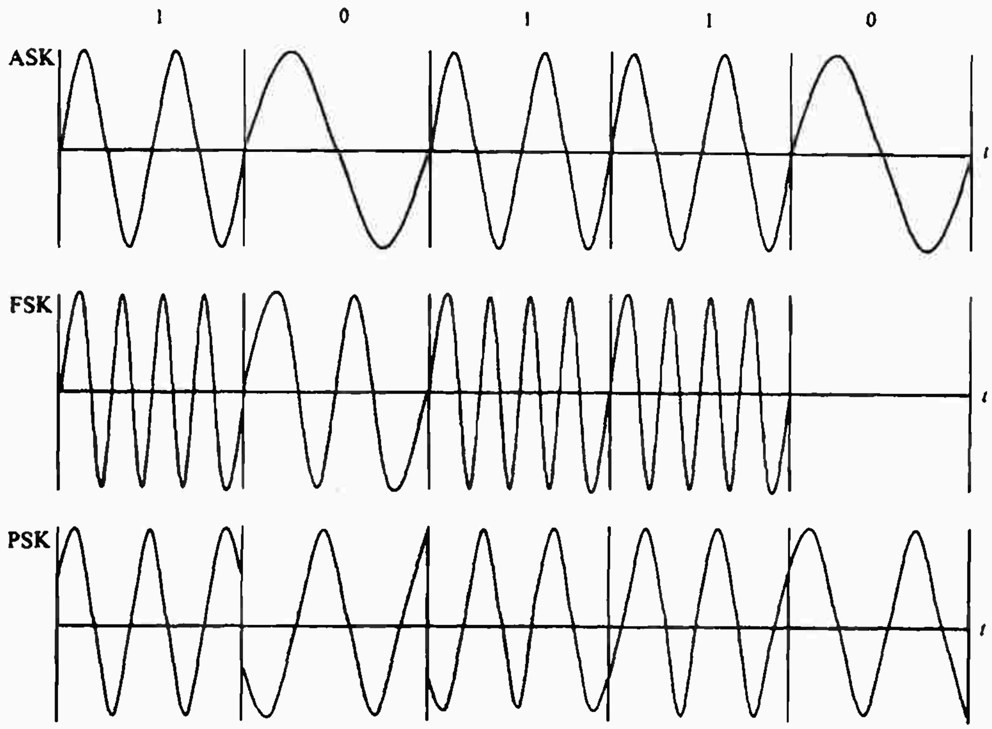


Fig. (8.1) Three basic bandpass modulation techniques
a) ASK b) FSK c) PSK

There are three primary criteria for choosing modulation schemes: power efficiency, bandwidth efficiency and system complexity. The power efficiency of a modulation scheme is the required E_b/η for a certain bit error probability over an AWGN channel. The bandwidth efficiency is defined as the number of bits per second that can be transmitted in one Hertz of system bandwidth. There are three bandwidth efficiencies. Nyquist bandwidth efficiency is used for ideal rectangular filtering at baseband, corresponding to minimum bandwidth required for ISI free transmission. The bandwidth at baseband is $B = R_s/2$ and DSB bandwidth is $B = R_s$. Since $R_s = R_b/\log_2 M$ for M-ary modulation, the DSB bandwidth efficiency is

$$\frac{R_b}{B} = \log_2 M \qquad (8-1)$$

Spectral efficiency ε_s depends on symbol or baud rate R_s and bandwidth B and the number of states M, for statistically independent, equiprobable symbols. Therefore

$$\varepsilon = \frac{R_b}{B} = \frac{R_s \log_2 M}{B} \quad bits/sHz \tag{8-2}$$

$$= \frac{\log_2 M}{T_s\ B} \quad bits/sHz \tag{8-3}$$

Thus, the spectral efficiency may be maximized by making the size M large and $T_s\ B$ product as small as possible. Pulse shaping or filtering may be used to decrease B for a given baud rate $R_s = 1/T_s$. For ISI free transmission, the minimum $T_s\ B$ product was limited to $T_s\ B \geq 0.5$ since the minimum (single sided Nyquist) bandwidth $B = 1/2T_s$. In IF (bandpass) modulation, the modulation process results in a double sided signal. The minimum ISI free $T_s\ B_{bp}$ product is $T_s\ B_{bp} \geq 1$ and the minimum bandwidth $B_{bp} = 1/T_s$ which is called double sided Nyquist bandwidth.

We may classify modulation schemes into two large categories: constant envelope and non constant envelope. Under constant envelope class, there are subclasses FSK and PSK. Under non constant envelope class there are subclasses ASK and QAM. There are other advanced systems which are variations of the main systems. PSK is usually used in most communication systems, e.g. satellite communication. Binary FSK was used in low rate control channels of first generation of cellular systems and voice channel modems. QAM is widely used is telephone networks such as computer modems.

8.2 Signal Space:

In baseband modulation, digital symbols take the form of shaped pulses. In bandpass modulation, those shaped pulses modulate a carrier which then be converted to electromagnetic radiation. Assigning different carrier frequencies to different channels is known as frequency division multiplexing (FDM). This means that we divide the frequency spectrum among different channels or users. The general form of the carrier wave is

$$s(t) = A(t)\cos\left[\omega_0 t + \theta(t)\right] \tag{8-4}$$

where ω_0 is the radian frequency of the carrier and $\theta(t)$ is the phase, and $A(t)$ is the amplitude. Thus, the modulator stores the information bits in either the amplitude, the frequency or the phase. (Fig. 8.1) shows the three basic types of modulation : amplitude shift keying (ASK), frequency shift keying (FSK) and phase shift keying (PSK). By modulating both the amplitude and phase of the carrier, we obtain quadrature amplitude modulation (QAM). The simplest case of ASK is binary ASK (BASK) where bit 1 is represented by $+A\cos[\omega t+\theta(t)]$ while bit 0 is represented by $-A\cos[\omega t+\theta(t)]$.

Fig. (8.2) shows what happens when bits 010 arrive at the BASK modulator. Table (8.1) shows the waveforms for BASK for the range $qT_b \le t \le (q+1)T_b$. The waveform can be expressed as the continuous carrier multiplied by gating pulse $g_{T_b}(t-T_b)$

Table (8.1) B-ASK

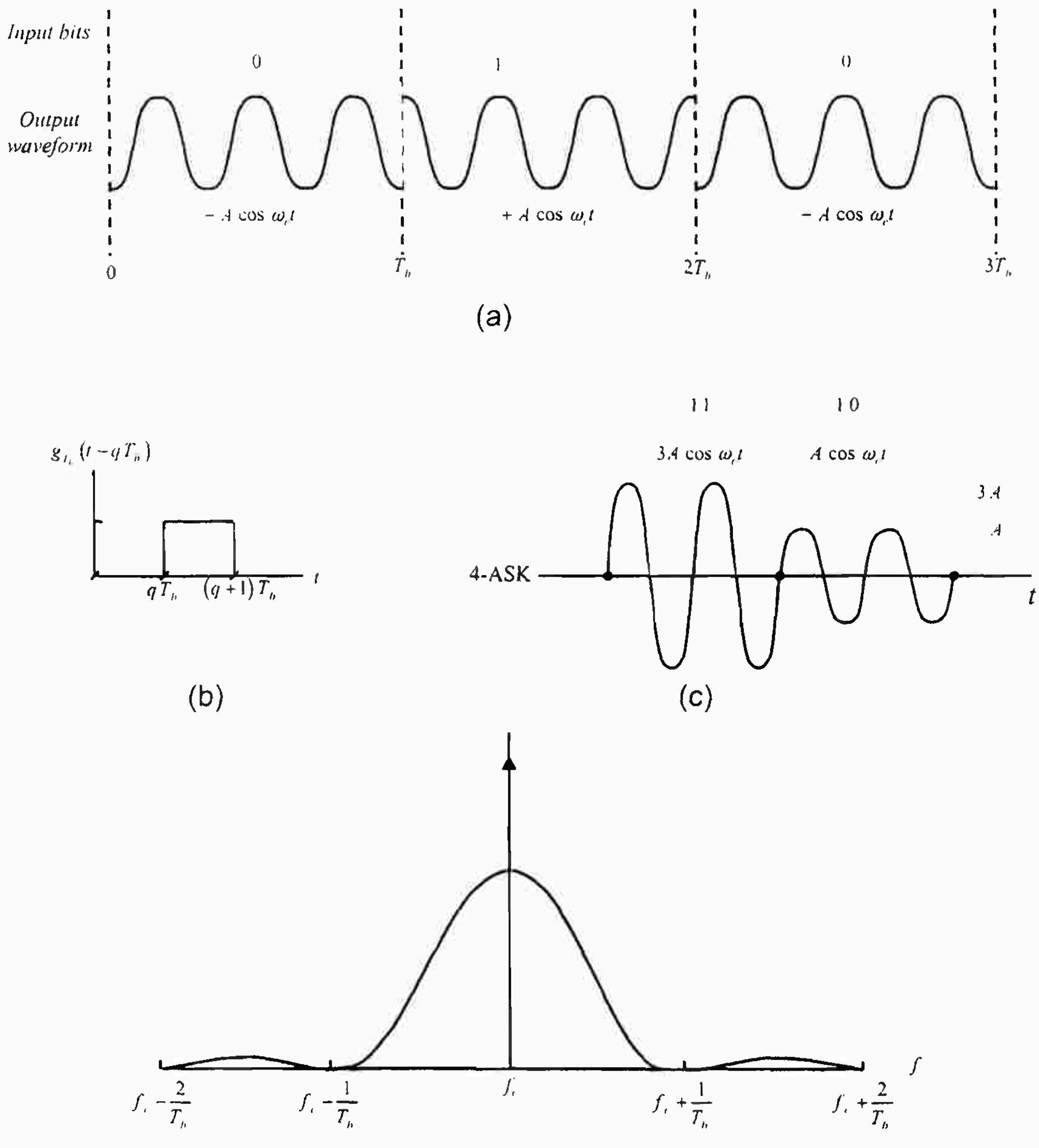
Input	*Output waveform*
0	$s_0(t) = -A\cos\omega_c t,\quad qT_b \le t \le (q+1)T_b$ $= -A\cos\omega_0 t\; g_{T_b}(t-q\,T_b)$
1	$s_1(t) = +A\cos\omega_c t\quad q\,T_b \le t \le (q+1)T_b$ $= +A\cos\omega_c t\; g_{T_b}(t-q\,T_b)$

A more complex ASK system is M-ary ASK (MASK) or M-ary amplitude modulation (MAM). Consider for simplicity 4-ASK (4-ary ASK). We let two bits enter the modulator at the same time. Rather than map bit 0 to one amplitude and bit 1 to another, the modulator sees every bit pair 0 0 , 0 1 , 1 0 or 1 1 . The modulator maps each set of two bits to a waveform with different amplitude. For example, for input 1110 we have the wave from shown (Fig. 8.2c). Table (8.2) gives the different cases for 4-ary ASK.

We notice that each waveform created in a 4-ASK modulator lasts twice as long as a waveform created in a BASK modulator. Thus, $T_s = 2T_b$. Table (8.2) also gives waveforms for 8-ASK when $T_s = 3T_b$ and so on.

The most popular of the bandpass modulators are the PSK modulators. The bit information is embedded in the phase $\theta(t)$. The simplest form of PSK is binary PSK (B-PSK) (Fig. 8.3). When 0 is inputted to the modulator, its output is $A\cos(\omega_c t+0^o)$. When 1 is inputted, the output is $A\cos(\omega_c t+180^o)$, Table (8.3) summarizes the operation.

In 4-PSK, (Fig. 8.4) the modulator works on two bits at a time. For every two bits at the input the modulator outputs a different waveform. The modulator maps 0 0 into $s_1(t) = A\cos(\omega_c t+0^o)$.



(d)

Fig. (8.2) ASK modulator

a) BASK waveform for 0 1 0 *b)* $g_{T_b}(t - qT_b)$

c)4-ASK *d) PSK*

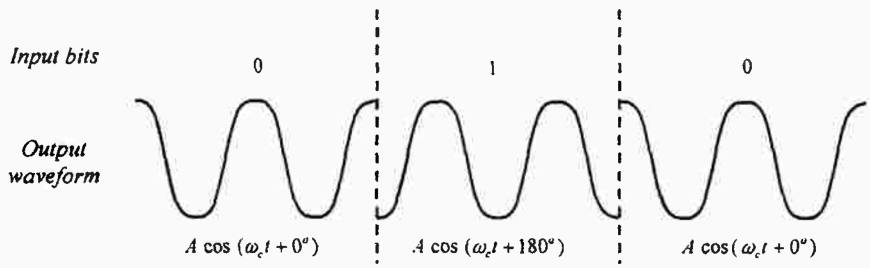


Fig. (8.3) BPSK

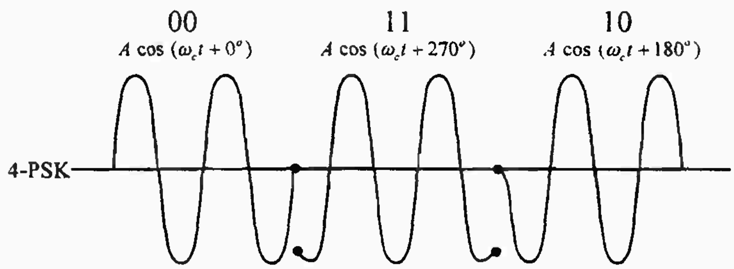


Fig. (8.4) 4PSK

For 1 1 it outputs $s_1(t) = A \cos\left(\omega_c t + 270^o\right)$(Fig. 8.4). It is common to call 4-PSK quadrature PSK (QPSK). Table (8.4) summarizes its operates, $T_s = 2T_b$ .For 8-PSK.

Table (8.5) shows that for every 3 bits $\left(T_s = 3t_b\right)$we have a different waveform. In FSK, the information is embedded in the frequency. In the simplest form of binary FSK (BFSK), if bit 0 is at the input, the modulator output will be $A \cos\left[\left(\omega_c + \Delta\omega_0\right)t + \theta\right]$, and if bit 1 is at the input, the modulator output will be $A \cos\left[\left(\omega_c + \Delta\omega_1\right)t + \theta\right]$, where $\Delta\omega_0$ and $\Delta\omega_1$ are frequency shifts (offsets) corresponding to bit 0 and bit 1, respectively (Fig. 8.5). In 4-FSK, we have four frequency offsets $\Delta\omega_0$, $\Delta\omega_1$, $\Delta\omega_2$, $\Delta\omega_3$ corresponding to input bit pairs 0 0, 0 1, 1 0, 1 1, respectively as shown in Table (8.6) where, $T_s = 2\,T_b$.

Table (8.2) 4-ASK / 8-ASK

	Input bit	*Output waveform*	
4 ASK	0 0	$s_0(t) = -3A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	0 1	$s_1(t) = -A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 0	$s_2(t) = A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 1	$s_3(t) = 3A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
8-ASK	0 0 0	$s_0(t) = -7A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	0 0 1	$s_1(t) = -5A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	0 1 0	$s_2(t) = -3A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	0 1 1	$s_3(t) = -A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 0 0	$s_4(t) = A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 0 1	$s_5(t) = 3A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 1 0	$s_6(t) = 5A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1 1 1	$s_7(t) = 7A\cos\omega_c t$	$g_{T_s}(t - qT_s)$

Table (8.3) BPSK

	Input	*Output waveform*	
	0	$s_0(t) = A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	1	$s_1(t) = -A\cos\omega_c t$	$g_{T_s}(t - qT_s)$

Table (8.4) 4-PSK (QPSK)

	Input	*Output waveform*	
	0 0	$s_0(t) = A\cos\omega_c t$	$g_{T_s}(t - qT_s)$
	0 1	$s_1(t) = A\cos(\omega_c t + 90^o)$	$g_{T_s}(t - qT_s)$
	1 0	$s_2(t) = A\cos(\omega_c t + 180^o)$	$g_{T_s}(t - qT_s)$
	1 1	$s_3(t) = \cos(\omega_c t + 270^o)$	$g_{T_s}(t - qT_s)$

Table (8.5) 8-PSK

Input bits	*Output waveform*	
0 0 0	$s_0(t) = A\cos(\omega_c t + 0^o)$	$g_{T_s}(t - qT_s)$
0 0 1	$s_1(t) = A\cos(\omega_c t + 45^o)$	$g_{T_s}(t - qT_s)$
0 1 0	$s_2(t) = A\cos(\omega_c t + 90^o)$	$g_{T_s}(t - qT_s)$
0 1 1	$s_3(t) = A\cos(\omega_c t + 135^o)$	$g_{T_s}(t - qT_s)$
1 0 0	$s_4(t) = A\cos(\omega_c t + 180^o)$	$g_{T_s}(t - qT_s)$
1 0 1	$s_5(t) = A\cos(\omega_c t + 225^o)$	$g_{T_s}(t - qT_s)$
1 1 0	$s_6(t) = A\cos(\omega_c t + 270^o)$	$g_{T_s}(t - qT_s)$
1 1 1	$s_7(t) = A\cos(\omega_c t + 315^o)$	$g_{T_s}(t - qT_s)$

Table (8.6) BFSK / 4-FSK

	Input bits	*output waveform*	
BFSK	0	$s_0(t) = A\cos(\omega_c + \Delta\omega_0)t$	$g_{T_s}(t - qT_s)$
	1	$s_1(t) = A\cos(\omega_c + \Delta\omega_1)t$	$g_{T_s}(t - qT_s)$
4-FSK	0 0	$s_0(t) = A\cos(\omega_c + \Delta\omega_0)t$	$g_{T_s}(t - qT_s)$
	0 1	$s_1(t) = A\cos(\omega_c + \Delta\omega_1)t$	$g_{T_s}(t - qT_s)$
	1 0	$s_2(t) = A\cos(\omega_c + \Delta\omega_2)t$	$g_{T_s}(t - qT_s)$
	1 1	$s_3(t) = A\cos(\omega_c + \Delta\omega_3)t$	$g_{T_s}(t - qT_s)$

Naturally, ASK is not the best choice since it picks up distortion and noise in the channel. FSK uses more bandwidth than ASK or PSK. It uses more than one frequency whereas ASK and PSK use one frequency only. PSK is the most common. Now should we uses BPSK, 4-PSK, or 8-PSK or higher? For BPSK, we have one sided bandwidth $B = 1/T_b$, whereas for 4-PSK the one sided bandwidth is $B = 1/2T_b$. Thus, as the modulation size gets bigger, the bandwidth gets smaller (Fig. 8.6). However, BPSK is more robust to noise than 4-PSK, since it is more difficult to make a waveform $A\cos\omega_c t$ look like $A\cos(\omega_c t + 180^o)$ due to noise than to make it look like $A\cos(\omega_c t + 90^o)$.

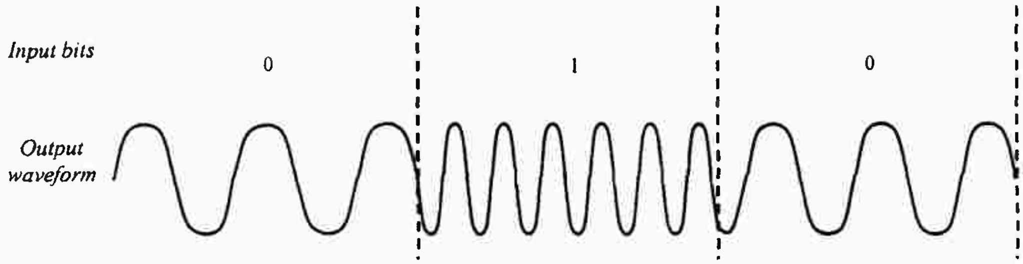


Fig. (8.5) BFSK modulation

8.3 Constellations:

We have seen before that a signal waveform can be represented as a point in the signal space. Fig. (8.7) shows a square wave. It can be represented as point $(1,1)$ if the x-axis is $\varphi_1(t)$ and the y-axis is $\varphi_2(t)$ as shown where $\varphi_1(t)$ and $\varphi_2(t)$ are two orthonormal functions. Let us now consider BPSK. We have two signals $\{s_0(t)\ ,\ s_1(t)\}$. It can be shown (Prob. 8.2) that the orthonormal function for BPSK is just one element set $\{\varphi_1(t)\}$. Using the set of equations

$$\phi_1(t) = s_1(t)/\sqrt{E_1} \qquad (8-5)$$

$$E_1 = \int_{-\infty}^{\infty} s_1(t)\ s_1(t)\ dt \qquad (8-6)$$

$$a_{ij} = \int_{-\infty}^{\infty} s_i(t)\ \phi_j(t)\ dt \qquad (8-7)$$

we find

$$\phi_1(t) = \sqrt{2/T_b}\ \cos\omega_c(t)\ \ g_{T_b}(t-qT_b) \qquad (8-8)$$

$$s_0(t) = a_{01}\ \phi_1(t) \qquad (8-9)$$

$$s_1(t) = a_{11}\ \varphi_1(t) \qquad (8-10)$$

where

$$a_{01} = A\ \sqrt{T_b/2} \qquad (8-11)$$

$$a_{11} = -A\ \sqrt{T_b/2} \qquad (8-12)$$

We note from eqn. (8 – 5) that $E = A^2\ T_b/2$ which checks with eqns. (8 – 11) and (8 – 12). Fig. (8.8) shows the two signals $s_0(t)$ and $s_1(t)$ in the signal space

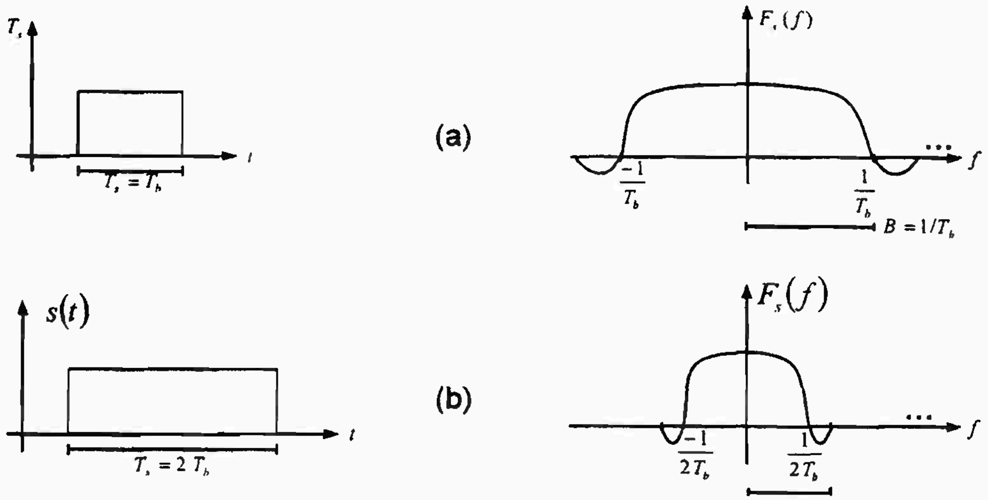


Fig. (8.6) Bandwidth for BPSK, 4-PSK
a) BPSK b) 4-PSK

Next consider 4-PSK. From Table (8.4), we have four 4-PSK signals $\{s_0(t)\ ,\ s_1(t)\ ,\ s_2(t)\ ,\ s_4(t)\}$. We may rewrite Table (8.4) in the form of Table (8.7)

It is clear from the table as well as from the standard Gram Schmitt procedure [Prob. (8.3)] that

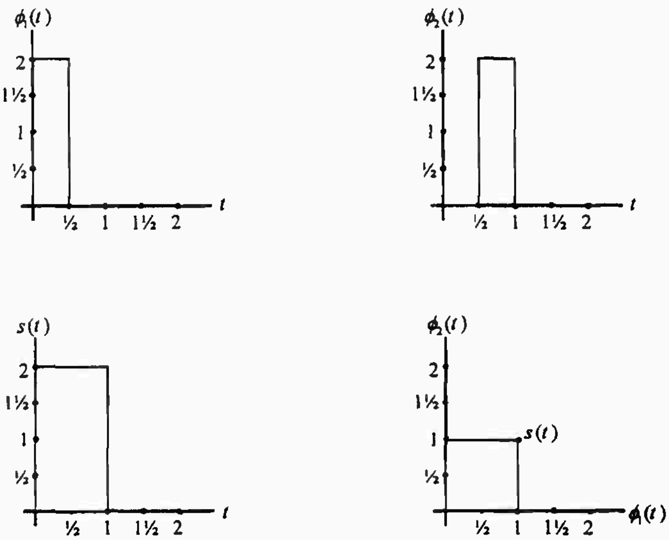
$$\phi_1(t) = \sqrt{2/T_s}\ \cos\omega_c t \cdot g_{T_s}(t - qT_b) \qquad (8-13)$$

$$\phi_2(t) = -\sqrt{2/T_s}\ \sin\omega_c t \cdot g_{T_s}(t - qT_s) \qquad (8-14)$$

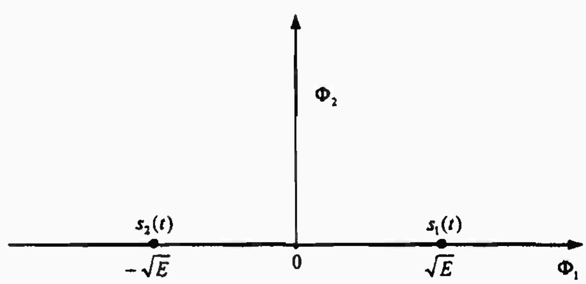
Noting that $\phi_2(t)$ has a minus sign, the coefficients $\{a_{i,j}\}$ are now given by

$$a_{01}\ ,\ a_{02} = A\sqrt{T_s/2}\ ,\ 0 \qquad (8-15)$$

$$a_{11}\ ,\ a_{12} = 0\ ,\ A\sqrt{T_s/2} \qquad (8-16)$$



(a)



(b)

Fig. (8.7) Signals in a signal space

a) a square wave *b) BPSK*

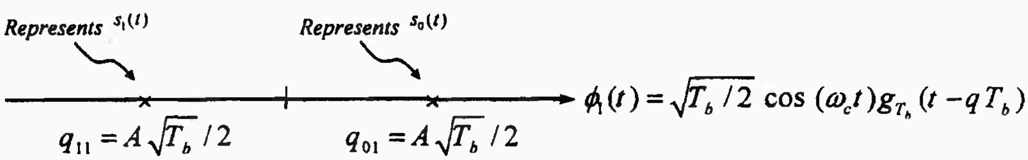


Fig. (8.8) BPSK signals in the signal space

Table (8.7) 4-PSK (QPSK) as sums of sines and cosines

	Input Bits	Output waveform		cosine term		sine term
4-PSK	0 0	$s_0(t)=A\cos(\omega_c t+0^\circ)\cdot g_{T_s}(t-qT_s)$	=	$A\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	0
	0 1	$s_1(t)=A\cos(\omega_c t+90^\circ)\cdot g_{T_s}(t-qT_s)$	=	0	−	$A\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
	1 0	$s_2(t)=A\cos(\omega_c t+180^\circ)\cdot g_{T_s}(t-qT_s)$	=	$-A\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	0
	1 1	$s_3(t)=A\cos(\omega_c t+270^\circ)\cdot g_{T_s}(t-qT_s)$	=	0	+	$A\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$

Fig. (8.9) shows a plot of 4-PSK signals in the signal space. Such a plot is called 4-PSK constellation.

For 8-PSK as in Table (8.8) we have

$$\phi_1(t)=\sqrt{2/T_s}\ \cos\omega_c t\ \cdot g_{T_s}(t-qT_s) \qquad (8-19)$$

$$\phi_2(t)=\sqrt{2/T_s}\ \sin\omega_c t\ \cdot g_{T_s}(t-qT_s) \qquad (8-20)$$

Table (8.8) shows the output waveforms as sums of cosines and sines and Fig. (8.9) shows the output waveforms represented on orthonormal basis:

$$a_{21}\ ,\ a_{22}=-A\ \sqrt{T_s/2}\ ,\ 0 \qquad (8-17)$$

$$a_{31}\ .\ a_{32}=0,-A\ \sqrt{T_s/2} \qquad (8-18)$$

Table (8.8) Output waveforms represented on orthonormal basis

	Output Waveform	Output waveform represented on orthonthonormal basis
8-PSK	$s_0(t)$	$s_0=(a_{01}\ ,\ a_{02})=\left(A\sqrt{T_s/2}\ ,\ 0\right)$
	$s_1(t)$	$s_1=(a_{11}\ ,\ a_{12})=\left(A\sqrt{T_s}/2\ ,\ A\sqrt{T_s}/2\right)$
	$s_2(t)$	$s_2=(a_{21}\ ,\ a_{22})=\left(0\ ,\ A\sqrt{T_s}/2\right)$
	$s_3(t)$	$s_3=(a_{31}\ ,\ a_{32})=\left(-A\sqrt{T_s}/2\ ,\ A\sqrt{T_s}/2\right)$
	$s_4(t)$	$s_4=(a_{41}\ ,\ a_{42})=\left(-A\sqrt{T_s/2}\ ,\ 0\right)$
	$s_5(t)$	$s_5=(a_{51}\ .\ a_{52})=\left(-A\sqrt{T_s}/2\ ,\ -A\sqrt{T_s}/2\right)$
	$s_6(t)$	$s_6=(a_{61}\ .\ a_{62})=\left(0\ ,\ -A\sqrt{T_s/2}\right)$
	$s_7(t)$	$s_7=(a_{71}\ ,\ a_{72})=\left(A\sqrt{T_s}/2\ ,\ -A\sqrt{T_s}/2\right)$

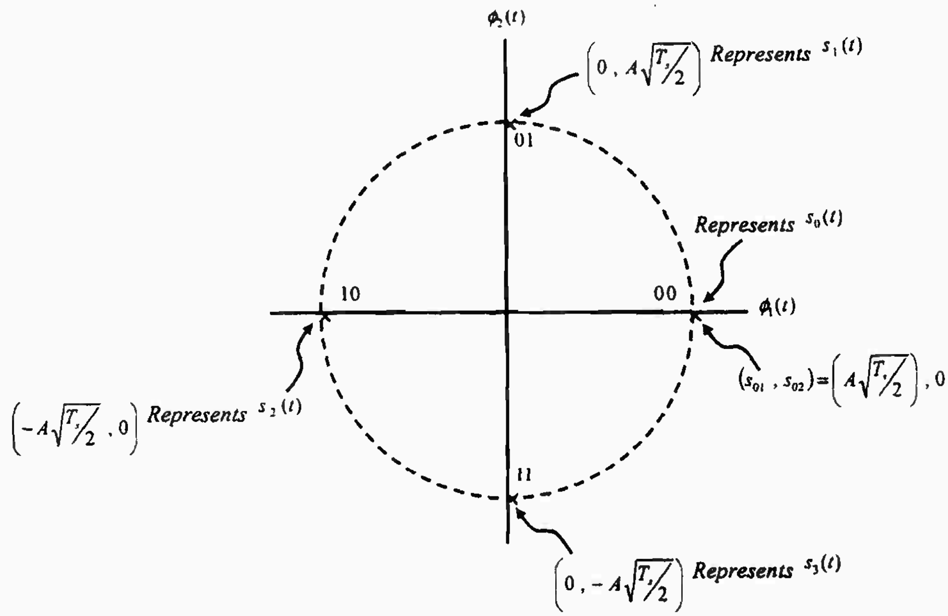


Fig. (8.9) Plot of 4-PSK signal in the signal space

Table (8.9) 8-PSK waveforms as sums of cosines and sines

Input Bits	*Output waveform*			
000	$s_0(t) = A\cos(\omega_c t + 0^o)\cdot p_{T_s}(t-qT_s)$	$= A\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	0
001	$s_1(t) = A\cos(\omega_c t + 45^o)\cdot p_{T_s}(t-qT_s)$	$= \frac{A}{\sqrt{2}}\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	-	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
010	$s_2(t) = A\cos(\omega_c t + 90^o)\cdot p_{T_s}(t-qT_s)$	= 0	-	$A\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
011	$s_3(t) = A\cos(\omega_c t + 135^o)\cdot p_{T_s}(t-qT_s)$	$= \frac{-A}{\sqrt{2}}\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	-	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
100	$s_4(t) = A\cos(\omega_c t + 180^o)\cdot p_{T_s}(t-qT_s)$	$= -A\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	0
101	$s_5(t) = A\cos(\omega_c t + 225^o)\cdot p_{T_s}(t-qT_s)$	$= \frac{-A}{\sqrt{2}}\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
110	$s_6(t) = A\cos(\omega_c t + 270^o)\cdot p_{T_s}(t-qT_s)$	= 0	+	$A\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$
111	$s_7(t) = A\cos(\omega_c t + 315^o)\cdot p_{T_s}(t-qT_s)$	$= \frac{A}{\sqrt{2}}\cos(\omega_c t)\cdot g_{T_s}(t-qT_s)$	+	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot g_{T_s}(t-qT_s)$

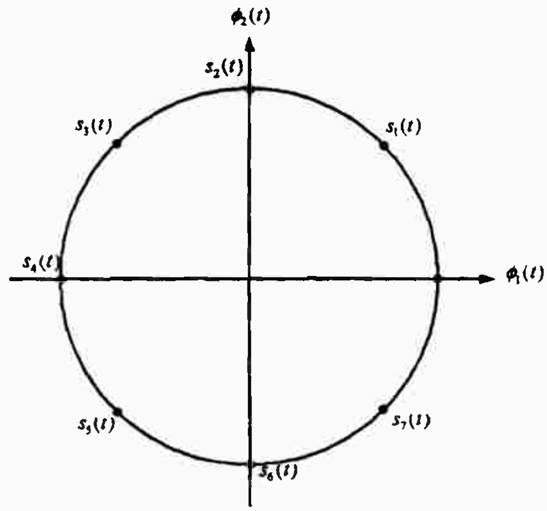


Fig. (8.10) 8-PSK constellation

Fig. (8.10) shows the 8-PSK constellation

Consider now ASK. All ASK output signals are simply $\cos \omega_c t . g_{T_s}(t - kT_s)$ terms, with different amplitudes.

$$\phi_1(t) = \sqrt{2/T_s} \cos \omega_c t \cdot g_{T_s}(t - qT_s) \qquad (8-21)$$

The amplitudes are given in Table (8.10) for BASK / 4-ASK / 8-ASK

Now we consider QAM. Here, the information is embedded in both the **phase** (θ) and the amplitude A of the cosine waveform.

$$s_i(t) = A_i \cos(\omega_c t + \theta_i) \cdot g_{T_s}(t - qT_s) \qquad (8-22)$$

$$= A_i \cos \theta_i \cos \omega_c t \cdot g_{T_s}(t - qT_s)$$

$$- A_i \sin \theta_i \sin \omega_c t \cdot g_{T_s}(t - qT_s) \qquad (8-23)$$

Again we have the orthonormal basis $\{\phi_1(t), \phi_2(t)\}$ as

$$\phi_1(t) = \sqrt{2/T_s} \cos \omega_c t \; g_{T_s}(t - qT_s) \qquad (8-24)$$

$$\phi_2(t) = \sqrt{2/T_s} \sin \omega_c t \; g_{T_s}(t - qT_s) \qquad (8-25)$$

Thus,

$$s_i(t) = a_{i1} \phi_1(t) + a_{i2} \phi_2 \qquad (8-26)$$

We end up with

$$a_{i1} = A_i \sqrt{T_s / 2} \cos \theta_i \qquad (8-27)$$

Table (8.10) BASK / 4-ASK / 8-ASK

	Output waveform	Output waveform represented on orthonthonormal basis
BASK	$s_0(t)$	$s_0 = a_{01} = -A\sqrt{T_s/2}$
	$s_1(t)$	$s_0 = a_{11} = A\sqrt{T_s/2}$
4-ASK	$s_0(t)$	$s_0 = a_{01} = -3A\sqrt{T_s/2}$
	$s_1(t)$	$s_1 = a_{11} = -A\sqrt{T_s/2}$
	$s_2(t)$	$s_2 = a_{21} = A\sqrt{T_s/2}$
	$s_3(t)$	$s_3 = a_{31} = 3A\sqrt{T_s/2}$
8-ASK	$s_0(t)$	$s_0 = a_{01} = -7A\sqrt{T_s/2}$
	$s_1(t)$	$s_1 = a_{11} = -5A\sqrt{T_s/2}$
	$s_2(t)$	$s_2 = a_{21} = -3A\sqrt{T_s/2}$
	$s_3(t)$	$s_3 = a_{31} = -A\sqrt{T_s/2}$
	$s_4(t)$	$s_4 = a_{41} = A\sqrt{T_s/2}$
	$s_5(t)$	$s_5 = a_{51} = 3A\sqrt{T_s/2}$
	$s_6(t)$	$s_6 = a_{61} = 5A\sqrt{T_s/2}$
	$s_7(t)$	$s_7 = a_{71} = 7A\sqrt{T_s/2}$

$$a_{i2} = A_i\sqrt{T_s/2}\sin\theta_i \qquad (8-28)$$

Thus,

$$s_i(t) \leftrightarrow \vec{s}_i = (a_{i1}\ ,\ a_{i2})$$
$$= \left(A_i\sqrt{T_s/2}\cos\theta_i\ ,\ A_i\sqrt{T_s/2}\sin\theta_i\right) \qquad (8-29)$$

A state $s_i(t)$ is shown (Fig. 8.11) and 16-QAM constellation is illustrated (Fig. 8.12) for states $(0000$ to $1111)$

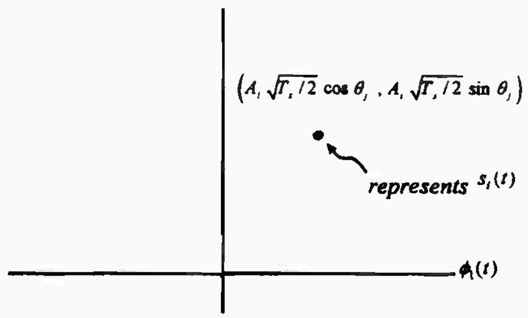


Fig. (8.11) Single QAM state

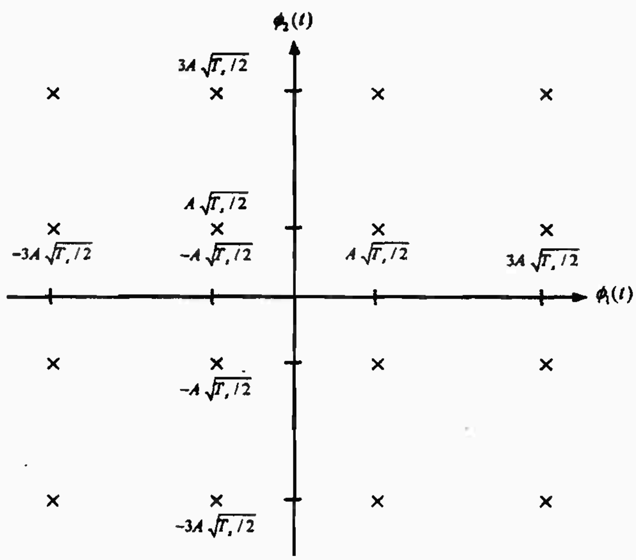


Fig. (8.12) 16-QAM constellation

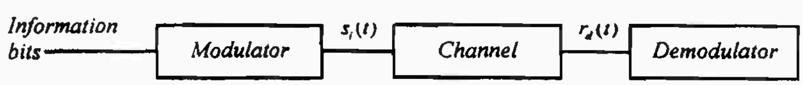


Fig. (8.13) Modulator and demodulator

8.4 Detection of Bandpass Signals:

The demodulator receives the signal sent across the channel and turns it back to bits (Fig. 8-13).

The received signal $r_d(t)$ is given by

$$r_d(t) = \hat{s}_i(t) + n(t) \qquad (8-30)$$

If for example one of the 4-PSK signals $\{s_0(t)\,,\, s_1(t)\,,\, s_2(t)\,,\, s_3(t)\}$ was sent, the demodulator's task is to figure out which one of these signals was sent, given $r_d(t)$. Then it translates this information back to bits. The main point in the design of the demodulator is to keep the error due to noise minimum so as to make the guess work as close to truth as possible. The first thing a demodulator does is to transform the continuous time function $r_d(t)$ to a vector using the orthonormal basis

$$r_d(t) = r_1\ \phi_1(t) + r_2\ \phi_2 + ..r_N\ \phi_N \qquad (8-31)$$

Thus, $r_d(t)$ can be represented as vector $\vec{r_d}$. For 4-PSK, the orthonormal set $\{\phi_1(t)\,,\, \phi_2(t)\}$ are

$$\phi_1(t) = \sqrt{2/T_s}\ \cos\omega_c t \cdot g_{T_s}(t - qT_s) \qquad (8-32)$$

$$\phi_2(t) = \sqrt{2/T_s}\ \sin\omega_c t \cdot g_{T_s}(t - qT_s) \qquad (8-33)$$

Suppose that the transmitted $s_i(t)$ is $s_m(t)$ which is one the 4-PSK set $\{s_0(t),\, s_1(t),\, s_2(t),\, s_3(t)\}$. We have seen that $s_m(t)$ may be given as projections on the two basis functions $\phi_1(t)$ and $\phi_2(t)$. The noise that counts is the noise projections on the same basis functions $\phi_1(t)$ and $\phi_2(t)$. Thus, $\phi_1(t)$ and $\phi_2(t)$ serve also as basis functions for $r_d(t)$. In general, referring to eqn. (8 – 31),

$$r_1 = \int_{qT_s}^{(q+1)T_s} r_d(t)\ \phi_1(t)\ dt \qquad (8-34)$$

$$= \int_{qT_s}^{(q+1)T_s} \left[\hat{s}_m(t) + n(t) \right] \phi_1(t)\ dt \qquad (8-35)$$

$$= \int_{qT_s}^{(q+1)T_s} \hat{s}_m(t)\ \phi_1(t)\ dt + n(t)\ \phi_1(t)\ dt \qquad (8-36)$$

$$= a_{m1} + n_1 \qquad (8-37)$$

where a_{m1} is the coefficient of $s_m(t)$ along $\phi_1(t)$ and n_1 is the noise projection along $\phi_1(t)$ and is given by

$$n_1 = \int_{qT_s}^{(q+1)T_s} n(t)\, \phi_1(t)\, dt \qquad (8-38)$$

$$a_{m1} = \int_{qT_s}^{(q+1)T_s} s_m(t)\, \phi_1(t)\, dt \qquad (8-39)$$

We note that $s(t)$ and $n(t)$ have units of volts, since $\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$. $\phi_1(t)$ has units of ($\sec^{-1/2}$) for 1Ω operation, a_{m1} or n_1 and r_1 all have the same units ($\sec^{-1/2}$). noting $\phi_1(t)$ is given by eqn. (8-32).

$a_{m1} = A\sqrt{T_s/2}$, $E = A^2\, T_s$, $s_1(t) = A\cos\omega_c t$, $s_1(t) = a_{m1}\, \phi_1(t)$. Thus, n_1^2 has units $V^2\, see^{-1}$. Noting $n_1^2 = \eta_1$ hence n_1 becomes another Gaussian random variable with zero mean and variance σ_n^2.

Similarly, we have

$$r_2 = a_{m2} + n_2 \qquad (8-40)$$

where a_{m2} is the coefficient of $s_m(t)$ along $\phi_2(t)$ and n_2 is the projection of noise along $\phi_2(t)$ and is a Gaussian random variable with zero mean and variance σ_n^2. Note that n_1 and n_2 are statistically independent. We have

$$a_{m2} = \int_{qT_s}^{(q+1)T_s} s_m(t)\, \phi_2(t)\, dt \qquad (8-41)$$

$$n_2 = \int_{qT_s}^{(q+1)T_s} n(t)\, \phi_2(t)\, dt \qquad (8-42)$$

Thus, the receiver will process only information and noise contained in r_1 and r_2 since in this case, we have only $\phi_1(t)$ and $\phi_2(t)$. The rest of the noise i.e. apart from whatever is intercepted along $\phi_1(t)$ and $\phi_2(t)$ will be discarded. That part of the receiver which does this job is called correlator receiver front end (Figs. 8.14, 15). The correlator calculates r_i i.e. it turns $r(t)$ into $\{r_i(t)\}$ or $\hat{r}$, where

$$r_i = \int_{qT_s}^{(q+1)T_s} r_d(t)\, \phi_i(t)\, dt \qquad (8-43)$$

Now it is up to the rest of the demodulator which is called the decision device to pick up which $\hat{s}_i$ which the input is closest to. For 4-PSK decision device we have

$$\vec{r} = (r_1 , r_2) \qquad (8-44)$$

$$r_1 = a_{m1} + n_1 \qquad (8-45)$$

$$r_2 = a_{m2} + n_2 \qquad (8-46)$$

$$\vec{r} = \hat{\vec{s}}_m + \vec{n} \qquad (8-47)$$

Thus out of $\vec{r}$ which is the noisy version of $\hat{\vec{s}}_m$, the decision device tries to figure out $\hat{s}_m(t)$ that was sent. Finally, it is up to a look up table to figure out the bits corresponding to this $\hat{\vec{s}}_m$. Fig. (8.16) shows (r_1 , r_2). If the decision device decides that $s_1(t)$ was sent then the corresponding bits must be $01(a_{11} = o, a_{12} = 1)$. The criterion for the operation of the decision device is to require minimum probability of error P_ϵ i.e., choosing the most likely $\hat{\vec{s}}_i$ given $\vec{r}$, i.e.,

$$\hat{\vec{s}}_i = \arg\max \left[f\left(\vec{s}_i \middle| \vec{r} \right) \right] \qquad (8-48)$$

where $\hat{\vec{s}}_i = \arg\max_{\vec{s}_i}$ means: let the decision device choose as output $\hat{\vec{s}}_i$ the value of $\vec{s}_i$ that maximizes what follows. What follows in this case is $f\left(\vec{s}_i \middle| \vec{r}\right)$ which is the probability that $\vec{s}_i$ occurs (was sent) given that $\vec{r}$ is received. Altogether, eqn. (8 – 48) states: let the decision device choose the output $\hat{\vec{s}}_i$ to be the value of $\vec{s}_i$ that is most likely to occur given $\vec{r}$. We may rewrite eqn. (8 – 48) using Bayes rule as

$$\hat{\vec{s}}_i = \arg\max \left[\frac{f\left(\vec{r} \middle| \vec{s}_i\right) f\left(\vec{s}_i\right)}{f\left(\vec{r}\right)} \right] \qquad (8-49)$$

Since the term $f\left(\vec{r}\right)$ in the denominator is not a function of $\vec{s}_i$, it does not affect the decision process; hence it can be dropped out. Thus, we require

$$\hat{\vec{s}}_i = \arg\max \left[f\left(\vec{r} \middle| \vec{s}_i\right) f\left(\vec{s}_i\right) \right] \qquad (8-50)$$

The term $f\left(\vec{r} \middle| \vec{s}_i\right)$ is the probability of having $\vec{r}$ given $\vec{s}_i$ was sent. This is equivalent to saying $f\left(\vec{r} \middle| \vec{s}_i\right) = P\left(\vec{n} = \vec{r} - \vec{s}_i\right)$, since to have $\vec{r}$ and $\vec{s}_i$ is the same as having $\vec{n} = \vec{r} - \vec{s}_i$ because we have defined $\vec{r} = \hat{\vec{s}}_i + \vec{n}$. Thus, eqn. (8 – 50) becomes

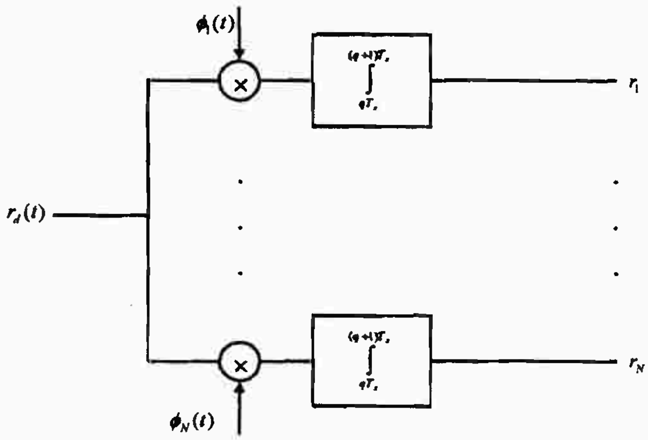


Fig. (8.14) Correlator receiver front end

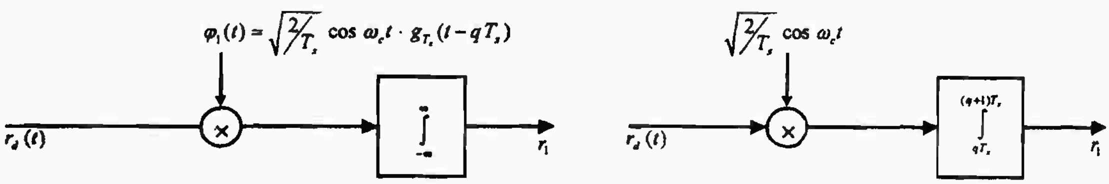


Fig. (8.15) Two equivalent implementations

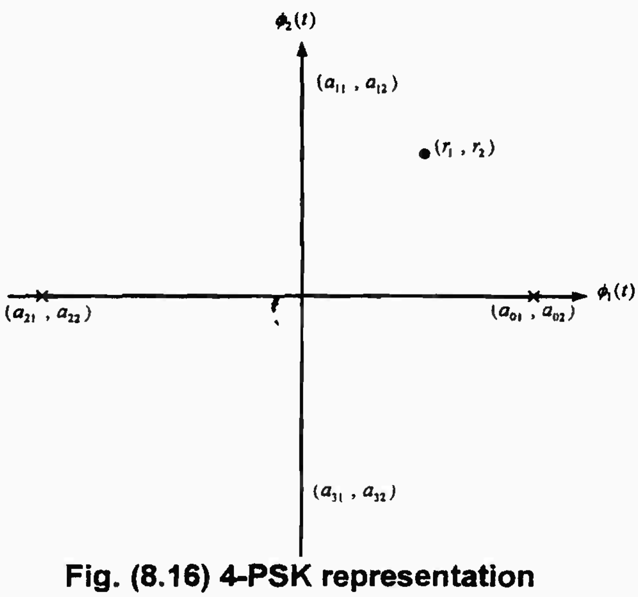


Fig. (8.16) 4-PSK representation

$$\hat{\bar{s}}_i = \arg\max\left[f\left(\bar{n} = \bar{r} - \bar{s}_i\right) f\left(\bar{s}_i\right)\right] \qquad (8-51)$$

The noise term $\bar{n}$ corresponds to $\bar{n} = (n_1, n_2 .. n_N)$. For 4-PSK, we have only two terms n_1, n_2, which are statistically independent, i.e.,

$$f\left(\bar{n}\right) = f\left(n_1\right) f\left(n_2\right) \qquad (8-52)$$

Hence, eqn. (8 – 51), becomes

$$\hat{\bar{s}}_i = \arg\max\left[f\left(n_1 = r_1 - a_{i1}\right) f\left(n_2 = r_2 - a_{i2}\right) f\left(\bar{s}_i\right)\right] \qquad (8-53)$$

Since the noise terms are Gaussian with zero mean

$$f\left(n_i\right) = \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-n_i^2/2\sigma_0^2} \qquad (8-54)$$

Using eqn. (8 – 53),

$$\hat{\bar{s}}_i = \arg\max\left[\left(\frac{1}{\sqrt{2\pi\sigma_0^2}}\right)^2 e^{-(r_1 - a_{i1})^2/2\sigma_0^2 - (r_2 - a_{i2})^2/2\sigma_0^2} f\left(\bar{s}_i\right)\right] \qquad (8-55)$$

We may take ℓn of the argument of the operator $\arg\max$ since what maximizes $f(x)$ maximizes $\ell n\ f(x)$.

$$\hat{\bar{s}}_i = \arg\max\left[\ell n\left(\frac{1}{2\pi\sigma_0^2}\right) - \frac{\left(r_1 - a_{i1}\right)^2}{2\sigma_0^2} - \frac{\left(r_2 - a_{i2}\right)^2}{2\sigma_0^2} + \ell n\left\{f\left(\bar{s}_i\right)\right\}\right] \qquad (8-56)$$

The constant may be removed since it will not affect the decision process

$$\hat{\bar{s}}_i = \arg\max\left[-\frac{\left(r_1 - a_{i1}\right)^2}{2\sigma_0^2} - \frac{\left(r_2 - a_{i2}\right)^2}{2\sigma_0^2} + \ell n\left\{f\left(\bar{s}_i\right)\right\}\right] \qquad (8-57)$$

Multiplying the argument by $2\sigma_0^2$

$$\hat{\bar{s}}_i = \arg\max\left[-\left(r_1 - a_{i1}\right)^2 - \left(r_2 - a_{i2}\right)^2 + 2\sigma_0^2\ \ell n\left\{f\left(\bar{s}_i\right)\right\}\right] \qquad (8-58)$$

The value that maximizes the argument is the value that minimizes minus the argument.

$$\hat{\bar{s}}_i = \arg\min\left[\left(r_1 - a_{i1}\right)^2 + \left(r_2 - a_{i2}\right)^2 - 2\sigma_0^2\ \ell n\left\{f\left(\bar{s}_i\right)\right\}\right] \qquad (8-59)$$

$$\hat{\bar{s}}_i = \arg\min\left[\left(\bar{r} - \bar{s}_i\right)^2 - 2\sigma_0^2\ \ell n\left\{f\left(\bar{s}_i\right)\right\}\right] \qquad (8-60)$$

If all $\bar{s}_i$ are equally likely the last term becomes a constant which may be discarded. Using the vector rotation, eqn. (8 – 59) becomes

$$\hat{\vec{s}}_i = \arg\min\left[\left|\vec{r} - \vec{s}_i\right|^2\right] \qquad (8-61)$$

Thus, the decision device chooses $\hat{\vec{s}}_i$ closest to $\vec{r}$. This is in agreement with minimum distance likelihood concept.

8.5 Implementation of Bandpass Demodulators:

Fig. (8.17) shows the complete demodulator consisting the receiver front end and the decision device. Such a demodulator is called the correlator receiver. The receiver front end transforms $r_d(t)$ to vector $\vec{r}$. The decision device contains a processor which receives $\vec{r}$ and outputs M values of $\left|\vec{r} - \vec{s}_i\right|^2_{i=1..M}$. For 4-PSK there are 4 values. Next M adders are used, one for each output which adds $-2\sigma_0^2\, \ell n\left[f\,(\vec{s}_i)\right]$. The third stage is to choose the minimum value of $\left|(\vec{r} - \vec{s}_i)\right|^2 - 2\sigma_0^2\, \ell n\left[f\,(\vec{s}_i)\right]$, $i = 1..M$. Hence, the output is $\hat{\vec{s}}_i$ given by eqn. (8 – 60) or eqn. (8 – 61) (for equiprobable states)

A second way of designing the receiver front end is shown (Fig. 8.18) where use is made of matched filters. Comparing Figs. (8.17) and (8.18), we want to show that the output of the matched version receiver front end is the same as the output of the correlator version receiver front end.

In (Fig. 8.18), the impulse response of the matched filter

$$h_i(t) = \phi_i(T_s - t)\,,\; i = 1...M \qquad (8-62)$$

Thus, the output of the matched filter A_i

$$R_i = h_i(t) * r_d(t)\;\Big|_{t=T_s} \qquad (8-63)$$

$$= \int_{-\infty}^{\infty} h_i(\tau)\, r_d(t-\tau)\, d\tau\;\Big|_{t=T_s} \qquad (8-64)$$

$$= \int_{-\infty}^{\infty} \phi_i(T_s-\tau)\, r_d(T_s-\tau)\, d\tau\;\Big|_{t=T_s} \qquad (8-65)$$

Putting $T_s - \tau = u$, $d\tau = -du$, refining to eqn.(8 -32),

$$R_i = \int_{-\infty}^{\infty} \phi_i(u)\, r_d(u)\, du = r_i \qquad (8-66)$$

Hence, the two receivers of Fig. (8.17) and Fig. (8.18) are exactly equivalent.

Another version of matched filter receiver can be conceived by considering eqns. (8 – 59), (8 – 60). Assuming $\vec{r} = (r_1\,,\, r_2)$

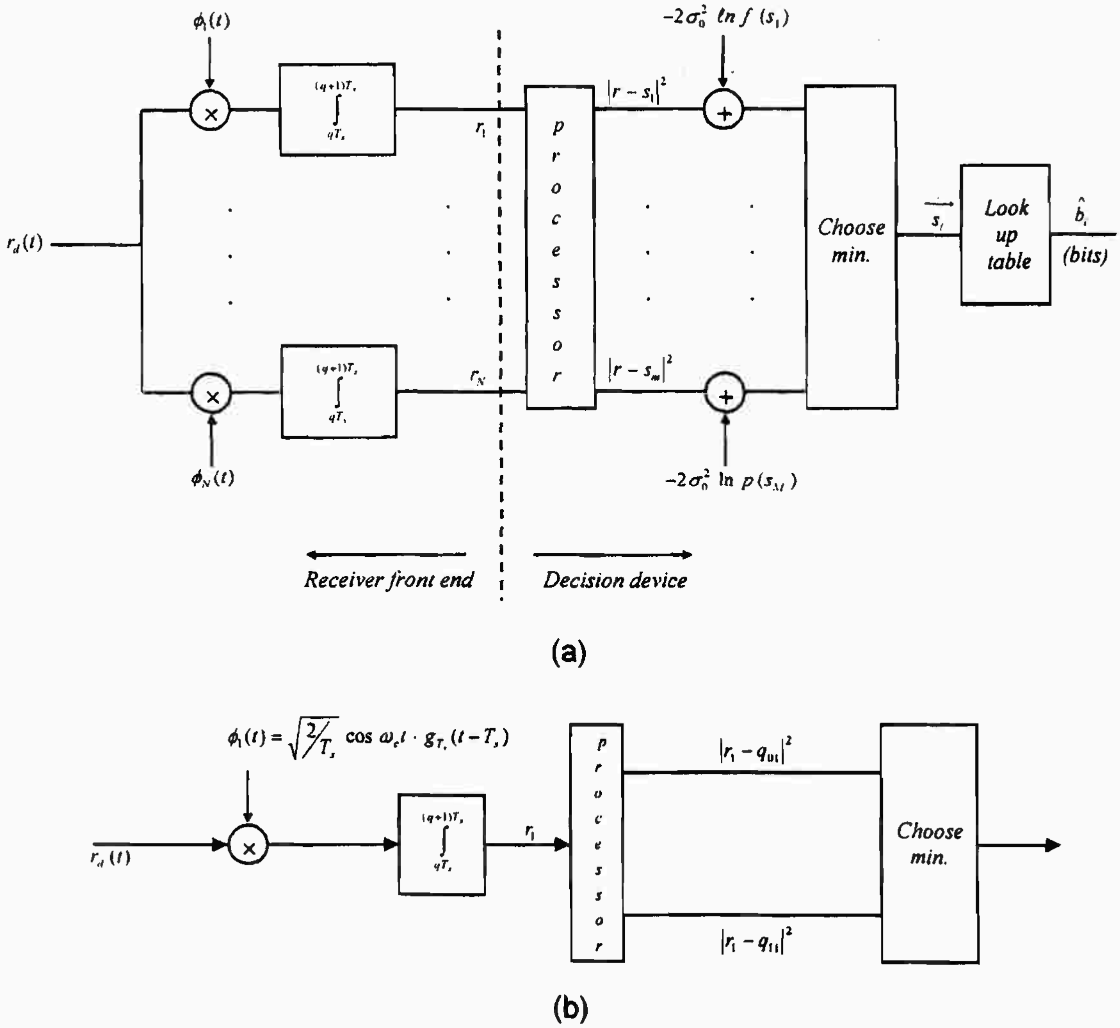


Fig. (8.17) Complete correlator receiver
a) General *b) BPSK*

$$\hat{\vec{s}}_i = \arg\min\left[|\vec{r}| - 2(r_1\ a_{i1} + r_2\ a_{i2}) + |\vec{s}_i|^2 - 2\sigma_0^2\ \ell n\{f\ (\vec{s}_i)\}\right] \qquad (8-67)$$

Now the $|\vec{r}|^2$ term is the same for all $\vec{s}_i$ and will not affect the decision process

$$\hat{\vec{s}}_i = \arg\min\left[-2(r_1\ a_{i1} + r_2\ a_{i2}) + |\vec{s}_i|^2 - 2\sigma_0^2\ \ell n\{f\ (\vec{s}_i)\}\right] \qquad (8-68)$$

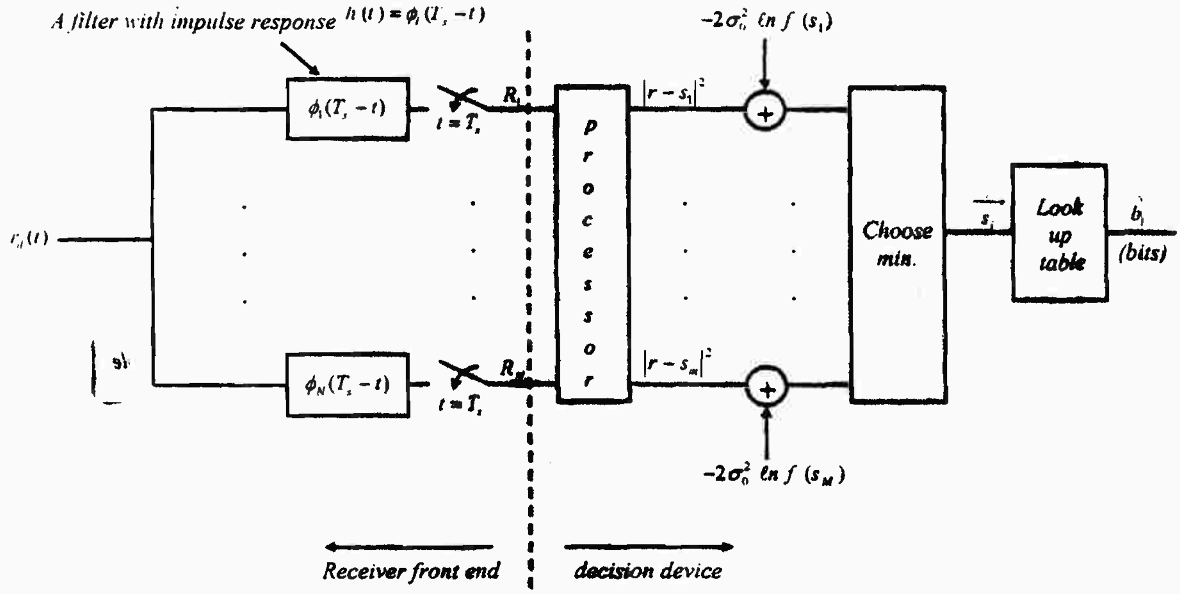


Fig. (8.18) Matched filter receiver-version

we may now use $\arg\max$ by inverting the sign and dividing by 2

$$\hat{\vec{s}}_i = \arg\max\left[(r_1\ a_{i1} + r_2\ a_{i2}) - \frac{1}{2}|\vec{s}_i|^2 + \sigma_0^2\ \ell n\{f\ (\vec{s}_i)\}\right] \qquad (8-69)$$

$$= \arg\max\left[\vec{r}\cdot\vec{s}_i + c_i\right] \qquad (8-70)$$

where

$$c_i = \frac{-1}{2}\left[|\vec{s}_i|^2 - 2\sigma_0^2\ \ell n\{f\ (\vec{s}_i)\}\right] \qquad (8-71)$$

Fig. (8.19) shows the implementation of eqn. (8 – 70) for two basis functions. The output R_1 (Fig. 8.19) is given by

$$R_1 = \int_{-\infty}^{\infty} r_d(t)\ s_1(t)\ dt \qquad (8-72)$$

$$= \int_{-\infty}^{\infty}\left[r_1\ \phi_1(t) + r_2\ \phi_2(t)\right]\cdot\left[a_{11}\ \phi_1(t) + a_{12}\ \phi_2(t)\right]dt$$

$$= \left[r_1\ a_{11}\int_{-\infty}^{\infty}\phi_1(t)\ \phi_1(t)\ dt + r_2\ a_{12}\int_{-\infty}^{\infty}\phi_2(t)\ \phi_2(t)\ dt + \right.$$

$$r_1\ a_{12}\int_{-\infty}^{\infty}\phi_1(t)\ \phi_2(t)\ dt + r_2\ a_{21}\int_{-\infty}^{\infty}\phi_1(t)\ \phi_2(t)\ dt \qquad (8-73)$$

Because $\phi_1(t)$ and $\phi_2(t)$ are orthonormal

$$R_1 = (r_1\, a_{11} + r_2\, a_{12}) = \vec{r} \cdot \vec{s}_1 \qquad (8-74)$$

The choose max block picks up the largest value of the argument in eqn. (8 – 70). Finally a look up table determines the output bits $\hat{\vec{b}}_i$.

8.6 Performance Measures:

To evaluate the modulator-demodulator pair, it is required to calculate P_ϵ, the probability that the demodulator makes an error when deciding what signal was, the smaller P_ϵ the better the demodulator. Consider BPSK demodulator 0 is mapped to $s_0(t)$ and 1 to $s_1(t)$ where $s_0(t) = A\sqrt{T_s/2}\,\phi_1(t)$ and $s_1(t) = -A\sqrt{T_s/2}\,\phi_1(t)$ and $\phi_1(t) = \sqrt{T_s/2}\,\cos\omega_c t \cdot g_{T_s}(t - qT_s)$. The receiver front end takes $r_d(t)$ and maps it to $\vec{r}$ and the decision device takes $\vec{r}$ and figures out which symbol $\vec{s}_m$ was sent (Fig. 8.20). The decision device outputs the symbol which is closest to the received $r_d(t)$. In accordance with eqn. (8 -60),(8 – 61),

$$\hat{\vec{s}}_i = \arg\max\left[(r_1 - a_{i1})^2 - \sigma_0^2\, \ell n\{f(\vec{s}_i)\}\right] \qquad (8-75)$$

For equally likely symbols

$$\hat{\vec{s}}_i = \arg\min(r_1 - a_{i1})^2 \qquad (8-76)$$

This result means that we output the symbol closest to the received $\vec{r}$. To calculate $P_\epsilon = P_B$ for BPSK we note that error occurs if the receiver interprets a symbol opposite to that which was sent

$$P_B = f\left(s_1(t)\,|\, s_0(t)\,sent\right) f\left(s_0(t)\right) + f\left(s_0(t)\,|\, s_1(t)\,sent\right) f\left(s_1(t)\right) \qquad (8-77)$$

If the symbols are equally likely

$$P_B = 0.5\left[f\left(s_1(t)\,|\, s_0(t)\,sent\right) + f\left(s_0(t)\,|\, s_1(t)\,sent\right)\right] \qquad (8-78)$$

Consider Fig. (8.21a), assuming that $s_1(t)$ is sent while $n_1 < A\sqrt{T_s/2}$. The decision device picks up correctly $r_1 = a_{11} + n_1$. In Fig. (8.21b) $n_1 > A\sqrt{T_s/2}$, then r_1 is closest to a_{01} and the demodulator will decide $s_0(t)$, hence making an error. Thus,

$$f\left(s_0(t)\,|\, s_1(t)\,sent\right) = f\left(n_1 > A\sqrt{T_s/2}\right) \qquad (8-79)$$

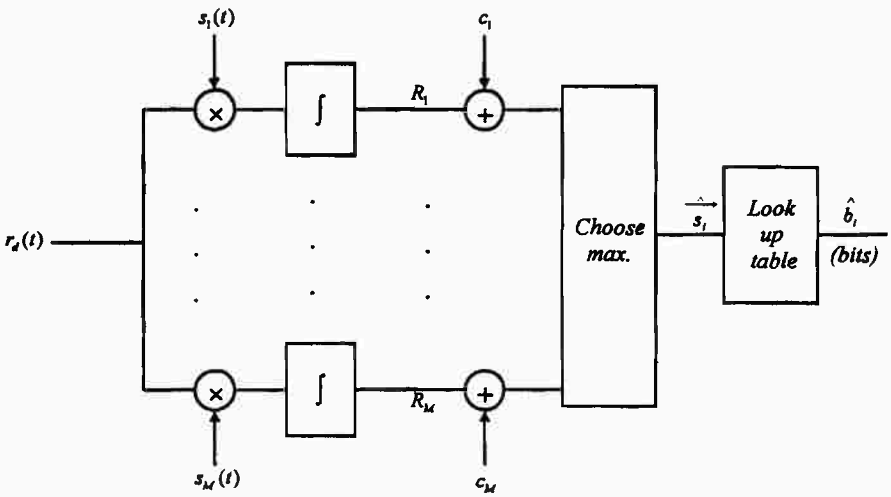


Fig. (8.19) Matched filter implementation-alternative version

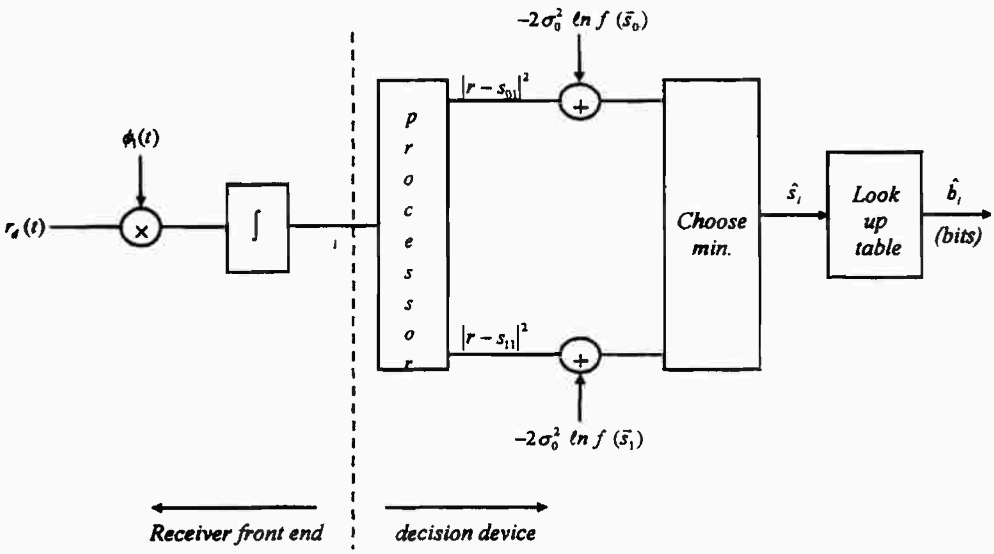


Fig. (8.20) BPSK demodulator

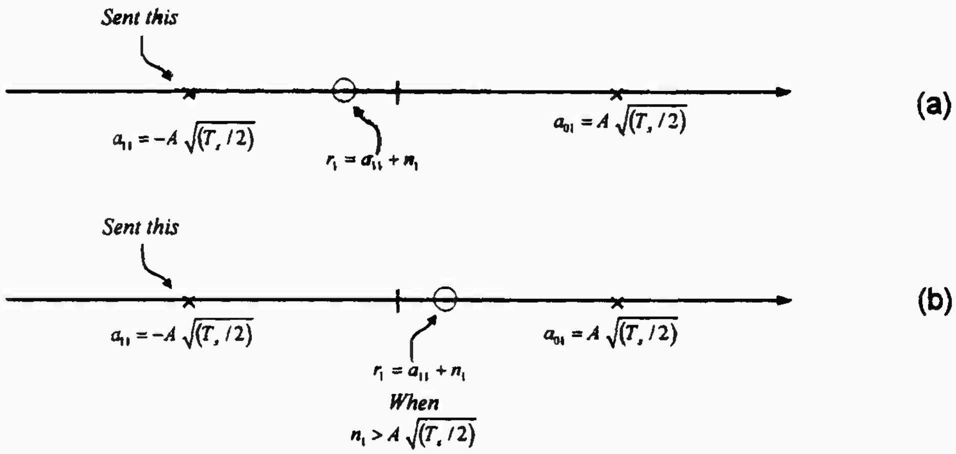


Fig. (8.21) BPSK decision process

a) $s_1(t)$ *sent* $n_1 < A\sqrt{T_s/2}$ b) $s_1(t)$ *sent* $n_1 > A\sqrt{T_s/2}$

Similarly,

$$f\left(s_1(t)\middle| s_0(t)\,sent\right)=f\left(n_1 < A\sqrt{T_s/2}\right) \qquad (8-80)$$

Therefore, from eqns. (8 – 78), (8 – 79), (8 – 80)

$$P_B = 0.5\left[P\left(n_1 > A\sqrt{T_s/2}\right)+P\left(n_1 < -A\sqrt{T_s/2}\right)\right] \qquad (8-81)$$

From Gaussian properties $P(x > A) = P(x < -A)$. Thus,

$$P_B = P\left(n_1 > A\sqrt{T_s/2}\right) \qquad (8-82)$$

$$= \int_{A\sqrt{T_s/2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-n_1^2/2\sigma_0^2} dn_1 \qquad (8-83)$$

Putting $u = n_1/\sigma_0$

$$P_B = \frac{1}{\sqrt{2\pi}} \int_{\frac{A\sqrt{T_s/2}}{\sigma_0}}^{\infty} e^{-u^2/2}\, du \qquad (8-84)$$

$$= Q\left(\frac{A\sqrt{T_s/2}}{\sigma_0}\right) \qquad (8-85)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2}\, du \qquad (8-86)$$

Now, the energy per symbol E_s (in this case E_b) BPSK is given by

$$E_b = \int_0^{T_b} s_i^2(t)\, dt \qquad (8-87)$$

$$= \int_0^{T_s} A^2 \cos^2 \omega_c t\, dt$$

$$= \frac{1}{2} A^2 T_b \qquad (8-88)$$

Referring to Ex 8.1,

$$\sigma_0^2 = \frac{kT}{2} = \frac{\eta}{2} \qquad (8-89)$$

Thus, eqn. (8 – 85) becomes

$$P_B = Q\left(\frac{A\sqrt{T_b/2}}{\sigma_n}\right)$$

$$= Q\left(\sqrt{2E_b/\eta}\right) \qquad (8-90)$$

Thus, P_B depends on the signal to noise ratio E_b/η which is actually the symbol energy divided by the noise PSD. As (E_b/η) increases, P_B decreases rapidly (Fig. 8.22),

Ex. 8.1

Obtain the variance of the projected noise along any basis function

Solution

For AWGN, we have PSD. $S_n(f) = \eta/2 = kT/2$ and the autocorrelation function is given by

$$R(\tau) = E[n(t)\, n(t-\tau)]$$

$$= \int_{-\infty}^{\infty} S_n(f)\, e^{j2\pi f\tau}\, df$$

$$= \int_{-\infty}^{\infty} \frac{\eta}{2} e^{j2\pi f\tau}\, df$$

$$= \frac{\eta}{2}\, \delta(\tau)$$

The amplitude of the noise $n(t)$ is given by

$$f(n) = \frac{1}{\sqrt{2\pi\sigma_0^2}}\, e^{-n^2/2\sigma_0^2} \qquad (8-91)$$

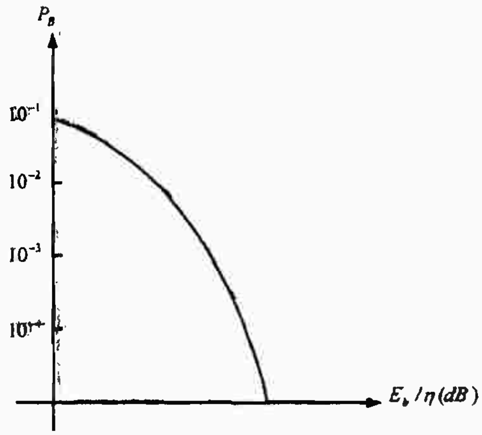


Fig. (8.22) P_B for BPSK

We note that $\sigma_0^2 = \eta B$ where B is the bandwidth. For unlimited bandwidth $\sigma_0^2 = \infty$, i.e., the noise power is infinite. However, when $r_d(t)$ is correlated with an orthonormal function $\phi(t)$ the noise in the output has a finite variance

$$r_j = \int_{-\infty}^{\infty} r_d(t)\,\phi_j(t)\,dt = a_{ij} + n_j \qquad (8-92)$$

$$a_{ij} = \int_{-\infty}^{\infty} s_j(t)\,\phi_j(t)\,dt \qquad (8-93)$$

$$n_j = \int_{-\infty}^{\infty} n(t)\,\phi_j(t)\,dt \qquad (8-94)$$

The variance of n_j is

$$\sigma_j^2 = E\left[n_j^2\right] = E\left\{\left[\int_{-\infty}^{\infty} n_j(t)\,\phi_j(t)\right]^2\right\} \qquad (8-95)$$

$$= E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n(t_1)\,\phi_j(t_1)\,n(t_2)\,\phi_j(t_2)\,dt_1\,dt_2\right\} \qquad (8-96)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left\{n(t_1)\,n(t_2)\right\}\phi_j(t_1)\,\phi_j(t_2)\,dt_1\,dt_2 \qquad (8-97)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\eta}{2}\delta(t_1 - t_2)\,\phi_j(t_1)\,\phi_j(t_2)\,dt_1\,dt_2$$

$$= \frac{\eta}{2} \int_{-\infty}^{\infty} \phi_i^2(t)\, dt = \frac{\eta}{2} \qquad (8-98)$$

Thus, the noise variance along a basis function is finite and equal to $\eta/2$. The probability density function for n_j is therefore:

$$f(n_i) = \frac{1}{\sqrt{\pi\eta}} e^{-n_i^2/\eta} \qquad (8-99)$$

8.7 BPSK:

Binary data are represented by two signals with different phase (antipodal) or phase reversal keying (PRK)

$s_1(t) = A \cos 2\pi f_c t \qquad 0 \le t \le T_b$ for 1

$s_2(t) = -A \cos 2\pi f_c t \qquad 0 \le t \le T_b$ for 0

They are not orthogonal but they have a correlation of -1. All BPSK signals can be graphically represented by a constellation in two dimensional coordinate system

$$\phi_1(t) = \sqrt{2/T_s} \cos 2\pi f_c t \qquad 0 \le t \le T_b \qquad (8-100)$$

$$\phi_2(t) = -\sqrt{2/T_s} \cos 2\pi f_c t \qquad 0 \le t \le T_b \qquad (8-101)$$

For BPSK we represent $s_1(t)$ and $s_2(t)$ as two points on the $\phi_1(t)$ axis $\sqrt{E_b}$ and $-\sqrt{E_b}$ when $E_b = A^2 T_b / 2$.

Fig. (8.23) shows BPSK waveforms. In general, the phase is not continuous at bit boundaries. If $f_c = m R_b = m / T_b$ where m is an integer the bit timing is synchronous with the carrier, then the initial phase at the bit boundary is either 0 or π.

Fig. (8.24) shows PRK modulator waveforms, spectra and phase states. It is seen that the double sided (null to null) bandwidth is $2/T_b = 2R_b$. The bipolar data stream $b(t)$ is formed from the binary data stream as

$$b(t) = \sum_{q=-\infty}^{\infty} b_q\, g_{T_b}(t - qT_b) \qquad (8-102)$$

Where b_q *is* $+1$ *or* -1. Then $b(t)$ is multiplied with a sinusoidal carrier $A\cos 2\pi f_c t$. The result is the BPSK signal.

$$s(t) = Ab(t)\cos 2\pi f_c t \qquad -\infty < t < \infty \qquad (8-103)$$

Fig. (8.25) illustrates a BPSK (PRK) modulator and a coherent demodulator, where the carrier is obtained by a carrier recovery (CR) circuit. In the absence of noise, setting $A = 1$ the output of the correlator at $t = (q+1)T_b$ is given by

$$z(qT_b) = \int_{qT_b}^{(q+1)T_b} r_d(t) \cos 2\pi f_C t \, dt \qquad (8-104)$$

$$= \frac{1}{2} \int_{qT_b}^{(q+1)T_b} a_i \cos^2 2\pi f_C t$$

$$= \frac{1}{2} \int_{qT_b}^{(q+1)T_b} a_i \, (1 + \cos 4\pi f_C t) \, dt$$

$$= \frac{T_b}{2} a_i + \frac{a_i}{8\pi f_C} [\sin 4\pi f_C (q+1) T_b - \sin 4\pi f_c q T_b] \qquad (8-105)$$

For $f_C >> R_b$ the second tem is negligible. The bit error probability can be derived from the formula for general binary signals as in eqn. (7 – 170)

$$P_B = Q\left[\sqrt{1-\rho} \; (E_b/\eta)^{1/2}\right] \qquad (8-106)$$

$$Q(u) = \frac{1}{2} erfc\left(u/\sqrt{2}\right) \qquad (8-107)$$

Since

$$erfc(u) = 1 - erf(u) \qquad (8-108)$$

We may rewrite eqn. (8 – 106) as

$$P_B = Q\left[\sqrt{1-\rho} \; (E_b/\eta)^{1/2}\right] \qquad (8-109)$$

For $\rho = -1$ (PRK)

$$P_B = Q\left(\sqrt{2E_b/\eta}\right) \qquad (8-110)$$

For binary system, we may write P_B according to eqn. (8-105)

$$P_B = \frac{1}{2}\left[1 - erf \; \frac{1}{2}(E_1/\eta)^{1/2}\right] \qquad (8-111)$$

Defining the avenge energy for symbol

$$< E > = \frac{1}{2}(E_1 + E_0). \qquad (8-112)$$

For OOK $E_0 = O$

$$P_B = \frac{1}{2}\left[1 - erf \left(1/\sqrt{2}\right)(< E > /\eta)^{1/2}\right] \qquad (8-113)$$

Defining carrier to noise C/N as

$$C = < E > /T_b \qquad (8-114)$$

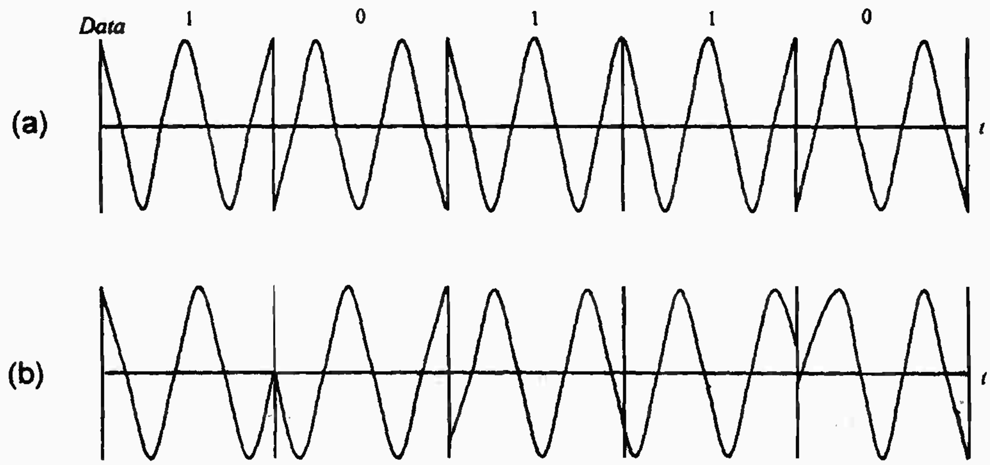


Fig. (8.23) BPSK

a) $f_C = 2/T_b$ b) $f_C = 1.8/T_b$

$$N = \eta B \tag{8 - 115}$$

$$\frac{<E>}{\eta} = T_b\, B\;\; C/N \tag{8 - 116}$$

where C is the received carrier power averaged over all symbol periods as N is the normalized power in bandwidth B

$$P_B = \frac{1}{2}\left[1 - erf\left\{\frac{(T_b B)^{1/2}}{\sqrt{2}}\left(\frac{C}{N}\right)^{1/2}\right]\right. \tag{8 - 117}$$

For minimum Nyquist bandwidth, $T_b\, B = 1$

$$\frac{<E>}{\eta} = C/N$$

For PRK, $E_b = <E>$ since $E_1 = E_0$

$$P_B = \frac{1}{2}\left[1 - erf\,(E_b/\eta)^{1/2}\right] \tag{8 - 118}$$

We may consider the case when the difference between phasor states is less than 180°. Then P_B performance is derived by resolving the allowed phasor states into a residual carrier and a reduced amplitude PRK signal (Fig. 8.26)

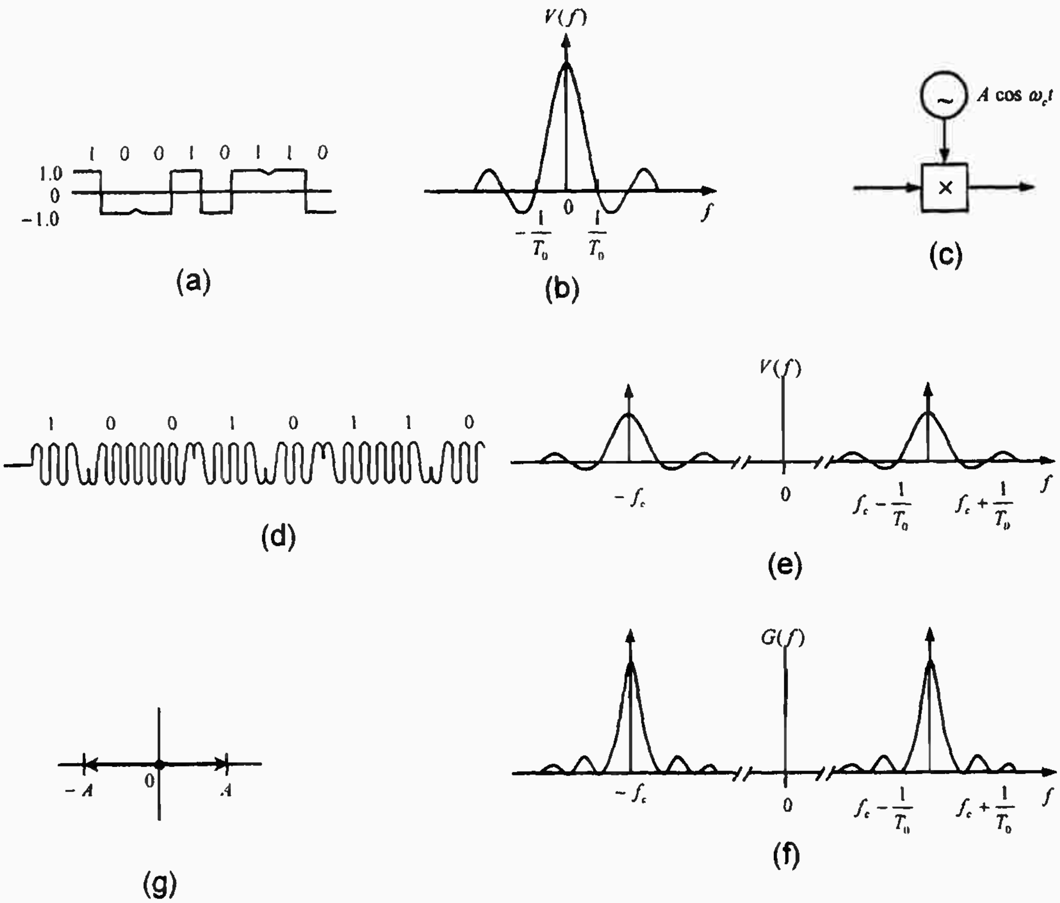


Fig. (8.24) PRK (band unlimited)

a) Baseband *b) Baseband voltage spectrum*
c) Modulator *d) PRK signal*
e) PRK voltage spectrum *f) PRK PSD*
g) PRK space state

$$= \frac{1}{2}\left[1 - erf\left\{(T_b B)^{1/2}(C/N)^{1/2}\right\}\right] \qquad (8-119)$$

The residual carrier which contributes nothing to symbol detection can be employed as a pilot transmission and used for carrier recovery purposes at the receiver. If the difference between phasor states is $\Delta\theta$, we define

$$g = \sin(\Delta\theta / 2) \qquad (8-120)$$

g Is the portion of the transmitted signal voltage which conveys information and the corresponding proportion of total symbol energy is

$$g^2 = \sin^2(\Delta\theta / 2) = \frac{1}{2}(1 - \cos\Delta\theta) \qquad (8-121)$$

where $\cos\Delta\theta$ is the scalar product of the two unit amplitude symbol phasors. Denoting this quantity by the normalized correlation coefficient

$$g = \sqrt{\frac{1-\rho}{2}} \qquad (8-122)$$

The BPSK probability of symbol error is now found by replacing E_b in eqn. (8 – 106) with $g^2 E_b$

$$P_B = \frac{1}{2}\left[1 - erf\left\{g\,(E/\eta)^{1/2}\right\}\right] \qquad (8-123)$$

$$= \frac{1}{2}\left[1 - erf\left\{\sqrt{(1-\rho)/2}\,(E/\eta)^{1/2}\right\}\right] \qquad (8-124)$$

8.8 Carrier Recovery:

Coherent detection requires a reference signal which replicates the phase of the signal carrier. A residual (pilot) carrier if present may be used to regenerate a full fledged carrier. PRK signals have suppressed carrier. To recover the carrier in this case, we may square the received PRK signal creating double frequency carrier with no phase transitions, since $\sin^2(2\pi f_C t)$ and $\sin^2(2\pi f_C t + \pi)$ are equal from which $\cos(2\pi f_C t)$ is regeneratel using a PLL and a divider (Fig. 8.27). A second technique is to use Costas loop (Fig. 8.28). It consists of two PLLs in phase quadrature. It locks to the suppressed carrier of a PRK signal.

8.9 MPSK:

The advantage of M-ary PSK (MPSK) is to increase the bandwidth efficiency of PSK modulation. In BPSK each data bit is a symbol. In MPSK, $k = \log_2 M$ data bits are represented by a symbol. Thus the bandwidth efficiency in increased k times. M-ary PSK signal set is defined as:

$$s_i(t) = A\cos(2\pi f_C t + \theta_i) \qquad \begin{matrix} 0 \le t \le T_s \\ i = 1, \ldots M \end{matrix} \qquad (8-125)$$

$$\theta_i = \frac{(2i-1)\pi}{M} \qquad i = 1, \ldots M \qquad (8-126)$$

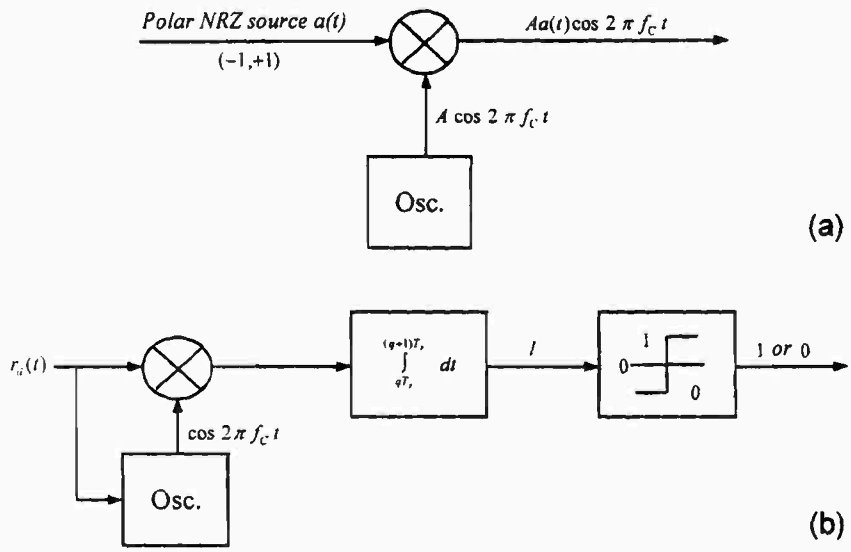


Fig. (8.25) BPSK modulator and demodulator
a) modulator b) coherent demodulator

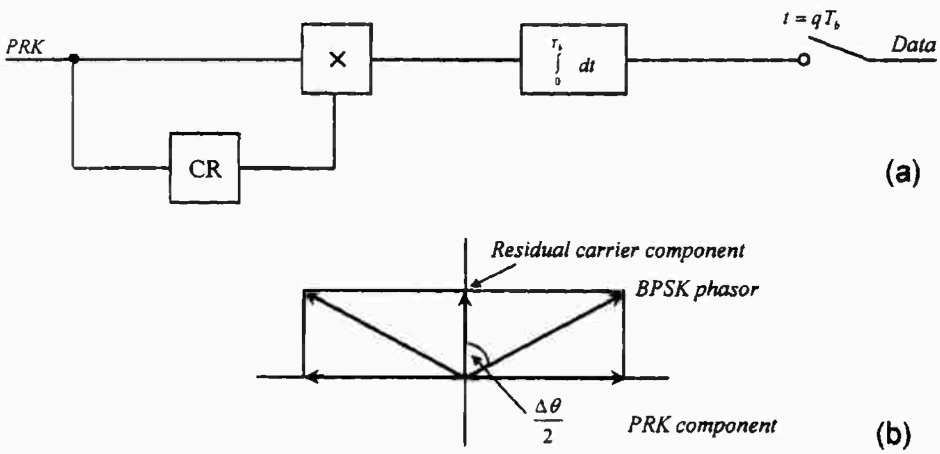


Fig. (8.26) Resolution of BPSK signal into PRK plus residual carrier
a) correlator detector b) residual carrier

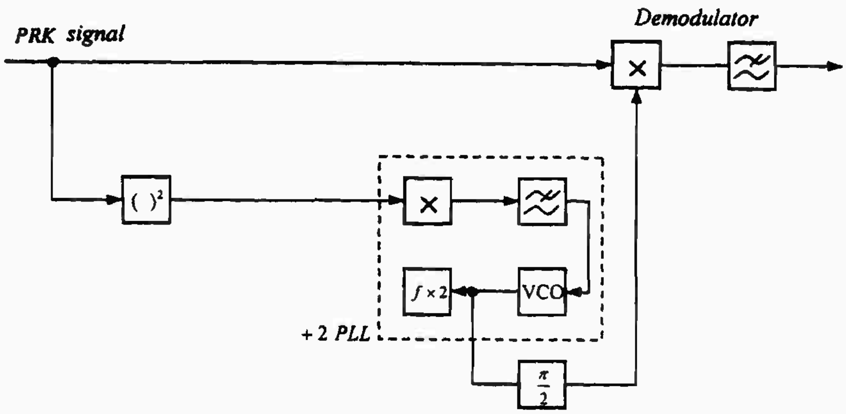


Fig. (8.27) Squaring loop for suppressed carrier recovery

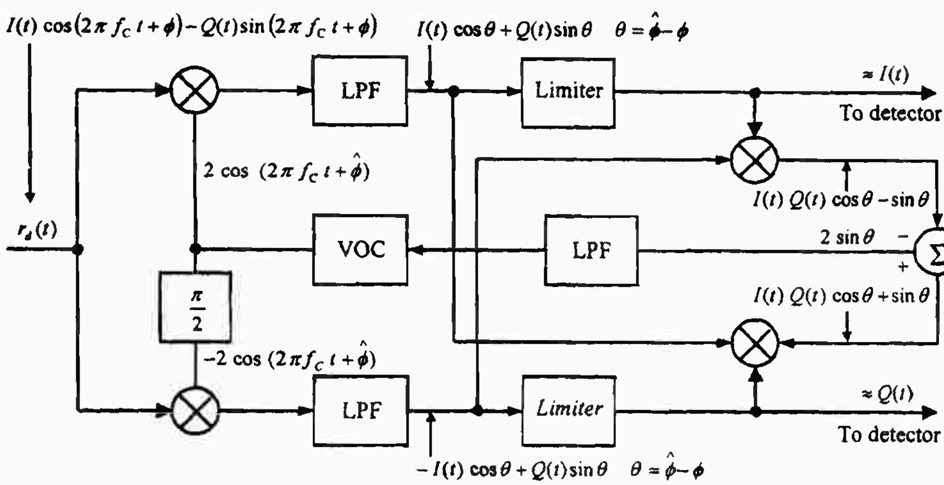


Fig. (8.28) Costas loop

$$\Delta\theta_i = \frac{2\pi}{M} \qquad (8-127)$$

The carrier frequency is chosen as multiple of the symbol rate. Therefore, within any symbol interval, we have only one of the initial phase states. M Is chosen as power of 2 $(M = 2, k = \log_2 M)$, i.e. binary data stream is divided into k-tuples each of them is represented by a symbol with an initial phase. From eqn. (8 – 125)

$$s_i(t) = A \cos\theta_i \cos 2\pi f_C t - A \sin\theta_i \sin 2\pi f_C t \qquad (8-128)$$

$$= a_{i1}\ \phi_1(t) + a_{i2}\ \phi_2(t) \tag{8–129}$$

where

$$\phi_1(t) = \sqrt{2/T_s}\ \cos 2\pi f_C\, t \qquad 0 \le t \le T_s \tag{8–130}$$

$$\phi_2(t) = -\sqrt{2/T_s}\ \sin 2\pi f_C\, t \qquad 0 \le t \le T_s \tag{8–131}$$

$$a_{i1} = \int_0^{T_s} s_i(t)\ \varphi_1(t)\,dt = \sqrt{E_s}\ \cos\theta_i \tag{8–132}$$

$$a_{i2} = \int_0^{T_s} s_i(t)\ \varphi_2(t)\,dt = \sqrt{E_s}\ \sin\theta_i \tag{8–133}$$

$$E_s = E_i = \frac{1}{2} A^2\ T_s \tag{8–134}$$

where E_s is the symbol energy. The phase is related to a_{i1} and a_{i2} as

$$\theta_i = \tan\left(a_{i2}/a_{i1}\right) \tag{8–135}$$

The MPSK signal constellation is two dimensional whose axes are $\varphi_1(t)$ and $\varphi_2(t)$. Each signal $s_i(t)$ is represented by a point $(a_{i1}\ , a_{i2})$. The polar coordinates of the signal are $\sqrt{E_s}\ , \theta_i$. Thus, the signal points are equally spaced on a circle of radius $\sqrt{E_s}$. Gray coding assigns k-tuples with only one bit difference to two adjacent signals in the constellation. When a symbol error occurs, it is likely that the signal is detected as the adjacent signal on the constellation, thus only one of the input bits is in error. Fig. (8.29) is an 8-PSK constellation where Gray coding is used for bit assignment. Note that BPSK is a special case of MPSK with $M = 2$ and QPSK is a special case of MPSK with $M = 4$. If we have a bit steam $b(t)$ given by

$$b(t) = \int_{q=-\infty}^{\infty} c_k\, g_{Ts}(t - q_{Ts}) \tag{8–136}$$

where g_{Ts} is a rectangular pulse with unit amplitude, We may express MPSK signal as a time function as

$$s(t) = A \sum_{q=-\infty}^{\infty} \cos\theta_q\ g_{T_s}(t - qT_s)\cos 2\pi f_C\, t$$

$$- A \sum_{q=-\infty}^{\infty} \sin\theta_q\ g_{T_s}(t - qT_s)\sin 2\pi f_C\, t \tag{8–137}$$

$$= s_1(t)\cos 2\pi f_C\, t - s_2\ \sin 2\pi f_C\, t \tag{8–138}$$

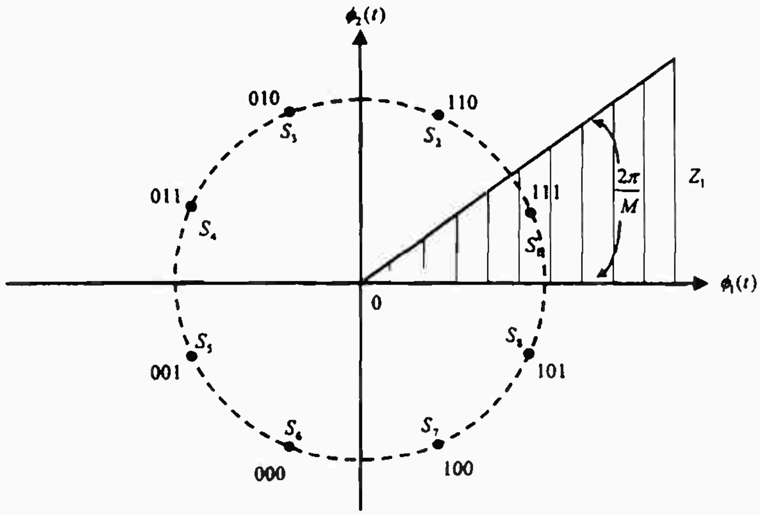


Fig.(8.29) 8-PSK constellation

where

$$s_1(t) = A \sum_{q=-\infty}^{\infty} \cos\theta_q \; g_{T_s}(t - qT_s) \qquad (8-139)$$

$$s_2(t) = A \sum_{q=-\infty}^{\infty} \sin\theta_q \; g_{T_s}(t - qT_s) \qquad (8-140)$$

where θ_q is one of the M phases determined by the input binary k-tuple, Eqn. (8-138) implies that the carrier frequency is an integer multiple of symbol timing so that the phase θ_i of the signal in any symbol period is θ_q.

MPSK uses quadrature modulator (Fig. 8.30). Each k-tuple of the input bits is used to control the level generator. It provides the I and Q channels the particular sign and level for the signal's horizontal and vertical coordinates.

MPSK signals may be digitally synthesized and fed to D/A converter whose output is the desired phase modulated signal.

The coherent demodulator for M-ary signals is shown (Fig. 8.31) where the receiver uses two correlators . Let us define

$$\ell_i = \int_0^{T_s} r_d(t)\; s_i(t)\; dt \qquad (8-141)$$

$$= \int_0^{T_s} r_d(t)\; \left[a_{i1}\; \phi_1(t) + a_{i2}\; \phi_2(t)\right] dt$$

$$= \int_0^{T_s} r_d(t) \left[\sqrt{E_s} \cos\theta_i \; \phi_1(t) + \sqrt{E_s} \sin\theta_i \; \phi_2(t)\right] dt$$

$$= \sqrt{E}\left[r_1 \cos\theta_i + r_2 \sin\theta_i\right] \tag{8-142}$$

where

$$r_1 = \int_0^{T_s} r_d(t) \; \varphi_1(t) \; dt \tag{8-143}$$

$$= \int_0^{T_s} \left[s(t) + n(t)\right] \varphi_1(t) \; dt = a_{i1} + n_1 \tag{8-144}$$

$$r_2 = \int_0^{T_s} r_d(t) \; \varphi_2(t) \; dt$$

$$= \int_0^{T_s} \left[s(t) + n(t)\right] \varphi_2(t) \; dt$$

$$= a_{i2} + n_2 \tag{8-145}$$

where r_1 and r_2 are independent Gaussian random variables with mean values a_{i1} and a_{i2}, respectively, and their variance is $\eta/2$.

Let

$$r_1 = r \cos\hat{\theta} \tag{8-146}$$

$$r_2 = r \sin\hat{\theta} \tag{8-147}$$

$$r = \sqrt{r_1^2 + r_2^2} \tag{8-148}$$

$$\hat{\theta} = \tan^{-1} \frac{r_2}{r_1} \tag{8-149}$$

Thus,

$$\ell_i = \sqrt{E}\left[r \cos\hat{\theta} \cos\theta_i + r \sin\hat{\theta} \sin\theta_i\right] \tag{8-150}$$

$$= \sqrt{E} \; r \cos(\theta_i - \hat{\theta}) \tag{8-151}$$

In the absence of noise, $\hat{\theta} = \tan^{-1} \frac{r_2}{r_1} = \tan^{-1} \frac{a_{i2}}{a_{i1}} = \theta_i$. With noise, $\hat{\theta}$ will deviate from θ_i. Since r is independent of any signal, then choosing the nearest ℓ_i is equivalent to choosing the smallest $\left|\theta_i - \hat{\theta}\right|$.

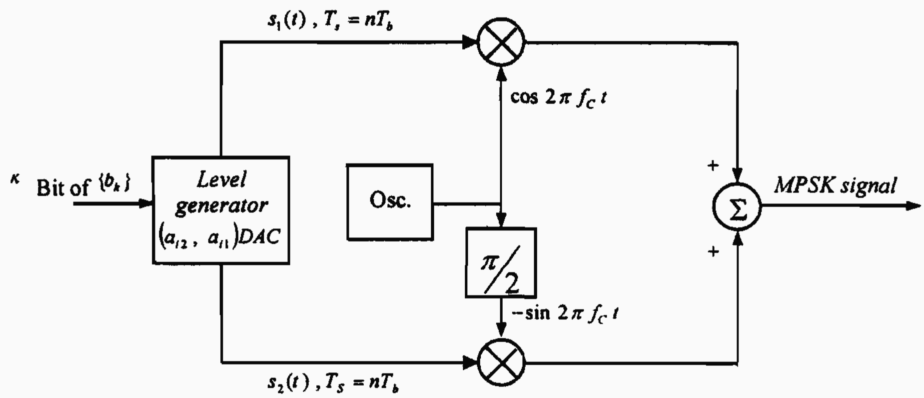


Fig. (8.30) MPSK quadrature modulator

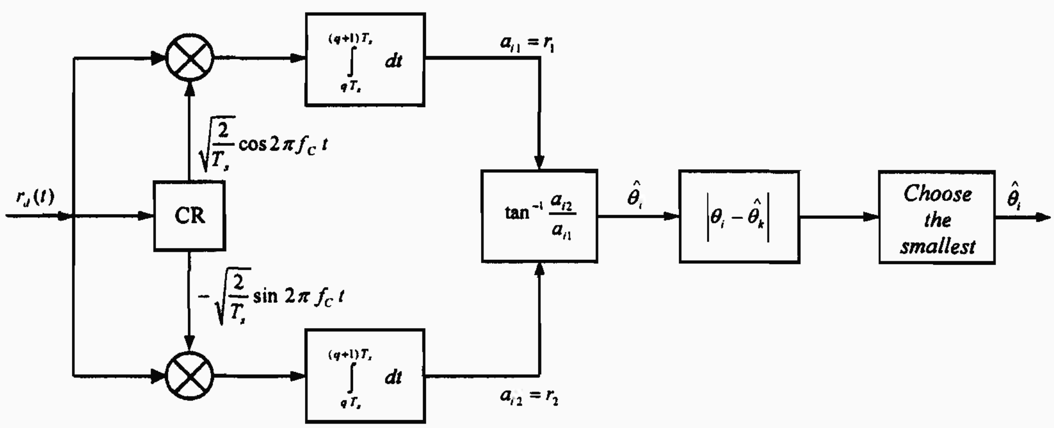


Fig. (8.31) MPSK demodulator

This rule is equivalent to choosing $s_i(t)$ when $\vec{r}_d = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ falls inside the pie-shaped decision region of the signal (Fig. 8.29). The demodulator in Fig. (8.31) is an implementation of this procedure.

The symbol error probability can be derived as follows. Given $s_i(t)$ is transmitted, the received vector $\vec{r}_d = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ is a point in the $\phi_1(t) - \varphi_2(t)$ plane. The channel bandwidth required for MPSK is B null to null bandwidth and for $R_b = 1/T_b$

$$B = 2/T_s \tag{8-152}$$

$$T_s = T_b \log_2 M \tag{8-153}$$

$$B = \frac{2R_b}{\log_2 M} \tag{8-154}$$

And bandwidth efficiency ε_s for MPSK is given by

$$\varepsilon_s = \frac{\log_2 M}{2} \tag{8-155}$$

Consider Fig. (8.32). Due to the symmetry of the signal constellation, P_ϵ is the error probability of detecting $\vec{s}_1$ which is the probability that the signal vector $\vec{r}_d$ does not fall in the decision region Z_1. The distance from s_1 to the nearest region signal is

$$d_{12} = d_{18} = 2\sqrt{E} \sin \frac{\pi}{M} \tag{8-156}$$

For s_1 to fall into $\hat{s}_2$

$$P\left(\hat{s}_2 | s_1\right) = \int_{-\infty}^{-d_{12}/2} \frac{1}{\sqrt{\pi\eta}} e^{-x^2/\eta}\, dx \tag{8-157}$$

$$= Q\left(\sqrt{2E_s/\eta} \sin \pi/M\right) \tag{8-158}$$

We have equally likely probability $P(\hat{s}_8 | s_1)$ which is identical due to symmetry

$$P_s = 2Q\left(\sqrt{2E_s/\eta} \sin \pi/M\right) \tag{8-159}$$

The bit error rate (BER) is $P_\epsilon\ R_b$ where

$$BER = \frac{P_\epsilon\ P_s}{\log_2 M} \tag{8-160}$$

Fig. (8.33) shows P_s for MPSK .

Ex 8.2

An MPSK, ISI free system is to operate with 2^k PSK symbols over a $120\ kHz$ channel. The minimum required bit rate is $900\ kb/s$, what is minimum CNR to maintain reception with P_B no worse than 10^{-6}:

Solution

Maximum (ISI free) baud rate

$$R_s = 1/T_s = B$$

$R_s \le 120k$ baud (k symbols/s)

$$\frac{R_b}{R_s} = \frac{900\times10^3}{120\times10^3} = 7.5 \text{ bits/symbol}$$

The minimum number of symbols required is given by

$$\frac{R_b}{R_s} =\le \log_2 M$$

$$M \ge 2^{7.5}$$

But M must be an integer power of 2

$$M = 2^8 = 256$$

$$\log_2 M = 8$$

$$P_s = P_b \log_2 M = 10^{-6}\times\log_2 256$$

$$= 8\times10^{-6}$$

Using eqns (8 – 117) and (8 – 159),

$$P_s = 1 - erf\left[(T_s\ B)^{1/2} \sin\frac{\pi}{M}(C/N)^{1/2}\right] \qquad (8-161)$$

$$R_s = \frac{R_b}{\log_2 M} = \frac{900\times10^3}{8} = 112.5\times10^3 \text{ baud}$$

$$T_s = \frac{1}{R_s} = 8.889\times10^{-6}s$$

$$T_s\ B = 8.889\times10^{-6}\times120\times10^3 = 1.067$$

$$\frac{C}{N} = \left[\frac{erf^{-1}(1-P_s)}{(T_s\ B)^{1/2}\sin(\pi/M)}\right]^2$$

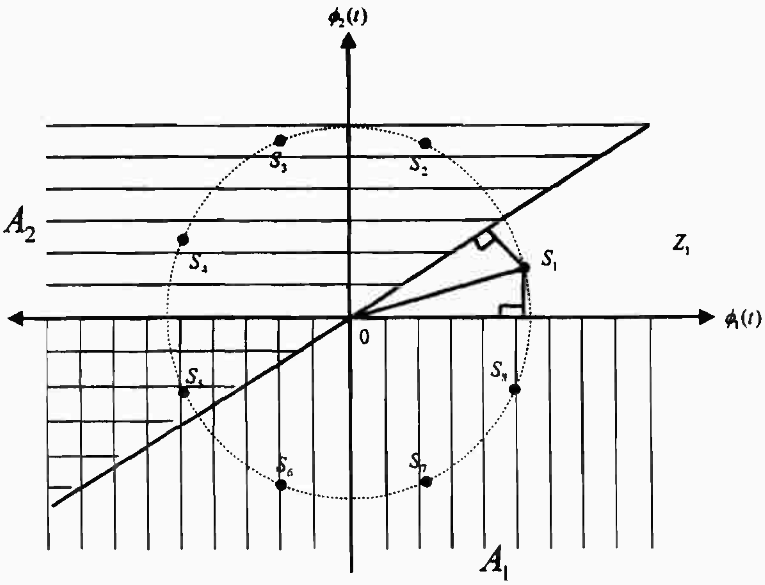


Fig. (8.32) Decision regions for MPSK signals

$$=\left[\frac{erf^{-1}\left(1-8\times10^{-6}\right)}{\left(1.067\right)^{1/2}\sin\left(\pi/256\right)}\right]^2$$

$$=\left[\frac{erf^{-1}\left(0.999\right)}{1.033\sin\left(\pi/256\right)}\right]^2$$

$$=\left(\frac{3.157}{0.01268}\right)^2=61988=47.9\ dB$$

8.10 QPSK:

This is most often used.

$$s_i(t)=A\cos\left(2\pi f_C t+\theta_i\right)\qquad 0\le t\le T_s\qquad i=1,2,3,4\qquad (8-162)$$

$$\theta_i=\frac{(2i-1)\pi}{4}\qquad (8-163)$$

The initial phases are $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$. The carrier frequency is an integer multiple of the symbol rate. Therefore in any symbol interval $[qT_s,(q+1)T_s]$, the signal initial phase is one of the four phases.

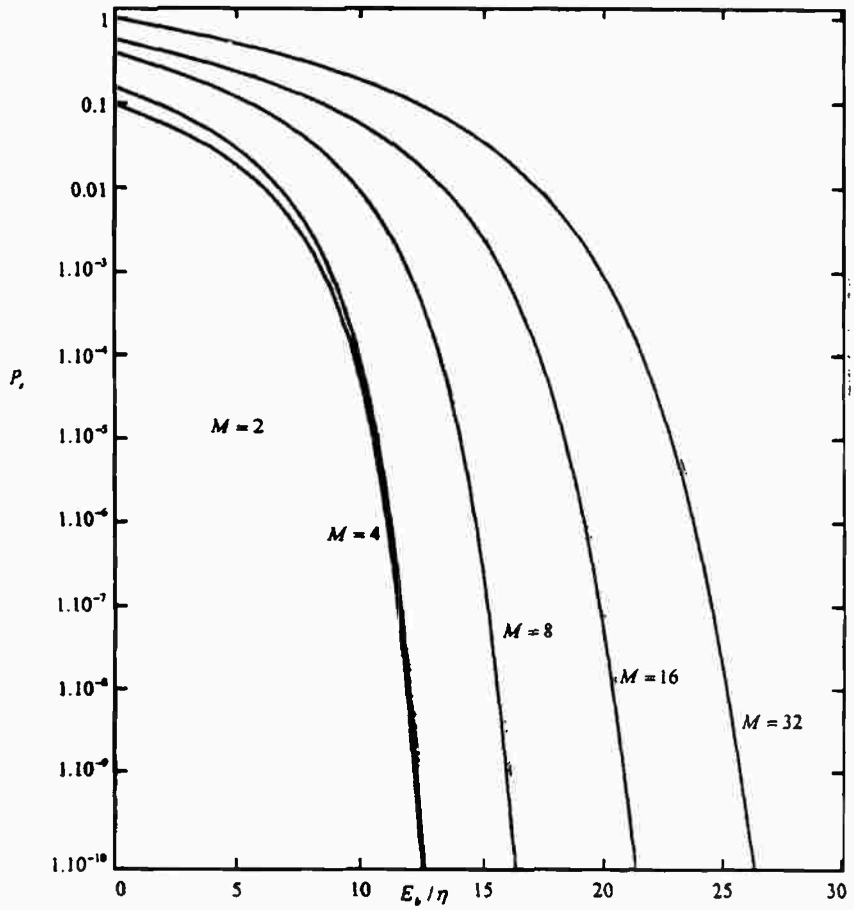


Fig. (8.33) P_s **for MPSK**

$$s_i(t) = A \cos\theta_i \cos 2\pi f_C t - A \sin\theta_i \sin 2\pi f_C t \tag{8-164}$$

$$= a_{i1}\,\phi_1(t) + a_{i2}\,\phi_2(t) \tag{8-165}$$

$$a_{i1} = \sqrt{E_s}\cos\theta_i \tag{8-166}$$

$$a_{i2} = \sqrt{E_s}\sin\theta_i \tag{8-167}$$

$$\theta_i = \tan^{-1}\frac{a_{i2}}{a_{i1}} \tag{8-168}$$

$$E_s = A^2 \frac{T_s}{2} \tag{8 –169}$$

Table (8.13) shows QPSK coordinates. Data bits are divided into groups of two (digits) 00, 01, 10, 11.

We can represent the four signals by four points or vectors $\vec{s}_i = \begin{bmatrix} a_{i1} \\ a_{i2} \end{bmatrix}$ $i = 1,2,3,4$. From eqns. (8 - 164) to (8 – 167) and Fig. (8.36a), we obtain the signal constellation as shown (Fig. 8.34) based on Gray coding.

We may express QPSK as

$$s(t) = \frac{A}{\sqrt{2}} I(t) \cos 2\pi f_c t - \frac{A}{\sqrt{2}} Q(t) \sin 2\pi f_c t, \quad -\infty < t < \infty \tag{8 –170}$$

where $I(t)$, $Q(t)$ are pulse trains determined by odd numbered bits and even numbered bits $I(t) = \sum_{q=-\infty}^{\infty} I_q \, g_{T_s}(t - qT_s)$ (8 –171)

$$Q(t) = \sum_{q=-\infty}^{\infty} Q_q \, g_{T_s}(t - qT_s) \tag{8 –172}$$

Where $I_q = \pm 1$ and $Q_q = \pm 1$ mapping $1 \to 1$ and $0 \to -1$. This QPSK waveform is shown (Fig. 8.35). Like BPSK, the waveform has constant envelope and discontinuous phases at the boundaries. Because $T_s = 2T_b$, QPSK transmits data twice as fast as BPSK. Since the distance of adjacent points of QPSK constellation is shorter than that of BPSK, we expect higher bit error rate than in BPSK. Surprisingly, we shall find shortly that the symbol probability error remains unchanged. The modulator based on eqn. (8 – 170) is shown (Fig. 8 .36a)

The channel with cosine reference is in-phase (I) and the channel with sine reference is quadrature (Q). The data sequence is separated by a serial to parallel (S/P) converter to form the odd numbered bit sequence for the I channel, and the even numbered bit sequence for the Q channel. The logic 1 is converted to a positive pulse and logic 0 to a negative pulse. Both have the same amplitude and duration T_s. Next the odd numbered bit pulse train is multiplied by $\cos 2\pi f_c t$, and the even numbered bit pulse train is multiplied by $\sin 2\pi f_c t$. Each of the I and Q channel signals is a BPSK signal with a symbol duration of $2T_b$. Finally a summer adds these two waveforms at various stages. Since QPSK is a special case of MPSK, the demodulator for MPSK is applicable to QPSK. The I and Q channel signals are demodulated separately as two individual BPSK signals.

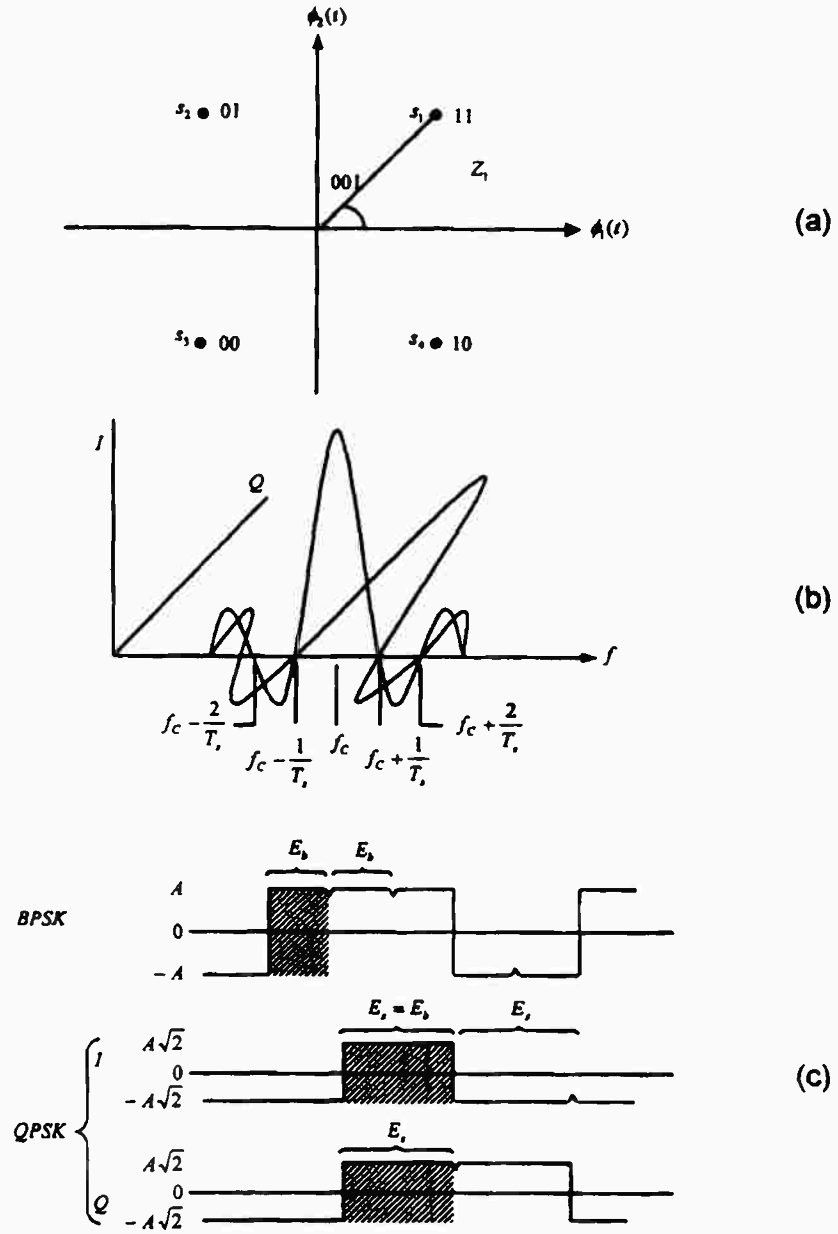


Fig. (8.34) QPSK (I and Q) equivalent to BPSK
a) constellation *b) orthogonal I/Q voltage spectra* *c) bit energy*

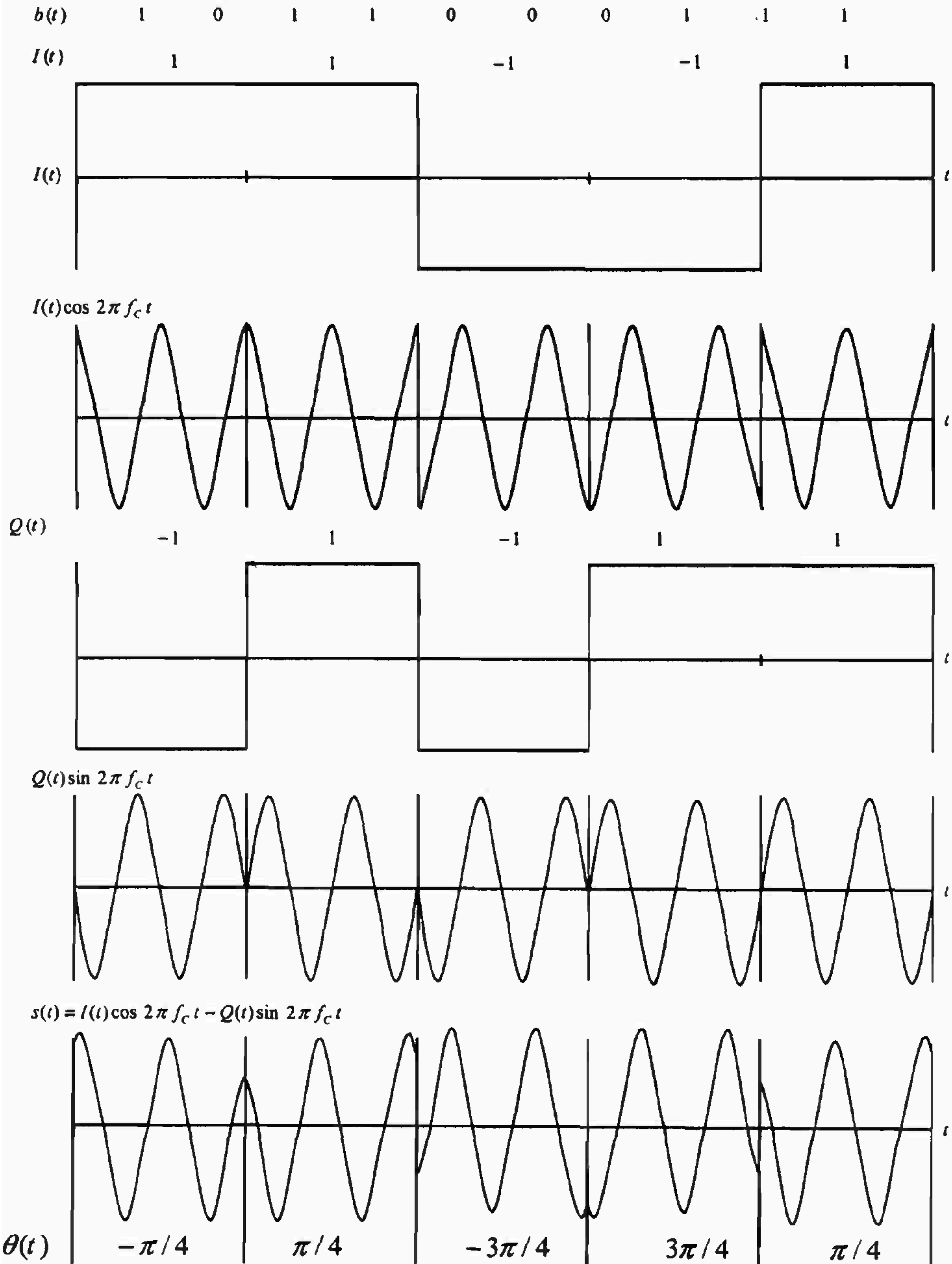
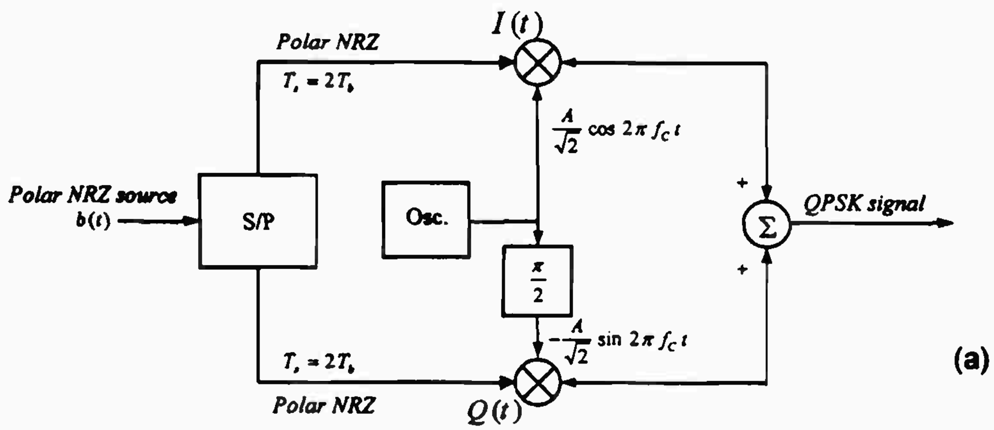


Fig. (8.35) QPSK waveforms



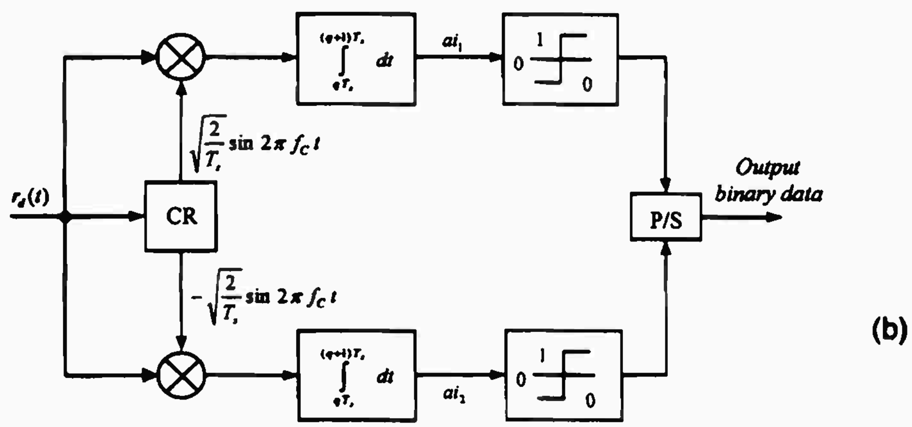


Fig. (8.36) QPSK modulator and demodulator

a) modulator *b)demodulator*

A parallel to serial (P/S) converter is used to combine the two sequences into a single sequence. This is possible because of the one to one correspondence between the data bits and the I and Q channel signals noting their orthogonality. The average bit error probability for each channel is

$$P_B = P(\in | 1sent) = P(\in | 0sent)$$

$$= Q\left(\sqrt{E_s/\eta}\right) = Q\left(\sqrt{2E_b/\eta}\right) \quad (8-173)$$

The final output of the demodulator is just the multiplexed I and Q channel outputs. Thus, the bit error rate for the final output is the same as that of each channel. A symbol represents two bits from the I and Q channels, respectively. A symbol error occurs if one of them is in error. Thus

$$P_s = 1 - P \text{ (Both bits are in error)} \quad (8-174)$$

$$= 1-(1-P_B)^2$$

$$= 2P_B - P_B^2$$

$$= 2Q\left(\sqrt{E_s/\eta}\right)-\left[Q\left(\sqrt{E_s/\eta}\right)\right]^2 \quad (8-175)$$

We note that for Gray coding a symbol error most likely causes the symbol being detected as the adjacent symbol, which is only one bit different out of two bits. Thus, for the case $E_s/\eta >> 1$ and $Q\left(\sqrt{E_s/\eta}\right) << 1$, we may neglect the second term in eqn. (8 – 175)

$$P_B = \frac{1}{2}P_s = Q\left(\sqrt{2E_s/\eta}\right) \quad (8-176)$$

The P_B curve is the same as that of BPSK. For QPSK, we note that the null to null bandwidth $B_{n-n} = 2\times\frac{1}{T_s} = 2\times\frac{1}{2T_s} \quad \frac{1}{T_b} = R_b$ since $T_s = 2T_b$

Ex 8.3

A 4-PSK modulator has an input bit rate of $2400 b/s$ and works on commercial speed band $300\ Hz - 3400\ Hz$. Determine

1. the number of possible symbols at the output
2. the symbol rate
3. the phase difference between the symbols
4. the maximum bit rate

Solution

$$M = 4\ ,\quad k = 2$$

$$R_s = \frac{R_b}{2} = 1200 \text{ symbols/s}$$

$$\Delta\theta = \frac{360}{4} = 90^0$$

$$B = 3400 - 300 = 3100Hz$$

This is $B = 1/T_s$, while the null to null bandwidth

$$B_{n-n} = 2/T_s$$

The bit rate $R_b = R_s \log_2 M = R_s k = \frac{1}{T_s}k = 2400\ b/s$

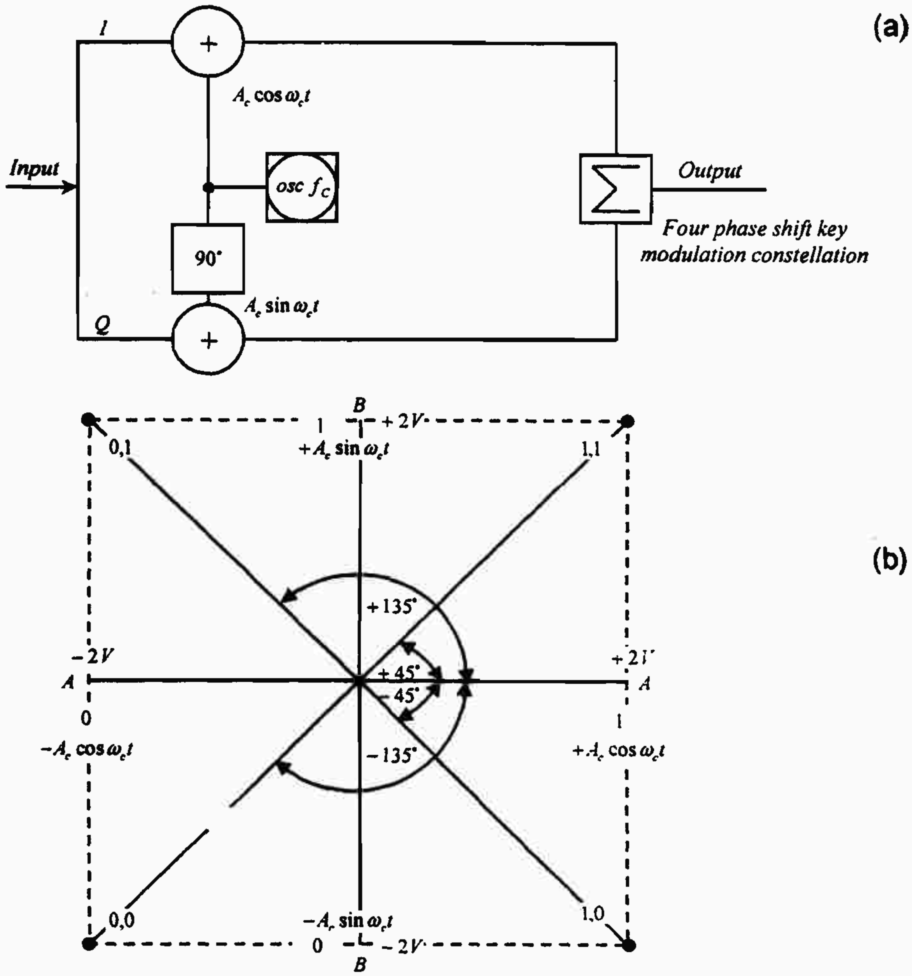


Fig. (8.37) (Ex. 8.3)
a) Circuit *b) constellation*

For ISI free operation,

$$R_b = 2 \times \frac{1}{2T_{s_{min}}} = \frac{1}{T_{s_{min}}} = \frac{1}{2T_{b\,min}} = \frac{R_{b\,max}}{2}$$

The maximum bit rate $R_{b\,max}$ and the maximum symbol rate $R_{s\,max}$ are related by

$$R_{b\,max} = R_{s\,max} \log_2 M = \frac{1}{T_{s_{min}}} \times k = k \times B_{n-n}$$

$$= 3100 \times 2 = 6200 \; b/s$$

If we exceed this rate, the received data would be in error.

Because the output bit rate is less than the input bit rate this results in a smaller bandwidth. A typical 4-PSK circuit is shown together with the constellation. The input is applied to a 2 bit splitter or $1-2$ demultiplexer. The output bit rate on each of the two outputs is $1/2$ that at the input to the splitter. Logic 0 is $-2V$ and logic 1 is $+2V$. The outputs from the two ring modulators are then summed together in a linear summer. The output of the linear summer produces the constellation. (Fig. 8.37).

Ex 8.4

A 8-PSK modulator has an input bit rate of $2400 \, b/s$ and works into a commercial speech band circuit. Determine:

1. the number of possible symbols at the output
2. the symbol rate
3. the phase difference between the symbols
4. the maximum bit rate

Solution

$$M = 8 \;\; , \;\; k = 3$$

$$R_s = \frac{R_b}{3} = \frac{2400}{3} = 800 \text{ symbols/s}$$

$$\Delta\theta = \frac{360}{8} = 45^{\circ}$$

$$R_{b\max} = B_{n-n} \log_2 M = 3100 \times 3 = 9300 \; b/s$$

Ex. 8.5

Consider a modulator using a carrier frequency of $140 \, MHz$ when 120 Mb/s bit stream is applied to it. Compare the bandwidth allocation for various modulation schemes

Solution

A rectangular pulse of width T_s has first null baseband one sided bandwidth of $1/T_s$ and double band bandpass bandwidth of $2/T_s$. If this rectangular pulse is inputted to a LPF of cutoff frequency $B = 1/T_s = R_s$, the first lobe is allowed. In time domain, however, if a signal is band limited by a LPF, the filtering action will result in a time *sinc* function with this first zero at $t_0 = 1/2B$, where B is the one sided baseband bandwidth (positive frequency only) i.e., $B = \frac{1}{2} t_0$ The effective double sided pulse width is thus $T_s' = 2t_0 = 1/B$. where T_s' is the null to null bandwidth in the time domain i.e. $R_s = B$. However, we may achieve the ISI free condition for minimum Nyquist bandwidth if we take $T_s = t_0$, so that a symbol peak coincides with the null of the preceding symbol. In this case $T_s = t_0 = 1/2B$ or $R_s = 2B$, $B = R_s/2$ coincides. This is in accordance with Nyquist criterion. We call this pulse an unfiltered rectangular pulse $(\alpha = 0)$. We call it unfiltered because we have not used pulse shaping for the time rectangular pulse except for the LPF. If we use a raised cosine filter, we eliminate the tailing effect of the sinc pulse $(\alpha = 1)$ In this case, $R_s = B$ Usually we settle for unfiltered pulse shaping to conserve the bandwidth. Thus, the DSB bandwidth extends from $f_C - 1/2T_s$ to $f_C + 1/2T_b$ i.e. $f_C - R_s/2$ to $f_C + R_s/2$ while if we use a raised cosine filter $(\propto = 1)$ we have $f_C - 1/T_s$ to $f_C + 1/T$ or $f_C - R_s$ to $f_C + R_s$.

Therefore for 2-PSK, the entire band stretches from $140\,MHz - 120\,MHz$ to $140\,MHz + 120\,MHz$. For Nyquist bandwidth we get only half of that i.e. $\pm 60MHz$-120MHz. The one sided bandwidth of the DSB bandwidth in this case is $120\,M\,Hz$.

For 4-PSK modulation the bandwidth is now divided by 2. Since $R_s = 1/T = R_b/2 = 60\,M\,b/s$, since $T_s = 2T_b$. Thus the bandwidth extends from $f_C - 1/T_s$ i.e. reduced R_s for the same B or increased B for the same R_s to $f_C + 1/T_s$ or $f_C - R_s$ to $f_C + R_s$ for raised cosine filter $(\propto = 1)$.

But for Nyquist bandwidth $f_C - 1/2T_s$ to $f_C + 1/2T_s$ or $f_C - R_s/2$ to $f_C + R_s/2$, i.e., the one sided bandwidth of the DSB bandwidth is $30\,MHz$.

For 8-PSK the one sided bandwidth of the double side band bandwidth is $15\,MHz$.

For 16-PSK the one sided bandwidth of the double side band bandwidth is $7.5\,MHz$ (Table 8-13). Fig. (8.38) shows the bandwidth reduction due to M-ary modulation. Thus, we have

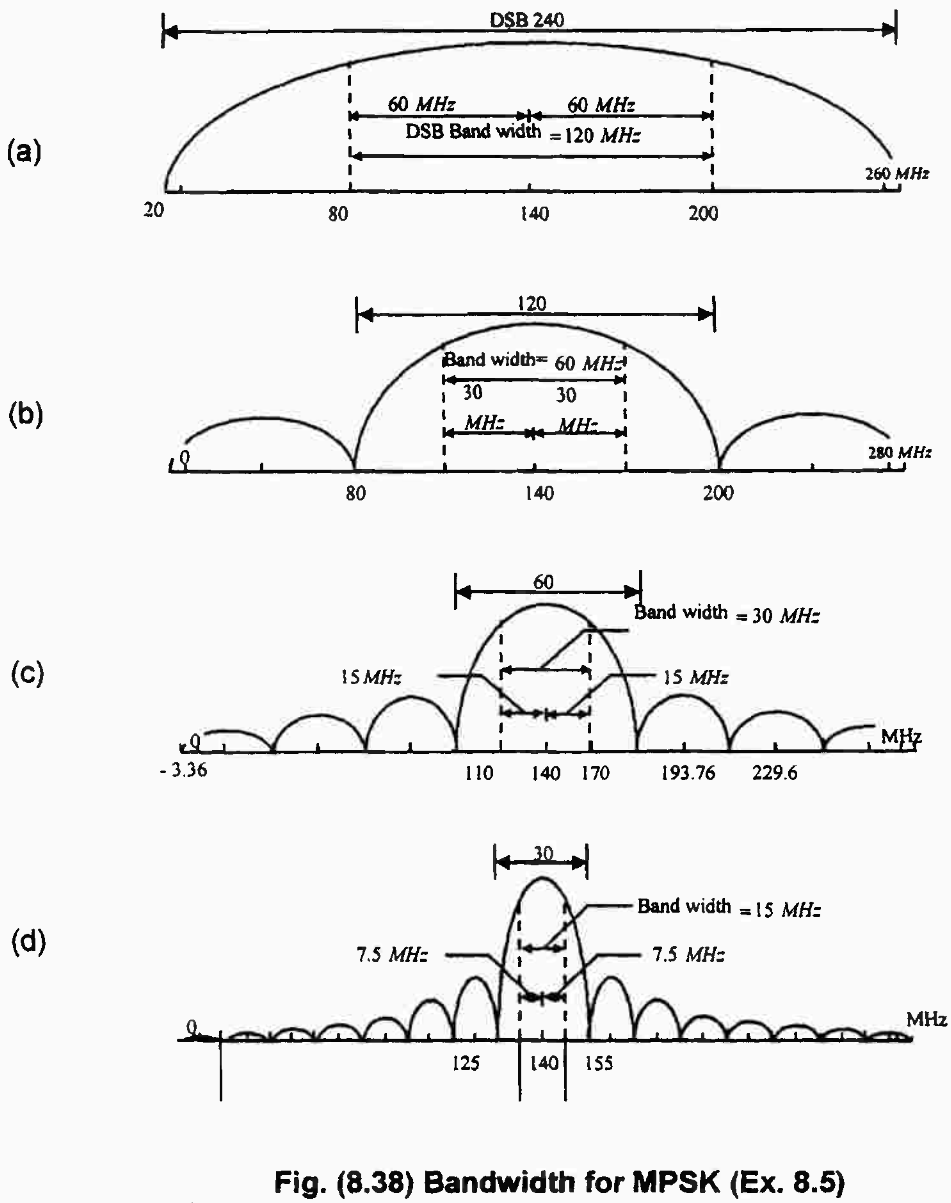


Fig. (8.38) Bandwidth for MPSK (Ex. 8.5)

a) 2 – $PSK(BPSK)$ b) 4 – PSK c) 8 – PSK d) 16 – PSK

Table (8.11) BW for M-PSK

	M	Raised cosine $BW(\propto -1)$	Nyquist bw
2-PSK	2	$\pm R_b$	$\pm R_b/2$
4-PSK	4	$\pm R_b/2$	$\pm R_b/4$
8-PSK	8	$\pm R_b/4$	$\pm R_b/8$
16-PSK	16	$\pm R_b/8$	$\pm R_b/16$
64-PSK	64	$\pm R_b/16$	$\pm R_b/32$

8.11 QAM:

All schemes studied so far have constant envelope. The constant envelope property is important to systems with power amplifiers which must operate in the nonlinear region of input output characteristic for maximum power efficiency, i.e., satellite transponders. Quadrature amplitude modulation (QAM) is the class for non constant envelope which is used in modems over telephone channels.

ASK can be made M-ary, hence called, MAM. QAM can be considered as two MAM components which can be demodulated in two separate channels. MAM signal can be expressed as

$$s_i(t) = a_i\ \phi(t) \qquad 0 \le t \le T_s \tag{8-177}$$

$$\int_0^{T_s} \phi^2(t)\ dt = 1 \qquad i = 1,...M \tag{8-178}$$

$$a_i^2 = \int_0^{T_s} [s_i(t)]^2\, dt = E_i \tag{8-179}$$

If $\phi(t)$ is a baseband pulse, then $s_i(t)$ is baseband MAM, usually called pulse amplitude modulation (PAM). If $\phi(t)$ is a high frequency sinusoidal carrier then $s_i(t)$ is bandpass MAM, which is ASK. The BPSK case can be viewed as a binary bandpass MAM with two antipodal a_i.

The baseband MAM signal set is

$$s_i(t) = A_i\ g_{Ts}(t)\cos 2\pi f_C t \qquad 0 \le t \le T_s \tag{8-180}$$

$$\phi(t) = \sqrt{2/E_b}\ g_{Ts}(t)\cos 2\pi f_C t \qquad i = 1,2..M \tag{8-181}$$

Note that $f_c > 1/T_s$ and E_b is the energy in the pulse of the shaping signal $g_{T_s}(t)$ within T_s

$$\int_0^{T_s} \phi^2(t)\ dt = 1 \tag{8-182}$$

Thus,

$$s_i(t) = a_i \ \phi(t) \tag{8 –183}$$

$$a_i = A_i \ \sqrt{E_b / 2} \tag{8 –184}$$

The modulator (Fig. 8.39a) is a direct implementation of eqn. (8 – 186). The level generator takes $k = \log_2 M$ bits from the binary data stream and maps them to an amplitude level $A_q \in \{A_i\}$ for the q^{th} symbol interval. Mapping is performed according to Gray code where k tuples represent the adjacent amplitudes differing only by one bit. The $g_{Ts}(t)$ may be replaced with a filter with an impulse response $g_{Ts}(t)$. In order to generate a pulse $A_i \ g_{Ts}(t)$ the input is impulse $A_i \ \delta(t)$. The output of the modulator is given by

$$s_i(t) = A_i g_{Ts}(t) \cos 2\pi f_C t \sqrt{2/E_b} \cos 2\pi f_C t$$

$$= \frac{1}{2} A_i \sqrt{2/E_b} \ g_{Ts}(t) \left[1 + \cos 4\pi f_C t\right] \tag{8 –185}$$

The integrator output for $f_C >> 1/T_s$ is

$$z(T_s) = \frac{1}{2} A_i \sqrt{2/E_b} \int_0^{T_s} \left[g_{Ts}(t)\right]^2 \left[1 + \cos 4\pi f_C t\right] dt$$

$$= A_i \sqrt{E_b / 2} = a_i \tag{8 –186}$$

The output noise of the integrator is

$$N_0 = \int_0^{T_s} n(t) \ \phi(t) \, dt \tag{8 –187}$$

where $n(t)$ is the noise in the received signal. The simplest MAM is OOK whose signal set is

$$s_1(t) = A \cos 2\pi f_C t \qquad \text{for } b = 1, \qquad 0 \le t \le T_s \tag{8 –188}$$

$$s_2(t) = 0 \qquad \text{for } b = 0, \qquad 0 \le t \le T_s \tag{8 –189}$$

The symbol error probability for coherent demodulation of OOK is

$$P_B = Q\left(\sqrt{E_b / 2\eta}\right) \tag{8 –190}$$

We note that in MAM schemes, signals have the same phase but different amplitudes in MPSK schemes signals, have the same amplitude but different phases. In QAM, we use both amplitude and phase modulations, i.e.,

$$s_i(t) = A_i \ \cos(2\pi f_C t + \theta_i) \qquad i = 1, \ldots M \tag{8 –191}$$

$$= A_{i1} \ g_{Ts}(t) \cos 2\pi f_C t - A_{i2} \ g_{Ts}(t) \sin 2\pi f_C t \tag{8 –192}$$

when

$$A_{i1} = A_i \cos\theta_i \tag{8-193}$$

$$A_{i2} = A_i \sin\theta_i \tag{8-194}$$

$$A_i = \sqrt{A_{i1}^2 + A_{i2}^2} \tag{8-195}$$

But

$$s_i(t) = a_{i1}\ \varphi_1(t) + a_{i2}\ \varphi_2(t) \tag{8-196}$$

Where

$$\varphi_1(t) = \sqrt{2/E_b}\ g_{T_s}(t)\cos 2\pi f_C t \qquad 0 \le t \le T_s \tag{8-197}$$

$$\varphi_2(t) = \sqrt{2/E_b}\ g_{T_s}(t)\sin 2\pi f_C t \qquad 0 \le t \le T_s \tag{8-198}$$

and

$$a_{i1} = \sqrt{E_b/2}\,A_{i1} = \sqrt{E_b/2}\,A_i \cos\theta_i \tag{8-199}$$

$$a_{i2} = \sqrt{E_b/2}\,A_{i2} = \sqrt{E_b/2}\,A_i \sin\theta_i \tag{8-200}$$

where E_b is the energy of $g_{T_s}(t)$ in $[0\ ,\ T_s]$, i.e., $E_b = \int_0^{T_s} [g_{T_s}(t)]^2\ dt$. We see that the basis functions $\varphi_1(t)$ and $\varphi_2(t)$ are virtually orthonormal for $f_C >> 1/T_s$ since $g_{T_s}(t)$ is a slowly varying envelope.

$$\int_0^{T_s} [\varphi_1(t)]^2 dt = \frac{2}{E_b}\int_0^{T_s} [g_{T_s}(t)]^2 (t)\cos^2 2\pi f_C t\ dt \tag{8-201}$$

$$= \frac{1}{E_b}\int_0^{T_s} [\,g_{T_s}(t)]^2[1+\cos\ 4\pi f_C\ t\,]\,dt \tag{8-202}$$

$$\sim 1 \text{ for } f_C >> 1/T_s$$

$$\int_0^{T_s} \varphi_1(t)\ \varphi_2(t)dt = \frac{-2}{E_b}\int_0^{T_s} [g_{T_s}(t)\,]^2 \cos\ 2\pi f_C\ t\cos 2\pi f_C t dt \tag{8-203}$$

$$= \frac{-2}{E_b}\int_0^{T_s} [g_{T_s}(t)\,]^2 \sin 4\pi f_C t\ dt \tag{8-204}$$

$$\sim 0 \text{ for } f_C >> 1/T_s \tag{8-205}$$

Thus, $\varphi_1(t)$ and $\varphi_2(t)$ are nearly orthonormel. When $g_{T_s}(t) = 1$ in $[0\ ,\ T_s]$, $E_b = T_s$. The energy of the i^{th} signal

$$E_i = \int_0^{T_s} s_i^2(t)dt = \frac{1}{2}A_i^2\ E_b \tag{8-206}$$

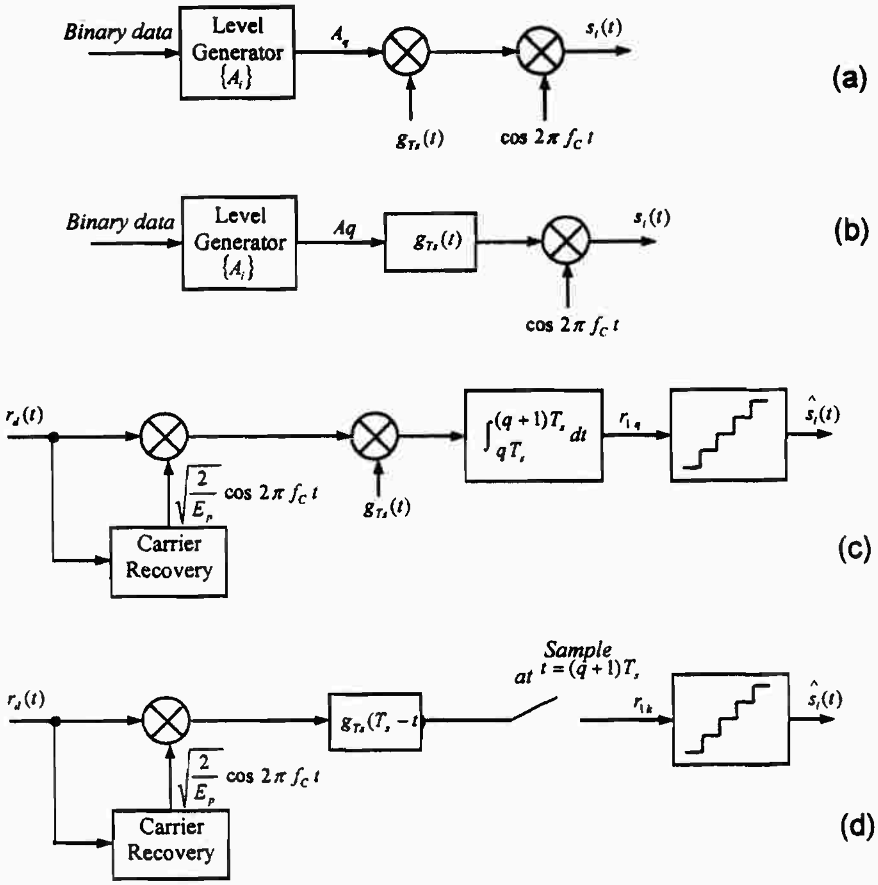


Fig. (8.39) Bandpass MAM modulator and demodulator

a) modulator version 1 *b) modulator version 2*

c) demodulator version 1 *d) demodulator version 2*

$$\langle E \rangle = \frac{1}{2} E_b \langle A_i^2 \rangle \qquad (8-207)$$

The average power $\langle S_p \rangle$ is given by

$$\langle S_p \rangle = \frac{\langle E \rangle}{T_s} \qquad (8-208)$$

$$\langle A_i^2 \rangle = \frac{2\langle E \rangle}{E_b}$$

$$= \frac{2\langle S_p \rangle T_s}{E_b}$$

$$A_{i_{rms}} = \sqrt{2\,\langle S_p \rangle} \qquad (8-209)$$

Since $E_b = T_s$ numerically for $g(T_s) = 1$ in $[0, T_s]$

A QAM signal is represented in the constellation plane as a point with coordinates $(a_{i1}\ ,\ a_{i2})$, where the two axes are $\varphi_1(t)$ and $\varphi_2(t)$. Alternatively, the two axes may be chosen as $g_{T_s}(t)\cos 2\pi f_C t$ and $-g_{T_s}(t)\sin 2\pi f_C t$. Then in this case the signal coordinates are $(A_{i1}\ ,\ A_{i2})$. In all cases, the axes are labelled I , Q. Fig. (8.40) shows 16-QAM constellation. Fig. (8.41) shows 16-QAM modulator, Fig. (8.42) shows 16-QAM demodulator. Assuming the axes are $\varphi_1(t)$ and $\varphi_2(t)$, then each signal is represented by the phasor $\vec{s}_i = (a_{i1}\ ,\ a_{i2})$

$$\left\|\vec{s}_i\right\| = \sqrt{a_{i1}^2 + a_{i2}^2} \qquad (8-210)$$

$$A_i = \sqrt{\frac{2}{E_b}}\ \left\|\vec{s}_i\right\| \qquad (8-211)$$

$$\langle E \rangle = \langle E_i \rangle = \langle \left\|s_i\right\|^2 \rangle \qquad (8-212)$$

$$\theta_i = \tan^{-1}\frac{a_{i2}}{a_{i1}} \qquad (8-213)$$

The distance between any pair of phasors is

$$d_{ij} = \sqrt{\left|\vec{s}_i - \vec{s}_j\right|^2} \qquad (8-214)$$

$$= \sqrt{(a_{i1} - a_{j1})^2 + (a_{i2} - a_{j2})^2} \qquad i, j = 1, ..M \qquad (8-215)$$

We should make the minimum Euclidean distance $d_{\min}$ among the phasors (signal points), the phase differences as large as possible and the average power of the phasors as small as possible. Such a square constellation. It can easily be decomposed to two MAM signals impressed on two phase quadrature carrier. Hence, it can be easily demodulated to yield two quadrature components. Each component can be individually detected by comparing it to a set of thresholds.

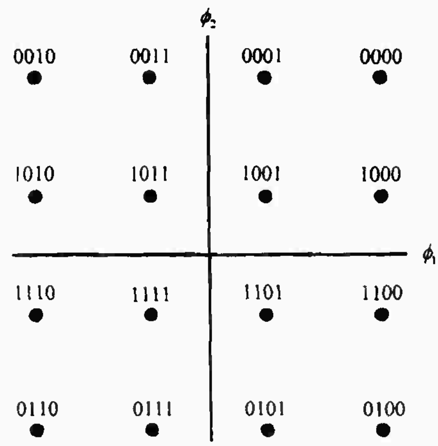


Fig. (8.40) Gray coded 16-QAM constellation

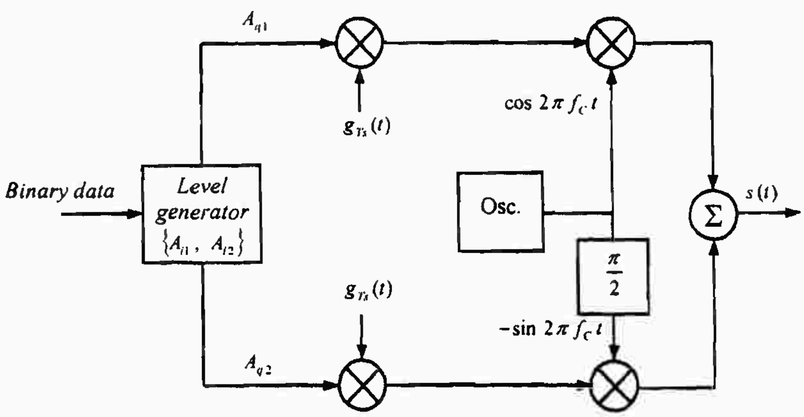


Fig. (8.41) QAM modulator

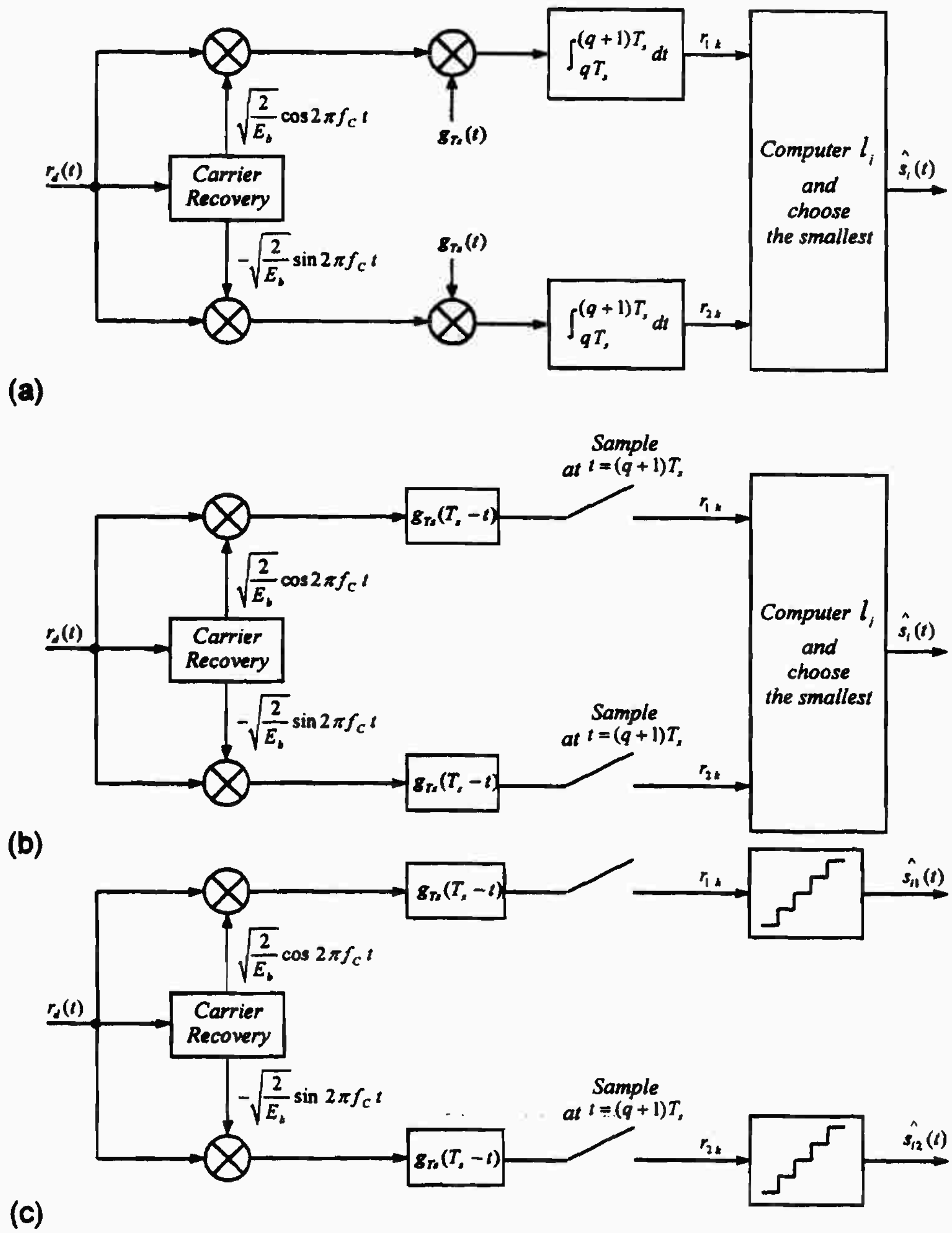


Fig. (8.42) QAM demodulator

a) Threshold detector *b) using a correlator* *c) Using a matched filter*

8.12 Synchronization:

Coherent demodulation requires a reference signal and symbol timing at the receiver to be synchronized in phase and frequency with the received signal. Synchronization can be achieved either by sending a pilot carrier or by using a carrier recovery circuit such as Costas loop which extracts phase and frequency information from the noisy received signal and uses it to generate a fresh reference signal. Symbol synchronization is achieved by a clock (symbol timing) recovery circuit, which uses the received signal to control the local oscillator. The clock or symbol timing recovery (STR) can be classified into two basic groups. One group is the open loop synchronizer which uses nonlinear devices. These circuits recover the clock signal directly from the data stream by nonlinear operations on the received data stream. The second group is the closed loop synchronizers which lock a local oscillator on to the received data stream by comparison measurements on the local and received signals. An example of open loop synchronizer is shown (Fig. 8.43) The clock is generated using differentiator rectifier combination. The LPF is used to counteract the effect of noise.

An example of closed loop synchronizer is the early late gate circuit (Fig. 8.44). A square wave clock is locally generated by the VCO. If the VCO square wave clock is in perfect synchronization with the demodulated signal $m(t)$ the early gate integrator and the late gate integrator will accumulate the same amount of signal energy so that the error signal ϵ is zero. If the VCO frequency is higher than that of $m(t)$, then $m(t)$ is late by $\Delta < d$ relative to the VCO clock. Thus, the integration time in the early gate integrator will be $T_b - d - \Delta$, while the integration time in the late gate integrator is $T_b - d$. The error signal will be proportional to $-\Delta$. This error signal will reduce the VCO frequency and retard the VCO timing to bring it back toward the timing of $m(t)$.

If the VCO frequency had been lower and the timing had been late the error signal would be proportional to $+\Delta$, and the reverse process would happen, i.e., the VCO frequency would be increased and its timing would be advanced toward that of the incoming signal.

Note that the early gate is designed to integrate from the start of the incoming data pulse to time $T_b - d$, while the late gate integrates from d to T_b, where d is fixed by design. At locking, the VCO and incoming frequencies are equal. In transients, however, differences in frequencies are interpreted as differences in phase which produce a corrective voltage for the VCO until locking takes place and the input to the VCO is zero.

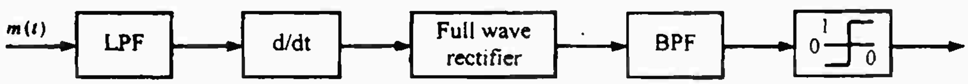


Fig. (8.43) Open loop synchronizer

8.13 Noncoherent Detector:

Assume a received signal in the form

$$r_d(t) = \sqrt{2E_s/T_s}\cos(2\pi f_i t + \theta)\ ,\ \ 0 \le t \le T_1$$

and θ is unknown. We may express

$$r_d(t) = \sqrt{2E_s/T_s}\left[\cos\theta\cos 2\pi f_i t - \sin\theta\sin 2\pi f_i t\right]\ ,\ \ 0 \le t \le T_1 \qquad (8-216)$$

Suppose that the received signal is applied to a pair of correlators one with the reference signal $\sqrt{2/T_s}\cos 2\pi f_i t$ and the other $\sqrt{2/T_s}\sin 2\pi f_i t$. The dependence on the unknown phase may be removed by summing the squares of the two correlator outputs and then taking the square root of the sum. This gives $\sqrt{E_s}$ which is independent of θ we may also replace each correlator by a matched filter. In one branch we have a filter matched to the signal $\sqrt{2/T_s}\cos 2\pi f_i t$, and in the other we have a filter matched to $\sqrt{2/T_s}\sin 2\pi f_i t$, both defined for $0 \le t \le T_1$. The filter outputs are sampled at $t = T_s$, squared and then added together.

Alternatively, we may have a filter matched to $s(t) = \sqrt{2/T_s}\cos(2\pi f_i t + \theta)$ for $0 \le t \le T_s$. The envelope of the matched filter output is unaffected by θ, we simply might as well use a matched filter with an impulse response $\sqrt{2/T_s}\cos\left[2\pi f_i (T_s - t)\right]$ corresponding to $\theta = 0$.

The output of such a filter is

$$z(T_s) = \sqrt{\frac{2}{T_s}}\int_0^{T_s} r_d(\tau)\cos\left[2\pi f_i\left(T_s - t + \tau\right)\right]d\tau \qquad (8-217)$$

$$= \sqrt{\frac{2}{T_s}}\cos\left[2\pi f_i\left(T_s - t\right)\right]\int_0^{T_s} r_d(\tau)\cos 2\pi f_i\tau\, d\tau$$

$$- \sqrt{\frac{2}{T_s}}\sin\left[2\pi f_i\left(T_s - t\right)\right]\int_0^{T_s} r_d(\tau)\sin 2\pi f_i\tau\, d\tau \qquad (8-218)$$

The envelope of the matched filter output is proportional to the square root of the sum of the squares of the integrals above. The envelope at $t = T_s$ is given by

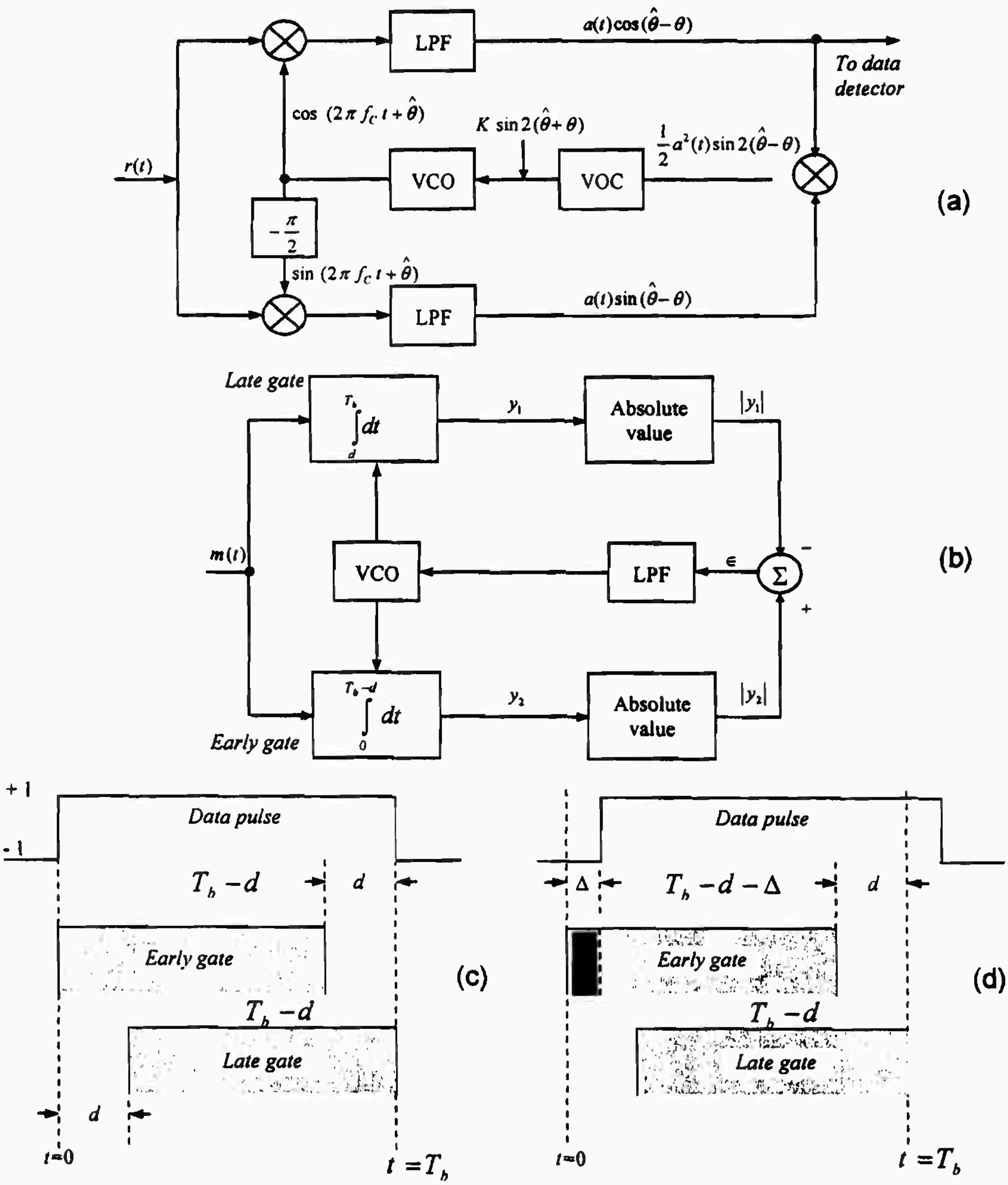


Fig. (8.44) Early late gate synchronizer

a) Costas loop b) early late circuit c) synchronized waveforms d) data late by Δ

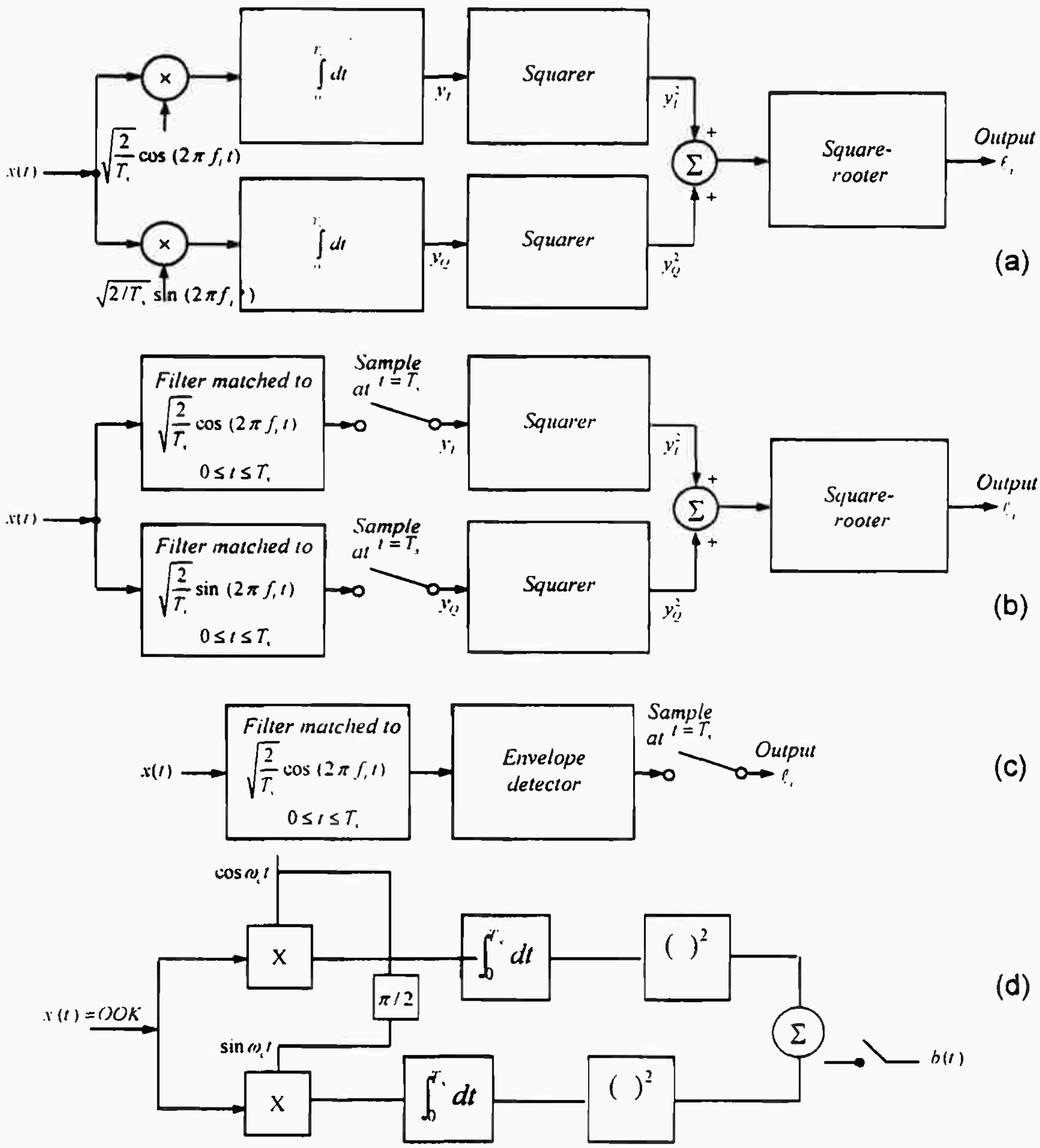


Fig. (8.45) Noncoherent demodulator

a) quadratute receiver using correlators

b) quadratute receiver using matched filter

c)noncoherent matched filter with integrate and dump or center point sampling at $t = T_s$

d) quadrature receiver for OOK.

$$\ell_i = \left\{ \left[\int_0^{T_s} r_d(\tau) \sqrt{2/T_s} \cos 2\pi f_i \tau \, d\tau \right]^2 + \left[\int_0^{T_s} r_d(\tau) \sqrt{2/T_s} \sin 2\pi f_i \tau \, d\tau \right]^2 \right\}^{1/2} \quad (8\text{-}219)$$

The combination of matched filter and envelope detector is called noncoherent matched filter (Fig. 8.45). To avoid using an envelope detector, we may use quadrature receiver. Although we need a carrier yet phase synchronization is not necessary. We may use noncoherent detection in OOK signals. In this case, an envelope detector is used to recover the baseband signal.

8.14 BFSK:

In BFSK we use two signals with different frequencies to represent $1, 0$

$$s_1(t) = A\cos(2\pi f_1 t + \theta_1),\ qT_b \le t \le (q+1)T_b \text{ For } 1 \quad (8\text{-}220)$$

$$s_2(t) = A\cos(2\pi f_2 t + \theta_2),\ qT_b \le t \le (q+1)T_b \text{ For } 0 \quad (8\text{-}221)$$

where θ_1 and θ_2 are initial phases at $t = 0$. The two signals are hence non coherent, since θ_1 and θ_2 are not the same. This form of FSK is called noncoherent FSK. It can be generated by switching the modulator output between different oscillators (Fig. 8.46). BFSK can be viewed as two OOK signals (Fig. (8.47).

The second type of FSK is coherent FSK where the two signals have the same initial phase θ at $t = 0$

$$s_1(t) = A\cos(2\pi f_1 t + \theta),\ qT_b \le t \le (q+1)T_b \text{ For } 1 \quad (8\text{-}222)$$

$$s_2(t) = A\cos(2\pi f_2 t + \theta),\ qT_b \le t \le (q+1)T_b \text{ For } 0 \quad (8\text{-}223)$$

The modulator shown in Fig. (8.46b) has a frequency synthesizer which generates two frequencies f_1 and f_2 which are synchronized. The binary input data controls the multiplexer. The bit timing must be synchronized with the carrier frequency. Note that $s_1(t)$ and $s_2(t)$ are always there regardless of the input data. For orthogonality

$$\int_{qT_b}^{(q+1)T_s} s_1(t)\, s_2(t)\, dt = 0 \quad (8\text{-}224)$$

In this case,

$$\phi_1(t) = \sqrt{2/T_b} \cos 2\pi f_i t \ ,\ 0 \le t \le T_b,\ i = 1, 2 \quad (8\text{-}225)$$

$$0 \qquad \textit{elsewhere}$$

$$a_{ij} = \sqrt{E_b}\ ,\ i = j \quad (8\text{-}226)$$

For this to happen, we must have

$$\int_{qT_b}^{(q+1)T_b} \cos(2\pi f_1 t + \theta) \cos(2\pi f_2 t + \theta) dt$$

$$=\frac{1}{2}\int_{qT_b}^{(q+1)T_b}\left[\cos\{2\pi(f_1+f_2)t+2\theta\}+\cos 2\pi(f_1-f_2)t\right]dt=0 \qquad (8-227)$$

This requires

$$2\pi(f_1+f_2)T_b=2n_1\pi \qquad (8-228)$$

$$2\pi(f_1-f_2)T_b=n_2\pi \qquad (8-229)$$

where n_1, n_2 are integers. Thus,

$$f_1=\frac{2n_1+n_2}{4T_b} \qquad (8-230)$$

$$f_2=\frac{2n_1-n_2}{4T_b} \qquad (8-231)$$

$$2\Delta f=f_1-f_2=\frac{n_1}{2T_b} \qquad (8-232)$$

Thus, we conclude that for orthogonality each of f_1 , f_2 must be an integer multiple of $1/4T_b$ and their difference must be an integer multiple of $1/2T_b$

Therefore

$$f_1=f_c+\Delta f \qquad (8-233)$$

$$f_2=f_c-\Delta f \qquad (8-234)$$

$$f_c=\frac{f_1+f_2}{2}=\frac{n_2}{2T_b} \qquad (8-235)$$

where f_c is the nominal (apparent) carrier frequency which must be an integer multiple of $1/2T_b$ for orthogonality. At bit transitions we may have a continuous phase if $2\Delta f=c/T_b$ where c is constant. This is called Sunde's FSK. Let us consider the phase of $s_i(t)$ at $t=mT_b$

$$\phi_1=2\pi f_1\, mT_b+\theta \qquad (8-236)$$

and the phase of $s_2(t)$

$$\phi_2=2\pi f_2\, mT_b+\theta \qquad (8-237)$$

$$=2\pi\left(f_1+\frac{c}{T_b}\right)mT_b+\theta \qquad (8-237)$$

$$=2\pi f_1\, mT_b+2\pi cm+\theta$$

$$=2\pi f_1 mT_b+\theta \qquad (8-238)$$

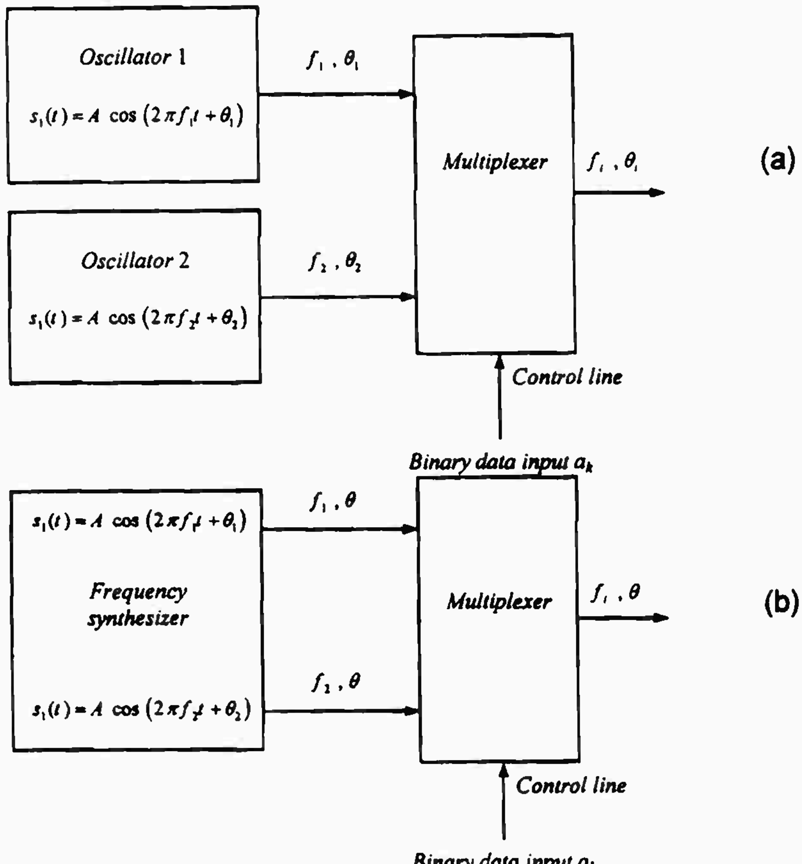


Fig. (8.46) FSK modulator

a) non coherent FSK modulator *b) coherent FSK modulator*

which is exactly the same as ϕ_1. Thus, the input switches from 1 to 0 smoothly and $s_2(t)$ will start at the same amplitude where $s_1(t)$has ended. We see then that the minimum separation for orthogonality between f_1 and f_2 is$1/2T_b$example of Sunde's FSK where bit 1 corresponds to f_1 and 0 to f_2 since f_1 and f_2 are multiple of $1/T_b$, the ending phase of the carrier is the same as the starting phase at the bit boundaries. Thus, Sunde's FSK is a continuous phase FSK. FSK with discontinuities (Fig. 8.48b) is an example of wavefroms with discontinuity of phase at bit boundaries as $f_1 = 9/4T_b$, $f_2 = 6/4T_b$ and $2\Delta f = 3/4T_b$. Since discontinuity of the waveform broadens the signal bandwidth continuous phase FSK (CPFSK) (Fig. 8.48) is desirable.

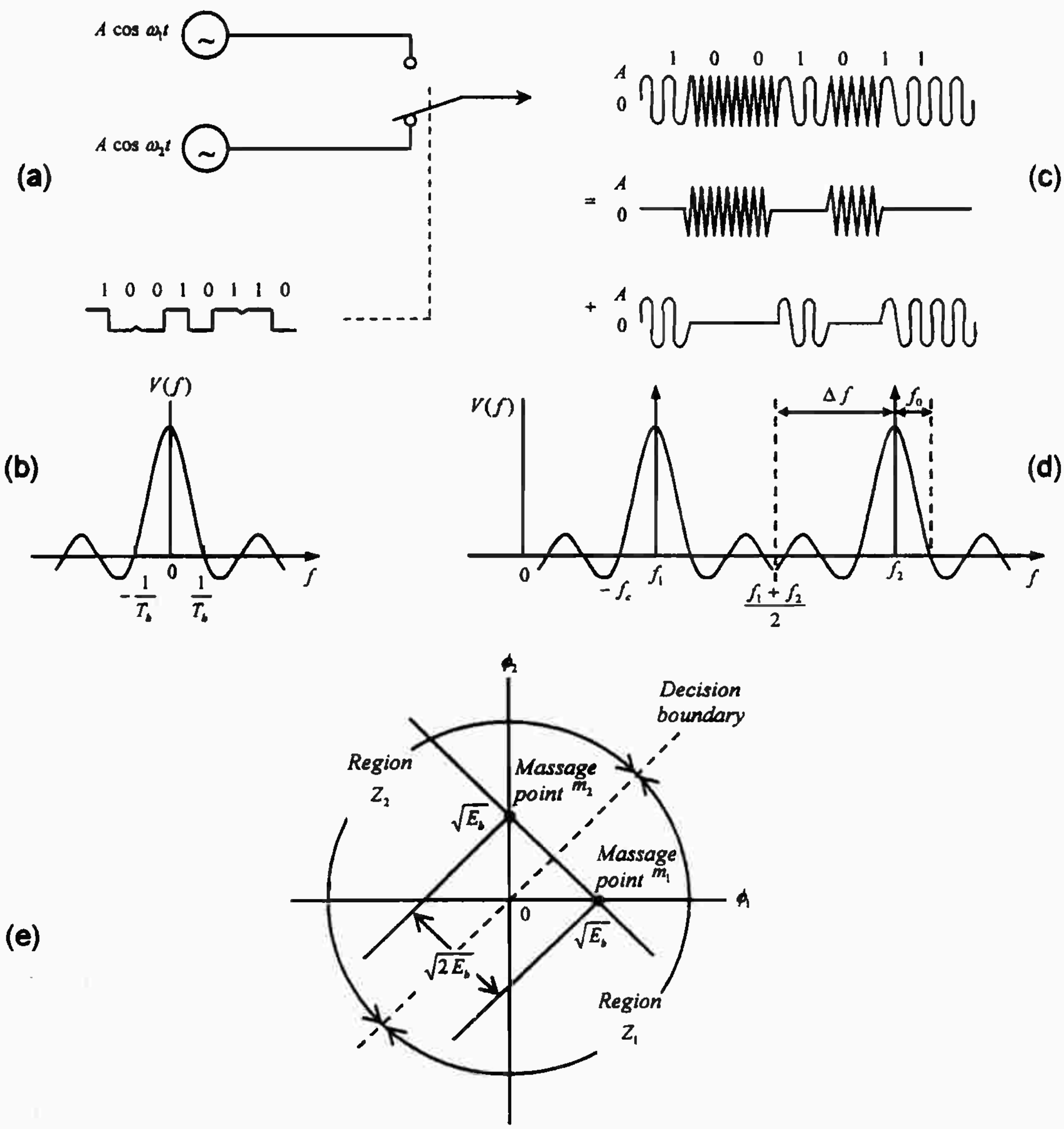


Fig. (8.47) BFSK

a) Baseband data

b) baseband voltage spectrum of a single symbol

c) BFSK signal and two component bandpass OOK signals

d) frequency spectnim of two OOK signals

e) Signal space for coherent BFSK

The FSK coherent demodulator can be implemented with two correlators (Fig. 8.49a) where two reference signals $\cos 2\pi f_1 t$ and $\cos 2\pi f_2 t$ synchronized with the input are used. Alternatively, one correlator may be used with the reference signal $\cos 2\pi f_1 t - \cos 2\pi f_2 t$ (Fig. 8.49b). The correlator may be replaced by a matched filter which matches $\cos 2\pi f_1 t - \cos 2\pi f_2 t$ (Fig. 8.49c). For equally likely binary signals of equal energy

$$P_B = Q\left[\sqrt{\frac{E_b}{\eta}(1-\rho)}\right] \tag{8-239}$$

For Sunde's signals $\rho = 0$ because the signals are orthogonal

$$P_B = Q\left[\sqrt{E_b/\eta}\right] \tag{8-240}$$

Where $E_B = A^2 T_b / 2$ is the average bit energy of the FSK signal. P_B For BFSK is shown in (Fig. 8.50). Coherent FSK signals can be noncoherently demodulated to avoid carrier recovery. It is a problem of detecting a signal with unknown phase. The received signal (ignoring noise for the moment) with an unknown phase may be written as

$$\begin{aligned} s_i(t,\theta) &= A\cos\left(2\pi f_i t + \theta\right) \qquad i = 1, 2 \\ &= A\cos\theta\cos 2\pi f_i t - A\sin\theta\sin 2\pi f_i t \end{aligned} \tag{8 –241}$$

The signal consists of an in phase component $A\cos\theta\cos 2\pi f_i t$ and a quadrature component $A\sin\theta\sin 2\pi f_i t$. Thus, the signal is partially correlated with $\cos 2\pi f_i t$ and partially correlated with $\sin 2\pi f_i t$. The outputs of the in phase and quadrature correlators will be $\frac{A T_b}{2}\cos\theta$ and $\frac{A T_b}{2}\sin\theta$. The square of the sum of these two signals is not dependent on the unknown phase

$$\ell_i^2 = \left(\frac{A T_b}{2}\cos\theta\right)^2 + \left(\frac{A T_b}{2}\sin\theta\right)^2 = \frac{A^2 T_b^2}{4} \tag{8 – 242}$$

This quantity is the mean value of the statistics ℓ_i^2 when signal $s_i(t)$ is transmitted. The comparator decides which signal is sent by checking ℓ_i^2 (Fig. 8.51a). A matched filter implementation is also shown (Fig. 8.51b). Another implementation uses BPFs centered at f_1 and f_2 (Fig. 8.51c).

For noncoherent detectors, the error probability P_B for orthogonal, equiprobable equal energy noncoherent signals is found to be

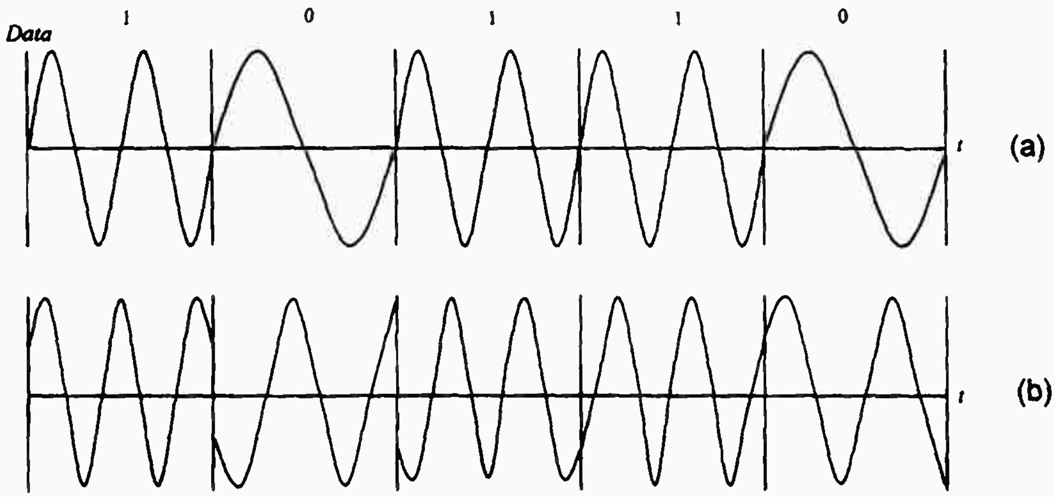


Fig. (8.48) FSK signal

a) Sunde's FSK $f_1 = 2/T_b\ ,\ f_2 = 1/T_b\ ,\ 2\Delta f\ = 1/T_b$

b) FSK with discontinuous phase $f_1 = 9/4T_b\ ,\ f_2 = 6/4T_b\ ,\ 2\Delta f\ = 3/4T_b$

$$P_B = \frac{1}{2}e^{-E_b/2\eta} \qquad (8-243)$$

It is seen (Fig. 8.50) that noncoherent FSK requires at most only $1dB$ more E_b/η than that for coherent FSK for $P_b \le 10^{-4}$. The noncoherent FSK demodulator is considerably easier to build since coherent reference signals need not be generated. Therefore, almost all FSK receivers use noncoherent demodulators.

We have shown that the minimum frequency separation for coherent signals is $1/2T_b$. We now show that the minimum separation for noncoherent signals is $1/T_b$ instead of $1/2T_b$. The two noncoherent FSK signals are $s_1(t) = \cos 2\pi f_1 t$ and $s_2(t) = \cos(2\pi f_2 t + \theta)$. For them to be orthogonal

$$\int_{qT_b}^{(q+1)T_b} \cos 2\pi f_1 t \cos(2\pi f_2 t + \theta)dt = 0 \qquad (8-244)$$

Thus,

$$\cos\theta \int_{qT_b}^{(q+1)T_b} \cos 2\pi f_1 t \cos 2\pi f_2 t\, dt - \sin\theta \int_{qT_b}^{(q+1)T_b} \sin 2\pi f_2 t \cos 2\pi f_1 t\, dt = 0$$

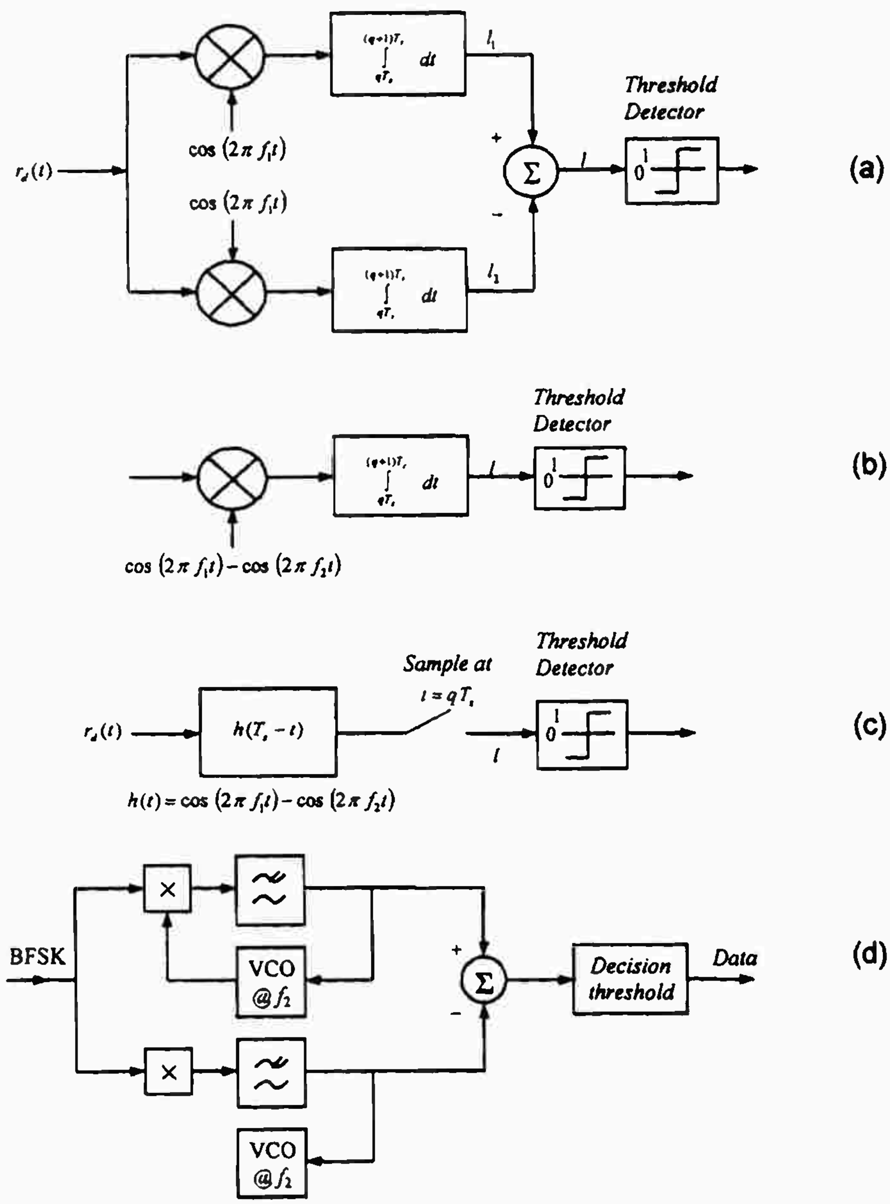


Fig. (8.49) FSK demodulator

a) Two coherent correlators demodulator *b) one coherent correlator*

c) Using a matched filter *d) using PLL*

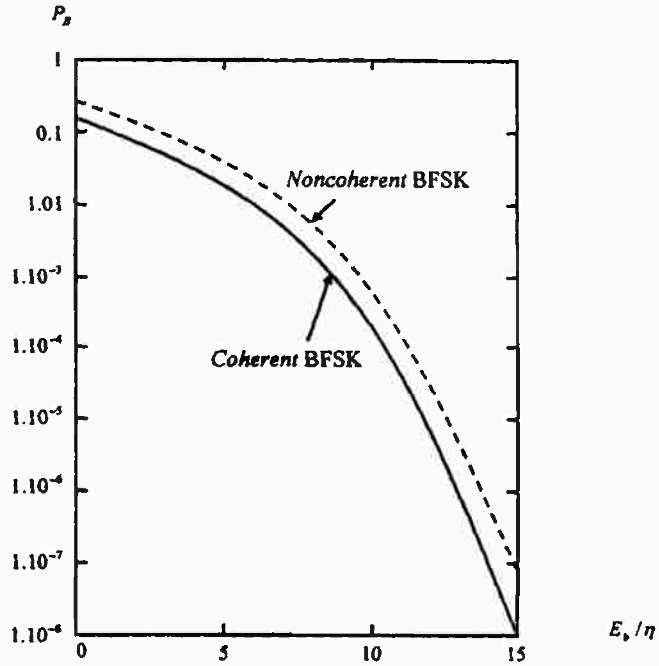


Fig. (8.50) P_B **of coherently and noncoherently demodulated FSK signal**

Thus,

$$\cos\theta\left[\frac{\sin 2\pi(f_1+f_2)\,t}{2\pi(f_1+f_2)}+\frac{\sin 2\pi(f_1-f_2)\,t}{-2\pi(f_1-f_2)}\right]_{qT_b}^{(q+1)T_b}$$

$$+\sin\theta\left[\frac{\cos 2\pi(f_1+f_2)\,t}{2\pi(f_1+f_2)}-\frac{\cos 2\pi(f_1-f_2)\,t}{2\pi(f_1-f_2)}\right]_{qT_b}^{(q+1)T_b}=0 \qquad (8\text{--}245)$$

For arbitrary θ, this requires that the sums inside the brackets be zero, i.e.

$$2\pi(f_1+f_2)T_b = m_1\pi \qquad (8\text{-}246)$$

$$2\pi(f_1-f_2)T_b = n_2\pi \qquad (8\text{-}247)$$

$$2\pi(f_1+f_2)T_b = n_3\pi \qquad (8\text{-}248)$$

$$2\pi(f_1-f_2)T_b = n_4\pi \qquad (8\text{-}249)$$

For m_1, m_2, m_3, m_4 integers where $m_1 > m_2$, $m_3 > m_4$. The $m_1\pi$ and $m_2\pi$ cases are included in the $2m_3\pi$ and $2m_4\pi$ respectively. Thus, the above requirements, are met if

$$2\pi(f_1 + f_2)T_b = 2m_3\,\pi \tag{8 -250}$$

$$2\pi(f_1 - f_2)T_b = 2m_4\,\pi \tag{8 -251}$$

$$f_1 = \frac{m_3 + m_4}{2T_b} \tag{8 -252}$$

$$f_2 = \frac{m_3 - m_4}{2T_b} \tag{8 -253}$$

$$f_1 - f_2 = \frac{m_3}{T_b} \tag{8 -254}$$

Thus, for two noncoherent FSK signals to be orthogonal, the two frequencies must be integer multiple of $1/2T_b$ and their separation must be multiple of $1/T_b$. For $m_3 = 1$, we have minimum separation which is double that for coherent FSK. Thus, more bandwidth is required for noncoherent FSK of the same symbol rate.

It is of course possible to choose symbol frequencies f_1 and f_2 and symbol duration T_b such that the symbols are not orthogonal

$$\int_0^{T_b} \cos\left(2\pi f_1 t\right)\cos\left(2\pi f_1 t\right) \neq 0 \tag{8 -255}$$

In this case, there will be nonzero sampling instant outputs from both BFSK receiver channels when either symbol is present at the receiver input. Denoting a general pair of symbols by $v_1(t)$ and $v_2(t)$, we define the correlation coefficient ρ as

$$\rho = \frac{< v_1(t)\, v_2(t) >}{\sqrt{\left\langle |v_1(t)|^2 \right\rangle}\,\sqrt{\left\langle |v_2(t)|^2 \right\rangle}} \tag{8 -256}$$

$$= \frac{1}{v_{1\,rms}\; v_{2\,rms}}\,\frac{1}{T_b}\int_0^{T_b} v_1(t)\; v_2(t)\; dt \tag{8 -257}$$

$$= \frac{2}{T_b}\int_0^{T_b} \cos\left(2\pi f_1 t\right)\cos\left(2\pi f_2 t\right) dt \tag{8 -258}$$

It can be shown that

$$\rho = \frac{\sin 2\pi(f_2 - f_1)T_b}{2\pi(f_2 - f_1)T_b} \tag{8 -259}$$

Fig. (8.52) shows ρ plotted against $2(f_2 - f_1)\,T_b$. The zero crossing point represents orthogonal signaling system, and $2(f_2 - f_1)\,T_b = 1.43$ represents a signaling system between orthogonal and antipodal.

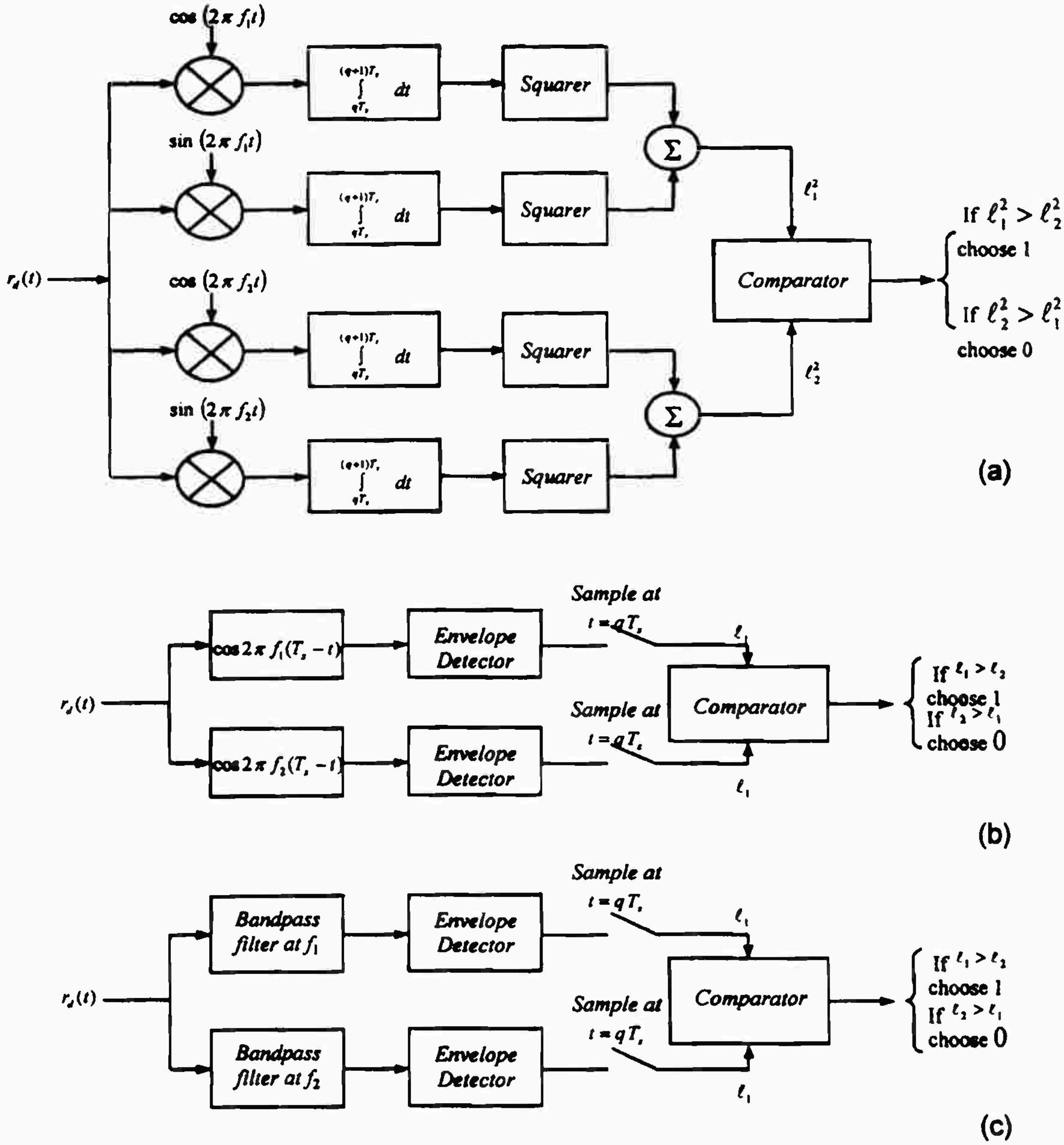


Fig. (8.51) FSK noncoherent demodulator

a) correlator implementation *b) matched filter implementation*

c) BPF implementation

It has optimum power efficiency. While zero crossing on the $\rho - T_b$ diagram corresponds to orthogonal BFSK. It is not possible to use the first zero, $2(f_2 - f_1)T_b = 1$. The minimum frequency separation for successful incoherent detection of orthogonal BFSK is given by the second zero
$2(f_2 - f_1)T_b = 2$ or $\Delta f = f_b/2$. The voltage and power spectra for the scheme are shown (Fig. 8.53). If the first spectral zero definition of bandwidth (Carson's rule) is applied to Sunde's FSK, the bandwidth is given by.

$$B = 2(\Delta f + f_{max}) = (f_2 - f_1) + \frac{2}{T_b} = \frac{3}{T_b} = 2\Delta f + R_b \qquad (8-260)$$

where R_b is the bit rate since $f_{max} = \frac{1}{2T_b} = \frac{1}{2}R_b$

Thus, the bandwidth is the tone separation plus the bit rate. Thus FSK is bandwidth expensive.

We may consider the voltage spectrum of the BFSK signal as the superposition of two OOK spectra, one representing the baseband data stream modulated onto a carrier with frequency f_1 and one representing the inverse data stream modulated onto a carrier with frequency f_2. We cannot, however, consider the PSD of the BFSK signal as superposition of two OOK PSDs because of the overlap of the spectral lines (Fig. 8.53c). When the separation of f_1 and f_2 is large this overlap is small then we may perform the superposition. We note that Sunde's FSK is a special case of continuous phase frequency shift keying (CPFSK) when the phase at the transition points appears continuous.

The overlapping lines of the component OOK signals result in cancellation giving a $1/f^4$ roll off in the envelope. For this reason we may take B as

$$B = f_2 - f_1 = \frac{1}{T_b} \qquad (8-261)$$

Minimum shift keying (MSK) not only has minimum separation but also continuous phase. We have seen that discontinuity in the waveform broadens the bandwidth that is why we resort to continuous phase operation. The frequency derivation for coherent FSK is given by

$$f_1 = f_c + \frac{k}{2T_b}, \; f_2 = f_c - \frac{k}{2T_b} \qquad (8-262)$$

Hence,

$$\Delta f = \frac{f_1 - f_2}{2} = \frac{k}{2T_b} = \frac{kR_b}{2}, \; k \text{ is an integer} \qquad (8-263)$$

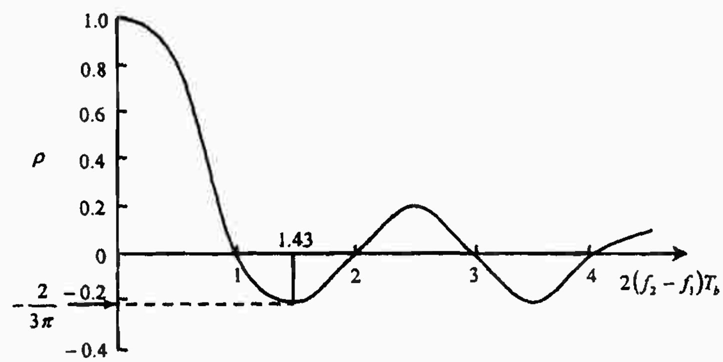


Fig. (8.52) $\rho - T_b$ **diagram for BFSK**

$$f_c = \frac{f_1 + f_2}{2} \qquad (8\text{-}264)$$

Then, using the first zero crossing points in the BFSK voltage spectrum to define its bandwidth B the maximum frequency for the baseband signal of rectangular form is $f_{max} = 1/2T_b$. Here again we usually use the relation

$$B = 2(\Delta f + f_{max}) = \text{Tone separation} + \text{Bit rate} \qquad (8\text{-}265)$$

We note that the binary symbols of BFSK are orthogonal because

$$\int_0^{T_b} \cos 2\pi f_1 t - \cos 2\pi f_2 t = 0 \qquad (8\text{-}266)$$

Thus when the output of a channel in the coherent BFSK receiver is maximum the output of the other channel is zero. After subtracting the post filtered signals from each receiver channel the orthogonal BFSK decision instant voltage (Fig. 8.6) is

$$z\,(qT_b) = \begin{cases} hE & digital\ 1 \\ -hE & digital\ 0 \end{cases} \qquad (8\text{-}267)$$

where h is a constant, The noise power at the receiver output is

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 = h^2\ E\eta \qquad (8\text{-}268)$$

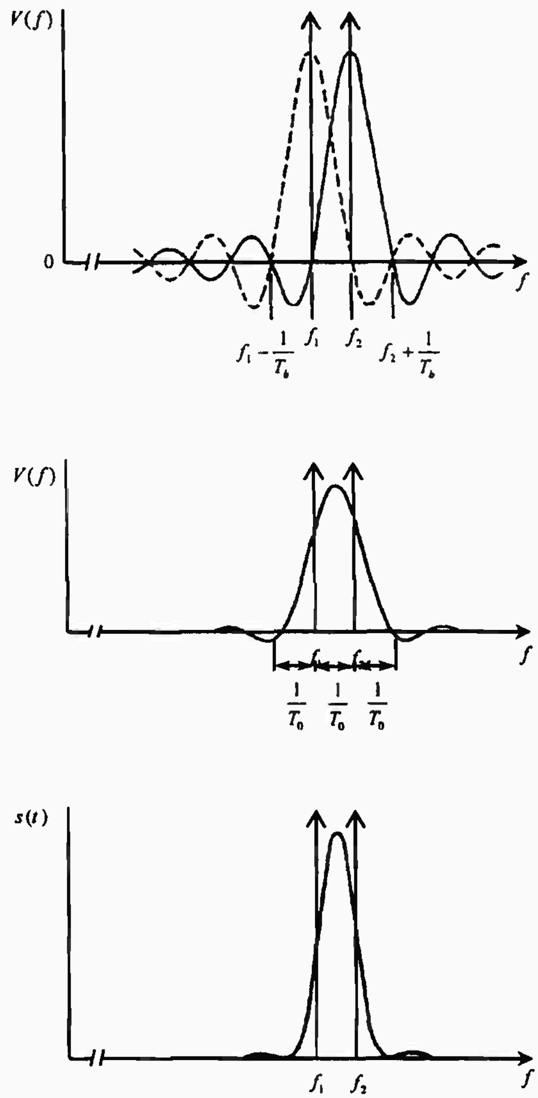
where

$$\sigma_1^2 = h^2\ E\eta/2 \qquad (8\text{-}269)$$

$$\sigma_2^2 = h^2\ E\eta/2 \qquad (8\text{-}270)$$

when σ_1^2 is the noise power in channel 1 and σ_2^2 is the noise power in channel 2 and they add up since they are uncorrelated.

Substituting $\Delta V = 2hE$ and noting $\sigma = h\sqrt{E\eta}$, we have



(a)
(b)
(c)

Fig. (8.53) Sunde's FSK
a) voltage spectrum as superposition of OOK spectrum
b) voltage spectrum
c) PSD

$$P_s = \frac{1}{2}\left[1 - erf\,\frac{1}{\sqrt{2}}(E/\eta)^{1/2}\right] \qquad (8\text{-}271)$$

Noting $E = \langle E \rangle$ we have the same result for OOK, Table (8.12) summarizes P_s results so far

Table (8.12) P_s in different schemes

		P_s	
Baseband signalling	Unipolar (OOK)	$\frac{1}{2}erfc\sqrt{\frac{1}{2}\frac{E}{\eta}}$	$\frac{1}{2}erfc\sqrt{\frac{1}{4}\frac{S}{N}}$
	Polar	$\frac{1}{2}erfc\sqrt{\frac{E}{\eta}}$	$\frac{1}{2}erfc\sqrt{\frac{1}{2}\frac{S}{N}}$
IF/RF signalling	OOK	$\frac{1}{2}erfc\sqrt{\frac{1}{2}\frac{E}{\eta}}$	$\frac{1}{2}erfc\sqrt{\frac{T_s B}{2}\frac{C}{N}}$
	BFSK (orthogonal)	$\frac{1}{2}erfc\sqrt{\frac{1}{2}\frac{E}{\eta}}$	$\frac{1}{2}erfc\sqrt{\frac{T_s B}{2}\frac{C}{N}}$
	PRK	$\frac{1}{2}erfc\sqrt{\frac{E}{\eta}}$	$\frac{1}{2}erfc\sqrt{T_s B\frac{C}{N}}$

Table (8.13) Relative power efficiencies in different schemes

	Bandpass OOK	*Orthogonal BFSK*	*PRK*
$\frac{E_1}{\eta}$	4	2	1
$\frac{\langle E \rangle}{\eta}$	2	2	1

Table (8.13) gives relative power efficiencies for binary bandpass modulation schemes. We note that OOK and orthogonal FSK systems have the same probability of symbol error for the same $\langle E \rangle/\eta$ since they are both orthogonal schemes. PRK is antipodal and is more power efficient i.e. it has better P_s performance. It is possible to define a nominal spectral efficiency using the signal pulse's main lobe bandwidth $B = 2/T_b$ (DSB). This does not correspond to the theoretical minimum channel bandwidth $B = 1/T_b$. Table (8.14) shows relative spectral efficiencies of binary bandpass modulation scheme. Table (8.15) shows a comparison of several PSK schemes

Table (8.14) Relative spectral efficiencies of binary bandpass modulation schemes

	Baseband BASK	Bandpass BASK	Orthogonal BASK $(n \geq 3)^*$	Sunde's BFSK $(n = 2)^*$	BPSK
Data rate (bit/s)	$1/T_s$	$1/T_s$	$1/T_s$	$1/T_s$	$1/T_s$
Nominal bandwidth (Hz)	$1/T_s$	$2/T_s$	$(n+4)/2T_s$	$1/T_s$ or $3/T_s$	$2/T_s$
Nominal spectral efficiency (bit/s/Hz	1	$1/2$	$2/(n+4)$	1 or $1/3$	$1/2$
Minimum (ISI free) bandwidth	$1/2T_s$	$1/T_s$	$(n+4)/2T_s$	$2/T_s$	$1/T_s$
Maximum spectral efficiency	2	1	$2/(n+2)$	$1/2$	1

Table (8.15) Comparison of several PSK modulation schemes

	Required E_b/N_0 for $P_b = 10^{-6}$	Minimum channel bandwidth for ISI free signalling (R_b = bit rate)	Max spectral efficiency (bit/s/Hz)	Required CNR in minimum channel bandwidth
PRK	10.6 dB	R_b	1	10.6 dB
QPSK	10.6 dB	0.5 R_b	2	13.6 dB
8-PSK	14.0 dB	0.33 R_b	3	18.8 dB
16-PSK	18.3 dB	0.25 R_b	4	24.3 dB

Ex. 8.8

Develop a generalized expression P_s which can be applied to all binary coherently detected modulation schemes.

Solution

For two channel coherent receiver the decision instant voltage for equiprobable equal energy symbol

$$z(qT_b) = h\left[E_b - \int_0^{T_b} V_1(t)\, V_2(t)\, dt\right] \quad \text{symbol } 0 \qquad (2\text{-}272)$$

$$z(qT_b) = h\left[-E_b + \int_0^{T_b} V_1(t)\, V_2(t)\, dt\right] \quad \text{Symbol } 1 \qquad (8\text{-}273)$$

$$= h[E_b - \rho E_b] \qquad \text{Symbol } 0 \qquad (8\text{-}274)$$

$$h[-E_b + \rho E_b] \qquad \text{Symbol } 1 \qquad (8\text{-}275)$$

$$\Delta V = h[2E_b - 2\rho E_b] = 2hE_b(1-\rho) \qquad (8\text{-}276)$$

The two channels have total noise power of

$$\sigma^2 = h^2\, E_b\, \eta\,(1-\rho) \qquad (8\text{-}277)$$

$$P_s = \frac{1}{2}\left[1 - erf\,\sqrt{\frac{1-\rho}{2}}\left(\frac{E_b}{\eta}\right)^{1/2}\right] \qquad (8\text{-}278)$$

For OOK and orthogonal BFSK, $\rho = 0$, for PRK $\rho = -1$

Alternatively we may write

$$P_s = \frac{1}{2}\left[1 - erf\,\sqrt{\frac{1-\rho}{2}}\,(T_b\, B)^{1/2}\left(\frac{C}{N_0}\right)^{1/2}\right] \qquad (8\text{-}279)$$

Assuming ISI free reception and nonfiltering (rectangular pulse)

$$BER = P_s\, R_s \qquad (8\text{-}280)$$

Ex. 8.9

Propose an FSK transmitter and receiver using VCO, and calculate the deviation ratio and bandwidth required to transmit a $1200\,b/s$ digital signal where logic 0 is represented by $2100\ Hz$ and logic 1 by $1300\ Hz$.

Solution

Fig. (8.54) shows FSK transmitter and receiver using VCO. The modulation index (deviation ratio) is denoted by

$$m_f = \frac{\Delta f}{f_{max}}$$

where Δf is the frequency deviation measured from the carrier frequency and f_{max} is the maximum baseband modulating frequency. The bandwidth is given by

$$B = 2(\Delta f + f_{max})$$

We note tone separation is $2\Delta f$. We also note for the baseband pulse of unfiltered of (rectangular frequency) pulse

$$f_{max} = \frac{1}{2T_b} = \frac{R_b}{2}$$

From eqns. (8 – 262), (8 – 263)

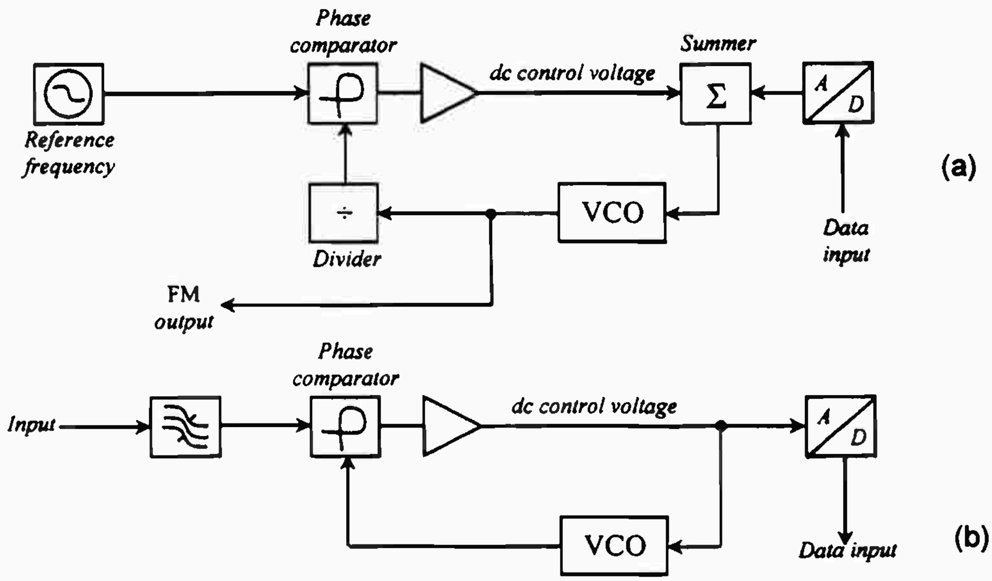


Fig. (8.54) FSK transmitter and receiver using VCO
a) transmitter *b) receiver*

$$f_1 = f_c + \frac{k}{2T_b}$$

$$f_2 = f_c - \frac{k}{2T_b}$$

Thus, $$\Delta f = \frac{f_1 - f_2}{2} = \frac{k}{2T_b}$$

$$Tone\ speration = 2100 - 1300 = 800\ Hz$$

$$B = Tone\ separation + Baud\ rate(bit/s)\ R_b$$

$$= 2000\ kHz$$

$$D = \frac{tone\ separation}{bit\ rate}$$

$$= \frac{800}{1200} = 0.67$$

8.15 MFSK:

In an M-ary FSK (MFSK), the binary data is divided into k-tuples of $k = \log_2 M$ bits. There are M signals with different frequencies

$$s_i(t) = A \cos\left(2\pi f_i t + \theta_0\right) \cdot qT_s \le t \le (q+1)T_s \qquad (8-281)$$

If the initial phase $\theta_0 = 0$ is the same for all i, the signal set is coherent. In this case, the demodulation could be coherent or noncoherent. Otherwise if $\theta_0 \ne 0$, the signal set is noncoherent and the demodulation must be noncoherent. FSK signals to be orthogonal, the frequency separation between any two of them must be $m/2T_s$ for coherent case and $1/T_s$ for noncoherent case. We express the separation in terms of modulation index $m_f = 2\Delta f\, T_s$. The frequency separation is $2\Delta f$.

The M-ary messages are

$$m_i = \pm 1, \pm 3, .. \pm (M-1) \qquad (8-282)$$

The M - ary signals are

$$s_i(t) = A \cos\left[2\pi f_C t + m_i \frac{m_f \pi}{T_s}(t - qT_s) + \theta_0\right] \quad qT_s \le t \le (q+1)T_s \qquad (8-283)$$

For coherent detection,

$$B = \frac{M}{2T_s} \qquad (8-284)$$

But

$$T_s = T_b \log_2 M$$

$$B = \frac{R_b M}{2\log_2 M} \qquad (8-285)$$

The bandwidth efficiency of M-ary FSK (8 -286)

$$\varepsilon_s = \frac{R_b}{B} \qquad (8-287)$$

$$= \frac{2\log_2 M}{M} \qquad (8-288)$$

Thus, while increasing the number of levels M tends to increase the bandwidth efficiency of MPSK, it tends to decrease the bandwidth efficiency of MFSK. Thus, MPSK signals are spectrally efficient while MFSK are not (Table 8.17)

Table (8.16) Bandwidth efficiency of MPSK and MFSK

	M	2	4	8	16	32	64
MPSK	ε_s $bits/s\ Hz$	0.5	1	1.5	2	2.5	3
MFSK	ε_s $bits/s\ Hz$	1	1	0.75	0.5	0.3125	0.1875

The noncoherent modulator and demodulator for BFSK (Fig. 8.25) and (Fig. 8.30) can be extended for noncoherent MFSK by simply increasing the number of independent oscillators to M. The frequency synthesizer generates M signals with designed frequencies and the multiplexer chooses one according to the n data bits (Fig. 8.55). The coherent modulator of BFSK (Fig. 8.46) can also be extended to MFSK (Fig. 8.56)

We assume that MFSK signals are equiprobable and of equal energy. The symbol error probability for the case when the signal set is of equal energy and orthogonal and when all distances between any two signals are equal $(d = \sqrt{2E_s})$ are shown (Fig. 8.57)

$$P_s = (M-1)\, Q\left(\sqrt{E_s/\eta}\right) \qquad (8-289)$$

since $k = \log_2 M\ bits$,

$$E_b = \frac{E_s}{\log_2 M} \qquad (8-290)$$

For equally likely orthogonal M-ary signals, all symbol errors are equiprobable. The demodulator may choose any one of $(M-1)$ erroneous orthogonal signals with equal probability

$$P_B = \frac{P_s}{M-1} = \frac{P_s}{2^k - 1} \qquad (8-291)$$

There are $\begin{pmatrix} k \\ u \end{pmatrix}$ ways, in which u bits out of k bits may be in error. Thus, the average number of bit errors per k bit symbol is

$$\sum_{k=1}^{n} k \begin{pmatrix} k \\ u \end{pmatrix} \frac{P_s}{2^k - 1} = k\, \frac{2^{k-1}}{2^k - 1} P_s \qquad (8-292)$$

Thus, the average bit error probability in the above divided by k

$$P_B = \frac{2^{k-1}}{2^k - 1} P_s \sim \frac{1}{2} P_s \qquad (8-293)$$

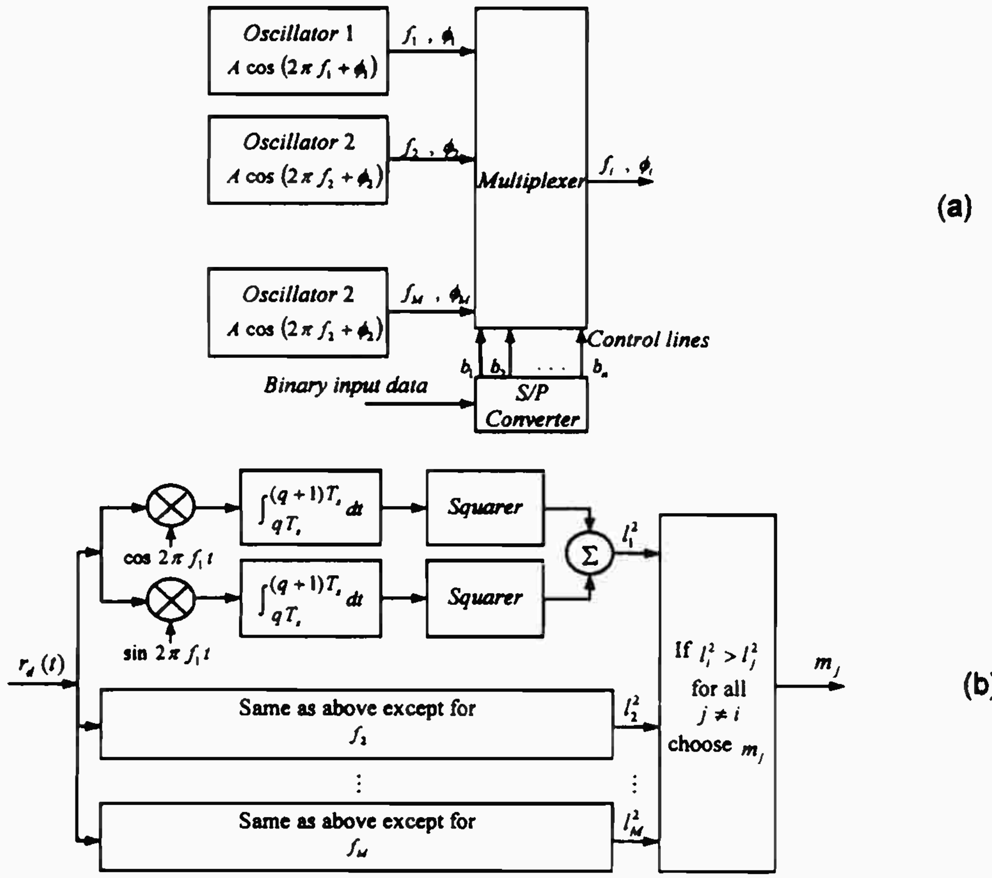


Fig. (8.55) MFSK noncoherert modulator and demodulator
a)modulator *b)demodulator (correlator – squarer)*

Fig. (8.58) shows P_s and P_B for noncoherent demodulator
From eqn. (8-292), (8-293) as M increases E_s increases for the same E_b.The Q term overrides the (M-1) term and as a result P_s decreases dramatically with increasing M at the same E_b/η (Figs. 8.57, 8.58, 8.59).

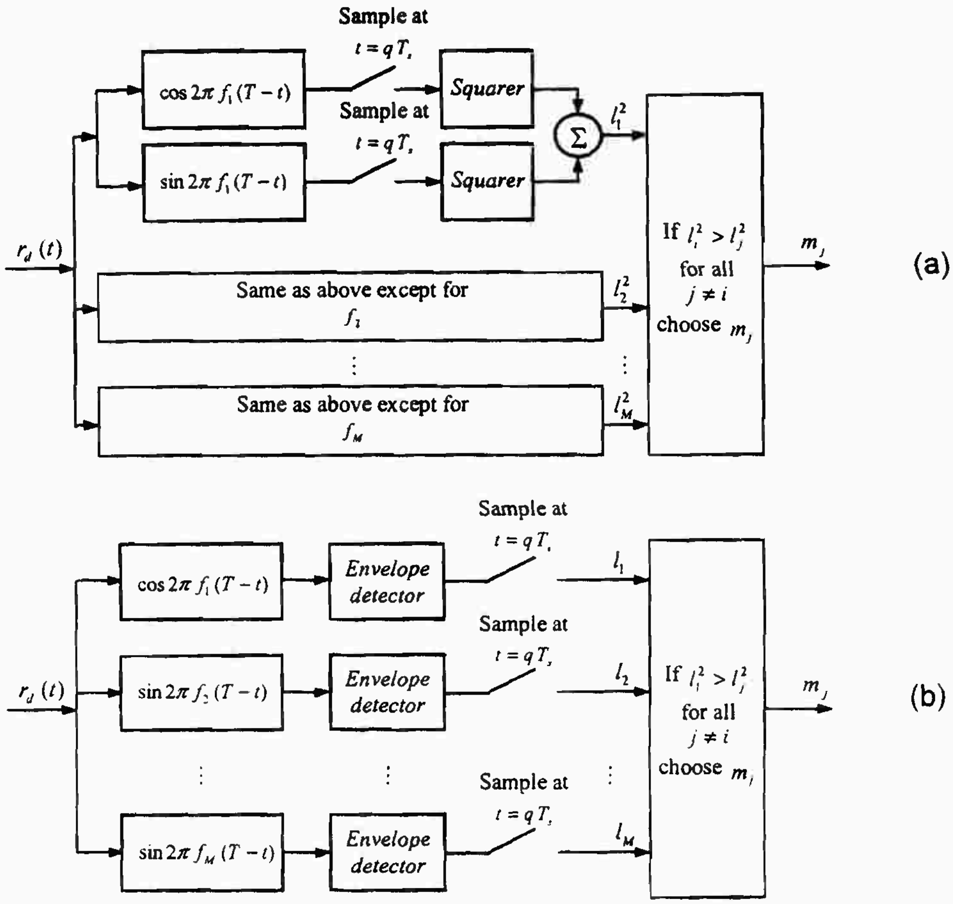


Fig. (8.56) MFSK coherent modulator and demodulator
a) matched filter-squarer b) envelope detector

8.16 Multidimensional Signaling:

We should note that in communication systems where large bandwidths are available but signal power is limited we use power efficient systems. Such a system is MFSK. In general, data rate can be improved by increasing the number of symbols at the transmitter. The symbols must remain as widely spaced in the constellation as possible. This can be achieved without increasing transmitted power by adding orthogonal axes to the constellation space i.e. multidimensional signaling. Also, the most significant power saving comes from optimizing the lattice pattern of constellation points. This is called symbol packing and results in an increased constellation point

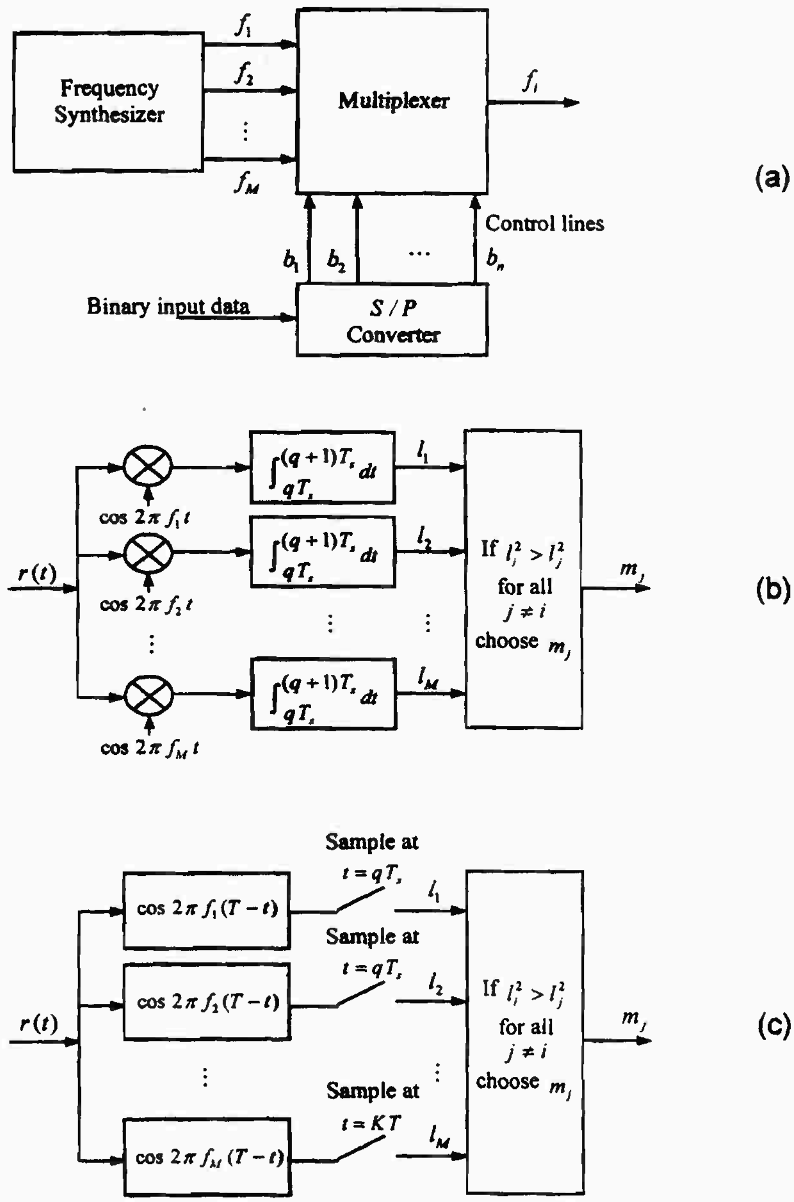


Fig. (8.57) MFSK coherent modulator demodulator
a) coherent modulator b) demodulator (correlator) c) demodulator (matched filter)

density without decreasing point separation. In MFSK symbols are mutually orthogonal. (Fig. 8.60) as an extension of Fig. (8.53) shows the voltage spectrum of an orthogonal MFSK signal as a superposition of OOK signals. The increased data rate is realized by MFSK signaling at the expense of increased bandwidth eqn. (8 – 286).

Multidimensional signaling can also be achieved using sets of orthogonally coded bit patterns. (Fig. 8.61) shows an example of orthogonal coded set for 8 symbols. Such symbols must be M binary digits long. The nominal bandwidth of such a symbol is $M/T_s\ Hz$ for baseband signaling and $2M/T_s\ Hz$ for bandpass signaling, while $T_s/M = T_b$. For equiprobable symbols, the nominal spectral efficiency for bandpass orthogonal code signaling is

$$\varepsilon_s = \frac{\log_2 M}{2M} \qquad bits/s\ Hz \tag{8 –294}$$

The maximum spectral efficiency for $T_s\ B = M$ is twice this. For coherent detection of any equal energy equiprobable, M-ary orthogonal symbol set (including MFSK) in the presence of AWGN

$$P_s = \frac{1}{2}(M-1)\left[1 - erf\ \sqrt{E_s/2\eta}\right] \tag{8 –295}$$

$$E_b = \frac{E_s}{\log_2 M} \tag{8 –296}$$

For any particular information bit associated with any given symbol only $M/2$ symbol errors out of a total of $M-1$ possible symbol errors will result in that bit being in error. Thus, for orthogonal signaling.

$$P_B = \frac{M/2}{M-1} P_s = \frac{2^{k-1}}{2^k - 1} \tag{8–297}$$

if M is ∞

$$\lim_{M\to\infty} P_B = \frac{P_s}{2} \tag{8–298}$$

Another system is developed for multicarrier broadcast to mobiles. It uses orthogonal FDM (OFDM) based on simultaneous MFSK, to reduce a single high data rate signal into many parallel low data rate signals, and is useful in overcoming problems in multipath channels where interference may degrade the received signal. Because of orthogonality such degradation is checked. This is achieved by transmitting data in parallel by using a large number of modulated carriers with sufficient frequency spacing so that the carriers are orthogonal. This provides resistance to data errors due to multipath channels.

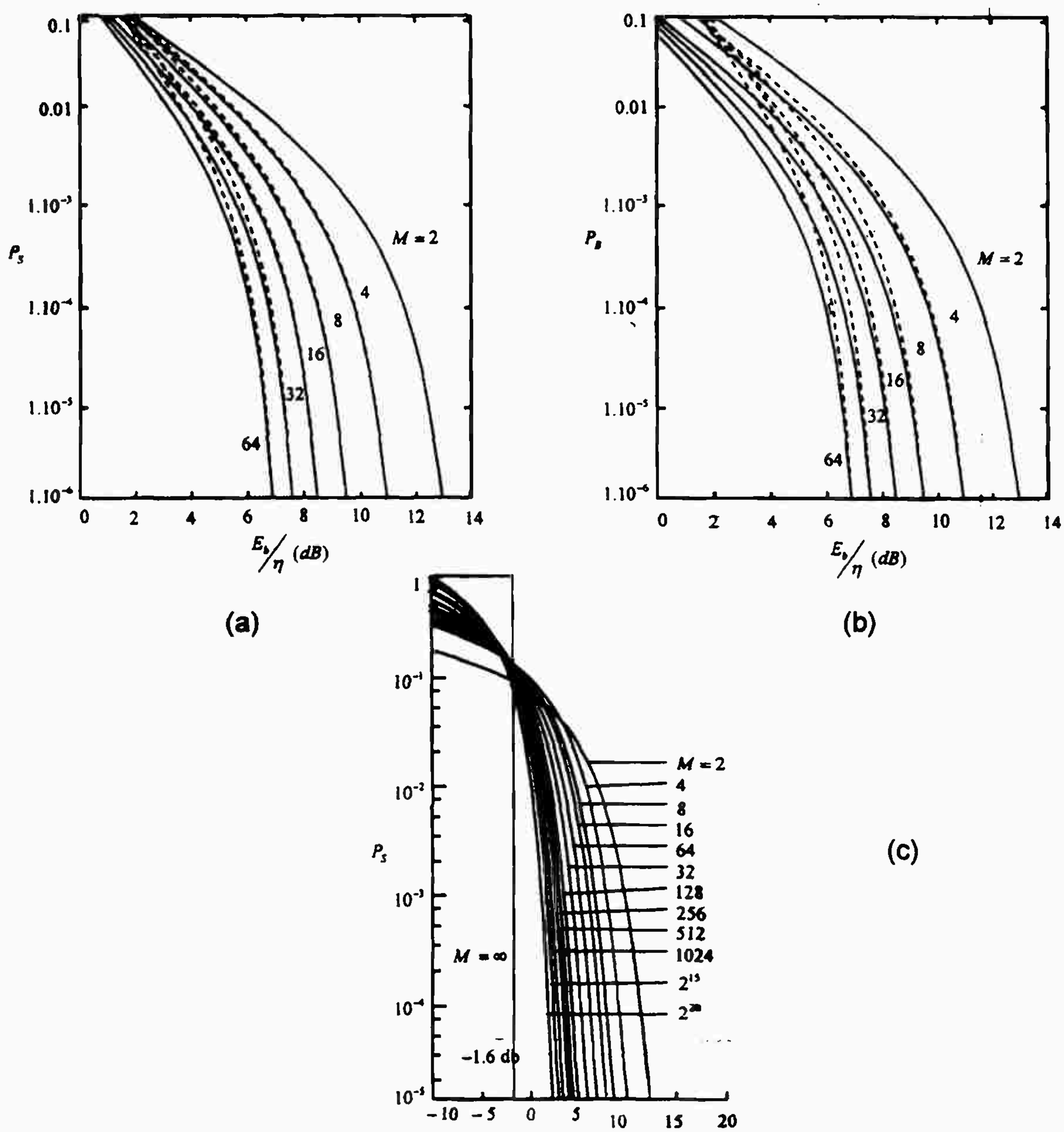


Fig. (8.58) P_s **and** P_b **for coherently demodulated equiprobable equal energy and orthogonal MFSK**

a) P_s b) P_B c) P_s as M is large

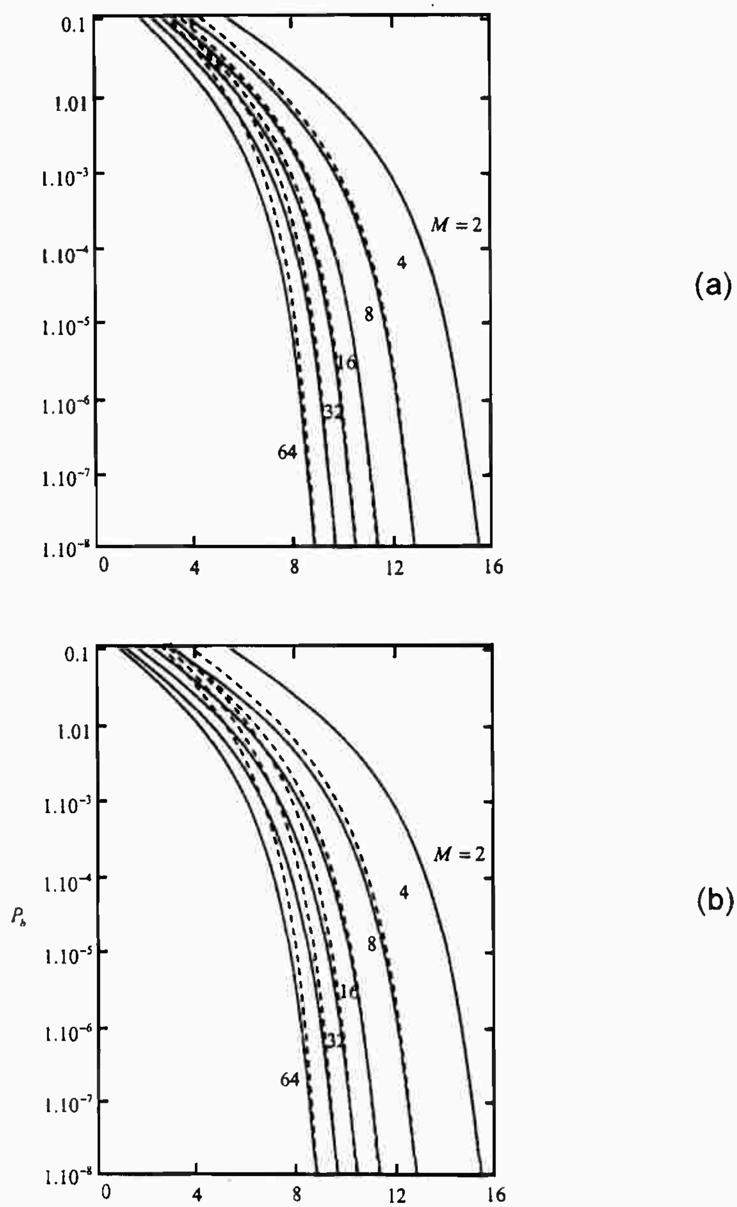


Fig. (8.59) P_s and P_B for noncoherently demodulated equiprobable equal energy and orthogonal MFSK

a) P_s b) P_B

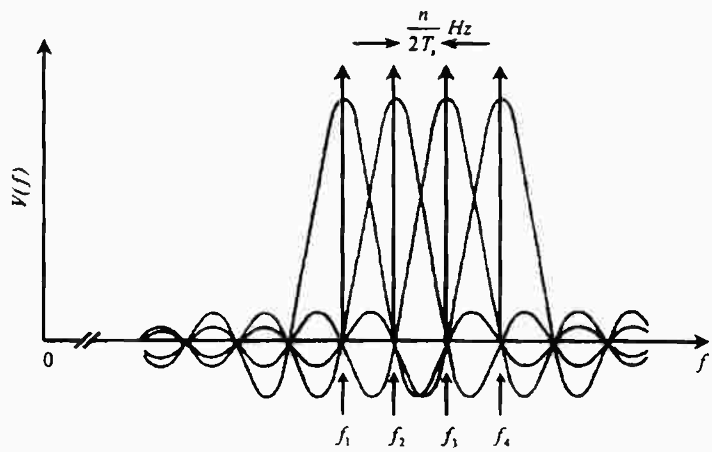


Fig. (8.60) Spectrum of orthogonal MFSK $(M = 2)$ as a superposition of OOK signal voltage spectra

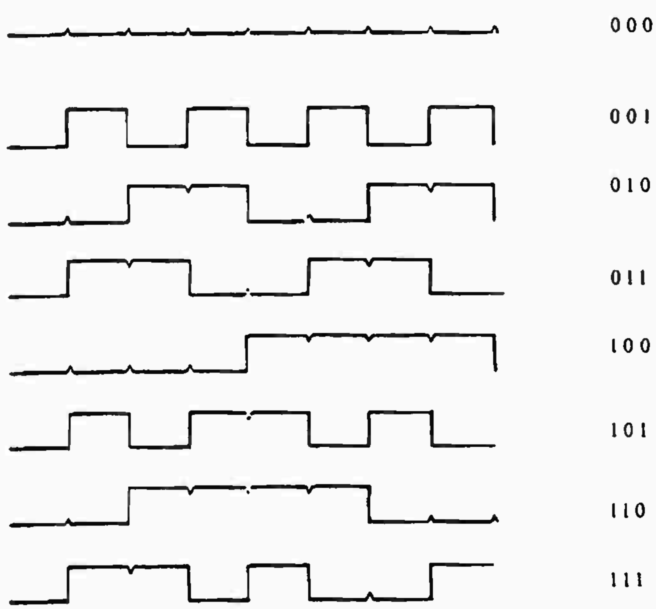


Fig. (8.61) Orthogonal code set for 8 symbols

Ex. 8.10
Compare bandwidth and spectral efficiency for MPSK and MFSK.

Solution:
The channel bandwidth for MPSK (for null to null bandwidth of the main spectral lobe) is

$$B = \frac{2}{T_s}$$

$$T_s = T_b \log_2 M$$

$$R_b = 1/T_b$$

$$B = \frac{2R_b}{\log_2 M}$$

$$\eta_{MPSK} = \frac{R_b}{B}$$

$$= \frac{\log_2 M}{2}$$

Consider MFSK that consists of an orthogonal set of M frequency shifted signals. When the orthogonal signals are detected coherently the adjacent signals need only be separated from each other by a frequency difference $\frac{1}{2}T_s$ so as to maintain orthogonality.

$$B = \frac{M}{2T_s}$$

$$R_b = \frac{1}{T_b}$$

$$B = \frac{R_b M}{2\log_2 M}$$

$$\eta_{MFSK} = \frac{R_b}{B}$$

$$= \frac{2\log_2 M}{M}$$

Increasing the number of levels M tends to increase the bandwidth efficiency of MPSK but tends to decrease the bandwidth efficiency of MFSK. Thus MPSK signals are spectrally efficient where MFSK signals are spectrally inefficient.

Problems

1. Sketch the output of a 4-ASK and 4-PSK system when the input bits are 001110.

2. Obtain the orthonormal basis for BPSK $\phi(t)$, and then express $s_0(t)$ and $s_1(t)$ is terms of $\phi(t)$.

3. Obtain the orthonormal basis for 4-PSK, and express $\{s_i(t)\}$ is terms of $\{\varphi_j(t)\}$.

4. Obtain the orthonormal basis for 8-PSK, and express $\{s_i(t)\}$ is terms of $\{\varphi_j(t)\}$.

5. Verify eqns. (8 – 24), (8 – 25).

6. Verify eqn. (8 – 91).

7. Verify eqn. (8 – 92).

8. Verify eqn. (8 – 93).

9. Design an 8-ASK demodulator, then calculate P_s and the symbol rate

10. An OOK IF modulated signal is detected by an ideal matched filter receiver. The 1 symbol at the matched filter input is a rectangular pulse with amplitude $100\ mV$ and duration $10\ ms$. The rms noise input is $140\ mV$ for noise bandwidth $10\ kHz$. Calculate the probability of bit error.

11. A $140\ Mb/s$ ISI free PRK signaling system uses pulse shaping to constrain its transmission to double sideband Nyquist bandwidth. The received signal power is $10\ mW$ and the one sided noise PSD is $6\ pW/Hz$. Find BER at the output of an ideal matched filter receiver. Find the residual power in the carrier and BER if the angle is 160°.

12. A rectangular pulse BFSK system operates at $f_2 - f_1 = 3f_0/2$. The maximum available transmitter power results in a carrier power at the receiver input of $60\ mW$. The one sided noise PSD is $0.1\ nW/Hz$. What is the maximum bit rate which the system can support if the probability of bit error is not to fall

below 10^{-6}. What is the nominal bandwidth of BFSK signal if the lower frequency symbol has frequency of $80\ M\,Hz$.

13. A 2 -PSK modulator has an input bit rate of $2400\ b/s$ and works on $300\ Hz - 3400\ Hz$ channel. Determine the number of possible symbols at the output, symbol rate, phase difference between the symbols, and the maximum bit rate.

14. Redo the above problem for 16-QAM.

15. Proper a 16 – QAM modulator and sketch its constellation based on Gray code.

16. Show that the error in BPSK is $3dB$ lower than in BFSK for the same E_b/η.

17. Verify Table (8 – 14).

18. Verify Table (8 – 15).

19. Verify Table (8 – 16).

20. Verify Table (8 – 17).

21. Compare P_B and P_s, B and η for coherent and noncoherent BPSK, What do you conclude.

22. Compare P_B, P_s/B and η for coherent and noncoherent BFSK, What do you conclude?

23. Compare P_B, P_s, B and η for coherent and noncoherent MPSK, What do you conclude.

24. Compare P_B, P_s, B and η for coherent and noncoherent MFSK, What do you conclude.

25. Compare P_B, P_s, B and η for coherent and noncoherent MPSK and MFSK what do you conclude?

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