Chapter 4

BASIC MODEL AND ANALYSIS

4.1 INTRODUCTION

In thesis, random-access protocols for the OCDMA network proposed in [3] for single-rate users are extended to be suitable for multi-rate users. More receiver detection models exist. The most famous ones are correlation receiver, and chip-level receiver [6]. The effect of both shot noise and thermal noise are added separately to the MAI, then the performance of the Pro1 and Pro2 is studied in the noisy environment. The obtained results are compared with noiseless system throughput in [44].

4.2 SYSTEM MODEL

The system model is composed of N users having same average activity A as shown in Fig. 4.1, we focus on slotted data transmission. Thus, after a successful control message, a user transmits a packet (with probability A) at the beginning of a time slot to the destination. The length of a packet is K bits and corresponds to a slot duration. An active user (one that is about to transmit a packet) is assigned a one coincidence frequency hop code/optical orthogonal code (**OCFHC/OOC**). The cardinality C of an OCFHC/OOC is given by [14]

$$\mathcal{C} = p^k(p^k-1)\cdot\frac{N_{ooc}-1}{w(w-1)}. \tag{4.1}$$

Note: We approximated the cardinality to effective part which is ϕ_{c_0} in Eq. 3.20 in Chapter 3, then the average number of hits between a codeword can be approximated to q which is:

$$q = \frac{\frac{w^2}{N_{OOC}}\cdot\frac{N_{OOC}-1}{w(w-1)}}{\phi_{c_0}-1}. \tag{4.2}$$

Acquiring the multi-coding technique to achieve multi-rate depends on offering each user a number of codes depending on the rate requested. This requires a large set of codes

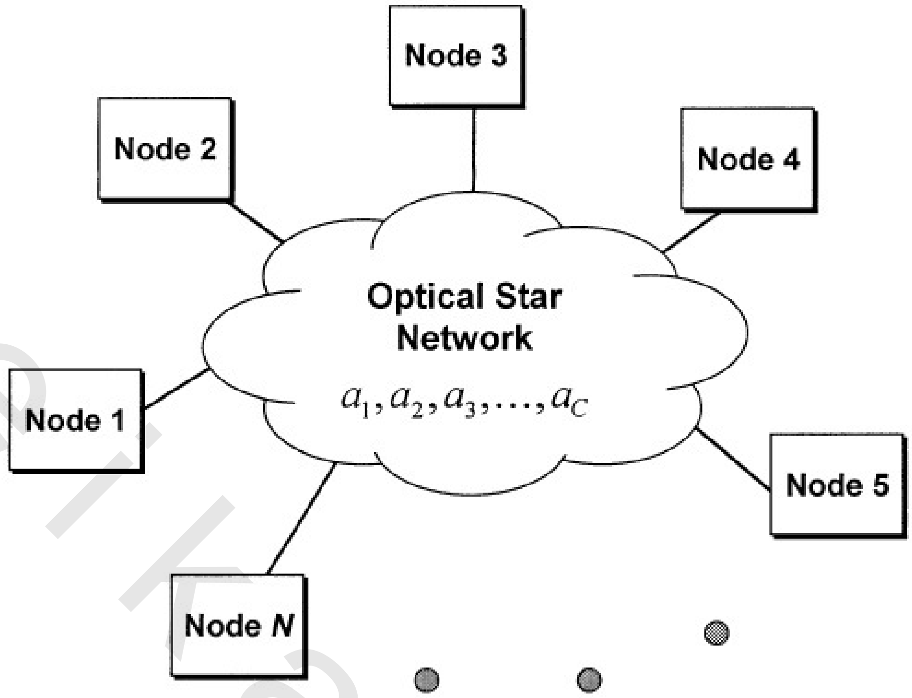

FIGURE 4.1 Optical CDMA network architecture [3]

which makes the **OOC** [7, 8] insufficient in such case; as increasing the cardinality means increasing the code length, which is not practical for high bit rate applications where the number of time slots is limited. That is the reason behind choosing the OCFHC/OOC. Furthermore, the **OCFHC/OOC** correlation constraints guarantee that the codes interfere by only one chip at most [14].

Multi-rate can be done by multi-coding technique, where a high rate user will be offered a number of codes from the pool depending on the rate requested (Number of codes $m_x = R_{new}/R_{basic}$), depending on the rule given in Pro 1 or 2 depending on the protocol used.

Two cases are studied; namely, two-class network and general number of classes where we divide users of system on classes according to required data rate. Also, available codes are divided into pools for each class, where an active user is assigned $m_x \in \{1, 2, \ldots\}$ code sequences selected from the pool (related to it's class) where $x \in \{1, 2, 3, \ldots\}$ is the number of class. The intended receiver, once it has received a packet, transmits an acknowledgment to the sending user, indicating whether the packet is received successfully or not. If not, the transmitter enters a backlog mode and retransmits the m_x packet after a random delay time with average time slots d. Let the number of backlogged users in class x be $n_x \leq N_x$. The probabilities that $i \in \{1, 2, \ldots, n_x\}$ backlogged and $j \in \{1, 2, \ldots, N_x - n_x\}$ thinking (transmitting new packets) users being active at a given time slot are given by [3]:

$$
\begin{aligned}
P_{bl}\left(i|n_x\right) &= \binom{n_x}{i}\left(\frac{1}{d}\right)^{i}\left(1-\frac{1}{d}\right)^{n_x-i}. \\
P_{th}\left(j|n_x\right) &= \binom{N_x-n_x}{j}A^{j}(1-A)^{N_x-n_x-j}.
\end{aligned}
\tag{4.3}
$$

2D OCFHC/OOC is used with correlation constraints equal one, the probabilities of one and w chip interference denoted by p_1 and p_w, respectively, between two users. Assuming chip-synchronous interference model among users, it is easy to show that the last two probabilities are given by:

$$
\begin{aligned}
p_1 &= \begin{cases} \frac{t}{\mathcal{C}-1}\cdot\frac{w^2}{N_{ooc}}; & \text{for Protocol 1.} \\ \frac{t}{\mathcal{C}-1}\cdot\left[\frac{w^2}{N_{ooc}}(\mathcal{C}-1)+\frac{w(w-1)}{N_{ooc}}\right]\cdot\frac{1}{\mathcal{C}}; & \text{for Protocol 2.} \end{cases} \\
p_w &= \begin{cases} 0; & \text{for Protocol 1.} \\ \frac{t}{\mathcal{C}-1}\cdot\frac{1}{N_{ooc}}\cdot\frac{1}{\mathcal{C}}; & \text{for Protocol 2.} \end{cases}
\end{aligned}
\tag{4.4}
$$

Here, $\mathcal{C}$ is the cardinality of OCFHC/OOC as given in (4.1) and $t=(N_{ooc}-1)/w(w-1)$ [14].

4.3 SYSTEM THEORETICAL ANALYSIS

4.3.1 Two-class Network

Each user belongs to one of two available classes: Class 1 and Class 2 where $x=\{1,2\}$. A Class 1 user sends $m_1 \in \{1,2,3,\dots\}$ packets/slot (high rate class), whereas a Class 2 user sends only one packet/slot($m_2=1$). In other words, a Class 1 message is composed of m_1 packets, while a Class 2 message is composed of 1 packet. That is, the transmission rate of a Class 1 user $= m_1R_0$, where R_0 is the basic data rate, which is equal to the transmission rate of a Class 2 user. Assuming that the number of users in Class 1 and Class 2 are N_1 and N_2, respectively, with $N_2 \geq N_1$. Indeed, normally users requesting lower rates are greater than those requesting higher rates. Of course, $N_1+N_2=N$. Available code sequences are divided equally among two pools. An active user is assigned m_1 code sequences selected from the first pool (if it belongs to Class 1) or assigned a code sequence selected from the other pool (if it belongs to Class 2) as shown in Fig. 4.2.

4.3.1.1 System Throughput

System throughput is evaluated following similar methodologies as in [3] and [45], where it is calculated by multiplying average transmitted packets, packet success probability; as a function of average transmitted packets, and probabilities of backlogged and thinking

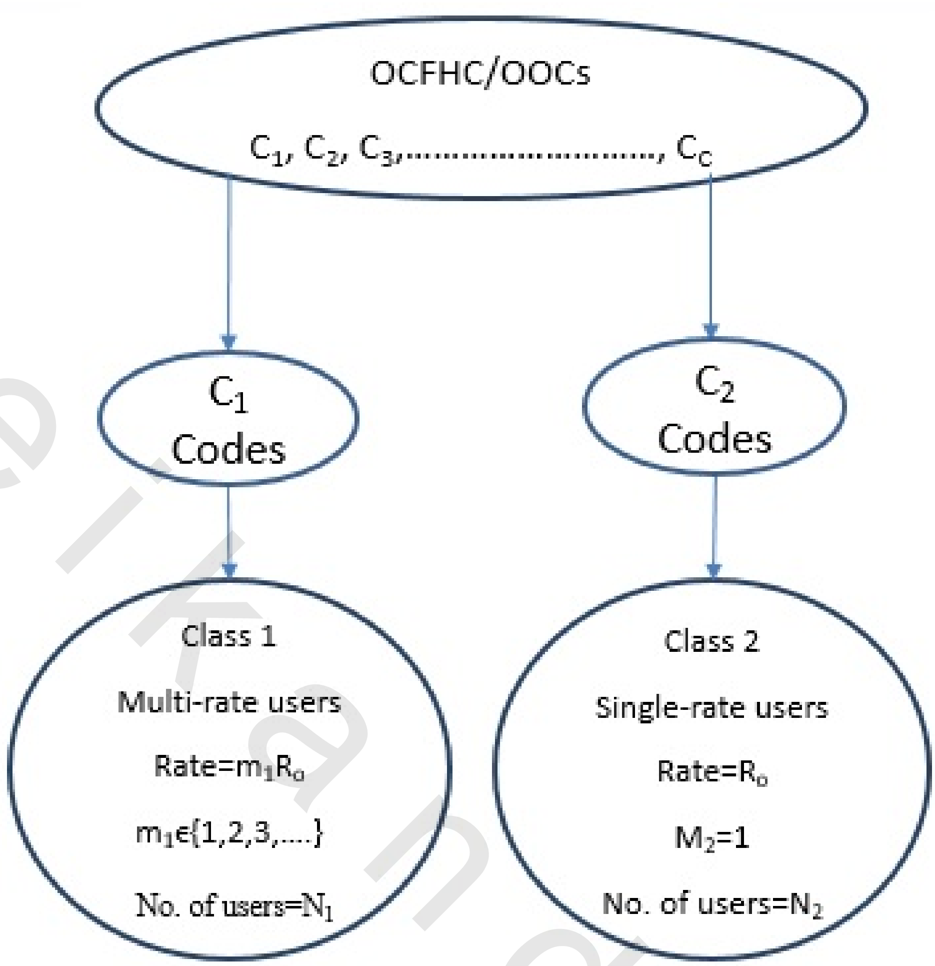

FIGURE 4.2 System Model for Two-class Network.

for two classes. In the analysis, focus is oriented towards multiple-access interference only, where the effects of both receivers' shot and thermal noises are neglected in order to have some insight on the problem. The effect of photodetector's shot and thermal noises can be considered easily by invoking the corresponding equations as give in [3] and [46].

The overall system throughput $\beta(n_1, n_2) = \beta_1(n_1, n_2) + \beta_2(n_1, n_2)$, at a given time slot, where β_1 and β_2 are given in Eq. (4.5) and (4.6), respectively, where n_1 and n_2 denote the number of backlogged users for classes 1 and 2, respectively,

$P_s(r)$ is the packet success probability given r active users, $\mathcal{C}_1$ and $\mathcal{C}_2$ are the number of codes available in Class 1 and Class 2, respectively, and the symbol $x \wedge y$ denotes the minimum of the two numbers x and y. Since we have r active users, there are $r-1$ interfering users to the desired one. Out of these $r-1$ users, let ν users interfere with the desired user at w chips and ℓ users interfere with it at 1 chip. Let $\mathcal{X} = \{1, 2, \ldots, w\}$ and ℓ_i, $i \in \mathcal{X}$, denote the number of users (out of ℓ users) that interfere with weighted chip i. Of course $\ell = \sum_{i=1}^{w} \ell_i$. Further, let $\overline{\boldsymbol{\ell}}$ be the vector $(\ell_1, \ell_2, \ldots, \ell_w)$. Assuming equally-likely binary data bits, the packet success probability given r active users is derived as in [3]

$$\beta_1(n_1,n_2)=\begin{cases}\displaystyle\sum_{l=0}^{N_2-n_2}\sum_{k=0}^{n_2}\sum_{j=0}^{N_1-n_1}\sum_{i=0}^{n_1}\left[\{m_1(i+j)\}\wedge\mathcal{C}_1\right] & \\ \qquad\times P_s\left[\{m_1(i+j)\}\wedge\mathcal{C}_1+(k+l)\wedge\mathcal{C}_2\right] & \text{for Protocol 1}\\ \qquad\times P_{bl}(i|n_1)P_{th}(j|n_1)P_{bl}(k|n_2)P_{th}(l|n_2); & \\ \displaystyle\sum_{l=0}^{N_2-n_2}\sum_{k=0}^{n_2}\sum_{j=0}^{N_1-n_1}\sum_{i=0}^{n_1}\left[m_1(i+j)\right]\cdot P_s\left[m_1(i+j)+k+l\right] & \text{for Protocol 2,}\\ \qquad\times P_{bl}(i|n_1)P_{th}(j|n_1)P_{bl}(k|n_2)P_{th}(l|n_2); & \end{cases} \tag{4.5}$$

$$\beta_2(n_1,n_2)=\begin{cases}\displaystyle\sum_{l=0}^{N_2-n_2}\sum_{k=0}^{n_2}\sum_{j=0}^{N_1-n_1}\sum_{i=0}^{n_1}\left[(k+l)\wedge\mathcal{C}_2\right] & \\ \qquad\times P_s\left[\{m_1(i+j)\}\wedge\mathcal{C}_1+(k+l)\wedge\mathcal{C}_2\right] & \text{for Protocol 1}\\ \qquad\times P_{bl}(i|n_1)P_{th}(j|n_1)P_{bl}(k|n_2)P_{th}(l|n_2); & \\ \displaystyle\sum_{l=0}^{N_2-n_2}\sum_{k=0}^{n_2}\sum_{j=0}^{N_1-n_1}\sum_{i=0}^{n_1}\left[k+l\right]\cdot P_s\left[m_1(i+j)+k+l\right] & \text{for Protocol 2,}\\ \qquad\times P_{bl}(i|n_1)P_{th}(j|n_1)P_{bl}(k|n_2)P_{th}(l|n_2); & \end{cases} \tag{4.6}$$

and is given by Eq. (4.7), which is dependent on the conditional bit-correct probability

$$P_s(r)=\sum_{\ell=0}^{r-1}\sum_{\nu=0}^{r-1-\ell}\frac{(r-1)!}{\ell!\nu!(r-1-\nu-\ell)!}\cdot p_1^{\ell}p_w^{\nu}(1-p_1-p_w)^{r-1-\ell-\nu}$$
$$\cdot\sum_{\substack{\ell_1,\ell_2,\ldots,\ell_w:\\ \ell_1+\ldots+\ell_w=\ell}}\frac{\ell!}{\ell_1!\ldots\ell_w!}\cdot\left(\frac{1}{w}\right)^{\ell}\left[P_{bc}(\nu,\overline{\boldsymbol{\ell}})\right]^K \tag{4.7}$$

$P_{bc}(\nu,\overline{\boldsymbol{\ell}})$, given by Eq. (4.8), and on both the probabilities of one and w chip interference of the 2D OCFHC/OOC.

$$P_{bc}(\nu,\overline{\boldsymbol{\ell}})=\begin{cases}\displaystyle\frac{1}{2}+\frac{1}{2^{\ell+\nu+1}}\sum_{i=0}^{w-1}\binom{\ell}{i}; & \text{for correlation receiver,}\\ \displaystyle\frac{1}{2}+\frac{1}{2^{\nu+1}}\left(\sum_{i=1}^{w}\frac{1}{2^{\ell_i}}-\sum_{i=1}^{w-1}\sum_{j=i+1}^{w}\frac{1}{2^{\ell_i+\ell_j}}+\ldots+(-1)^{w-1}\frac{1}{2^{\ell}}\right); & \text{for chip-level receiver.}\end{cases} \tag{4.8}$$

4.3.1.2 Steady-State Performance

To obtain the steady-state throughput and average packet delay, the above system can be described by two independent discrete-time Markov chains [47]. The chains consist of N_1+1 and N_2+1 states depending on the number of backlogged users $n_1 \in \{0, 1 \ldots, N_1\}$ and $n_2 \in \{0, 1 \ldots, N_2\}$, respectively. In a given chain $i \in \{1, 2\}$, the transition from a state to another occurs on a slot-by-slot basis. We determine the transition probability $P^i_{n_i m_i}$ from state n_i to state m_i, where $n_i, m_i \in \{0, 1 \ldots, N_i\}$, of backlogged users as follows. Let k and l denote the number of thinking and backlogged users, respectively, being active at state n_i. Following [3], it is easy to check that for any $i \in \{1, 2\}$, the transition probability $P^i_{n_i m_i}$ is given by Eq. (4.9), at the top of next page, where

$$P^i_{n_i m_i} = \begin{cases} \displaystyle\sum_{l=0\vee(n_i-m_i)}^{n_i} \sum_{k=0\vee(m_i-n_i)}^{(N_i-n_i)\wedge(\mathcal{C}_i+m_i-n_i)} P_{bl}(l|n_i) P_{th}(k|n_i) \\ \quad \times \binom{(k+l)\wedge\mathcal{C}_i}{k-m_i+n_i} P_s^{k-m_i+n_i}((k+l)\wedge\mathcal{C}_i) & \text{for Protocol 1.} \\ \quad \times \Big[1-P_s((k+l)\wedge\mathcal{C}_i)\Big]^{(k+l)\wedge\mathcal{C}_i-k+m_i-n_i}; \\ \displaystyle\sum_{l=0\vee(n_i-m_i)}^{n_i} \sum_{k=0\vee(m_i-n_i)}^{N_i-n_i} P_{bl}(l|n_i) P_{th}(k|n_i) \\ \quad \times \binom{k+l}{k-m_i+n_i} P_s^{k-m_i+n_i}(k+l) & \text{for Protocol 2.} \\ \quad \times \Big[1-P_s(k+l)\Big]^{l+m_i-n_i}; \end{cases} \tag{4.9}$$

$$\mathcal{C}_i = \begin{cases} \mathcal{C}_1; & \text{if } i=1, \\ \mathcal{C}_2; & \text{if } i=2 \end{cases} \tag{4.10}$$

and the symbol $x \vee y$ denotes the maximum of the two numbers x and y. For any $i \in \{1, 2\}$, stationary probability distributions π^i_n, $n \in \{0, 1, \ldots, N_i\}$, always exists for the above irreducible Markov chains. They can be obtained from the following set of equations:

$$\begin{aligned} \sum_{n=0}^{N_i} \pi^i_n &= 1 \\ \sum_{n=0}^{N_i} \pi^i_n P^i_{nm} &= \pi^i_m; \qquad \forall m \in \{0, 1, \ldots, N_i\}. \end{aligned} \tag{4.11}$$

Finally, the steady-state system throughput β can be calculated using:

$$\begin{aligned} &\beta = \beta_1 + \beta_2 = \\ &\sum_{n_2=0}^{N_2} \sum_{n_1=0}^{N_1} \beta_1(n_1, n_2)\pi_{n_1}^1 \pi_{n_2}^2 + \sum_{n_2=0}^{N_2} \sum_{n_1=0}^{N_1} \beta_2(n_1, n_2)\pi_{n_1}^1 \pi_{n_2}^2. \end{aligned} \tag{4.12}$$

and the average packet delay D which represents the average number of time slots a packet will be received successfully after them. D can be obtained using:

$$\begin{aligned} &D = 1 + \frac{1}{\beta} \cdot E\left\{m_1 n_1 + n_2\right\} = \\ &1 + \frac{1}{\beta} \sum_{n_2=0}^{N_2} \sum_{n_1=0}^{N_1} \left(m_1 n_1 + n_2\right) \pi_{n_1}^1 \pi_{n_2}^2. \end{aligned} \tag{4.13}$$

where $E\{\cdot\}$ denotes the expected value.

4.3.2 General number of classes network

To generalize the analysis, users of different rates are allowed to access the network. They are divided into classes; each operating on a different rate (different values for $m_x \in \{1, 2, 3, \ldots, \mathcal{C}\}$ where $x \in \{1, 2, 3, \ldots, \varphi\}$ is the number of class and φ is the maximum number of classes in the system which is depend on required data rates) as shown in Fig. 4.3. Equations for system throughput$\beta(n_1, n_2, \ldots, n_x) = \beta_1(n_1, n_2, \ldots, n_x) + \ldots + \beta_x(n_1, n_2, \ldots, n_x)$, steady-state system throughput β and average packet delay D will be changed as Eq.(4.14), Eq.(4.15) and Eq.(4.16), respectively, where n_x denotes the number of backlogged users for class x, $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_x$ are the number of codes available form Class 1 to Class x, respectively.

$$\begin{aligned} &\beta = \beta_1 + \beta_2 + \ldots + \beta_x = \\ &\sum_{n_x=0}^{N_x} \ldots \sum_{n_1=0}^{N_1} \beta_1(n_1, \ldots, n_x)\pi_{n_1}^1 \ldots \pi_{n_x}^x + \ldots \\ &\quad + \sum_{n_x=0}^{N_x} \ldots \sum_{n_1=0}^{N_1} \beta_x(n_1, \ldots, n_x)\pi_{n_1}^1 \ldots \pi_{n_x}^x. \\ &= \sum_{n_x=0}^{N_x} \ldots \sum_{n_1=0}^{N_1} \beta(n_1, \ldots, n_x)\pi_{n_1}^1 \ldots \pi_{n_x}^x. \end{aligned} \tag{4.15}$$

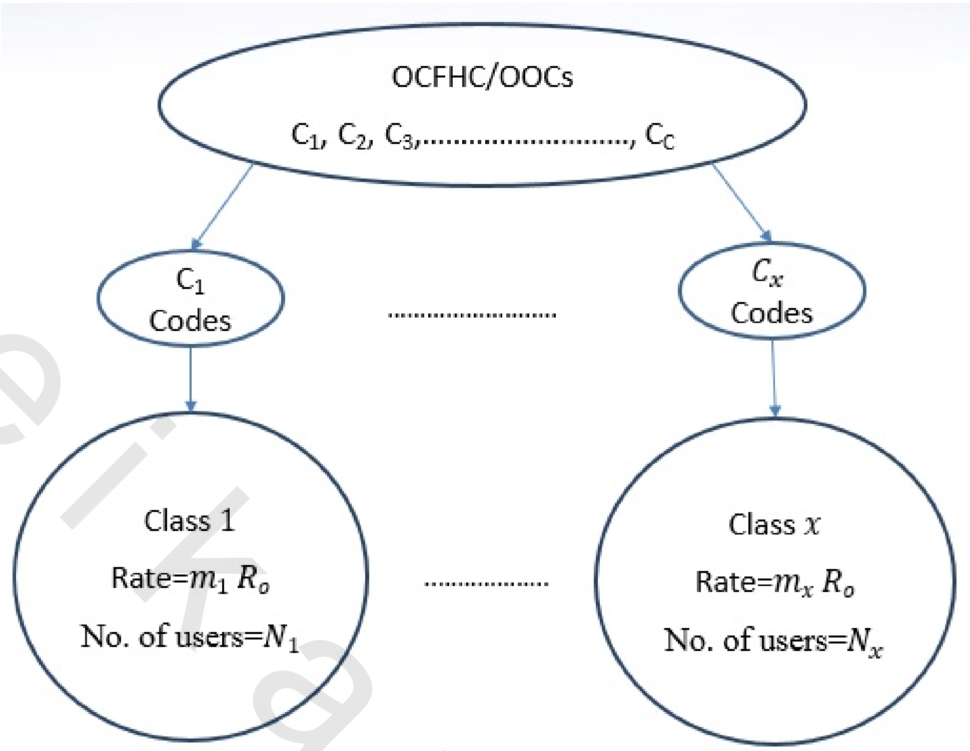

FIGURE 4.3 System Model for General Number of Classes Network.

$$\beta(n_1, n_2, \ldots, n_x) = \begin{cases} \displaystyle\sum_{l_x=0}^{N_x-n_x} \sum_{k_x=0}^{n_x} \cdots\cdots \sum_{l_1=0}^{N_1-n_1} \sum_{k_1=0}^{n_1} \\ \quad \big[\{m_x(k_x+l_x)\} \wedge \mathcal{C}_x + \ldots\ldots + \{m_1(k_1+l_1)\} \wedge \mathcal{C}_1\big] & \text{for Protocol 1.} \\ \quad \times P_s\big[\{m_x(l_x+k_x)\} \wedge \mathcal{C}_x + \ldots\ldots + \{m_1(k_1+l_1)\} \wedge \mathcal{C}_1\big] \\ \quad \times P_{bl}(k_x|n_x)P_{th}(l_x|n_x)\ldots\ldots P_{bl}(k_1|n_1)P_{th}(l_1|n_1); \\ \displaystyle\sum_{l_x=0}^{N_x-n_x} \sum_{k_x=0}^{n_x} \cdots\cdots \sum_{l_1=0}^{N_1-n_1} \sum_{k_1=0}^{n_1} \\ \quad \big[m_x(l_x+k_x) + \ldots\ldots + m_1(k_1+l_1)\big] & \text{for Protocol 2.} \\ \quad \times P_s\big[m_x(l_x+k_x) + \ldots\ldots + m_1(k_1+l_1)\big] \\ \quad \times P_{bl}(k_x|n_x)P_{th}(l_x|n_x)\ldots\ldots P_{bl}(k_1|n_1)P_{th}(l_1|n_1); \end{cases} \tag{4.14}$$

, and

$$D = 1 + \frac{1}{\beta} \cdot E\left\{m_1 n_1 + \ldots + m_x n_x\right\} = \\ 1 + \frac{1}{\beta} \sum_{n_x=0}^{N_x} \ldots \sum_{n_1=0}^{N_1} \left(m_1 n_1 + \ldots + m_x n_x\right) \pi_{n_1}^{1} \ldots \pi_{n_x}^{x}, \tag{4.16}$$

4.3.3 Effect of Noise

4.3.3.1 Poisson Shot-Noise-Limited Photodetectors.

In this section, we study the effect of a photodetectors shot noise on throughput performance of chip-level receivers. The only change in the throughput evaluation as derived in the last two sections is in the calculation of the conditional bit-correct probability $P_{bc}(m, \overline{\ell})$ of equation 3.9.

Assuming a Poisson shot noise at the receivers photodiode, $P_{bc}(m, \overline{\ell})$ is modified as follows. Let the number of photons collected from weighted chip $i \in \mathcal{X} = \{1, 2, \ldots, w\}$ be Y_i. Every Y_i is modeled as a Poisson random variable with mean QZ_i, where Q is the average received photons per pulse. A suboptimal, but good, decision rule is: decide data bit 1 was transmitted for every $i \in \mathcal{X}$, $Y_i > 0$; otherwise decide a data bit 0 was transmitted [3]. Defining $\ell, \overline{\ell} = \{\ell_1, \ell_2, \cdots, \ell_w\}$ as before, we have

$$\begin{aligned} & P_{bc}(m, \overline{\ell}) \\ & = \frac{1}{2} Pr\{a \quad bit \quad success | m, \overline{\ell}, 1 \quad was \quad sent\} \\ & \qquad + \frac{1}{2} Pr\{a \quad bit \quad success | m, \overline{\ell}, 0 \quad was \quad sent\} \\ & = \frac{1}{2} Pr\{Y_i \geq 1 \forall i \quad \in \mathcal{X} | m, \overline{\ell}, 1 \quad was \quad sent\} \\ & \qquad + \frac{1}{2} Pr\{Y_i = 0, some \quad i \in \mathcal{X} | m, \overline{\ell}, 0 \quad was \quad sent\} \\ & = \frac{1}{2} - \frac{1}{2} Pr\{Y_i = 0, some \quad i \in \mathcal{X} | m, \overline{\ell}, 1 \quad was \quad sent\} \\ & \qquad + \frac{1}{2} Pr\{Y_i = 0, some \quad i \in \mathcal{X} | m, \overline{\ell}, 0 \quad was \quad sent\} \end{aligned} \tag{4.17}$$

where the last two probabilities can be evaluated as follows. For $b \in \{0,1\}$

$$
\begin{aligned}
&Pr\{Y_i = 0, some \quad i \in \mathcal{X} | m, \bar{\ell}, bwassent\} \\
&\quad = \sum_{i=1}^{w} Pr\{Y_i = 0 | m, \bar{\ell}, b\} - \sum_{i=1}^{w-1} \sum_{j=i+1}^{w} Pr\{Y_i = Y_j = 0 | m, \bar{\ell}, b\} \\
&\qquad + \sum_{i=1}^{w-2} \sum_{j=i+1}^{w-1} \sum_{k=j+1}^{w} Pr\{Y_i = Y_j = Y_k = 0 | m, \bar{\ell}, b\} + \cdots \\
&\qquad + (-1)^{w-1} Pr\{Y_1 = Y_2 = \cdots = Y_w = 0 | m, \bar{\ell}, b\}
\end{aligned}
\tag{4.18}
$$

and

$$
\begin{aligned}
&Pr\{Y_{i_1} = Y_{i_2} = \cdots = Y_{i_t} = 0 | m, \bar{\ell}, b\} \\
&\quad = \quad e^{-Qbt} \cdot \sum_{u=0}^{m} \binom{m}{u} \left(\frac{1}{2}\right)^{u} \left(\frac{1}{2}\right)^{m-u} e^{-Qut} \\
&\qquad \cdot \sum_{v=0}^{\ell_{i_1}} \binom{\ell_{i_1}}{v} \left(\frac{1}{2}\right)^{v} \left(\frac{1}{2}\right)^{\ell_{i_1}-v} e^{-Qv} \\
&\qquad \cdots \sum_{y=0}^{\ell_{i_t}} \binom{\ell_{i_t}}{y} \left(\frac{1}{2}\right)^{y} \left(\frac{1}{2}\right)^{\ell_{i_t}-y} e^{-Qy} \\
&\quad = e^{-Qbt} \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-Qt}\right)^{m} \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{\ell_{i_1} + \ell_{i_2} + \cdots + \ell_{i_t}}
\end{aligned}
\tag{4.19}
$$

Combining the last three equations, we obtain

$$
\begin{aligned}
&P_{bc}(m, \bar{\ell}) \\
&= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{2} e^{-Q}\right) \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{m} \cdot \sum_{i=1}^{w} \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{\ell_i} \\
&\quad - \left(\frac{1}{2} - \frac{1}{2} e^{-2Q}\right) \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-2Q}\right)^{m} \cdot \sum_{i=1}^{w-1} \sum_{j=i+1}^{w} \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{\ell_i + \ell_j} \\
&\quad + \left(\frac{1}{2} - \frac{1}{2} e^{-3Q}\right) \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-3Q}\right)^{m} \cdot \sum_{i=1}^{w-2} \sum_{j=i+1}^{w-1} \sum_{k=j+1}^{w} \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{\ell_i + \ell_j + \ell_k} \\
&\quad + \cdots \cdots + (-1)^{w-1} \left(\frac{1}{2} - \frac{1}{2} e^{-Qw}\right) \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-Qw}\right)^{m} \cdot \left(\frac{1}{2} + \frac{1}{2} e^{-Q}\right)^{\ell}
\end{aligned}
\tag{4.20}
$$

Here, Q is the average received photons per pulse, which is relevant to the average photons/bit μ by $Q = \frac{2\mu}{w}$ as in [3]. Noting that as $Q \to \infty$ Eq. (4.20) converges to Eq. (3.9).

4.3.3.2 Thermal-Noise-Limited Case.

In this section, we study the effect of thermal noise on throughput performance of chip-level receivers. The only change in the throughput evaluation is in the calculation of the conditional bit-correct probability $P_{bc}(m, \overline{\ell})$ as before. Assuming a thermal noise at the receiver,$P_{bc}(m, \overline{\ell})$ is modified as follows.
Let the number of photons collected per marked chip positions $i \in \mathcal{X} = \{1, 2, \ldots, w\}$ be Y_i. Every Y_i is modeled as a Gaussian distribution, and the decision threshold θ. Considering u users out of m users interfering in w chips and v_i users out of ℓ_i users making interference at the weighted chip i, the conditional mean and variance m_{bi} and σ_{bi}^2, respectively [46], are expressed as follows:

$$m_{bi} = G_{APD}((u + b + v_i)Q + Q_d), \quad \sigma_{bi}^2 = FGm_{bi} + \sigma_n^2 \tag{4.21}$$

where $b \in \{0, 1\}$ is the data bit, G_{APD} denotes the average APD gain, Q and Q_d are the average number of absorbed photons per received single-user pulse and the photon count due to the APD dark current within a chip interval T_c, respectively, and are given by [36]:

$$Q = \frac{RP_{av}T}{ew}, \quad Q_d = \frac{I_d T_c}{e} \tag{4.22}$$

where P_{av} is the received average peak laser power (of a single user), T is the bit duration, R is the APD responsivity at unity gain, I_d is the APD dark current, and $e = 1.6 \times 10^{-19} C$ is the magnitude of the electron charge. The variance of the thermal noise within a chip interval σ_n^2 is given by [36]:

$$\sigma_n^2 = \frac{2k_B T^o T_c}{e^2 R_L} \tag{4.23}$$

where $k_B = 1.38 \times 10^{-23} J/K$ is Boltzmann's constant, T^o is the receiver noise temperature, and R_L is the receiver load resistor. Defining k_{eff} as the APD effective ionization ratio, the APD excess noise factor F can be written as

$$F = k_{eff} G_{APD} + (2 - \frac{1}{G_{APD}})(1 - k_{eff}) \tag{4.24}$$

then, $P_{bc}(m,\overline{\ell})$ will be:

$$\begin{aligned}
&P_{bc}(m,\overline{\ell})\\
&\quad=\frac{1}{2}Pr\{a \quad bit \quad success|m,\overline{\ell},1 \quad was \quad sent\}\\
&\qquad\quad+\frac{1}{2}Pr\{a \quad bit \quad success|m,\overline{\ell},0 \quad was \quad sent\}\\
&\quad=\frac{1}{2}Pr\{Y_i \geq \theta foralli \in \mathcal{X}|m,\overline{\ell},1 \quad was \quad sent\}\\
&\qquad\quad+\frac{1}{2}Pr\{Y_i < \theta forsomei \in \mathcal{X}|m,\overline{\ell},0 \quad was \quad sent\}\\
&\quad=\frac{1}{2}-\frac{1}{2}Pr\{Y_i < \theta forsomei \in \mathcal{X}|m,\overline{\ell},1 \quad was \quad sent\}\\
&\qquad\quad+\frac{1}{2}Pr\{Y_i < \theta forsomei \in \mathcal{X}|m,\bar{l},0 \quad was \quad sent\}\\
&\quad=\frac{1}{2}+\frac{1}{2}\sum_{b=0}^{1}(-1)^b * Pr\{Y_i < \theta \quad forsomei \in \mathcal{X}|m,\overline{\ell},b\}\\
&\quad=\frac{1}{2}-\frac{1}{2}\sum_{b=0}^{1}(-1)^b \sum_{i=1}^{w}(-1)^i \begin{pmatrix} w \\ i \end{pmatrix} Pr\{Y_1,Y_2,\cdots,Y_w < \theta|m,\overline{\ell},b\}
\end{aligned} \tag{4.25}$$

The last probability can be expressed as follows:

$$\begin{aligned}
&Pr\{Y_1,Y_2,\cdots,Y_i < \theta|m,\overline{\ell},b\} =\\
&\sum_{u=0}^{m}\begin{pmatrix} m \\ u \end{pmatrix}\left(\frac{1}{2}\right)^m \cdot \sum_{v_1}^{\ell_1}\begin{pmatrix} \ell_1 \\ v_1 \end{pmatrix}\left(\frac{1}{2}\right)^{\ell_1} \cdots \sum_{v_i}^{\ell_i}\begin{pmatrix} \ell_i \\ v_i \end{pmatrix}\left(\frac{1}{2}\right)^{\ell_i}
\end{aligned} \tag{4.26}$$

then,

$$\begin{aligned}
&Pr\{Y_1 < \theta|u,v_1,b\}Pr\{Y_2 < \theta|u,v_2,b\}\cdots Pr\{Y_i < \theta|u,v_i,b\}\\
&=\sum_{u=0}^{m}\begin{pmatrix} m \\ u \end{pmatrix}\left(\frac{1}{2}\right)^m \cdot \prod_{j=1}^{i}\sum_{v_j}^{\ell_j}\begin{pmatrix} \ell_j \\ v_j \end{pmatrix}\left(\frac{1}{2}\right)^{\ell_j} \cdot Q\left(\frac{m_{bj}-\theta}{\sigma_{bj}}\right)
\end{aligned} \tag{4.27}$$

where $Q(x)$ is the normalized Gaussian tail probability.

$$Q(x)=\frac{1}{\sqrt{2\Pi}}\int_x^{\infty} e^{-s^2/2}ds \tag{4.28}$$

Combining the last equations, one can get an expression for the bit correct probability as follows:

$$\begin{aligned}
P_{bc}(m,\bar{\ell},b) &= \frac{1}{2}-\frac{1}{2}\sum_{b=0}^{1}(-1)^b\sum_{i=1}^{w}(-1)^i\begin{pmatrix} w \\ i \end{pmatrix}\\
&\times\sum_{u=0}^{m}\begin{pmatrix} m \\ u \end{pmatrix}\left(\frac{1}{2}\right)^m \times \prod_{j=1}^{i}\sum_{v_j=0}^{\ell_j}\begin{pmatrix} \ell_j \\ v_j \end{pmatrix}\left(\frac{1}{2}\right)^{\ell_j} Q\left(\frac{m_{bj}-\theta}{\sigma_{bj}}\right)
\end{aligned} \tag{4.29}$$

4.4 CONCLUSIONS

In this chapter, two random-access protocols have been proposed for multi-rate OCDMA networks. Multi-coding technique is utilized to support different-classes (or different rates) in the network. Steady state system throughput and average packet delay have been derived for two cases ; namely, two class network and general number of classes in presence of MAI. Then, the relations in case of shot noise and thermal noise separately are derived.