CHAPTER 1

INRODUCTION

1.1 WHAT IS STRUCTURAL ANALYSIS

Structural analysis is the science used to determine the internal actions (forces and moments) and deformation of any structure of known geometry, and physical properties, due to the applied loading. Since in the structural design process the designer looks for the proper geometry and physical properties which safely sustain the applied loading on the structure, one realizes that structural analysis and structural design are tied together in any structural engineering design problem. The structural design process of any structure starts by visualizing the geometry of the structure, estimating the applied loading, whether dead loading or movable loading, and then determining the internal actions in the structure elements using structural analysis. In some structures, it is simple to determine the internal actions without the need to know completely the physical properties of the structural elements and those structures are called statically determinate structures. Other structures, it is impossible to determine the internal actions in the structures members without knowing the physical properties of each member, and those structures are called statically indeterminate structures. Therefore, the design of a statically indeterminate structure is a cyclic process which starts with assuming the physical properties of the structural members according to certain assumptions or rules, and then analyzing the structure in order to determine the internal actions associated with the predetermined physical properties. The internal actions are then used to redesign the structural members. This cyclic process should end when the internal actions obtained from the structural analysis are within the capability of the designed physical properties of the structural members. The designer may include in the design process other criteria in addition to the strength criteria, such as the serviceability or human comfort criteria, which specify, respectively, a limited deformation, or acceleration in the structure. In conclusion, structural analysis determines the response of the structure to the applied loading, whether this loading is static or dynamic. By evaluating the structural response, the designer can modify his design to satisfy the strength, and serviceability criteria, to guarantee the safety of the structure. The design process for statically indeterminate structure is illustrated in the flow chart shown in Figure 1.1.

1.2 BRIEF HISTORY OF STRUCTURAL ANALYSIS

Engineering mechanics principles have long been applied in many structures still standing today, like the pyramids of Egypt. The principles of structural analysis in use today can be accredited historically to three different groups of scientists. The first group introduced principles such as the concepts of centre of gravity, the fundamental principles of statics and equilibrium, the concept of forces and moments, the theory of cantilever beams, and Hooke's stress-strain law, to name a few. Belonging to this group are Archimedes (287-212 B.C.), Al-Farghani (860), Leonardo Da Vinci (1452-1519), Galileo (1564-1642), Mimar Sinan (1490-1588), Thabit Ibn Qurrah (1546-1601), and Hooke (1635-1703).

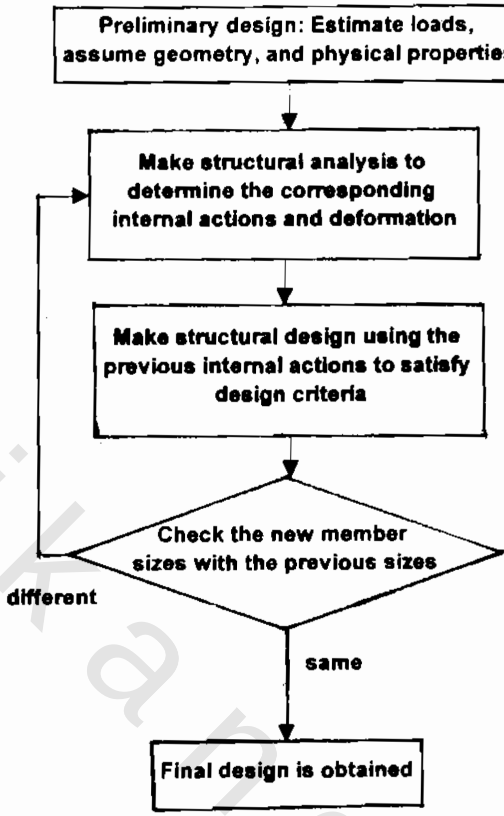

Figure 1.1 Design process of structures

The second group applied mathematical theorems in engineering, like Euler (1707-1783) introduced the theory of columns buckling; Lagrange (1736-1813) introduced the partial differential equation of plates; and the energy theorems were developed due to the efforts of Navier (1785-1836), Saint-Venant (1797-1886), Maxwell (1831-1879), Castigliano (1847-1884), Mohr (1835-1918), and Muller-Breslau (1851-1925).

In the third group, the slope deflection, equation was developed by G.A. Maney (1888-1947); the moment distribution method by H. Cross (1885-1959); the matrix methods by J.H. Argyris and W. Weaver, and others who have contributed to the development of the finite element method and computerized structural analysis.

1.3 TYPES OF STRUCTURES

Any structure can be regarded as composed of interconnected elements. These elements are connected together in different fashions. They could be one dimensional, two dimensional, or three dimensional elements. The one dimensional elements are those which have a length much greater than the other two dimensions, such as the beam and truss elements shown in Figure 1.2. These elements form structures like frames, beams, trusses, and arches, as shown in Figure 1.3. The elements are joined at joints which are also called nodes as shown in Figure 1.4.

Figure 1.2 One dimensional elements

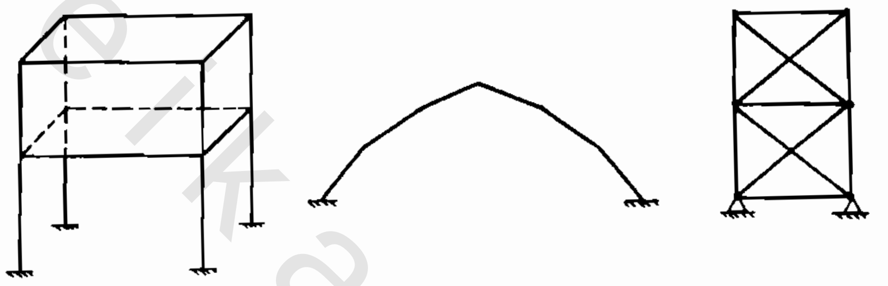

Figure 1.3 Skeletal structures

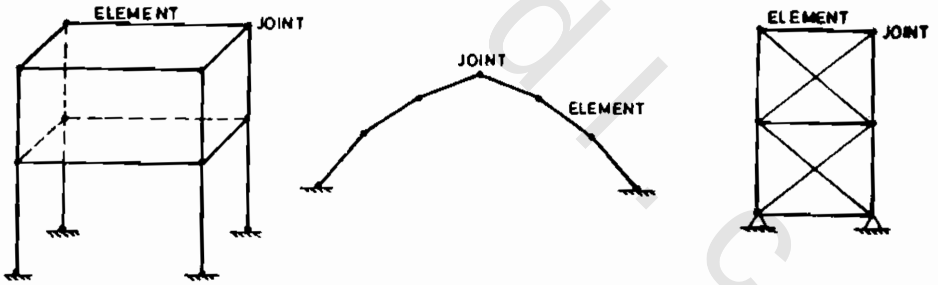

Figure 1.4 One dimensional structures

The two dimensional elements are those which have a small thickness as compared with the other two dimensions, such as plate and shell elements, as shown in Figure 1.5. These elements form structures like slabs, folded plates, domes,

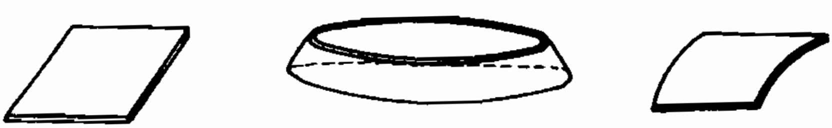

Figure 1.5 Two dimensional elements

shell structures, as shown in Figure 1.6. The elements are joined at the nodes as shown in Figure 1.7.

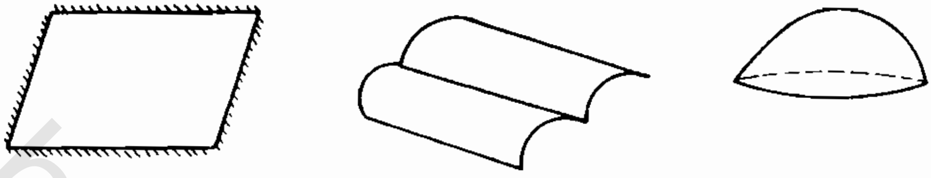

Figure 1.6 Plate and shell structures

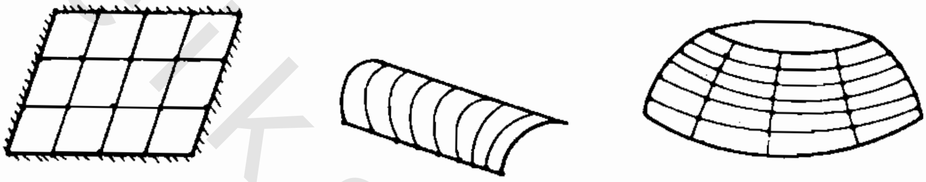

Figure 1.7 Two dimensional structures

The three dimensional elements are those which have three comparable dimensions, as in cube, prism, and tetrahedron elements as shown in Figure 1.8. These elements are the constituent of block type structures such as dams, footings, and retaining walls, as shown in Figure 1.9. The connection between the elements looks like Figure 1.10.

It is obvious that most structures in practice are three dimensional continuous systems, as they are composed, in general, from three dimensional infinitesimal elements joined together. However, one can discretize the structure into one dimensional, two dimensional, or three dimensional elements connected together at the nodes. In this case, the structure is treated as a discretized system approximating the actual one. For example, the skeletal structures shown in Figure 1.3 are discretized into one dimensional elements connected together at the joints as shown in Figure 1.4. Since the analysis of continuous structural systems is difficult, it is customary to deal with the discretized structural system as a model for the actual structure.

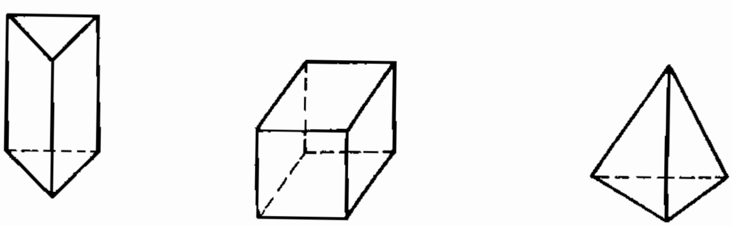

Figure 1.8 Three dimensional elements

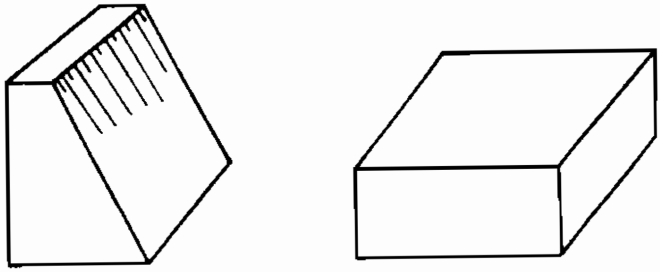

Figure 1.9 Solid structures

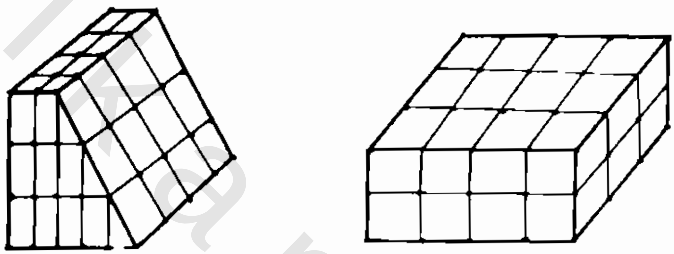

Figure 1.10 Three dimensional structures

1.4 METHODS OF STRUCTURAL ANALYSIS

In analyzing the discretized structural system which models the actual structure, one deals with the nodes or joints which connect the structural elements. Some nodes are on the boundary supporting the structure and usually called supports, but the other nodes are free from any restraints. In order to analyze any structure, one may determine first the internal actions at the nodes and then determine the displacements of the free nodes. In this case, one is using what is called the force method. The other method is to determine first the displacements of the free joints or nodes and then determine the internal actions. This method is called the displacement method. One may also use a combination of the two methods.

Thus, in general, there are two methods to analyze a structure. The force method is concerned with determining, first, the internal actions at the nodes or joints so that the internal actions in the structural members can be calculated. If these internal actions can not be determined directly from the static equilibrium principles, the structure is called statically indeterminate. The degree of static indeterminancy represents the number of unknown actions which exceeds the number of static equilibrium equations. In order to analyze these types of structures, one must use the physical properties of the structural elements and their integrity, connectivity, or compatibility at the nodes. The obtained compatibility equations which need to be solved have coefficients of displacement per unit action, and that is why the force method is also called the flexibility or the compatibility method.

The displacement method is concerned with determining, first, the displacements of the free nodes in the structure. If all nodes of the structure are fixed in position, the structure is kinematically determinate. This means its deformed shape is known. In the presence of free joints in the structure, the number of displacements which need to be determined is called the degree of freedom or degree of kinematic indeterminancy. In order to analyze these structures one must use the physical properties of the elements and the equilibrium between the internal actions and the external actions at each node. The obtained equilibrium equations have coefficients of action per unit displacement, and that is why the displacement method is also called the stiffness or the equilibrium method.

Therefore, a statically determinate structure is a kinematically indeterminate because it has unknown displacements at the free joints. Similarly, a kinematically determinate structure is a statically indeterminate because of the unknown actions at the fixed joints. In general, a statically indeterminate structure is also kinematically indeterminate. To analyze any structure, one, therefore, has to determinate the degree of static indeterminancy and the degree of kinematic indeterminancy (degree of freedom) in order to decide the suitable method which should be used in the analysis; the force method or the displacement method. Naturally, one should choose the method which is associated with the least number of unknowns. However, the systematic formulation of the displacement method and the possibility of completely programming the structural analysis process through the computer makes the stiffness method both preferable and more popular, than the flexibility method.

1.5 THE CLASSICAL AND MATRIX METHODS

Several methods have been developed which belong to either the force method or the displacement method. Until two or three decades ago, when the computer had not yet extensively been used, the structural engineer depended on hand calculations, manual slide rules, or electronic calculators. Several methods have been developed during this era in order to ease solving the structural analysis problems. By the extensive use of computer, from large frame computers to micro computers, the classical methods have been updated to suit the computer utilization. It is thus necessary to show the background of the modern methods which depend primarily on using matrices. The main objective of this text is to show how the modern matrix structural analysis methods were developed from the classical methods.

Classical force methods in the analysis of skeletal structures depend on using of virtual load, elastic centre, column analogy, three moment equation, and Castigliano's second theorem. Classical displacement methods in the analysis of skeletal structures depend on using the slope deflection equation, moment distribution, and Castigliano's first theorem.

The development of matrix methods shall be presented through two matrix approaches. The Matrix Approach I is developed directly from the classical methods. The Matrix Approach II is made to be more suitable for complete computer utilization, and to serve in developing general structural analysis software programs.