CHAPTER 6

ADDITIONAL TOPICS

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CHAPTER 6 ADDITIONAL TOPICS

6.1 INTRODUCTION

This chapter presents some topics of interest to the structural engineers. First, an introduction to the finite element method is given as an outcome of the stiffness matrix method approach II of Chapter 5. Second, the influence lines of statically indeterminate structures are presented. Finally, some examples are given utilizing the computer softwares to verify some of the examples presented in this book.

6.2 AN INTRODUCTION TO THE FINITE ELEMENT METHOD

The finite element method is a tool to solve one-dimensional, two-dimensional, and three-dimensional structures with approximation instead of solving complicated partial differential equations. The structure is discretized into a set of elements joined together at some points called nodes or nodal points. These nodes are similar to the joints in the one-dimensional structures which were investigated in the previous chapters. The nodes could be the common corners between the elements, or chosen between the boundaries of the elements, as shown in Figure 6.1. The similarity in the concept between one-dimensional skeleton structures and two- or three-dimensional structures, in terms of discretization is also shown in Figure 6.1.

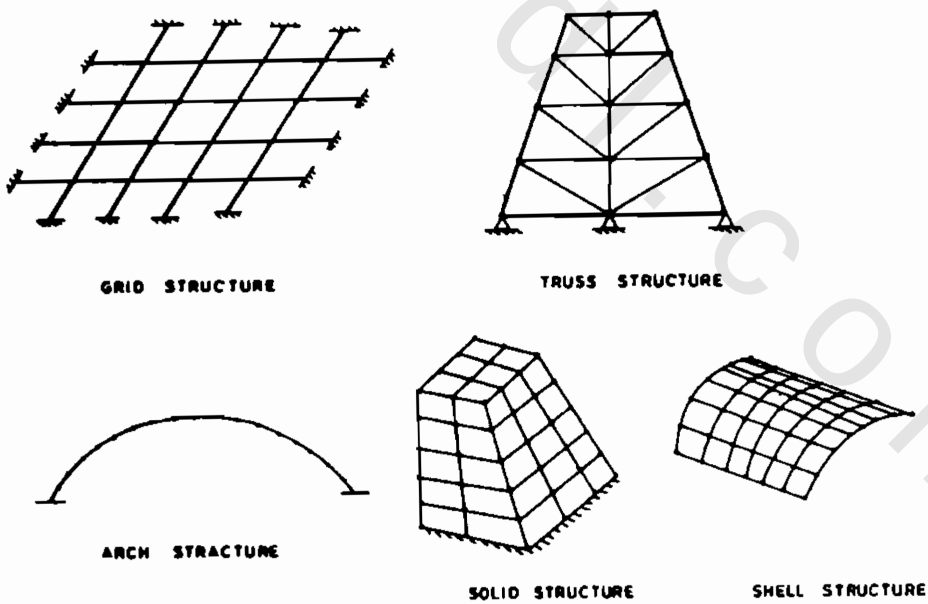

Figure 6.1

It is obvious that in one-dimensional structures the element is a one-dimensional member, as was analyzed in Chapters 3, 4, and 5. In two-dimensional structures, the element is a two-dimensional plate or shell element. The three-dimensional element could be a cube, prism, or a tetrahedron, either with straight sides or curved sides, as shown in Figure 6.2

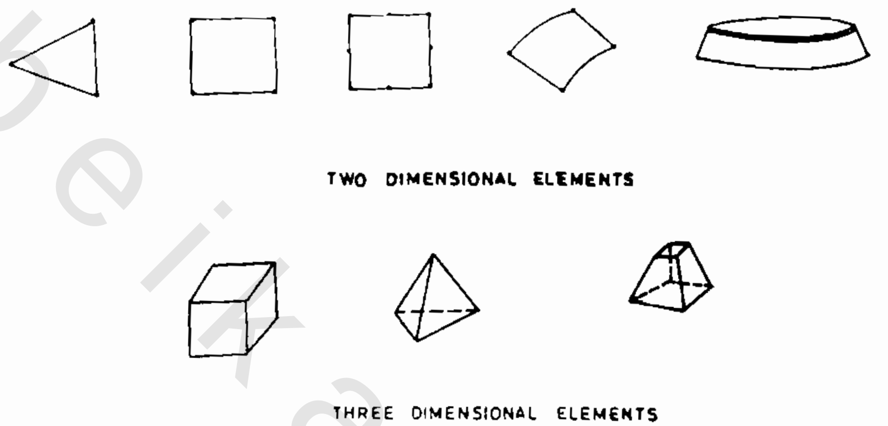

Figure 6.2

The solution of the finite element method is almost the same as the direct stiffness matrix method presented in Chapter 5. Once the elements' stiffness matrices are found, these matrices are augmented according to the compatibility and equilibrium conditions at every node. The free nodal displacements can be determined after specifying the boundary conditions at the boundary nodes. However, what interests the analyst in two-dimensional and three-dimensional structures is the stresses and strains not forces and displacements. Therefore, it is necessary to relate the strains at a point within the element with the nodal displacements.

6.2.1 The Relationship Between Strains and Nodal Displacements

The displacements $\underline{\mathbf{d}}$ at any point of coordinates (x,y) in a plane element can be related to the nodal displacement $\underline{\mathbf{D}}_c$ by

$$\underline{\mathbf{d}} = \underline{\mathbf{N}}\,\underline{\mathbf{D}}_c \qquad (6.1)$$

where the elements of matrix $\underline{\mathbf{N}}$ are function of the location of the point (x,y).

An easy approach to determine matrix $\underline{\mathbf{N}}$ is to express $\underline{\mathbf{d}}$ in terms of some displacement functions $\underline{\mathbf{P}}$ and weighting parameters $\underline{\alpha}$, as follows:

$$\underline{\mathbf{d}} = \underline{\mathbf{P}}\,\underline{\alpha} \qquad (6.2)$$

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The elements of $\underline{P}$ differ from one finite element to another. The dimension of the weighting parameters $\underline{\alpha}$ is preferably equal the number of the nodal displacements of the finite element.

Equation 6.2 can, be applied at every node in the element to obtain

$$\begin{bmatrix} D_{c1} \\ \vdots \\ D_{ci} \\ \vdots \\ D_{cn} \end{bmatrix}_k = \begin{bmatrix} P_1^T \\ \vdots \\ P_i^T \\ \vdots \\ P_n^T \end{bmatrix}_k \underline{\alpha}_k = \underline{C}_k \, \underline{\alpha}_k \qquad (6.3)$$

which indicates that the nodal displacements $\underline{D}_c$ for the finite element number k, which have nodes, 1, ..., i, ..., n, are related to the parameters $\underline{\alpha}_k$ by the matrix $\underline{C}_k$, whose elements depend on the locations of the nodes 1, ..., i, ..., n, in the global coordinates. For proper choice of global coordinates and the elements, one can relate $\underline{d}$ to $\underline{D}_c$ for any element using Equations 6 2 to 6.3 as follows:

$$\underline{\alpha}_k = \underline{C}_k^{-1} \, \underline{D}_{ck} \qquad (6.4)$$

$$\underline{d} = \underline{P} \, \underline{C}^{-1} \, \underline{D}_c \qquad (6.5)$$

Thus $\underline{N}$ for any element is obtained from

$$\underline{N} = \underline{P} \, \underline{C}^{-1} \qquad (6.6)$$

The inverse of matrix $\underline{C}$ exists when choosing appropriate displacement functions for each specific element. For the present introduction we shall deal only with plate elements.

Example 6.1

Show how to determine $\underline{N}$ for the triangular membrane element shown in Figure 6.3.

Solution

Any point of coordinates (x,y) within the element is subjected to the displacement u in the x-direction and displacement v in the y-direction of the global coordinates. Therefore, the nodal displacements at i, j, and k are given by

$$\underline{D}_c^T = \begin{bmatrix} u_i & v_i & u_j & v_j & u_k & v_k \end{bmatrix}$$

The weighting parameters $\underline{\alpha}$ are chosen to be 6 × 1 as follows:

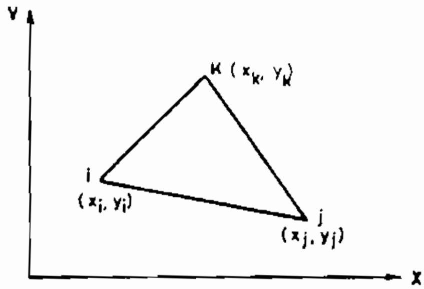

Figure 6.3

$$\underline{\alpha}_e^T = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \end{bmatrix}$$

The displacement functions which relate $\underline{d}$ to $\underline{\alpha}$ are chosen to be in the form

$$\underline{P} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix}$$

Substituting into Equation 6.3 one has

$$\begin{bmatrix} u_i \\ v_i \\ \hdashline u_j \\ v_j \\ \hdashline u_k \\ v_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ \hdashline 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_j & y_j \\ \hdashline 1 & x_k & y_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix}$$

Thus, the matrix $\underline{N}$ is determined from the following equation:

$$\underline{N} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_j & y_j \\ 1 & x_k & y_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_k & y_k \end{bmatrix}^{-1}$$

The relationships between strains and displacements can be obtained using Figure 6.4 which can be generalized into y-z and z-x planes as follows:

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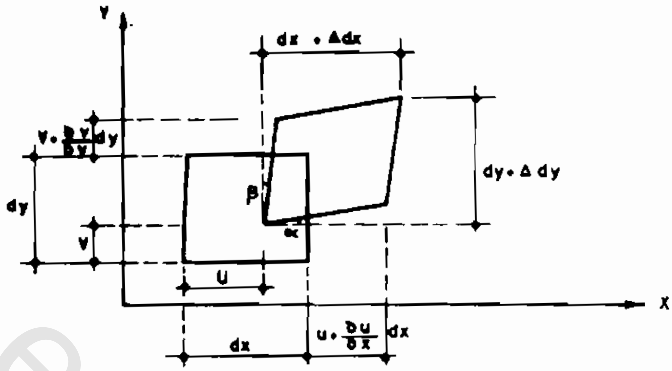

Figure 6.4

$$
\begin{aligned}
\varepsilon_x &= \frac{\Delta\,dx}{dx} = \frac{u + \dfrac{\partial u}{\partial x}dx - u}{dx} = \frac{\partial u}{\partial x} \\
\varepsilon_y &= \frac{\partial v}{\partial y} \\
\varepsilon_z &= \frac{\partial w}{\partial z} \\
\gamma_{xy} &= \gamma_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\gamma_{xz} &= \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
\gamma_{yz} &= \gamma_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
\end{aligned}
\qquad (6.7)
$$

where w is the displacement in z-direction.

Since the displacements $\underline{d}$ are related to the nodal displacements $\underline{\mathbf{D}}_e$, it is possible to relate the strains $\underline{\varepsilon}$ to the nodal displacements $\underline{\mathbf{D}}_e$. Such a relationship is expressed in general by

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$$\underline{\varepsilon} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \underline{\mathbf{N}}\,\underline{\mathbf{D}}_c = \underline{\mathbf{B}}\,\underline{\mathbf{D}}_c \qquad (6.8)$$

Example 6.2

Show how to determine matrix $\underline{B}$ for the triangular membrane element considered in Example 6.1.

Solution

The strains of interest in the membrane elements are $\varepsilon_x, \varepsilon_y$, and γ_{xy}, since $w = 0$. One thus has

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

But $\underline{\mathbf{d}}$ is given by

$$\underline{\mathbf{d}} = \begin{bmatrix} u \\ v \end{bmatrix} = \underline{\mathbf{P}}\,\underline{\mathbf{C}}^{-1}\,\underline{\mathbf{D}}_c = \underline{\mathbf{N}}\,\underline{\mathbf{D}}_c$$

The strains $\underline{\varepsilon}$ are thus obtained from

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \underline{\mathbf{C}}^{-1}\,\underline{\mathbf{D}}_c$$

The matrix $\underline{\mathbf{B}}$, is therefore, given by the following equation:

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$$\underline{\mathbf{B}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_j & y_j \\ 1 & x_k & y_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_k & y_k \end{bmatrix}^{-1}$$

6.2.2 Stresses and Strains Relationships

For a general infinitesimal element dv = dx dy dz, the stresses corresponding to the strains of Equation 6.7 are given as

$$\underline{\sigma}^T = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{yz} & \tau_{zx} \end{bmatrix} \qquad (6.9)$$

where σ_x, σ_y and σ_x indicate normal stresses and τ_{xy}, τ_{yz} and τ_{zx} indicate shear stresses as shown in Figure 6.5. For the linear elastic materials of our interest, the stress-strain relationships are

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \qquad (6.10)$$

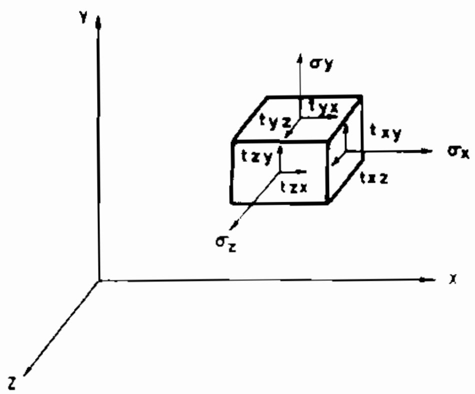

Figure 6.5

where E is the modulus of elasticity; G is the shearing modulus; and ν is the Poisson's ratio. The shearing modulus is related to the modulus of elasticity by

$$G = \frac{E}{2(1+\nu)} \tag{6.11}$$

Expressing the stresses $\underline{\sigma}$ in terms of strains $\underline{\varepsilon}$ one has, in a general expression, the following equation:

$$\underline{\sigma} = \underline{\mathbf{K}}\,(\underline{\varepsilon} - \underline{\varepsilon}_0) + \sigma_0 \tag{6.12}$$

in which $\underline{\varepsilon}_o$ represents initial strains, $\underline{\sigma}_o$ represents initial stresses, and $\underline{\mathbf{K}}$ is given by

$$\underline{\mathbf{K}} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{6.13}$$

6.2.3 Plane-Stress and Plane-Strain for Isotropic Materials

If the element under consideration is in x-y plane and subjected to plane-stresses or plane-strains, the stresses and strains in this case are

$$\underline{\sigma}^T = \begin{bmatrix} \sigma_x & \sigma_y & \tau_{xy} \end{bmatrix} \tag{6.14}$$

$$\underline{\varepsilon}^T = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \tau_{xy} \end{bmatrix} \tag{6.15}$$

In case of plane stresses, the element is subjected to $\sigma_z = 0$. In this case, the stress-strain relationship is obtained from Equation 6.10 as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau_{xy} \end{bmatrix} \tag{6.16}$$

If the element is also subjected to a change in temperature T, the initial strains become

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$$\underline{\varepsilon}_0^T = [\alpha T \quad \alpha T \quad o] \tag{6.17}$$

where α is the coefficient of thermal expansion.

In case of plane strains, the element strain ε_z is zero but $\sigma_z \neq 0$. In this case, the stress-strain relationship is given by

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{6.18}$$

If the plain strains element is subjected to a change in temperature T, the initial strains become

$$\underline{\varepsilon}_0^T = [(1+\nu)\alpha T \quad (1+\nu)\alpha T \quad 0] \tag{6.19}$$

6.2.4 Element Stiffness Matrix

In order to determine the stiffness matrix of any element, the principle of virtual work is applied. This principle, as presented in Chapter 2, states that the work done due to a virtual displacement is equal to the strain energy due to this virtual displacement. This can be expressed as

$$\delta \underline{\mathbf{D}}_e^T \underline{\mathbf{A}}_e = \int_{vol.} \delta \, \underline{\varepsilon}^T \underline{\sigma} \; d\,(vol) \tag{6.20}$$

where $\underline{\mathbf{A}}_e$ is the actions applied at the nodes of the element. Substituting Equations 6.8 and 6.12 into Equation 6.20, one has

$$\delta \underline{\mathbf{D}}_e^T \underline{\mathbf{A}}_e = \int_{vol.} \delta \underline{\mathbf{D}}_e^T \underline{\mathbf{B}}^T [\underline{\mathbf{K}} (\underline{\varepsilon} - \underline{\varepsilon}_0) + \underline{\sigma}_0] d\,(vol) \tag{6.21}$$

Equation 6.21 can be written as

$$\underline{\mathbf{A}}_e = \left[\int_{vol.} \underline{\mathbf{B}}^T \underline{\mathbf{K}} \, \underline{\mathbf{B}} \, d\,(vol) \right] \underline{\mathbf{D}}_e - \int_{vol.} \underline{\mathbf{B}}^T \underline{\mathbf{K}} \, \underline{\varepsilon}_0 \; d\,(vol) + \int_{vol.} \underline{\mathbf{B}}^T \underline{\sigma}_0 \; d\,(vol) \tag{6.22}$$

It is obvious that the equivalent nodal actions due to initial strains and initial stresses are, respectively, given by

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$$\underline{A}_{c\varepsilon_o} = + \int_{vol.} \underline{B}^T \, \underline{K} \, \underline{\varepsilon}_0 \, d\,(vol) \tag{6.23}$$

$$\underline{A}_{c\sigma_o} = - \int_{vol.} \underline{B}^T \, \underline{\sigma}_0 \, d\,(vol) \tag{6.24}$$

The relationship between nodal actions and nodal displacements is, in general,

$$\underline{A}_c = \left[\int_{vol.} \underline{B}^T \, \underline{K} \, \underline{B} \, d\,(vol) \right] \underline{D}_e \tag{6.25}$$

From which it is obvious that the element stiffness matrix can be obtained from

$$\underline{S} = \int_{vol.} \underline{B}^T \, \underline{K} \, \underline{B} \, d\,(vol) \tag{6.26}$$

6.2.5 Equivalent Nodal Actions

Equations 6.23 and 6.24 gave, respectively, the equivalent nodal actions due to initial strains and initial stresses. In this section, the nodal actions due to distributed forces will be found. In continuum structures, the forces could be distributed within the body of the structure or on its surface. We shall indicate the body forces by $\underline{g}$ kN/m^3 and the surface forces by $\underline{q}$ kN/m^2. The work done by these forces, due to virtual nodal displacements $\delta \, \underline{D}_e$ in an element, is given by

$$W = \int_{vol.} \delta \, \underline{d}^T \underline{g} \; d\,(vol) \; + \int_{surface} \delta \; \underline{d}^T \; \underline{q} \; d\,(area) \tag{6.27}$$

Substituting Equation 6.5 into Equation 6.27 one has

$$W = \int_{vol.} \delta \, \underline{D}_e^T \, N^T \underline{g} \; d\,(vol) \; + \int_{surface} \delta \; \underline{D}_e^T \; \underline{N}^T \; \underline{q} \; d\,(area) \tag{6.28}$$

Equating Equation 6.28 with the strain energy due to virtual displacements, one realizes that the equivalent nodal forces due to body and surface forces are, respectively, given by

$$\underline{A}_{c_{body}} = \int_{vol.} \underline{N}^T \underline{g} \; d\,(vol) \tag{6.29}$$

$$\underline{A}_{c_{surface}} = \int_{surface} \underline{N}^T \underline{q} \; d\,(area) \tag{6.30}$$

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6.2.6 Example 6.3

Analyze the plate shown in Figure 6.6, considering plane stresses, $E = 10^7$ N.cm^2 and $\nu = 0$.

Solution

For clear illustration, the plate is divided into only two triangular elements as shown in Figure 6.7. Element 1 has nodes number 1, 3 and 4. Element 2 has nodes number 1, 2 and 4. The two elements are joined at nodes 1 and 4.

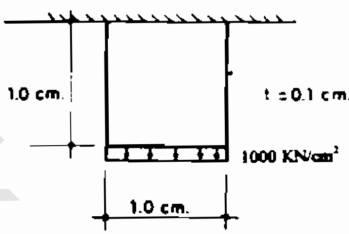

Figure 6.6

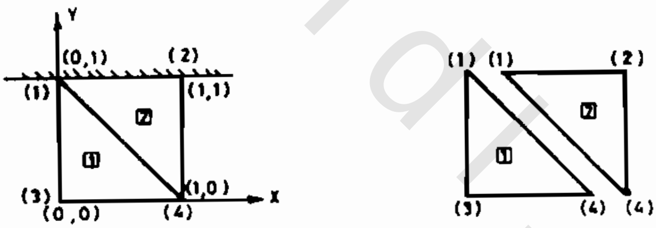

Figure 6.7

According to the coordinates of the nodes with respect to global x-y axes shown in Figure 6.7, the matrices $\underline{C}_1$ and $\underline{C}_2$ are determined as

$$\underline{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} \}\text{node 3} \\ \\ \}\text{node4} \\ \\ \}\text{node 1} \end{matrix}$$

$$\underline{\mathbf{C}}_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} \}\text{node 4} \\ \\ \}\text{node 2} \\ \\ \}\text{node 1} \\ \end{matrix}$$

The matrices $\underline{\mathbf{B}}_1$ and $\underline{\mathbf{B}}_2$ are, respectively, obtained as

$$\underline{\mathbf{B}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \underline{\mathbf{C}}^{-1}$$

$$\underline{\mathbf{B}}_1 = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad ; \quad \underline{\mathbf{B}}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

The stress-strain relationship for the plane stresses problem with $\nu = 0.0$ is thus

$$\underline{\mathbf{K}} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

The elements' stiffness matrices are determined from

$$\underline{\mathbf{S}}_1 = \int \underline{\mathbf{B}}_1^T \, \underline{\mathbf{K}} \, \underline{\mathbf{B}}_1 \, d\,(\text{vol})$$

$$= \frac{E}{40} \begin{bmatrix} \overbrace{\begin{matrix} 3 & 1 \end{matrix}}^{\text{node 3}} & \overbrace{\begin{matrix} -2 & -1 \end{matrix}}^{\text{node 4}} & \overbrace{\begin{matrix} -1 & 0 \end{matrix}}^{\text{node 1}} \\ \begin{matrix} 1 & 3 \end{matrix} & \begin{matrix} 0 & -1 \end{matrix} & \begin{matrix} -1 & 0 \end{matrix} \\ \begin{matrix} -2 & 0 \end{matrix} & \begin{matrix} 2 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} -1 & -1 \end{matrix} & \begin{matrix} 0 & 1 \end{matrix} & \begin{matrix} 1 & 0 \end{matrix} \\ \begin{matrix} -1 & -1 \end{matrix} & \begin{matrix} 0 & 1 \end{matrix} & \begin{matrix} 1 & 0 \end{matrix} \\ \begin{matrix} 0 & -2 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 2 \end{matrix} \end{bmatrix} \begin{matrix} \\ \}\text{node 3} \\ \\ \}\text{node 4} \\ \\ \}\text{node 1} \end{matrix}$$

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$$\underline{\mathbf{S}}_2 = \int \underline{\mathbf{B}}_2^T \, \underline{\mathbf{K}} \, \underline{\mathbf{B}}_2 \, d\,(\mathrm{vol})$$

$$= \frac{\mathbf{E}}{40} \begin{bmatrix} \overbrace{\begin{matrix} 1 & 0 \\ 0 & 2 \end{matrix}}^{\text{node } 4} & \overbrace{\begin{matrix} -1 & -1 \\ 0 & -2 \end{matrix}}^{\text{node } 2} & \overbrace{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}}^{\text{node } 1} \\ \begin{matrix} -1 & 0 \\ -1 & -2 \end{matrix} & \begin{matrix} 3 & 1 \\ 1 & 3 \end{matrix} & \begin{matrix} -2 & -1 \\ 0 & -1 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} & \begin{matrix} -2 & 0 \\ -1 & -1 \end{matrix} & \begin{matrix} 2 & 0 \\ 0 & 1 \end{matrix} \end{bmatrix} \begin{matrix} \}\ \text{node } 4 \\ \\ \}\ \text{node } 2 \\ \\ \}\ \text{node } 1 \end{matrix}$$

The structure stiffness matrix is obtained by superposition to have

$$\underline{\mathbf{S}} = \frac{\mathbf{E}}{40} \begin{bmatrix} \overbrace{\begin{matrix} 3 & 0 \\ 0 & 3 \end{matrix}}^{\text{node } 1} & \overbrace{\begin{matrix} -2 & 0 \\ -1 & -1 \end{matrix}}^{\text{node } 2} & \overbrace{\begin{matrix} -1 & -1 \\ 0 & -2 \end{matrix}}^{\text{node } 3} & \overbrace{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}}^{\text{node } 4} \\ \begin{matrix} -2 & -1 \\ 0 & -1 \end{matrix} & \begin{matrix} 3 & 1 \\ 1 & 3 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} -1 & 0 \\ -1 & -2 \end{matrix} \\ \begin{matrix} -1 & 0 \\ -1 & -2 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 3 & 1 \\ 1 & 3 \end{matrix} & \begin{matrix} -2 & -1 \\ 0 & -1 \end{matrix} \\ \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & \begin{matrix} -1 & -1 \\ 0 & -2 \end{matrix} & \begin{matrix} -2 & 0 \\ -1 & -1 \end{matrix} & \begin{matrix} 3 & 0 \\ 0 & 3 \end{matrix} \end{bmatrix} \begin{matrix} \}\ \text{node } 1 \\ \\ \}\ \text{node } 2 \\ \\ \}\ \text{node } 3 \\ \\ \text{node } 4 \end{matrix}$$

The equivalent nodal forces are determined from

$$\underline{\mathbf{A}}_{e1}^T = t \begin{bmatrix} \underbrace{0 \quad -500}_{\text{node } 3} & \underbrace{0 \quad -500}_{\text{node } 4} & \underbrace{0 \quad 0}_{\text{node } 1} \end{bmatrix}$$

The boundary conditions are $\underline{\mathbf{D}}_{c1} = \underline{\mathbf{D}}_{c2} = \underline{\mathbf{0}}$. Therefore, the final stiffness relationship is

$$\begin{bmatrix} 0 \\ -500 \\ 0 \\ -500 \end{bmatrix} = \frac{\mathbf{E}}{40} \begin{bmatrix} 3 & 1 & -2 & -1 \\ 1 & 3 & 0 & -1 \\ -2 & 0 & 3 & 0 \\ -1 & -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$

The solution is obtained as

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$$\left[\underline{D}_{e3}^{T} \quad \underline{D}_{e4}^{T}\right] = 10^{-4}\,[0 \quad -1 \quad 0 \quad -1]\ \text{cm}$$

Now, the stresses at any point within any element can be determined as follows:

$$\underline{\sigma} = \underline{K}\,\underline{\varepsilon} = \underline{K}\,\underline{B}\,\underline{D}_e$$

$$\underline{\sigma}_1 = \underline{K}\,\underline{B}_1\left[\underline{D}_e\right]_1 = [0 \quad 1000 \quad 0]$$

$$\underline{\sigma}_2 = \underline{K}\,\underline{B}_2\left[\underline{D}_e\right]_2 = [0 \quad 1000 \quad 0]$$

where $\left[\underline{D}_e\right]_1^T = 10^{-4}\,[0 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0]$

$\left[\underline{D}_e\right]_2^T = 10^{-4}\,[0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0]$

The results indicate that the elements are subjected to tensile stress $\sigma_y = 1000$ kN/cm^2.

6.2.7 Displacement Functions of Some Membrane Elements

It was pointed out, in the previous sections, that the displacement functions for a triangular membrane element are given by

$$\underline{P} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix} \tag{6.31}$$

where the weighting parameters $\underline{\alpha}$ are chosen to be 6 parameters. Another element used for membrane structures is shown in Figure 6.8. The triangle in this case has six nodes. The weighting parameters $\underline{\alpha}$ are thus chosen to be 12 parameters. The displacement functions in this case are chosen as follows:

$$\underline{P} = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & y & x^2 & xy & y^2 \end{bmatrix} \tag{6.32}$$

One also may use a rectangular element with four nodes as shown in Figure 6.9. The displacement functions in this case could be bilinear as

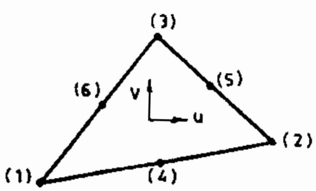

Figure 6.8

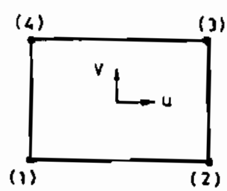

Figure 6.9

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$$\underline{P} = \begin{bmatrix} 1 & x & y & xy & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x & y & xy \end{bmatrix} \tag{6.33}$$

If the rectangular element has 8 nodes as shown in Figure 6.10, the displacement functions in this case could be chosen as

$$\underline{P} = \begin{bmatrix} 1 & x & y & xy & y^2 & xy^2 & y^3 & xy^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & y & xy & x^2 & x^2y & x^3 & x^3y \end{bmatrix} \tag{6.34}$$

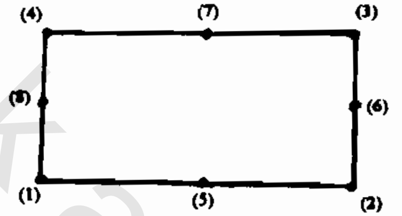

Figure 6.10

6.2.8 Stiffness Matrix of Rectangular Plate Bending Element

The displacements of interest at any node of the rectangular plate element shown in Figure 6.11 are given by

$$\underline{D}_{ei}^{T} = \begin{bmatrix} w_i & \theta_{x_i} & -\theta_{y_i} \end{bmatrix} = \begin{bmatrix} w_i & \dfrac{\partial w_i}{\partial y} & \dfrac{\partial w_i}{\partial x} \end{bmatrix} \tag{6.35}$$

The nodal displacements of the element are thus

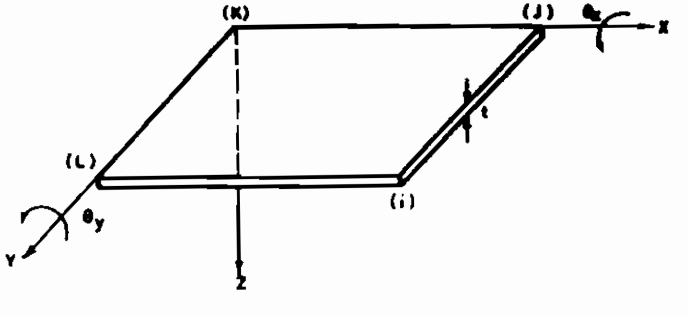

Figure 6.11

$$\underline{\mathbf{D}}_c^T = \left[\underline{\mathbf{D}}_{ci}^T \quad \underline{\mathbf{D}}_{cj}^T \quad \underline{\mathbf{D}}_{ck}^T \quad \underline{\mathbf{D}}_{c\ell}^T\right]_{12\times 1} \tag{6.36}$$

The strains at any point (x,y) within the element consist of the curvatures and are given by

$$\underline{\varepsilon}^T = \left[\frac{-\partial^2 w}{\partial x^2} \quad \frac{-\partial^2 w}{\partial y^2} \quad \frac{-2\partial^2 w}{\partial x\,\partial y}\right] \tag{6.37}$$

The corresponding stresses represent the bending and twisting moments. They are expressed as

$$\underline{\sigma}^T = D\left[-\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad -\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad (-1+\nu)\,\frac{\partial^2 w}{\partial x\,\partial y}\right] \tag{6.38}$$

where

$$D = \frac{E\,t^3}{12(1-\nu^2)} \tag{6.39}$$

Thus, by using Equation 6.37, one can write Equation 6.38 as follows:

$$\underline{\sigma} = D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}\underline{\varepsilon} \tag{6.40}$$

The displacement function is taken as follows:

$$\mathbf{P} = \left[1 \quad x \quad y \quad x^2 \quad xy \quad y^2 \quad x^3 \quad x^2y \quad xy^2 \quad y^3 \quad x^3y \quad xy^3\right] \tag{6.41}$$

where the deflection $\underline{\mathbf{w}}$ is expressed as

$$\underline{\mathbf{w}} = \underline{\mathbf{P}}\,\underline{\alpha} \tag{6.42}$$

Now, substituting Equation 6.42 into Equation 6.35 and 6.36 one obtains $\underline{\mathbf{C}}$. The displacements at any node i are, for example,

$$\underline{\mathbf{D}}_{ei} = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2y & xy^2 & y^3 & x^3y & xy^3 \\ 0 & 0 & 1 & 0 & x & 2y & 0 & x^2 & 2xy & 3y^2 & x^3 & 3xy^2 \\ 0 & 1 & 0 & 2x & y & 0 & 3x^2 & 2xy & y^2 & 0 & 3x^2y & y^3 \end{bmatrix}\underline{\alpha} \tag{6.43}$$

in which x and y are coordinates of joint i which are substituted by x_i and y_i.

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The relation between $\underline{\varepsilon}$ and $\underline{D}_e$ is obtained from Equation 6.37 and 6.41 as follows:

$$\underline{\varepsilon} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6x & -2y & 0 & 0 & -6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2x & -6y & 0 & -6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x & 6y \end{bmatrix} \underline{\alpha} \quad (6.44)$$

Matrix $\underline{B}$ is obtained from the inverse of matrix $\underline{C}$ and Equation 6.44, as follows:

$$\underline{B} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6x & -2y & 0 & 0 & -6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2x & -6y & 0 & -6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x & 6y \end{bmatrix} \underline{C}^{-1} \quad (6.45)$$

The element stiffness matrix can then be determined from Equation 6.26.

6.2.9 Elements for General Solids

The stress-strain relationships for general solids has been given in Equations 6.10 – 6.13. In this section, the displacement functions of some popular solid elements are given.

Tetrahedra with Constant Strain

The element in this case has four nodes as shown in Figure 6.12. The displacements at any point (x, y, z) are

$$\underline{d}^T = [u \quad v \quad w] \quad (6.46)$$

The displacement function in this case is

$$\underline{P} = \begin{bmatrix} 1 & x & y & z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x & y & z & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & y & z \end{bmatrix} \quad (6.47)$$

Tetrahedra with Linear Strain

The number of nodes in each element is 10, as shown in Figure 6.13. The displacement function is

$$\underline{P} = \begin{bmatrix} 1 & x & y & z & xy & yz & zx & x^2 & y^2 & z^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{\text{similar}} & \cdots\cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots\cdots & \boxed{\text{similar}} \end{bmatrix} \quad (6.48)$$

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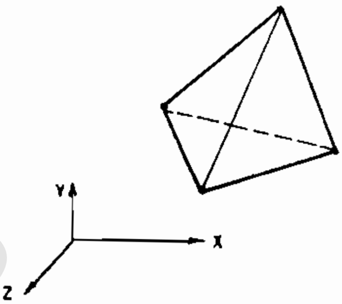

Figure 6.12

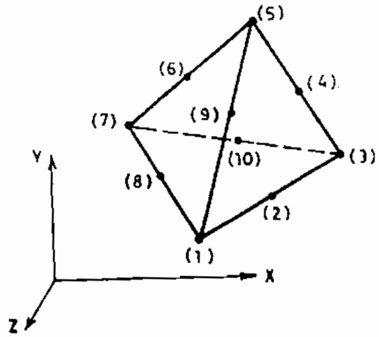

Figure 6.13

which has a dimension of 3 × 30.

Rectangular Solid

The number of nodes in this case is 8 as shown in Figure 6.14. The displacement function is trilinear and given by

$$\underline{\mathbf{P}} = \begin{bmatrix} 1 & x & y & z & xy & yz & zx & xyz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{\text{similar}} & \ldots\ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots\ldots & \boxed{\text{similar}} \end{bmatrix} \qquad (6.49)$$

which has a dimension of 3 × 24.

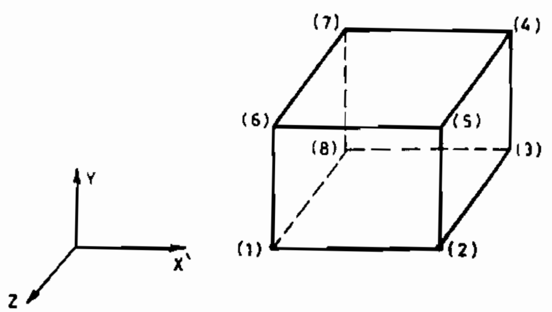

Figure 6.14

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6.3 INFLUENCE LINES

6.3.1 Introduction

In the design of a structural element, the designer looks for the greatest stresses applied on the element in order to provide sufficient strength to withstand the maximum stresses. Structures subjected to static or stationary loading are easy to design since the critical section can be found directly from the internal action diagrams. However, for structures subjected to moving loads, as in bridges, it is necessary to investigate the positions of the moving loads which result in maximum stresses. Moreover, the positions of the moving loads which cause maximum normal stresses could be different from those which cause maximum shearing stresses. Instead of solving such problems by rigeorous analysis considering all possible moving loads positions, the problem can greatly be simplified by studying a unit moving load. The variation of a specific internal action at a certain section can be plotted according to the position of the unit load. From these diagrams, which are called influence lines, one can determine the position of the moving loads which cause maximum stresses at a specific section.

6.3.2 Definition

The influence line is a diagram, its ordinate at any point along the structure, gives the magnitude of a specific force function due to a unit load at that point.

For example, the influence line of the reaction R_a of a simple beam is shown in Figure 6.15. The ordinate at any point along this diagram gives the reaction R_a when a unit load is applied at that point. For statically determinate structures, it is very easy to construct the influence lines, as will be illustrated by some numerical examples. However, the construction of influence lines for statically indeterminate structures is more involved.

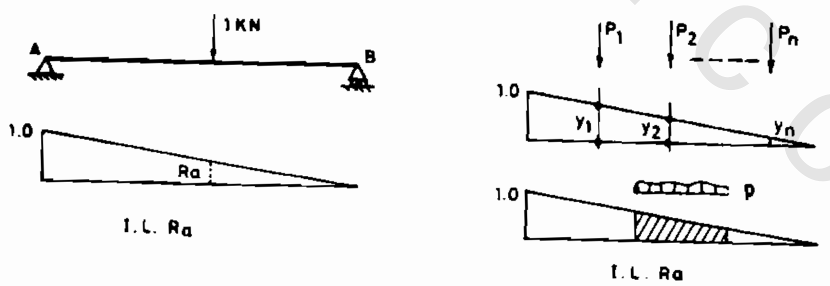

Figure 6.15

The process of determining the maximum internal action caused by the moving loads comes after the construction of the influence lines. If a set of concentrated loads is moving along the beam as shown in Figure 6.15, then for a selected position, the reaction at A is obtained from

$$R_a = P_1\, y_1 + P_2\, y_2 + \ldots + P_n\, y_n \qquad (6.50)$$

On the other hand, if the moving load is distributed as shown in Figure 6.15, then the reaction for a certain load position would be

$$R_a = \int (P\, dx)\, y \qquad (6.51)$$

For a uniformly distributed moving load, the reaction is the load intensity times the occupied area from the influence line.

6.3.3 Examples for Statically Determinate Structures

In this section, numerical examples on the construction of influence lines for statically determinate structures are given. These examples serve as bases for the next section.

Example 6.4

Determine the influence lines of the reactions at A, B, and C for the beam shown in Figure 6.16. From these influence lines, determine the influence lines of the shearing force and bending moment at the middle of span AB.

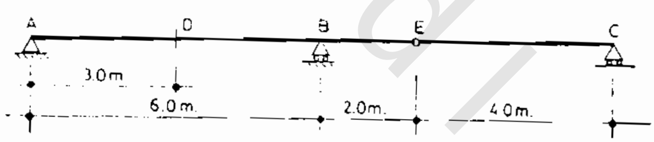

Figure 6.16

Solution

The influence lines of the reactions are determined by finding the reactions when a unit load takes various positions on the beam. If, for example, the unit load is located at A, the value of R_a is unity. If the unit load is located at B, the value of R_a is zero. If the unit load is positioned at E, the value of R_a is $(-\frac{1}{3})$. When the unit load is positioned at C, the value of R_a is zero. The influence lines of R_a, R_b, and R_c are shown in Figure 6.17.

The construction of the influence lines for the reactions eases the determination of the influence lines for the internal actions. The shear force at section D equal to R_a if the unit load is moving between D and C, and equals to $(-R_b)$ if the unit load is moving between A and D. Similarly, the moment at D equals to $(3R_a)$ if the unit load is moving between D and C, and equals to $(3R_b)$ if the load is moving between A and D. The influence lines of A_y and M_z at section D are shown in Figure 6.18.

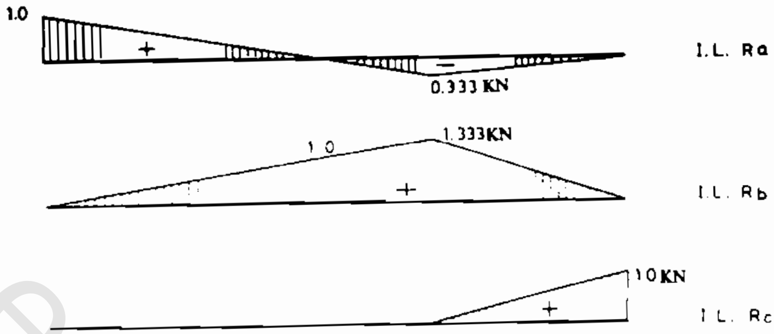

Figure 6.17

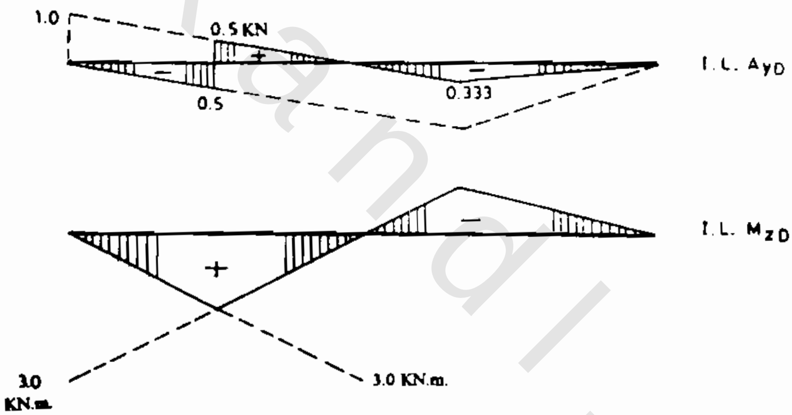

Figure 6.18

Example 6.5

Determine the influence lines for the shear force and bending moment at section s for the simple span bridge shown in Figure 6.19.

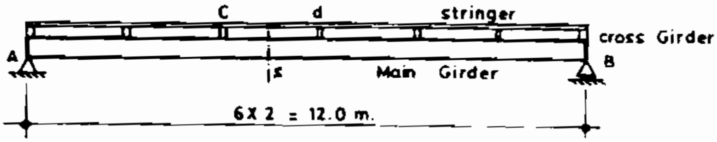

Figure 6.19

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Solution

This is a typical structural system for simple span bridges. The moving loads' effects are transmitted to the main girders through the stringers and cross girders. Therefore, the influence of the unit load on the main girder is only through these cross girders. The influence lines for shear force and bending moment are determined from the influence lines of the reactions at A and B.

When the unit load is moving between d and B, $A_{yc} = 2R_a$, and $M_{zc} = 4R_a$. When the unit load is moving between A and C, $A_{yd} = R_b$, and $M_{zd} = 6R_b$. The influence lines of A_{ys}, and M_{zs} are as shown in Figure 6.20.

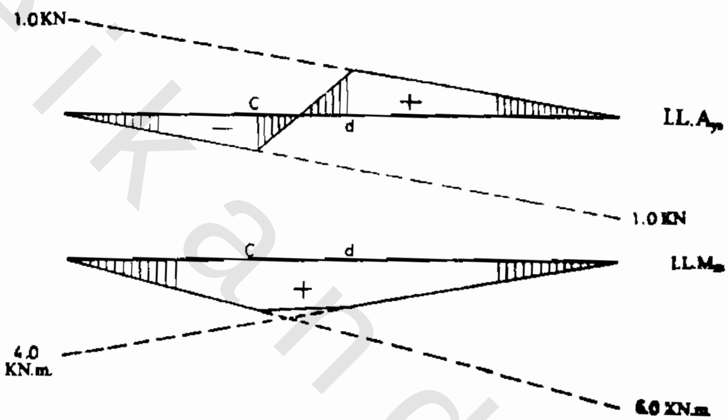

Figure 6.20

Example 6.6

Determine the influence lines for members 1-2, 3-5, 4-6, and 3-6 for the plane truss shown in Figure 6.21.

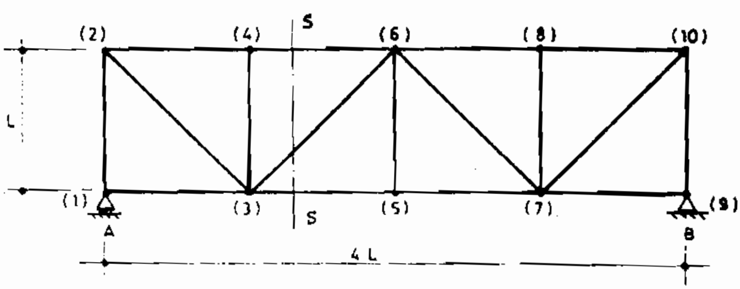

Figure 6.21

Solution

The effect of the moving unit load is transmitted through the floor system to the truss joints. If the unit load is moving along the lower chord of the truss, then its effect is transmitted to joints 1, 3, 5, 7, and 9.

From the equilibrium of joint (1) it is obvious that the influence line of member 1-2 is the same as R_a. By studying the equilibrium of the part on the left hand side of section s-s, it is obvious that when the unit load is moving along the right side of joint s, one has

$A'_{x3-5} = +2R_a$ (tension)

$A'_{x4-6} = -R_a$ (compression)

$A'_{x3-6} = -1.414\, R_a$ (compression)

If the unit load is moving towards the left side of joint 3, the forces in the members are

$A'_{x3-5} = +2R_b$ (tension)

$A'_{x4-6} = -3R_b$ (compression)

$A'_{x3-6} = +1.414\, R_b$ (compression)

The influence lines can then be constructed as shown in Figure 6.22.

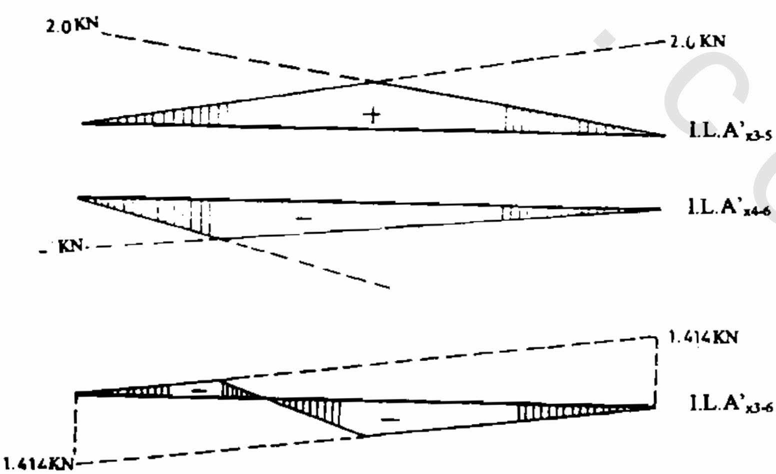

Figure 6.22

6.3.4 Examples for Statically Indeterminate Structures

It was shown in the previous section that the influence line of any internal action can easily be constructed after determining the influence lines of the reactions. A similar process is used for statically indeterminate structures, except that the reactions are usually statically indeterminate. In some other structures, internal members could be statically indeterminate. Therefore, one first has to determine the influence lines of the redundants in the statically indeterminate structures.

Example 6.7

Determine the influence lines of the reactions R_a, R_b and R_c for the beam shown in Figure 6.23.

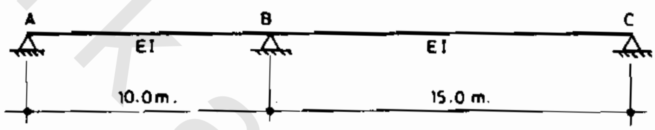

Figure 6.23

Solution

If R_a is considered as a redundant then the equation of consistent deformation in the force method is

$$\Delta_{10} + R_a f_{11} = 0 \quad ; \quad R_a = -\frac{\Delta 10}{f_{11}}$$

where Δ_{10} is the deflection at A due to the unit load at position x, and f_{11} is the deflection at A due to the unit load at A. However, from the reciprocal theorem one has Δ_{10} is the same as f_{x1}. Therefore, one only needs to place a unit load at A and calculate the deflections f_{x1} at various locations along the primary structure.

The deflection at various locations along the beam can be calculated by any suitable method. The conjugate beam method is perhaps the fastest method in this example. The calculated deflection is given in Figure 6.24. The influence line of R_a can now be determined by dividing all deflections by f_{11} = 833.333/EI.

In order to determine the influence lines of R_b and R_c one uses the equilibrium conditions. When R_b is written in terms of R_a one has

$$R_b = \frac{25 - x}{15} - \frac{25}{15} R_a$$

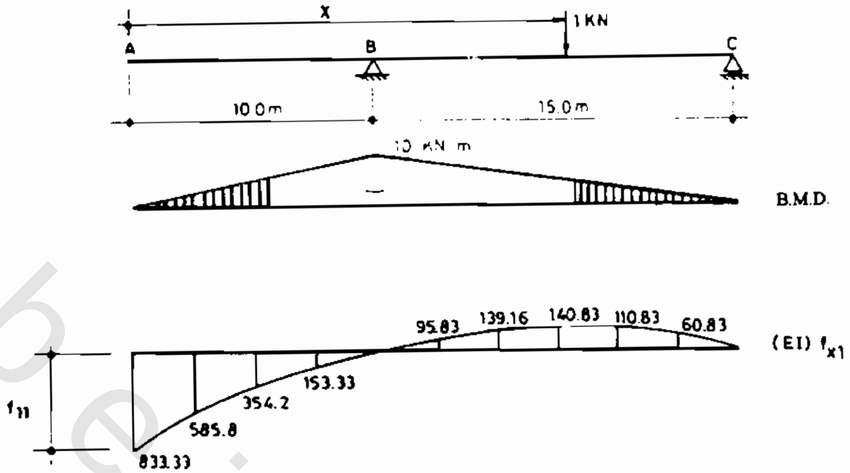

Figure 6.24

Similarly, R_c is obtained as

$$R_c = \frac{x - 10}{15} + \frac{10}{15} R_a$$

The influence lines of the reactions are shown in Figure 6.25.

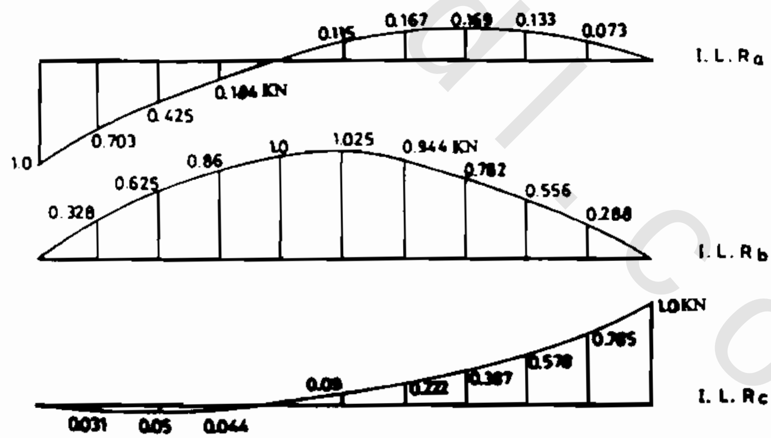

Figure 6.25

Example 6.8

Determine the influence lines of R_b and the forces in members 3-5 and 5-6 in the truss shown in Figure 6.26 (EA = 252000 kN for horizontal members, and EA = 126000 kN for diagonal members).

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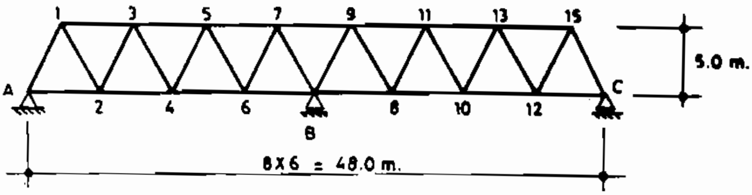

Figure 6.26

Solution

D.S.I. = $m + r - 2j = 31 + 4 - 2(17) = 1$

Select the reaction at B as the redundant. Calculating the deflection at the lower joints due to a unit load at B, the influence line of R_b can be constructed as shown in Figure 6.27.

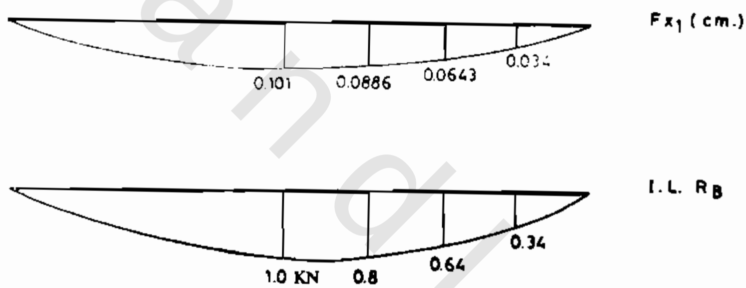

Figure 6.27

In order to determine the influence lines for the forces in members 3-5 and 5-6, one finds the value of these forces in terms of the reactions, as follows:

For member 3-5 one has

$$A'_{x3-5} = -\frac{12}{5} R_a \qquad \text{(if the load moves towards the right side of joint 4)}$$

$$A'_{x3-5} = -\frac{12}{5} R_b - \frac{36}{5} R_c \qquad \text{(if the load moves towards the left of joint 4)}$$

The relationships between R_a, R_c, and R_b are

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$$R_a = -\frac{48 - x}{48} - \frac{R_b}{2} \qquad \text{for} \quad 0 \le x \le 48$$

$$R_c = -\frac{x}{48} - 2R_b \qquad \text{for} \quad 0 \le x \le 48$$

The influence lines of the members forces can thus be constructed as shown in Figure 6.28.

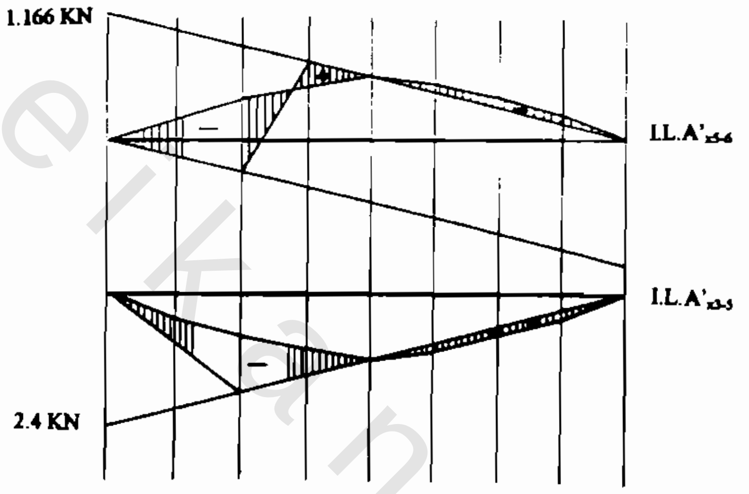

Figure 6.28

6.4 STRUCTURAL ANALYSIS USING COMPUTER

The principles of matrix structural analysis presented in chapters 5 and 6 can easily be programmed for computer use. One has to be familiar with the programming language of communicating with the computer machine, the numerical techniques needed to program matrix operations like addition, subtraction, multiplication, inverse and the solution of linear simultaneous equations. At present, the substantial skills in software techniques have resulted in many professional structural analysis and/or design softwares able to solve complicated structural analysis problems using personal computers. Programs like STAAD-III/ISDS, SAP90, ETAB among many others are characterized by easy use. In these programs, the user introduces the data of geometry, members properties, boundary conditions, applied loading, and the type of analysis or design required. It became possible by these softwares to solve large scale problems for different cases of loading in very short times. Most of the structural analysis programs have also excellent graphic capabilities to enable the user to check out the input geometry and to provide plots of deformations, internal actions, and stresses contours in the structure.

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In this section, some examples are solved using program STAAD-III/ISDS to show first the verification of the results which have previously been obtained manually. Secondly, to show the effect of the current practice in assuming the distribution of floors loading on the surrounding beams on the accuracy of the structural analysis of skeleton structures.

Example 6.9

This example is the same as example 5.14 of chapter 5. The geometry of the structure is shown by the computer plot of Figure 6.29. The input file is shown in Figure 6.30. The shear force and bending moment diagrams are given in the computer plots of Figures 6.31 and 6.32. It is obvious that the results are in close agreement with the results of example 5.14.

Example 6.10

This example is the same as example 5.17. The inclined roller support can either be introduced as an inclined link as shown in Figure 6.33, or by rotating the structure such that the new x-axis coincide with the rolling plane as shown in Figure 6.34. The input files of both cases are shown in Figures 6.35 and 6.36, respectively. The shear force and bending moment diagrams of both cases are, respectively, given in Figures 6.37 and 6.38. The results are the same as obtained in example 5.17.

Example 6.11

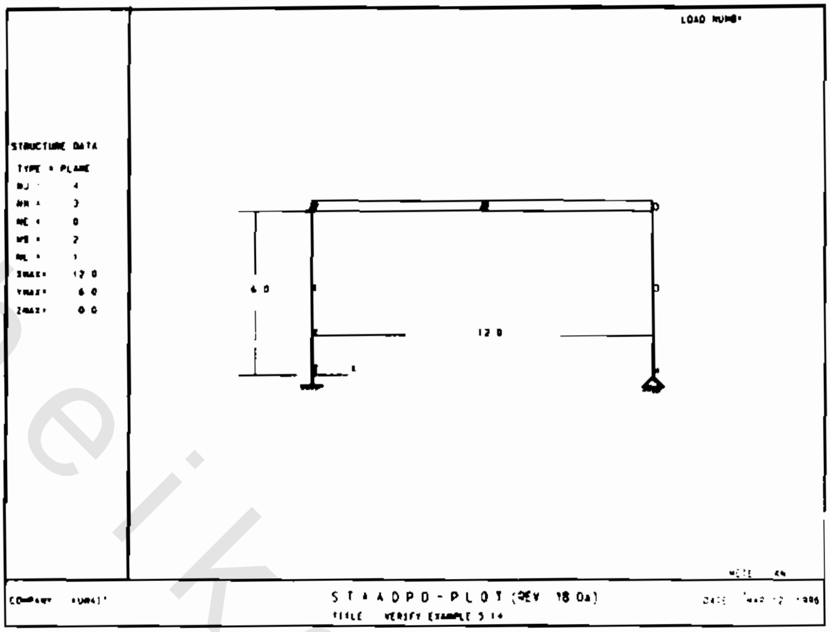
This example is the same as example 5.19. The geometry of the structure is shown in Figure 6.39. The input file is given in Figure 6.40. The shear force and bending moment diagrams are shown, respectively, in Figures 6.41 and 6.42 which are the same as obtained in example 5.19.

Example 6.12

The space frame with a roof slab shown in Figure 6.43 can be analyzed by distributing the slab weight on the surrounding girders using the attributed slab areas as shown in Figure 6.44. In this case, the problem becomes a one dimensional skeleton space frame. The input file of this problem is as shown in Figure 6.45. The bending moment about z-axis is shown in Figure 6.46. The bending moment along girder 1-2-3 is shown in Figure 6.47.

This structure can also be solved by discretizing the slab into finite elements as shown in Figure 6.48. The input file is given in Figure 6.49. The bending moment about z-axis is shown in Figure 6.50. The bending moment along the girder of nodes 1-2-3-4-5-6-7 is shown in Figure 6.51.

By comparing Figures 6.46 and 6.50 one finds out that in the second case, the girders are subjected to torsional moment which was not discovered in the first case due to neglecting the slab-girders interaction. Moreover, these are difference in the results of the bending moment of the girder and columns.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4
3
0
2
1
12.0
6.0
0.0
6.0
12.0
STAADPD-PLOT
TITLE VERIFY EXAMPLE 5.14

Figure 6.29

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*                                           *
*           S T A A D - III                 *
*           Revision 18.0a                  *
*           Proprietary Program of          *
*           RESEARCH ENGINEERS, Inc.        *
*           Date=    MAR  9, 1996           *
*           Time=    23:27:40               *
*                                           *
*********************************************

   1. STAAD PLANE  VERIFY EXAMPLE 5.14
   2. UNITS METER KN
   3. JOINT COORDINATES
   4. 1 0.0 0.0 ; 2 0.0 6.0 ; 3 12.0 6.0 ; 4 12.0 0.0
   5. MEMBER INCIDENCES
   6. 1 1 2 3
   7. MEMBER PROPERTY
   8. 1 3 PRIS  AX 100000.0 IZ 100000.0
   9. 2 PRIS YD 0.6 ZD 1.0 AX 200000.0 IZ 200000.0
  10. CONSTANTS
  11. E 1.0 ALL
  12. ALPHA 0.00001
  13. SUPPORTS
  14. 1 FIXED ; 4 PINNED
  15. LOADING 1
  16. JOINT LOAD
  17. 2 FX 36.0
  18. MEMBER LOAD
  19. 2 UNIFORM GY -2.5
  20. TEMPERATURE LOAD
  21. 2 TEMP 30.0 20.0
  22. PERFORM ANALYSIS

        P R O B L E M   S T A T I S T I C S
        -----------------------------------

  NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =      4/     3/     2
  ORIGINAL/FINAL BAND-WIDTH =     1/     1
  TOTAL PRIMARY LOAD CASES  =     1, TOTAL DEGREES OF FREEDOM =       7
  SIZE OF STIFFNESS MATRIX =      42 DOUBLE PREC. WORDS
  TOTAL REQUIRED DISK SPACE =    12.01 MEGA-BYTES


++ PROCESSING ELEMENT STIFFNESS MATRIX.             23:27:41
++ PROCESSING GLOBAL STIFFNESS MATRIX.              23:27:41
++ PROCESSING TRIANGULAR FACTORIZATION.             23:27:41
++ CALCULATING JOINT DISPLACEMENTS.                 23:27:41
++ CALCULATING MEMBER  FORCES.                      23:27:41

  23. PRINT ANALYSIS RESULTS
```

Figure 6.30

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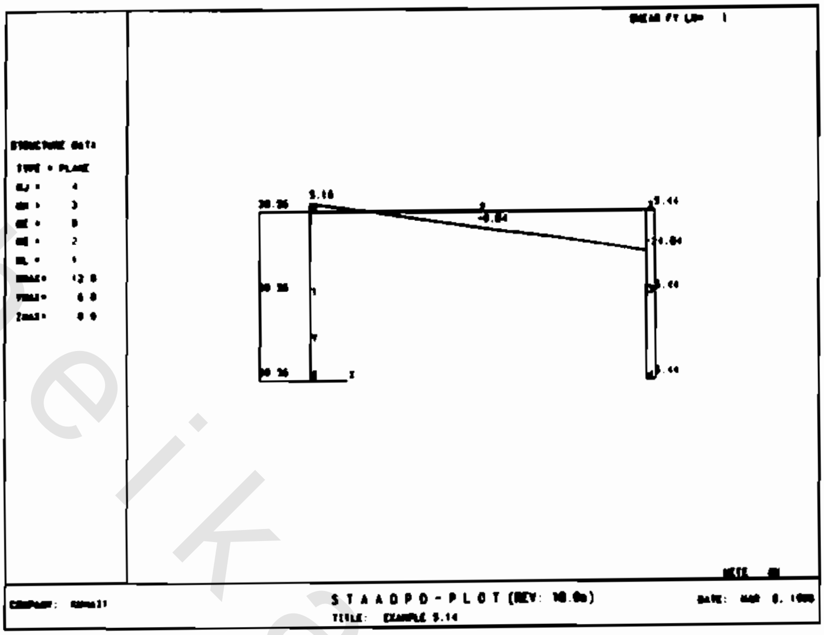

Figure 6.31

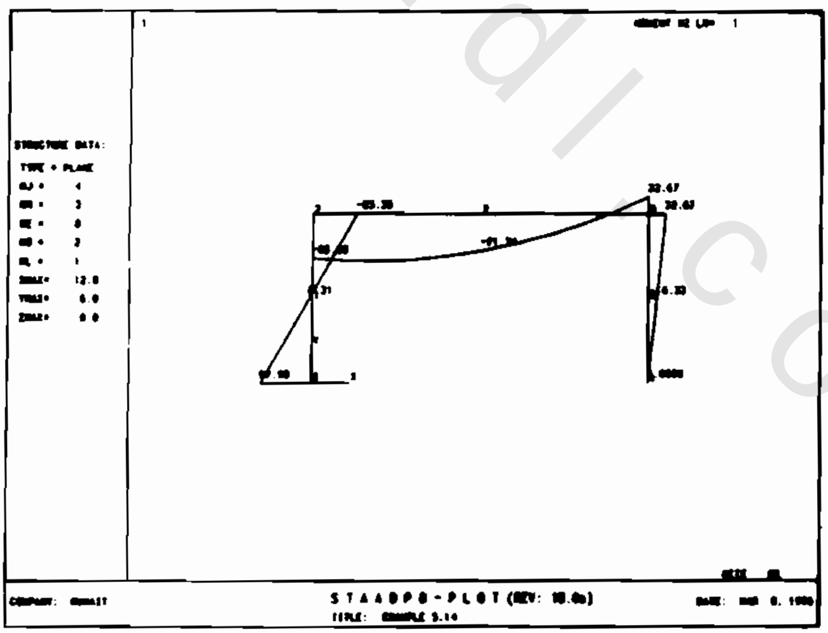

Figure 6.32

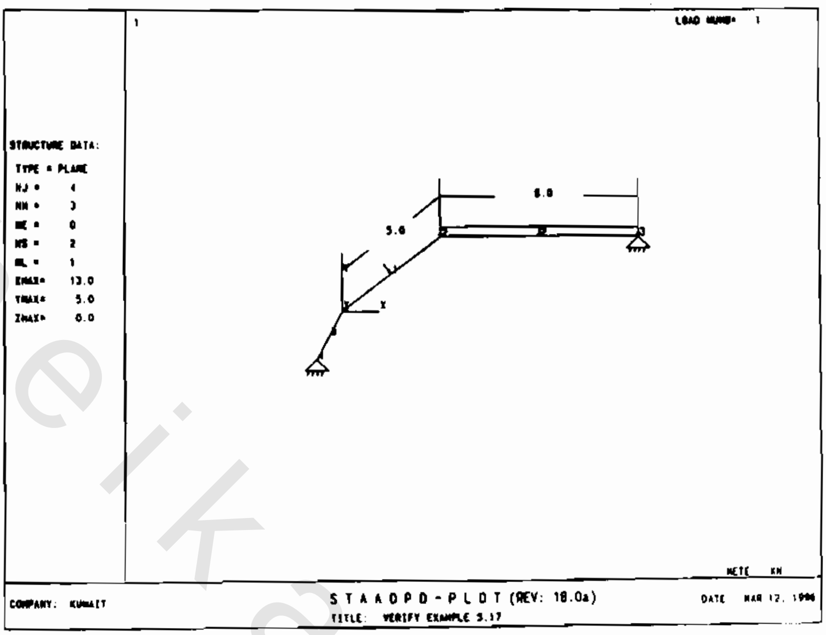

Figure 6.33

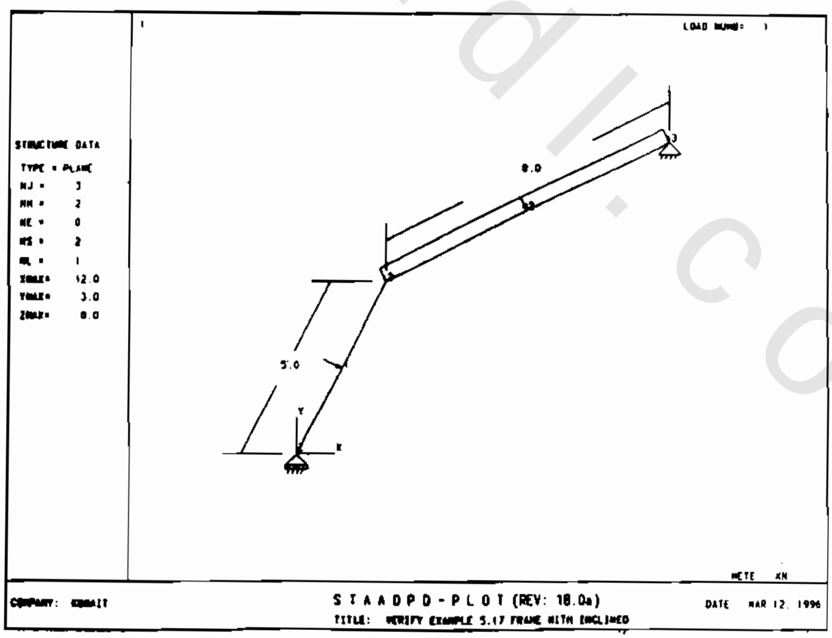

Figure 6.34

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 **************************************************
 *                                                *
 *            S T A A D - III                     *
 *            Revision 18.0a                      *
 *            Proprietary Program of              *
 *            RESEARCH ENGINEERS, Inc.            *
 *            Date=    MAR  9, 1996               *
 *            Time=    23:26:38                   *
 *                                                *
 **************************************************

   1. STAAD PLANE VERIFY EXAMPLE 5.17
   2. UNITS METER KN
   3. JOINT COORDINATES
   4. 1 0.0 0.0 ; 2 4.0 3.0 ; 3 12.0 3.0 ; 4 -1.0 -2.0
   5. MEMBER INCIDENCES
   6. 1 1 2 ; 2 2 3 ; 3 1 4
   7. MEMBER RELEASE
   8. 1 3 START MZ
   9. MEMBER PROPERTY
  10. 1 TO 3 PRIS AX 100000.0 IZ 1.0
  11. CONSTANTS
  12. E 2500000000.0 ALL
  13. SUPPORTS
  14. 4 PINNED
  15. 3 PINNED
  16. LOADING 1
  17. MEMBER LOADING
  18. 2 UNIFORM GY -2.0
  19. 1 CONCENTRATED Y -8.0 2.5
  20. PERFORM ANALYSIS

        P R O B L E M   S T A T I S T I C S
        -----------------------------------

   NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =      4/     3/     2
   ORIGINAL/FINAL BAND-WIDTH =     3/     1
   TOTAL PRIMARY LOAD CASES  =     1, TOTAL DEGREES OF FREEDOM =      8
   SIZE OF STIFFNESS MATRIX =      48 DOUBLE PREC. WORDS
   TOTAL REQUIRED DISK SPACE =   12.01 MEGA-BYTES

++ PROCESSING ELEMENT STIFFNESS MATRIX.          23:26:38
++ PROCESSING GLOBAL STIFFNESS MATRIX.           23:26:38
++ PROCESSING TRIANGULAR FACTORIZATION.          23:26:38
++ CALCULATING JOINT DISPLACEMENTS.              23:26:38
++ CALCULATING MEMBER  FORCES.                   23:26:38

  21. PLOT BENDING FILES
  22. PRINT ANALYSIS RESULTS
```

Figure 6.35

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**************************************************
*                                                *
*           S T A A D - III                      *
*           Revision 18.0a                       *
*           Proprietary Program of               *
*           RESEARCH ENGINEERS, Inc.             *
*           Date=    MAR  9, 1996                *
*           Time=    23:24:53                    *
*                                                *
**************************************************

  1. STAAD PLANE VERIFY EXAMPLE 5.17 FRAME WITH INCLINED ROLLER SUPPORT
  2. UNITS METER KN
  3. JOINT COORDINATES
  4. 1 0.0 0.0 ; 2 4.0 3.0 ; 3 12.0 3.0
  5. MEMBER INCDENCES
  6. 1 1 2 ; 2 2 3
  7. PERFORM ROTATION Z 26.56
  8. MEMBER PROPERTY
  9. 1 2 PRIS AX 100000.0 IZ 1.0
 10. CONSTANTS
 11. E 2500000000.0 ALL
 12. SUPPORTS
 13. 1 FIXED BUT FX MZ
 14. 3 PINNED
 15. LOADING 1
 16. MEMBER LOADING
 17. 2 UNIFORM Y -2.0
 18. 1 CONCENTRATED Y -8.0 2.5
 19. PERFORM ANALYSIS

        P R O B L E M   S T A T I S T I C S
        -----------------------------------

   NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =      3/     2/     2
   ORIGINAL/FINAL BAND-WIDTH =     1/     1
   TOTAL PRIMARY LOAD CASES  =     1, TOTAL DEGREES OF FREEDOM =      6
   SIZE OF STIFFNESS MATRIX =       30 DOUBLE PREC. WORDS
   TOTAL REQUIRED DISK SPACE =    12.00 MEGA-BYTES

++ PROCESSING ELEMENT STIFFNESS MATRIX.                23:24:54
++ PROCESSING GLOBAL STIFFNESS MATRIX.                 23:24:54
++ PROCESSING TRIANGULAR FACTORIZATION.                23:24:54
++ CALCULATING JOINT DISPLACEMENTS.                    23:24:54
++ CALCULATING MEMBER  FORCES.                         23:24:54

  20. PLOT BENDING FILES
  21. PRINT ANALYSIS RESULTS
```

Figure 6.36

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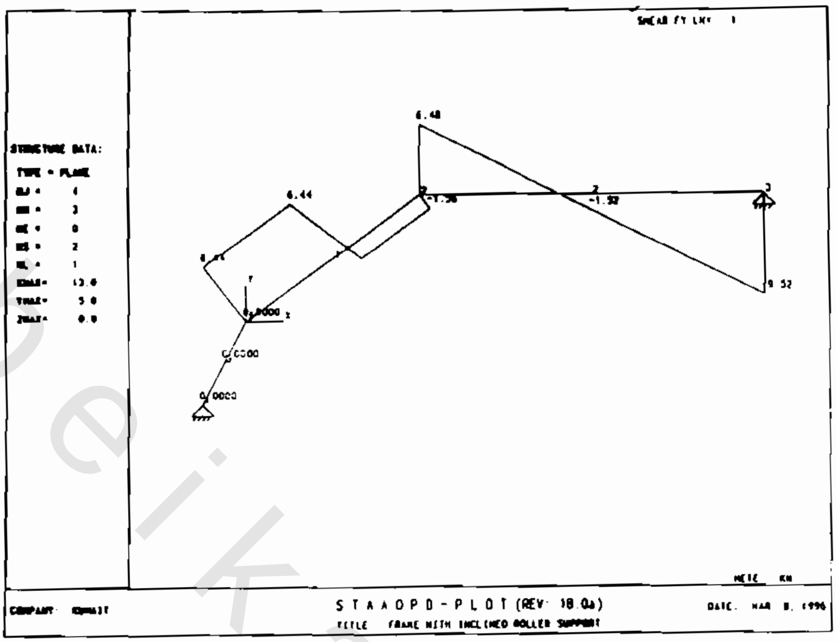

SHEAR FY LN: 1
STRUCTURE DATA:
TYPE = PLANE
6.48
6.44
-1.96
-1.52
9.52
STAADPD-PLOT (REV: 18.0a)
TITLE FRAME WITH INCLINED ROLLER SUPPORT

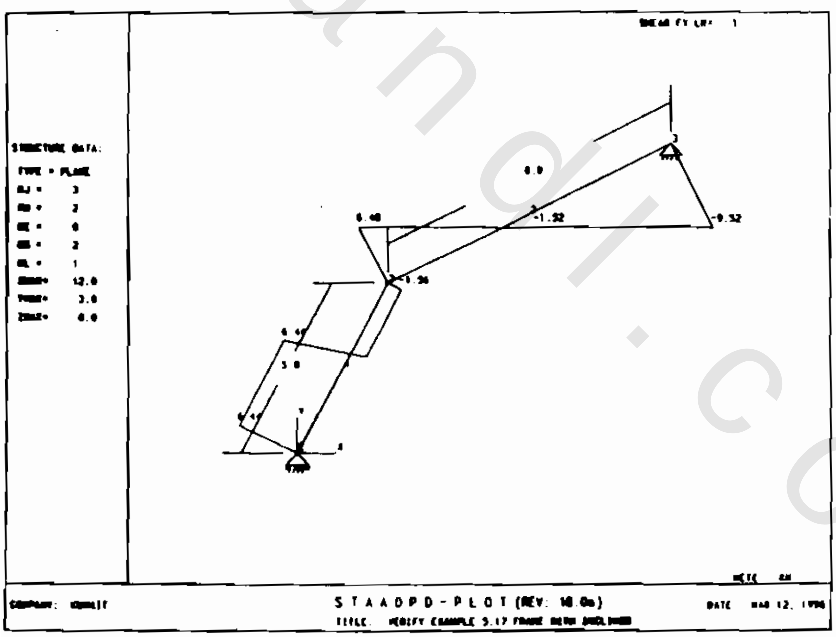
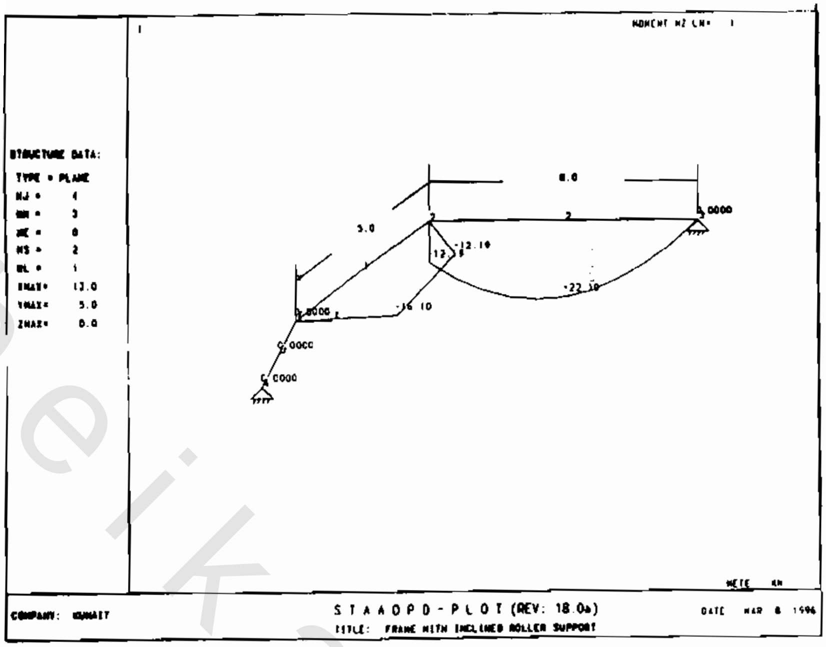

SHEAR FY LN: 1
STRUCTURE DATA:
TYPE = PLANE
6.48
-1.52
-9.52
-1.96
STAADPD-PLOT (REV: 18.0a)

Figure 6.37



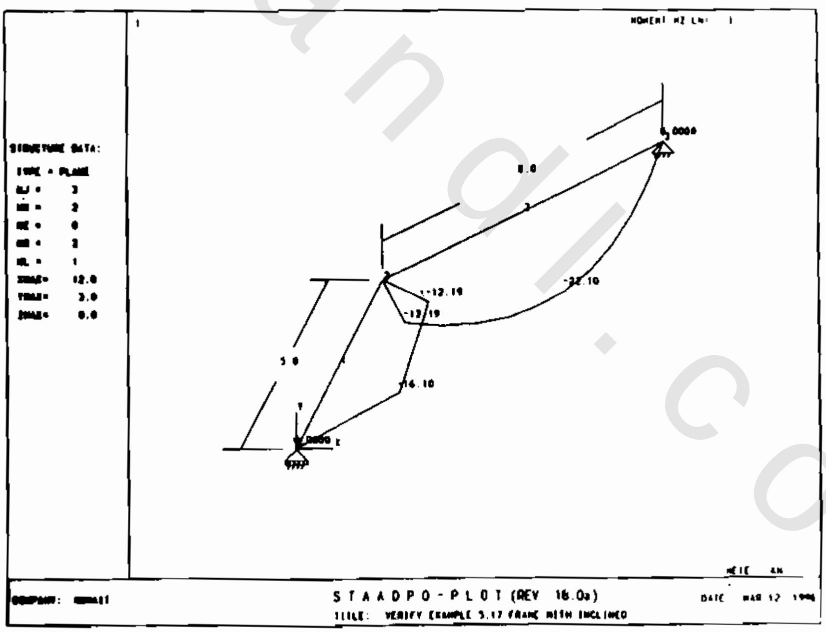


Figure 6.38

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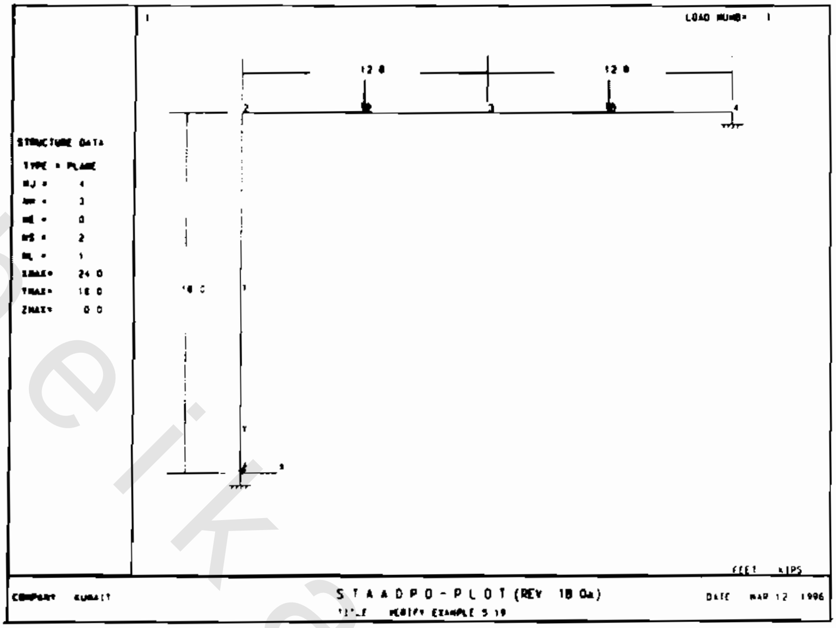

Figure 6.39

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**************************************************
*                                                *
*             S T A A D - III                    *
*             Revision 18.0a                     *
*             Proprietary Program of             *
*             RESEARCH ENGINEERS, Inc.           *
*             Date=     MAR  8, 1996             *
*             Time=     21:13:22                 *
*                                                *
**************************************************

  1. STAAD PLANE EXAMPLE 5.19
  2. UNITS FEET KIPS
  3. JOINT COORDINATES
  4. 1 0.0 0.0 ; 2 0.0 18.0 ; 3 12.0 18.0 ; 4 24.0 18.0
  5. MEMBER INCIDENCES
  6. 1 1 2 3
  7. MEMBER PROPERTY
  8. 1 3 PRIS AX 1.0 IZ 100.0
  9. 2 PRIS AX 2.0 IZ 300.0
 10. CONSTANTS
 11. E 1000.0 ALL
 12. SUPPORTS
 13. 1 4 FIXED
 14. LOAD 1
 15. MEMBER LOAD
 16. 2 CONCEN GY -30.0 6.0
 17. 3 CONCEN GY -30.0 6.0
 18. PERFORM ANALYSIS

       P R O B L E M   S T A T I S T I C S
       -----------------------------------

  NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =      4/     3/     2
  ORIGINAL/FINAL BAND-WIDTH =     1/     1
  TOTAL PRIMARY LOAD CASES  =     1, TOTAL DEGREES OF FREEDOM =      6
  SIZE OF STIFFNESS MATRIX  =      36 DOUBLE PREC. WORDS
  TOTAL REQUIRED DISK SPACE =    12.00 MEGA-BYTES


++ PROCESSING ELEMENT STIFFNESS MATRIX.              21:13:22
++ PROCESSING GLOBAL STIFFNESS MATRIX.               21:13:22
++ PROCESSING TRIANGULAR FACTORIZATION.              21:13:22
++ CALCULATING JOINT DISPLACEMENTS.                  21:13:23
++ CALCULATING MEMBER  FORCES.                       21:13:23

 19. PRINT ANALYSIS RESULTS
```

Figure 6.40

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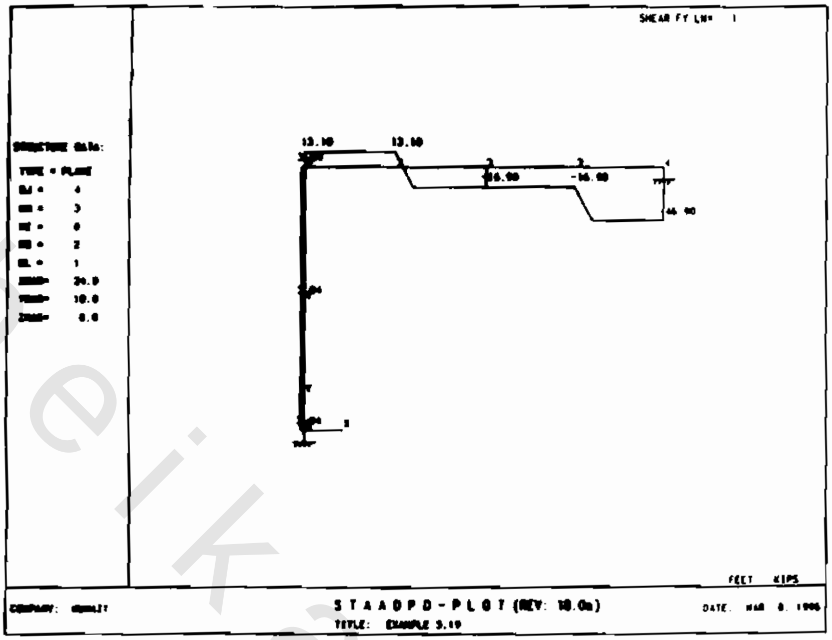

Figure 6.41

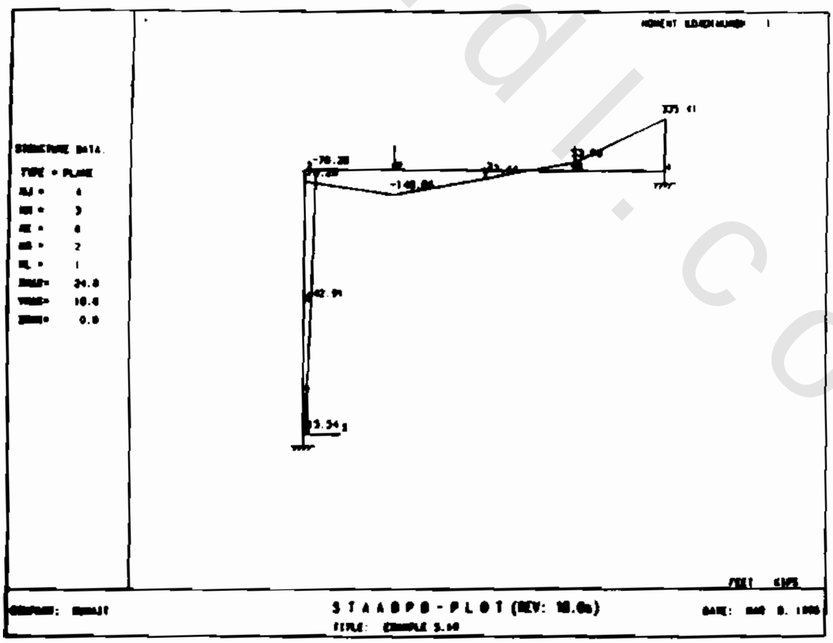

Figure 6.42

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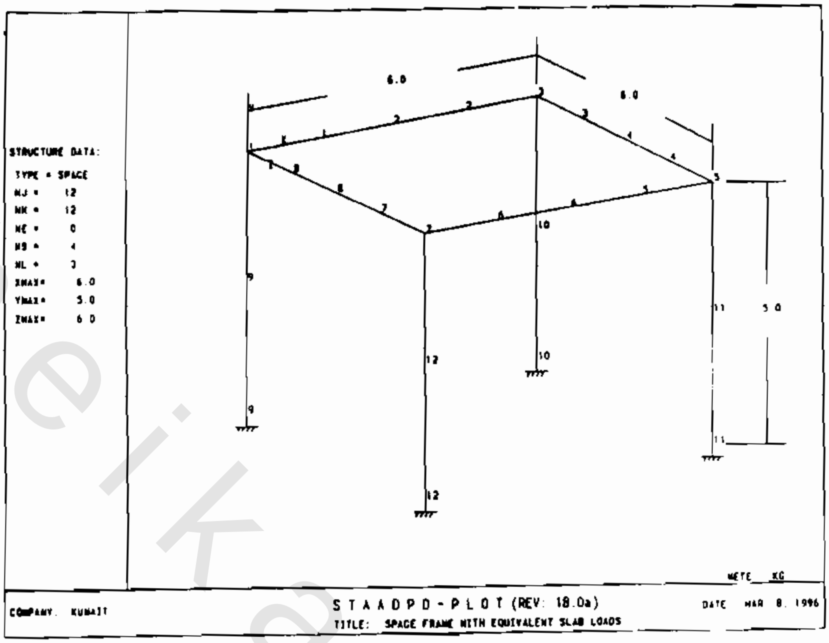

Figure 6.43

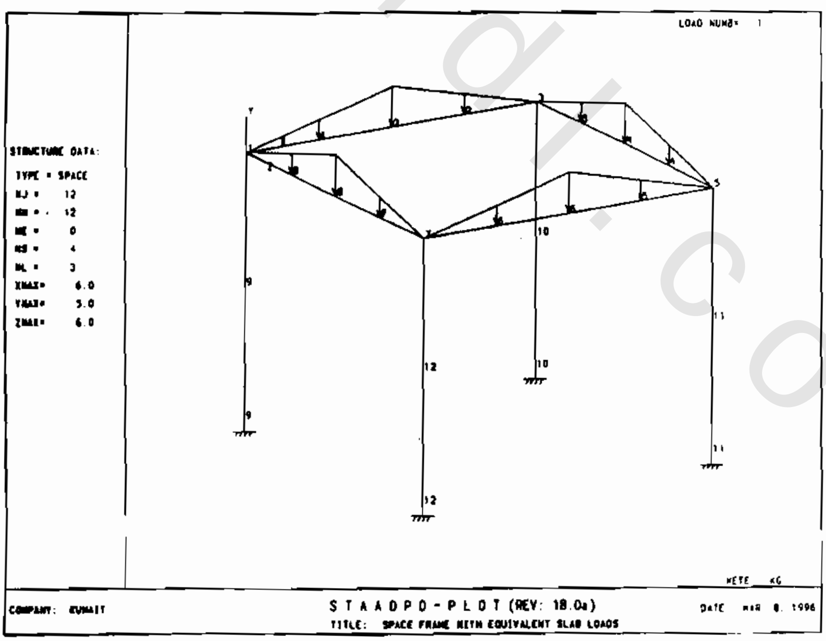

Figure 6.44

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**************************************************
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*           S T A A D - III                      *
*           Revision 18.0a                       *
*           Proprietary Program of               *
*           RESEARCH ENGINEERS, Inc.             *
*           Date=     MAR  4, 1996               *
*           Time=      1: 3:26                   *
*                                                *
**************************************************

  1. STAAD SPACE TESTING SLAB EFFECT
  2. UNIT METER KG
  3. JOINT COORDINATES
  4. 1  0.0 0.0 0.0 3 6.0 0.0 0.0
  5. 4 6.0 0.0 3.0 ; 5 6.0 0.0 6.0
  6. 6 3.0 0.0 6.0 ; 7 0.0 0.0 6.0 ; 8 0.0 0.0 3.0
  7. 9  0.0  -5.0  0.0
  8. 10 6.0  -5.0  0.0
  9. 11 6.0  -5.0  6.0
 10. 12 0.0  -5.0  6.0
 11. MEMBER INCIDENCES
 12. 1 1 2 7; 8 8 1
 13. 9 1 9 ; 10 3 10 ; 11 5 11 ; 12 7 12
 14. MEMBER PROP
 15. 1 TO 8 PRIS YD 0.5 ZD 0.4
 16. 9 TO 12 PRIS YD 0.6 ZD 0.6
 17. CONSTANT
 18. E 2500000000.0 ALL
 19. DEN 2450.0 ALL
 20. SUPPORTS
 21. 9 TO 12 FIXED
 22. LOAD 1
 23. SELFWEIGHT Y -1.0
 24. MEMB LOAD
 25. 1 3 5 7 LIN GY 0.0 -1335.0
 26. 2 4 6 8 LIN GY -1335.0 0.0
 27. LOAD 2
 28. MEMB LOAD
 29. 1 3 5 7 LIN GY 0.0 -1500.0
 30. 2 4 6 8 LIN GY -1500.0 0.0
 31. LOAD COMBINATION  3
 32. 1 1.0  2 1.0
 33. LOAD COMBINATION 4
 34. 1 1.4  2 1.7
 35. PERFORM ANALYSIS

         P R O B L E M   S T A T I S T I C S
         -----------------------------------

  NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =     12/    12/     4
  ORIGINAL/FINAL BAND-WIDTH =     8/     4
  TOTAL PRIMARY LOAD CASES  =     2, TOTAL DEGREES OF FREEDOM =     48
  SIZE OF STIFFNESS MATRIX  =     864 DOUBLE PREC. WORDS
  TOTAL REQUIRED DISK SPACE =    12.03 MEGA-BYTES

++ PROCESSING ELEMENT STIFFNESS MATRIX.                  1: 3:28
++ PROCESSING GLOBAL STIFFNESS MATRIX.                   1: 3:28
++ PROCESSING TRIANGULAR FACTORIZATION.                  1: 3:29
++ CALCULATING JOINT DISPLACEMENTS.                      1: 3:29
++ CALCULATING MEMBER  FORCES.                           1: 3:29

 36. PRINT ANALYSIS RESULTS
```

Figure 6.45

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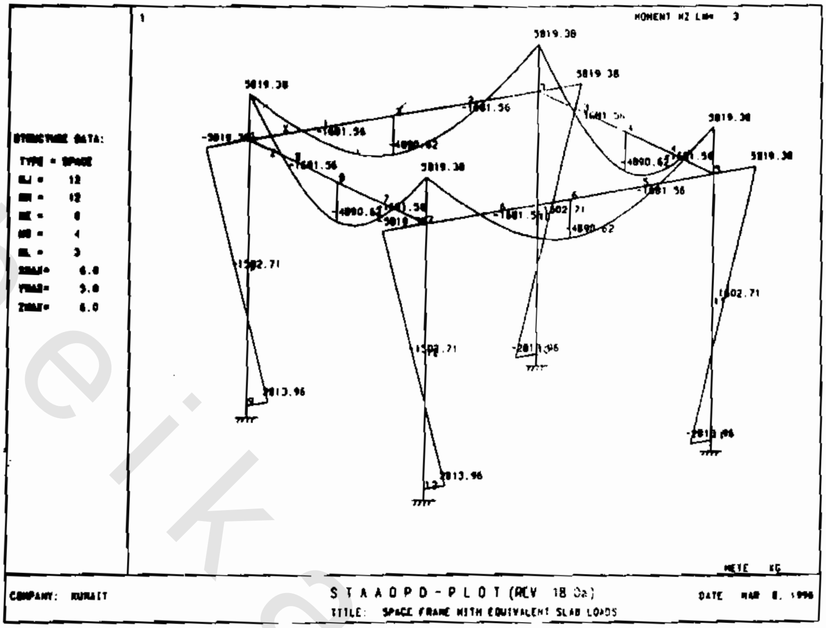

Figure 6.46

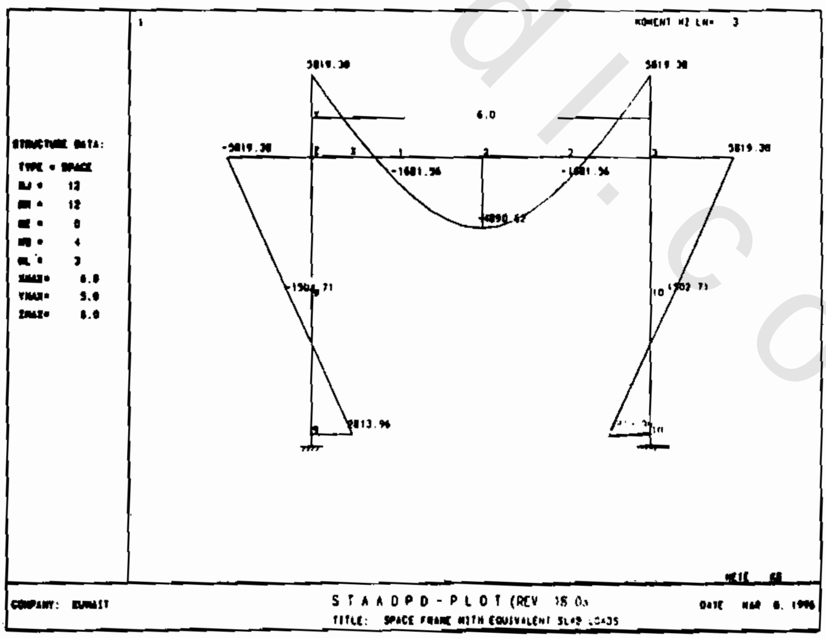

Figure 6.47

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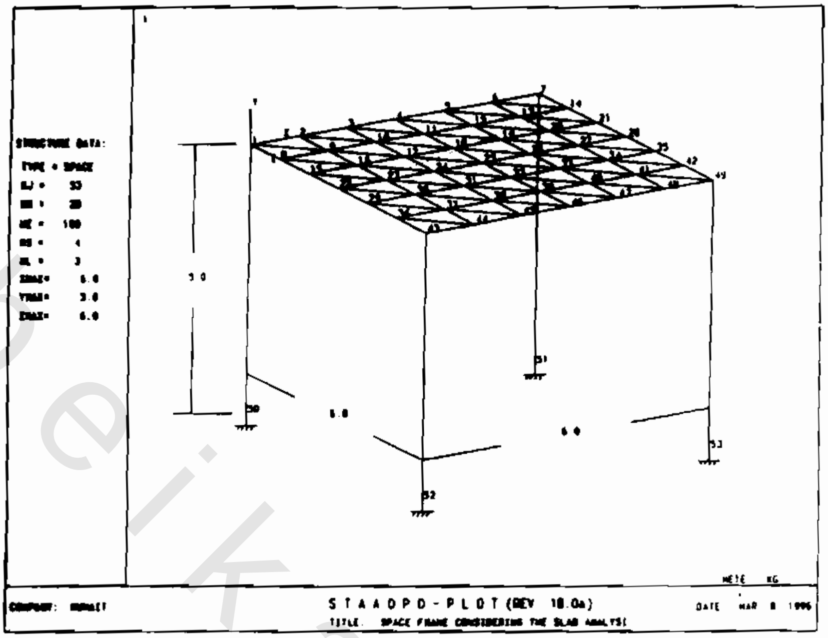

Figure 6.48

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****************************************************
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*            Revision 18.0a                        *
*            Proprietary Program of                *
*            RESEARCH ENGINEERS, Inc.              *
*            Date=    MAR  8, 1996                 *
*            Time=    17:45:36                     *
*                                                  *
****************************************************

 1. STAAD SPACE  SPACE FRAME CONSIDERING THE SLAB ANALYSIS
 2. UNITS METER KG
 3. JOINT COORDINATE
 4. 1 0.0 0.0 0.0 7 6.0 0.0 0.0 ; 8 0.0 0.0 1.0 14 6.0 0.0 1.0
 5. 15 0.0 0.0 2.0 21 6.0 0.0 2.0 ; 22 0.0 0.0 3.0 28 6.0 0.0 3.0
 6. 29 0.0 0.0 4.0 35 6.0 0.0 4.0 ; 36 0.0 0.0 5.0 42 6.0 0.0 5.0
 7. 43 0.0 0.0 6.0 49 6.0 0.0 6.0
 8. 50 0.0 -5.0 0.0 ; 51 6.0 -5.0 0.0 ; 52 0.0 -5.0 6.0 ; 53 6.0 -5.0 6.
 9. MEMBER INCIDENCES
10. 1 1 2 6 ; 7 7 14 12 1 7 ; 13 1 8 18 1 7 ; 19 43 44 24
11. 25 1 50 ; 26 7 51 ; 27 43 52 ; 28 49 53
12. ELEMENT INCIDENCES
13. 29 1 2 9 TO 34
14. 35 8 1 9 TO 40
15. 41 8 9 16 TO 46
16. 47 15 8 16 TO 52
17. 53 15 16 23 TO 58
18. 59 22 15 23 TO 64
19. 65 22 23 30 TO 70
20. 71 29 22 30 TO 76
21. 77 29 30 37 TO 82
22. 83 36 29 37 TO 88
23. 89 36 37 44 TO 94
24. 95 43 36 44 TO 100
25. CONSTANTS
26. E 2500000000.0 ALL
27. DEN 2450.0 ALL
28. MEMB PROP
29. 1 TO 24 PRIS YD 0.5 ZD 0.4
30. 25 TO 28 PRIS YD 0.6 ZD 0.6
31. ELEM PROP
32. 29 TO 100 THICK 0.10
33. SUPPORTS
34. 50 TO 53 FIXED
35. LOAD 1 DEAD LOAD
36. SELFWEIGHT Y -1.0
37. ELEM LOAD
38. 29 TO 100 PR GY -200.0
39. LOAD 2 LIVE LOAD
40. ELEM LOAD
41. 29 TO 100 PR GY -500.0
42. LOAD COMBINATION 3
43. 1 1.0 2 1.0
44. PERFORM ANALYSIS

          P R O B L E M   S T A T I S T I C S
          -----------------------------------

   NUMBER OF JOINTS/MEMBER+ELEMENTS/SUPPORTS =     53/   100/      4
   ORIGINAL/FINAL BAND-WIDTH =    49/     8
   TOTAL PRIMARY LOAD CASES  =     2, TOTAL DEGREES OF FREEDOM =    294
   SIZE OF STIFFNESS MATRIX =    14112 DOUBLE PREC. WORDS
   TOTAL REQUIRED DISK SPACE =     12.69 MEGA-BYTES

++ PROCESSING ELEMENT STIFFNESS MATRIX.                     17:45:38
++ PROCESSING GLOBAL STIFFNESS MATRIX.                      17:45:38
++ PROCESSING TRIANGULAR FACTORIZATION.                     17:45:39
++ CALCULATING JOINT DISPLACEMENTS.                         17:45:40
++ CALCULATING MEMBER  FORCES.                              17:45:40

  45. PRINT ANALYSIS RESULTS
```

Figure 6.49

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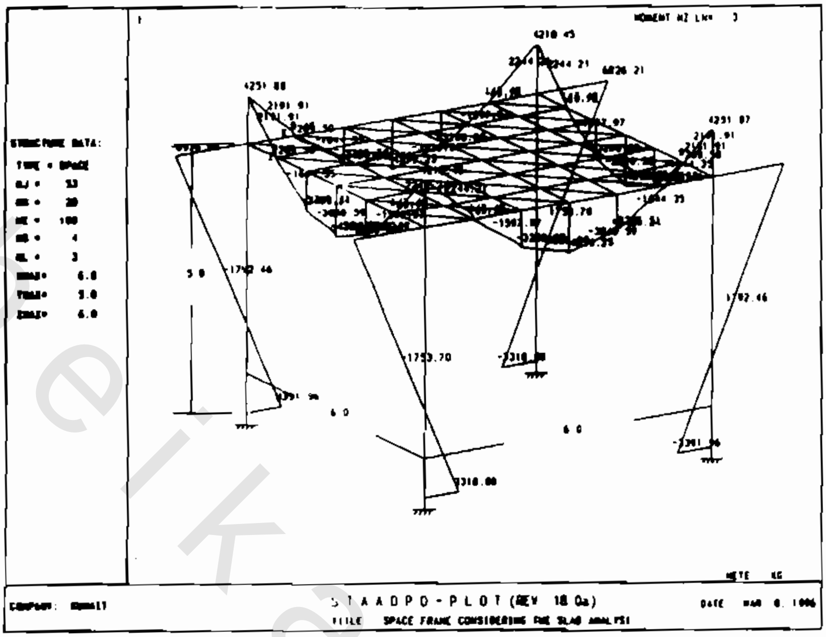

Figure 6.50

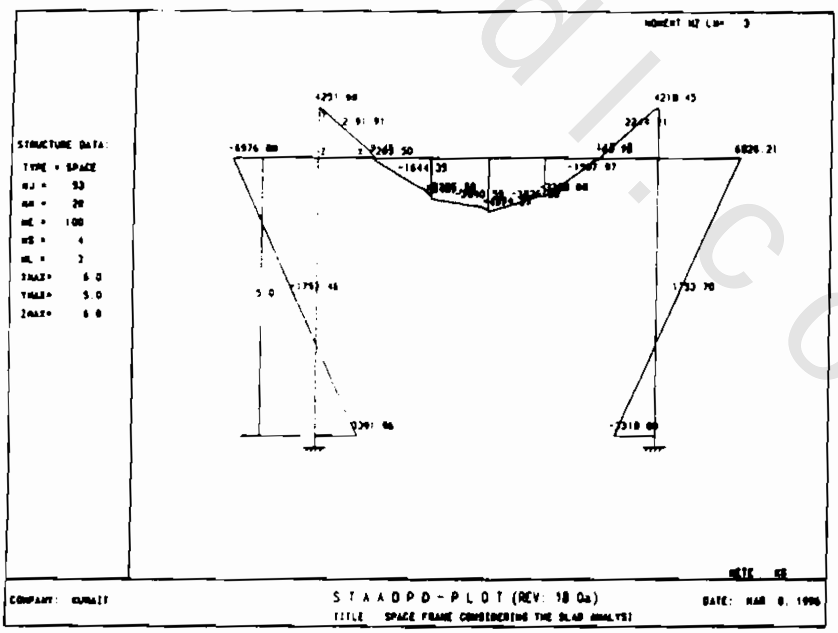

Figure 6.51

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Answers to Exercises

Chapter 2

2. (a) 60.355×10^{-5} m (↓) (b) 306.274×10^{-5} m (↓)

3. (a) $\frac{PL^3}{3}$ (b) -2.8336 (↓) (c) -0.70833 PL^3 (↓)

9. 0.00779 m (↓)

10. $\theta_D = 26.67 \times 10^{-5}$ rad , $\Delta_C = 938.67 \times 10^{-5}$ m (↓)

Chapter 3

2. Elastic Centre at D

I_x = 4096/EI ; I_y = 2730.67/EI ; I_{xy} = -2048/EI
M_A = -166.4 kN.m ; M_B = +35.46 kN.m ; M_C = -54.13 kN.m.

5. R_B = 2.829 kN (↓) ; Δ_B = 0.2829 cm (↓)

6. (a) M_B = 46.42 kN.m (b) M_B = 47.695 kN.m (c) M_B = -0.0766 kN.m

7. F_{BD} = 22.937 kN ; M_C = +1.2 kN.m
F_{AX} = 1.45 kN (→) ; M_B = -8.7 kN.m
Δ_C = 0.0028 cm

8. x_1 = R_A = 9.43 Kips ; Δ_{10} = -0.2 ft ; f_{11} = 0.03 ft ; Δ_1 = 1 inch

10. M_B = 3525.82 kN.m ; M_C = -4598.7 kN.m

11. x_1 = F_{CF} = -2/59 kN ; x_2 = R_c = 23.87 kN (↑)
$\Delta_{10} = -\frac{34.15 \times 5}{EA}$; $\Delta_{20} = -\frac{93.23 \times 5}{EA}$
F_B = 10 kN ; F_{BC} = 10 Kn ; F_{CB} = 20 kN

12. x_1 = R_{AX} = -1.42 kN (←) ; x_2 = R_{AY} = -0.284 kN (↓)
Δ_{10} = 0.0045 ft ; Δ_{20} = 0.00225 ft

13. (a) Δ_B = 1.867 in (↓) ; Δ_C = 1.15 in (↓)
R_C = 1.133 kips (↑) ; R_B = 3.734 Kips (↑)
(b) M_B = -6.44 K.ft ; M_C = -26.59 K.ft

14. x_1 = F_{BG} ; x_2 = F_{GD}
x_3 = R_{CX} ; x_4 = R_{CY}
F_{AB} = 0.32 kN ; F_{BC} = 5.09 kN
F_{CD} = -0.64 kN ; F_{DE} = -0.32 kN etc...

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16. $x_1 = A_{yCD}$; $x_2 = A_{xCD}$; $x_3 = M_{zCD}$
$x_4 = A_{yBE}$; $x_5 = A_{xBE}$; $x_6 = M_{zBE}$

18. Elastic centre at 7.5 ft from AD.

$A = 16/EI$; $I_x = 252/EI$; $M_i = 16.25 + 3.124\,y$
$M_A = -7.85$ K.ft ; $M_B = -29.28$ K.ft

19. $x_1 = F_{BD} = -8.536$ Kips
$\Delta_{10} = 617.95/EA$; $f_{11} = 72.39/EA$
$F_{AD} = -13.97$ Kips ; $F_{AB} = 6.03$ Kips

24. $x_1 = R_C = 50$ kN (←)
$\Delta_{10} = \dfrac{-1207.1}{EA}$; $f_{11} = \dfrac{24.142}{EA}$; $F_{AB} = -30$ kN

25. $x_1 = R_{BX} = -6.653$ Kips ; $x_2 = R_{By} = 0.54$ Kips
$M_C = -47.03$ K.ft ; $M_A = -13.98$ K.ft

27. $M_B = 570.9$ kN.m. ; $M_C = -1030.46$ kN.m.

29. At elastic centre $x_1 = 4.581$ kN (← →) ; $x_2 = 0$; $x_3 = -25.9$ kN.m
$I_x = 268.19/EI$; $I_y = 320/EI$
$M_A = +5.613$ kN.m ; $M_D = -11.035$ kN.m.

30. For $x_1 = F_{AD}$; $x_2 = R_E$

$$\underline{\Delta}_0 = \begin{bmatrix} -1866.67/EI \\ -864/EI \end{bmatrix} \quad ; \quad [f] = \frac{1}{EI}\begin{bmatrix} 146.88 & 28.8 \\ 28.8 & 34.56 \end{bmatrix}$$

$F_{AB} = 1.34$ kN ; $F_{AC} = 10.33$ kN ; $F_{CD} = -1.34$ kN
$F_{DE} = +12.44$ kN ; $F_{BE} = -15.55$ kN ; $F_{BC} = -17.22$ kN
$F_{AD} = +17.22$ kN

31. For $x_1 = F_{DX} = R_{DX} = -16.495$ kN ; $x_2 = R_{Dy} = 46.726$ kN
$x_3 = M_D = -169.695$ kN.m

$$\underline{\Delta}_0 = \begin{bmatrix} -28.6 \\ -48.67 \\ 2.5 \end{bmatrix} , \quad [\underline{f}] = \begin{bmatrix} 1.3533 & 0.8 & -0.08 \\ 0.8 & 1.1067 & -0.06 \\ -0.08 & -0.06 & 0.006 \end{bmatrix}$$

32. $$[f] = \frac{20^3}{EI}\begin{bmatrix} 0.4167 & 0.25 \\ 0.25 & 1.333 \end{bmatrix} \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3.662 \\ 19.43 \end{bmatrix} \begin{matrix} (\leftarrow) \\ (\uparrow) \end{matrix}$$

$x_1 = R_{Dx}$, $x_2 = R_{Dy}$, $M_A = 25.35$ K.ft , $R_{Ay} = 20.56$ K.ft (↑)

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34. $x_1 = R_B$, $x_2 = R_C$
$R_B = 0.768$ Kips (↑) , $R_C = 0.44$ Kips (↑)
$R_A = 0.12$ Kips (↓) , $R_D = 0.089$ Kips (↓)

35. $x_1 = R_D = 1.0285$ Kip (↑) , $x_2 = F_{FC} = -6.465$ Kips
$\Delta_{10} = -22.4 \times 10^{-3}$, $\Delta_{20} = 22.8 \times 10^{-3}$

36. (a) $K_{AB} = K_{BA} = 0.447$, $C_{AB} = C_{BA} = 0.5928$

(b) $M_{FAB} = -19.9$ K.ft , $M_{FBA} = -26$ K.ft

37. $A = \dfrac{24}{EI}$, $I_x = 160/EI$, $I_y = 197.33/EI$
$M_i = 2.22 + 0.23\,y$
$M_B = M_E = -5.78$ kN.m ; $M_A = M_F = 1.42$ kN.m
$M_C = M_D = 3.01$ kN.m

40. $F_{AD} = 0.747$ kN , $F_{CB} = -5.5$ kN , $\Delta_C = 38.65/EA$
$R_{AY} = 6.25$ kN ; $R_{AX} = 0.6$ kN (←)

41. $M_A = 30.125$ K.ft ; $M_B = -60.25$ K.ft
$R_B = 30.295$ Kips , $\theta_A = 0$ from B.M.D.

42. $x_1 = F_{BD} = 1.65$ Kips , $x_2 = F_{AC} = 4.128$ Kips
$M_B = -15.62$ K.ft , $M_C = -15.84$ K.ft
$\Delta_{10} = -0.81778$ ft , $\Delta_{20} = -1.3308$ ft

43. $M_B = -10.97$ kN.m ; $M_C = -5.197$ kN.m
$M_A = -19.133$ kN.m

44. Elastic center at A

$I_x = 369/EI$; $I_y = 1584/EI$, $I_{xy} = 378/EI$
$M_i = -2.496\,y + 2.436\,x$
$M_B = -14.976$ kN.m ; $M_C = 14.256$ kN.m
$M_D = -17.28$ kN.m

Chapter 4

1. $M_{AB} = 1865.53$ kN.m $\theta_B = -0.00603$
$M_{BA} = 397.32$ kN.m $\theta_C = 0.003455$
$M_{CB} = -20$ kN.m $\theta_D = 0.0033569$ $\Delta_D = 0.00676$ m (↑)

2. $M_{AB} = 14.76$ kN.m , $M_{BA} = -22.26$ kN
$M_{BC} = -0.233$ kN.m , $M_{CB} = -24.99$ kN.m
$M_{DC} = -21.24$ kN.m , $M_{DE} = 11.24$ kN.,

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4. M_{AB} = -68.65 kN.m , M_{BA} = -142.92 kN.m
M_{BC} = 112.92 kN.m , M_{CB} = 265.47 kN.m
M_{CD} = -265.47 kN.m

5. M_{AB} = 11.98 kN.m , M_{BA} = -12.56 kN.m
M_{BC} = 12.56 kN.m , M_{CB} = -10.58 kN.m
M_{CE} = -7.7 kN.m , M_{EC} = -25.33 kN.m
M_{CD} = 18.28 kN.m

6. M_{DC} = -27.9 kN.m , M_{DA} = -21.84 kN.m
M_{AD} = -10.77 kN.m , M_{DE} = 49.74 kN.m
M_{ED} = -44.65kN.m , M_{EB} = 25.7 kN.m
M_{BE} = 12.53 kN.m , M_{EF} = 18.95 kN.m
M_{FE} = -22.5 kN.m , M_{FG} = 22.5 kN.m

7. M_{DC} = -18.917 kN.m
M_{DA} = 9.7 kN.m , M_{AD} = 4.77 kN.m
M_{DF} = 6.2 kN.m , M_{DE} = 3.01 kN.m
M_{ED} = -0.415 kN.m , M_{BE} = -8.92 kN.m

11. M_{AB} = -91.67 kN.m , M_{BA} = -126.1 kN.m
M_{CB} = -96.17 kN.m , M_{CD} = 51.44 kN.m
M_{DC} = -2.92 kN.m , M_{CE} = 44.73 kN.m
M_{EC} = -213.98 kN.m , M_{FE} = -218.85 kN.m , M_{GF}= 119.75 kN.m

13. M_{BA} = -56.26 kN.m
M_{BE} = 2.72 kN.m
M_{EB} = 1.36 kN.m
M_{BC} = 54.54 kN.m
M_{CB} = -16.36 kN.m , M_{DC} = 8.18 kN.m

14. (a) S_A = 0.446 EI , C_{AB} = 0.593

(b) K_{AB} = 0.5643 EI , C_{AB} = 0.59276

15. θ_B = -6.25×10^{-4} rad , θ_E = θ_D = 3.125×10^{-4} rad
M_{AB} = -2.09 K.ft , M_{BA} = -7.29 K.ft
M_{BC} = 16.66 K.ft , M_{CB} = -21.35 K.ft
M_{BD} = -4.68 K.ft

16. M_{AB} = 19.09 kN.m ; M_{BA} = 11.04 kN.m
M_{CB} = -5.187 kN.m , M_{CD} = 5.187 kN.m

18. EI = 10000 $K.ft^2$
Δ_B = 0.5 inch (↓)
θ_B = -0.00844 rad
θ_C = 0.022544 rad
M_{AB} = 17 K.ft , M_{BA} = -43.13 K.ft

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19. $\theta_B = -6.365/EI$; $\theta_C = 4.52/EI$, $\Delta = 18.17/EI$
$M_{AB} = 9.316$ kN.m ; $M_{BA} = -9.947$ kN.m , $M_{CB} = -11.33$ kN.m
$M_{DC} = 9.07$ kN.m

20. $M_{AB} = 9.86$ K.ft ; $M_{BA} = -46.46$ K.ft
$M_{CB} = -36.71$ K.ft

21. Unknowns θ_B , θ_C , θ_E , θ_D , θ_G , Δ_G , Δ_E
Conditions $M_B = M_E = M_C = M_D = M_G = 0$
$H_{BC} + H_{ED} + H_{HG} = 0$
$H_{AB} + H_{FE} + H_{HG} = 0$

22. $\theta_C = 0$, $\theta_D = 0$, $\Delta = 720/EI$
$M_{AC} = 30$, $M_{FD} = -30$, $M_{CD} = 0$

23. (a) $M_{FAB} = -23.41$ K.ft

(b) $S_A = 0.447\ EI$, $C_{AB} = 0.592$

26. $M_{FBA} = -18.1$ kN.m , $M_{FAB} = 18.1$ kN.m
$S_A = 16.824$ kN.m , $C_{AB} = 0.6238$
$M_{AB} = 13.71$ kN.m , $M_{BA} = -24.97$ kN.m

27. $M_{AB} = -10.36$ kN.m , $M_{BA} = -65.73$ kN.m
$M_{BC} = 65.73$ kN.m

28. $A_{BC} = -43.78$ kN , $A_{DC} = 26.21$ kN , $A_{DA} = 0$
$A_{AC} = -8.78$ kN

29. $M_{AB} = 21.48$ kN.m , $M_{BA} = 34.35$ kN.m , $M_{BC} = 15.64$ kN.m
$M_{CB} = 4.91$ kN.m

Chapter 5

1. (a) $\underline{D}_C = \begin{bmatrix} 0.012 \\ 0 \\ 0.00016 \end{bmatrix}$, $\underline{D}_B = \begin{bmatrix} 0.0026 \\ -0.00034 \\ 0.00174 \end{bmatrix}$

$\underline{A}'^T_{AB} = [81.2 \quad -74.24 \quad 146.9]$, $\underline{A}'^T_{BA} = [81.2 \quad 5.76 \quad 127.1]$

$\underline{A}'^T_{BC} = [-5.73\ 81.2\ -127.25]$, $\underline{A}'^T_{CB} = [-5.73\ -38.77\ 0]$

$\underline{A}'^T_{AC} = 48.48$ kN

(b) $A'^T_{AC} = 58.93$ kN

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2. $D_{Ay} = -0.152 \times 10^{-3}$ m , $D_B = 10^{-3}\begin{bmatrix}0.252\\0.237\end{bmatrix}$, $D_D = 10^{-3}\begin{bmatrix}0.273\\0.262\end{bmatrix}$

$D_{CX} = 10^{-3} \times 0.489$ m

$A'_{AB} = 168$ kN , $A'_{AD} = 98.8$ kN , $A'_{AE} = -79$ kN , $A'_{BC} = 158$ kN

$A'_{BD} = 12.5$ kN , $A'_{BE} = 15.4$ kN

4. $D_2 = 10^{-3}\begin{bmatrix}3.071\\-8.93\end{bmatrix}$, $D_4 = 10^{-3}\begin{bmatrix}-1.2\\-16\end{bmatrix}$, $D_{3y} = -1.6 \times 10^{-2}$ m

$D_{5x} = -6 \times 10^{-4}$ m , $A'_{12} = -585.8$ kN , $A'_{15} = -120$ kN , $A'_{52} = 214.2$ kN

$A'_{54} = -120$ kN , $A'_{24} = 282.9$ kN , $A'_{23} = -614.18$ kN

6. $\underline{D}_2 = \frac{1}{EI}\begin{bmatrix}-68.38\\-48.95\end{bmatrix}$, $\underline{D}_3 = \frac{1}{EI}\begin{bmatrix}-115.9\\-10.59\end{bmatrix}$, $\theta_4 = 83.63/EI$

$M_{AB} = 164.9$ kN.m

7. $A'_{23} = 49.45$ kN , $A'_{24} = 0$, $A'_{25} = -55.12$ kN

$A'_{12} = 24.4$ kN , $A'_{14} = 9.56$ kN , $A'_{15} = 0$

$A'_{16} = 35.6$ kN , $A'_{35} = 5.52$ kN , A34 = 7.85 kN

$A'_{36} = -9.56$ kN , $A'_{56} = 43.45$ kN , $A'_{46} = -105.6$ kN

8. $\underline{A}'_{12} = \begin{bmatrix}-5.8\\-8.45\\79.04\end{bmatrix}$, $\underline{A}'_{21} = \begin{bmatrix}4.18\\1.54\\18.71\end{bmatrix}$, $\underline{A}'_{23} = \begin{bmatrix}-1.54\\4.18\\-18.7\end{bmatrix}$

$\underline{A}'_{32} = \begin{bmatrix}-11.54\\-5.8\\41.6\end{bmatrix}$, $\underline{D}_2 = 10^{-5}\begin{bmatrix}63.7\\-81.68\\3.39\end{bmatrix}$

9. $\underline{D}_2 = \begin{bmatrix}2.256\\-1.785\\3.176\end{bmatrix}$, $A'_{21} = 6.86$ kN , $A'_{23} = -3.448$ kN , $A'_{24} = -15$ kN

11. A'_{AB} = -19.51 kN , A'_{BC} = -1.13 kN , A'_{CD} = 12.74 kN

A'_{DF} = -18.02 kN , A'_{FE} = -18.36 kN , A'_{EA} = -6.34 kN

$\underline{D}_C = \begin{bmatrix}-82.56\\-263.81\end{bmatrix}$, $\underline{D}_F = \begin{bmatrix}-86.7\\-199.3\end{bmatrix}$

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12. $$\underline{S} = \frac{EA}{L}\begin{bmatrix} 1.353 & -0.353 & -1 & 0 \\ -0.353 & 1353 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

15. $$\underline{D}_B = \begin{bmatrix} -0.13 \\ -0.353 \\ \theta \end{bmatrix}, \quad D'_C = -0.853 \qquad \underline{A}'^T_{AB} = \begin{bmatrix} -4.42 & 0.76 & -6.09 \end{bmatrix}$$

16. $$\underline{D}_B = \begin{bmatrix} 3.75 \\ -1.52 \\ 0.232 \end{bmatrix}, \quad \underline{A}'_{AB} = \begin{bmatrix} -12.38 \\ 91.27 \\ 219.33 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} 109.53 \\ -14 \\ 453.57 \end{bmatrix}$$

17. $$\underline{A}'_{AB} = \begin{bmatrix} -25.5 \\ 10.28 \\ -10.97 \end{bmatrix}, \quad \underline{A}'_{BC} = \begin{bmatrix} -75.59 \\ -60.28 \\ 214.33 \end{bmatrix}, \quad \underline{A}'_{ED} = \begin{bmatrix} -75.79 \\ -60.28 \\ 214.33 \end{bmatrix}$$

18. $$\underline{A}_A = \begin{bmatrix} -25.52 \\ -22 \\ 188.14 \end{bmatrix}, \quad \underline{A}'_E = \begin{bmatrix} 24.48 \\ 22 \\ 200.27 \end{bmatrix}$$

19. $$\underline{A}_{AB} = \begin{bmatrix} -35.26 \\ -24.77 \\ 307.2 \end{bmatrix}, \quad \underline{A}'_{ED} = \begin{bmatrix} -14.72 \\ 24.77 \\ 0 \end{bmatrix}$$

20. $$\underline{D}'_A = \begin{bmatrix} -81.26 \\ 0 \\ -12.19 \end{bmatrix}, \quad \underline{D}'_B = \begin{bmatrix} 0 \\ -32.5 \\ -12.19 \end{bmatrix}, \quad \underline{D}'_C = \begin{bmatrix} -65 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{A}'_{AB} = \begin{bmatrix} -3.519 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} -3.519 \\ 0 \\ 1.52 \end{bmatrix}, \quad \underline{A}'_{DB} = \begin{bmatrix} 0 \\ 9.9 \\ -16.76 \end{bmatrix}$$

21. $$\underline{D}'_B = \begin{bmatrix} -54.75 \\ -235.82 \\ 7.727 \end{bmatrix}, \quad \underline{A}'_{AB} = \begin{bmatrix} -3.04 \\ 13.1 \\ -15.55 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} 3.04 \\ 46.90 \\ -335.26 \end{bmatrix}$$

22. $\underline{D}'_{AS} = 10^{-4}\begin{bmatrix} 3.829 \\ 0 \\ -1.977\times 10^{-3} \end{bmatrix}$, $\underline{D}_B = 10^{-4}\begin{bmatrix} 2.08 \\ -2.4 \\ 0.1265 \end{bmatrix}$, $\underline{D}_C = 10^{-4}\begin{bmatrix} 0 \\ 0 \\ 0.437 \end{bmatrix}$

$\underline{A}'_{AB} = \begin{bmatrix} -12.87 \\ -6.4 \\ 0 \end{bmatrix}$, $\underline{A}'_{BA} = \begin{bmatrix} -12.87 \\ 1.59 \\ 12.12 \end{bmatrix}$, $\underline{A}'_{23} = \begin{bmatrix} -11.24 \\ -6.48 \\ -12.17 \end{bmatrix}$

$\underline{A}'_{32} = \begin{bmatrix} -11.24 \\ 9.52 \\ 0 \end{bmatrix}$

23. $\underline{D}_B = 10^{-3}\begin{bmatrix} 0 \\ 0 \\ -0.378 \end{bmatrix}$, $A'_{AB} = \begin{bmatrix} 0 \\ -18.7 \\ 24.9 \end{bmatrix}$, $A'_{BC} = \begin{bmatrix} 0 \\ -17.8 \\ 62.7 \end{bmatrix}$

$\underline{A}'^{T}_{CB} = [0 \quad 2.2 \quad 0]$

24. $\underline{D}_A = \begin{bmatrix} 0.134 \\ -00293 \\ 0.0213 \end{bmatrix}$, $\underline{D}_B = \begin{bmatrix} 0.0014 \\ -0.0295 \\ 0 \end{bmatrix}$

$\underline{A}'_{AB} = \begin{bmatrix} -1.05 \\ -4.875 \\ -0.75 \end{bmatrix}$, $\underline{A}'_{BA} = \begin{bmatrix} -1.05 \\ 5.125 \\ 0 \end{bmatrix}$, $\underline{A}'_{BC} = \begin{bmatrix} -5.25 \\ -1.056 \\ 0 \end{bmatrix}$

$\underline{A}_{CB} = \begin{bmatrix} -5.25 \\ 18.94 \\ -71.48 \end{bmatrix}$

25. $\underline{D}_B = 10^{-3}\begin{bmatrix} -6.98 \\ -0.134 \\ -96.58 \end{bmatrix}$, $\underline{D}_E = 10^{-3}\begin{bmatrix} -6;54 \\ -0.09 \\ -926.52 \end{bmatrix}$

$\underline{A}_{AB} = \begin{bmatrix} 42.65 \\ -232.5 \\ 2.68 \end{bmatrix}$, $\underline{A}_{CB} = \begin{bmatrix} 47.65 \\ 196.76 \\ 2.086 \end{bmatrix}$

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29. $$\underline{A}'_{23} = \begin{bmatrix} 29.22 \\ 9.39 \\ -1246.9 \end{bmatrix}, \quad \underline{A}'_{43} = \begin{bmatrix} -105.4 \\ 37.18 \\ -1227.65 \end{bmatrix}, \quad \underline{A}'_{13} = 8.89\,K$$

$$\underline{D}_3 = \begin{bmatrix} 0.0389 \\ -0.0703 \\ -0.042 \end{bmatrix}, \quad \underline{D}_5 = \begin{bmatrix} 0.0389 \\ -4.95 \\ -0.125 \end{bmatrix}$$

31. $$\underline{A}'_{AB} = \begin{bmatrix} -28.517 \\ -12.55 \\ 31.97 \end{bmatrix}, \quad \underline{A}'_{BA} = \begin{bmatrix} -28.517 \\ 27.44 \\ -180.919 \end{bmatrix}$$

$$\underline{A}'_{BC} = \begin{bmatrix} -51.28 \\ 20.84 \\ -319.08 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} -51.28 \\ 20.84 \\ -202.1 \end{bmatrix}, \quad \underline{D}_B = \begin{bmatrix} -0.019 \\ -0.0965 \\ -0.0146 \end{bmatrix}$$

35. $$\underline{A}'_{AB} = \begin{bmatrix} -4.03 \\ -2.21 \\ 20.14 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} -4.03 \\ 2.2 \\ -20.14 \end{bmatrix}, \quad \underline{A}'_{DA} = \begin{bmatrix} -1.95 \\ 0 \\ -15.03 \end{bmatrix}$$

$$\underline{D}'_B = \begin{bmatrix} 5.1317 \\ 0.0442 \\ 0 \end{bmatrix}$$

36. $$\underline{A}'_{AB} = \begin{bmatrix} 3.25 \\ -8.46 \\ 39.37 \end{bmatrix}, \quad \underline{A}_{BD} = \begin{bmatrix} 3.25 \\ -8.46 \\ 39.37 \end{bmatrix}, \quad \underline{D}_B = \begin{bmatrix} 0.516 \\ -0.1726 \\ -0.00163 \end{bmatrix}$$

37. $$\underline{A}'_{AB} = \begin{bmatrix} -9.55 \\ -1.35 \\ 10.12 \end{bmatrix}, \quad \underline{A}'_{CB} = \begin{bmatrix} -3.65 \\ 0.44 \\ -6.448 \end{bmatrix}, \quad \underline{D}_B = \begin{bmatrix} 0.0328 \\ -0.086 \\ 0.0036 \end{bmatrix}$$

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