CHAPTER (3)

ANALYTICAL STUDY

3.1 Introduction

Stress-strain or constitutive relations for composite tube will be treated first, and in particular the behavior of woven-roving glass fiber reinforced epoxy (GFRE) with different fiber orientations and subjected to different loads. Then, using transformation matrix, the expressions for changing axis from global direction (*x-y*) to local direction (*1-2*) in terms of stresses and strains will be given in detail. The latter includes theory of multilayer thick-walled tubes, the hoop stress and strain for each layer of three-layer composite tube will be derived with two different layers manufacturing method.

3.2 The Global Stress State

The case studied consists of completely reversed pure bending, completely reversed pure torsion, pure static internal pressure or a combined completely reversed bending moment and internal pressure with different internal pressure values acting on tubular specimens made of woven- roving glass fiber reinforced epoxy with two fiber orientations $[0,90°]_{3s}$ and $[\pm45°]_{3s}$.

Global stress resulted from bending moment, torsional moment and internal pressure is shown in the figure 3.1, there global stresses (σ_x), (σ_y) and (τ_{xy}) may be found from equation (3.1):

$$\left.\begin{aligned} \sigma_x &= \sigma_b + \sigma_l = \frac{My}{I} + \sigma_l \\ \sigma_y &= \sigma_H \\ \tau_{xy} &= \frac{Tr}{J} \end{aligned}\right\} \qquad (3.1)$$

Where:

M : applied bending moment ($M = M_m + M_a \sin(\omega t)$).

M_m and M_a: mean and amplitude bending moments, respectively.

I: second moment of area for tube; $I = \frac{\pi}{64}(d_o^4 - d_i^4)$.

T : applied Torque ($T = T_m + T_a \sin(\omega t)$).

T_m and T_a: mean and amplitude Torques, respectively.

J: Polar second moment of area for tube; $J = \frac{\pi}{32}(d_o^4 - d_i^4)$.

σ_l: Longitudinal stress (MPa),

$\left(\sigma_l = \frac{P_i d_i}{4t}\right)$ for thin tube and $\left(\sigma_l = \frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2}\right)$for thick tube.

σ_H: Hoop stress (MPa),

$\left(\sigma_H = \frac{P_i d_i}{2t}\right)$ for thin tube and $\left(\sigma_H = \frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2}\left(1 + \frac{{r_o}^2}{r^2}\right)\right)$for thick tube.

P_i : Internal pressure.

t : Tube total thickness $t = \frac{d_o - d_i}{2}$

d_o and d_i : Outer and inner diameters of the specimen, respectively.

$y = r = \frac{d_o}{2}$

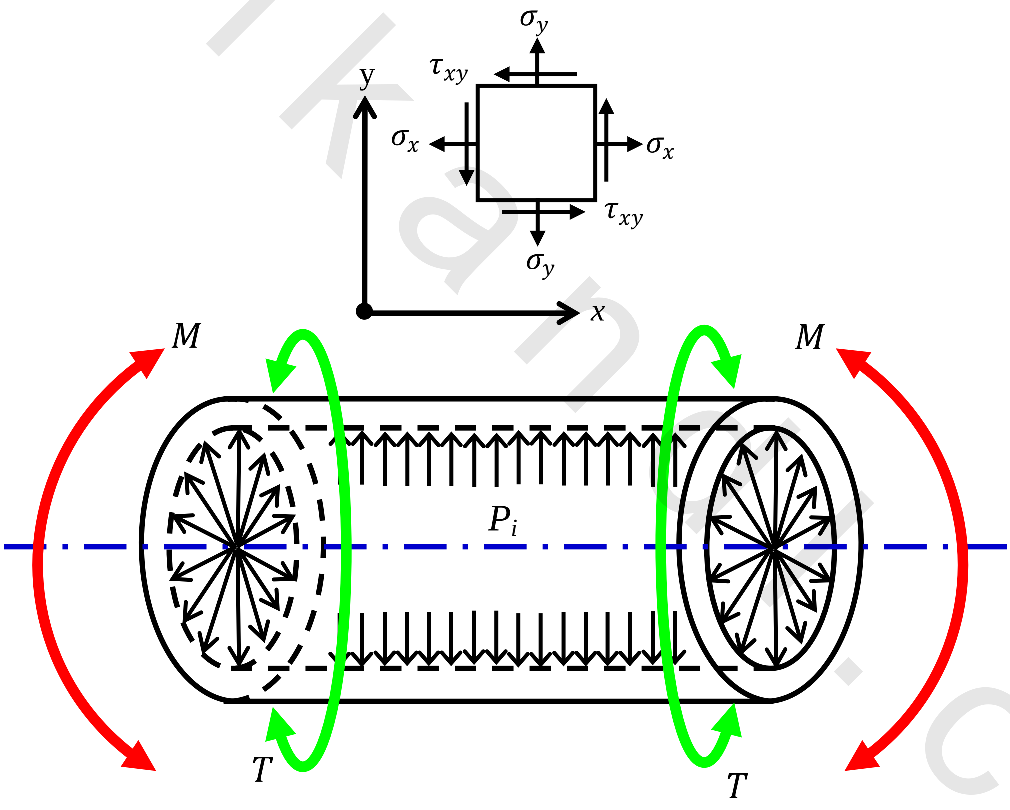


Figure 3.1 The Global Stress State

Then,

$$\left.\begin{aligned} \sigma_x &= \sigma_l + \left(\frac{M_m y}{I} + \frac{M_a y}{I}\sin(\omega t)\right) \\ \sigma_y &= \sigma_H \\ \tau_{xy} &= \frac{T_m r}{J} + \frac{T_a r}{J}\sin(\omega t) \end{aligned}\right\} \quad (3.2)$$

Or

$$\left.\begin{aligned}\sigma_x &= E + C + A\sin(\omega t)\\ \sigma_y &= F\\ \tau_{xy} &= D + B\sin(\omega t)\end{aligned}\right\} \quad (3.3)$$

Where:

$$A = \frac{M_a y}{I},\ B = \frac{T_a r}{J},\ C = \frac{M_m y}{I},\ D = \frac{T_m r}{J},\ E = \sigma_l \ \ and\ \ F = \sigma_H \quad .$$

Since, the experimental data are to be presented using maximum normal stress (σ_{max}), minimum normal stress (σ_{min}) and the stress ratio (R).

The stress ratio R is defined as: $= \frac{\sigma_{min}}{\sigma_{max}} = \frac{\tau_{min}}{\tau_{max}}$.

Where:

$$\left.\begin{aligned}\sigma_{max} &= \sigma_m + \sigma_a\\ \sigma_{min} &= \sigma_m - \sigma_a\end{aligned}\right\} \quad (3.4)$$

Then, $\sigma_m = \frac{(1+R)}{(1-R)}\sigma_a$ and $\tau_m = \frac{(1+R)}{(1-R)}\tau_a$

Equations (3.3) can be expressed for fluctuating cyclic loading as given into the following equations:

$$\left.\begin{aligned}\sigma_x &= E + A\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\\ \sigma_y &= F\\ \tau_{xy} &= B\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\end{aligned}\right\} \quad (3.5)$$

3.3 The local Stress State

It is important to note that the damage mechanisms responsible for fatigue failure occur locally and are driven by the local stress fields. The same globally applied multiaxial loads will generally produce different local stress states depending on the configuration. Thus, fatigue failure criteria formulated on the basis of global stresses have little chance of succeeding in predicting fatigue life for other than the particular laminate under consideration. The only rational way forward for developing predictive criteria of general validity is to base these on systematic studies of damage mechanisms **[88]**.

Consider an isolated infinitesimal element in the local material coordinate system (*1-2* system) that will be transformed into the *x-y* global coordinate system as shown in Figure 3.2. The fibers are oriented at angle θ with respect to the +*x-axis* of the global system. The fibers are parallel to the *x-y plane*. The orientation angle θ will be considered positive when the fibers rotate counterclockwise from the *+x-axis* toward the *+y-axis*.

Therefore, the applied global stresses are to be transformed to their corresponding local stress components for the fiber orientations, *[0,90°]* and *[±45°]* specimens.

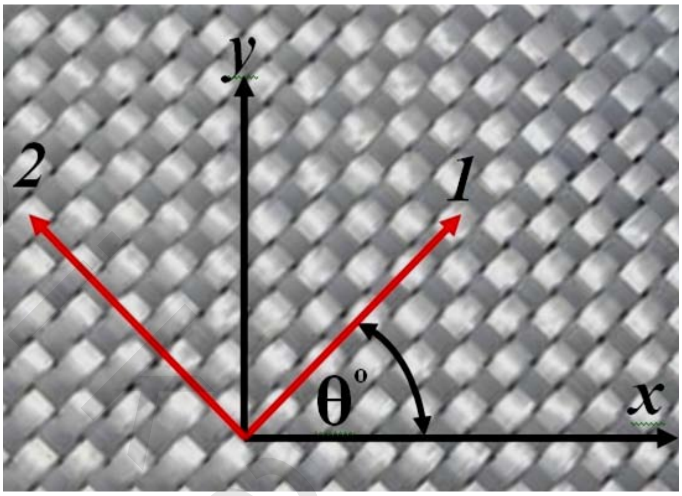


Figure 3.2 the global (*x-y*) and local (*1-2*) coordinate system for specimen

This is to be done for one layer using the transformation matrix given into the following equation:

$$\begin{bmatrix}\sigma_1\\\sigma_2\\\sigma_6\end{bmatrix}=\begin{bmatrix}cos^2(\theta) & sin^2(\theta) & 2\,sin(\theta)\,cos(\theta)\\ sin^2(\theta) & cos^2(\theta) & -2\,sin(\theta)\,cos(\theta)\\ -\,sin(\theta)\,cos(\theta) & sin(\theta)\,cos(\theta) & cos^2(\theta)-sin^2(\theta)\end{bmatrix}\begin{bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{bmatrix} \quad (3.6)$$

Where:

(σ_x) *and* (σ_y) : Global normal stress components in *x- direction* and *y- direction*, respectively,

(τ_{xy}) : Global shear stress components in *plane (x-y)*,

(θ) : Angle measured from direction (*x*) to direction (*1*).

The local stresses components are given in the following equations:

$$\left.\begin{aligned}\sigma_1 &= \sigma_x\,cos^2(\theta)+\sigma_y\,sin^2(\theta)+2\tau_{xy}\,sin(\theta)\,cos(\theta)\\ \sigma_2 &= \sigma_x\,sin^2(\theta)+\sigma_y\,cos^2(\theta)-2\tau_{xy}\,sin(\theta)\,cos(\theta)\\ \sigma_6 &= -\sigma_x\,sin(\theta)\,cos(\theta)+\sigma_y\,sin(\theta)\,cos(\theta)+\tau_{xy}\,(cos^2(\theta)-sin^2(\theta))\end{aligned}\right\} \quad (3.7)$$

Substituting from equation (3.5) into equation (3.7), we can get:

$$\left.\begin{array}{l}\sigma_1 = \left[E + A\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] cos^2(\theta) + F\, sin^2(\theta) + 2\left[B\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] sin(\theta)\, cos(\theta) \\ \sigma_2 = \left[E + A\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] sin^2(\theta) + F\, cos^2(\theta) - 2\left[B\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] sin(\theta)\, cos(\theta) \\ \sigma_6 = -\left[E + A\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] sin(\theta)\, cos(\theta) + F\, sin(\theta)\, cos(\theta) + \left[B\left(\frac{(1+R)}{(1-R)} + \sin(\omega t)\right)\right] (cos^2(\theta) - sin^2(\theta)) \end{array}\right\} \quad (3.8)$$

3.3.1 The *[0,90°]₃ₛ* specimens

Substituting $\theta = 0°$ into equation (3.9), we will find that the *$[0,90°]_{3s}$* specimen will have the following local stress state:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}$$

$$\left.\begin{array}{l}\sigma_1 = \sigma_x\\ \sigma_2 = \sigma_y\\ \sigma_6 = \tau_{xy}\end{array}\right\} \quad (3.10)$$

3.3.2 The [±45°]3s specimens

Substituting $\theta = +45°$ in equation (3.9), we will find that the *$[\pm 45°]_{3s}$* specimen will have the following local stress state:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{bmatrix} = \begin{bmatrix}0.5 & 0.5 & -1\\ 0.5 & 0.5 & 1\\ 0.5 & -0.5 & 0\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}$$

$$\left.\begin{array}{l}\sigma_1 = 0.5\sigma_x + 0.5\sigma_y - \tau_{xy}\\ \sigma_2 = 0.5\sigma_x + 0.5\sigma_y + \tau_{xy}\\ \sigma_6 = 0.5\sigma_x - 0.5\sigma_y\end{array}\right\} \quad (3.11)$$

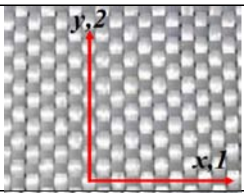
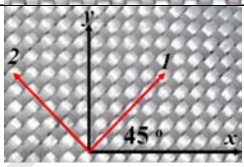
3.3.3 The local stresses for completely reversed pure bending or completely reversed pure torsion moment

For completely reversed pure bending (R= -1), according to equation (3.5), we can get:

$$\left.\begin{array}{l}\sigma_x = A = \frac{M_a y}{I}\\ \sigma_y = 0\\ \tau_{xy} = 0\end{array}\right\} \quad (3.12)$$

And substituting into equations (3.10) and (3.11), the local stresses for fiber orientations, *[0,90°]*and *[±45°]* are as given in Table 3.1.

Table 3.1: The local stresses for completely reversed pure bending

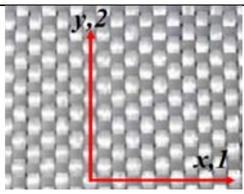
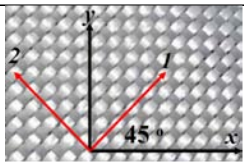
load	The stress equations					
Pure bending (Completely reversed)	Globa	σ_x	A			
		σ_y	0			
		τ_{xy}	0			
	local	Fiber orientation	*[0, 90]*		σ_1	A
					σ_2	0
					σ_6	0
			[±45]		σ_1	$(1/2)A$
					σ_2	$(1/2)A$
					σ_6	$(1/2)A$

For completely reversed pure torsion (R= -1), according to equation (3.5), we can get:

$$\left.\begin{aligned} \sigma_x &= 0 \\ \sigma_y &= 0 \\ \tau_{xy} &= B = \frac{T_a y}{I} \end{aligned}\right\} \qquad (3.13)$$

And substituting into equations (3.10) and (3.11), the local stresses for fiber orientations, *[0,90°]*and *[±45°]* are as given in Table 3.2.

Table 3.2: The local stresses for completely reversed pure torsion

load	The stress equations					
Pure torsion (Completely reversed)	Global	σ_x	0			
		σ_y	0			
		τ_{xy}	B			
	local	Fiber orientation	*[0, 90]*		σ_1	0
					σ_2	0
					σ_6	B
			[±45]		σ_1	B
					σ_2	$-B$
					σ_6	0

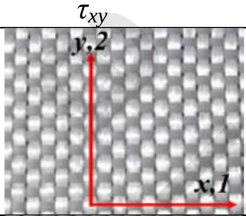
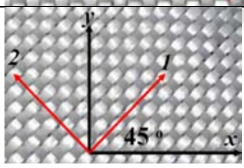
3.3.4 The local stresses for pure internal pressure (open and closed cylinder)

For internal pressure (open or closed cylinder), according to equation (3.5), we get:

$$\left.\begin{aligned} \sigma_x &= E = \sigma_l \\ \sigma_y &= F = \sigma_H \\ \tau_{xy} &= 0 \end{aligned}\right\} \qquad (3.14)$$

And substituting into equations (3.10) and (3.11), the local stresses for fiber orientations, *[0,90°]* and *[±45°]* are as given in Table 3.3.

Table 3.3: The local stresses for internal pressure only

load	The stress equations					
					closed cylinder	**open cylinder**
Internal Pressure stress only	**Global**	σ_x			$E = \sigma_l$	0
		σ_y			$F = \sigma_H$	$F = \sigma_H$
		τ_{xy}			0	0
	local	**Fiber orientation** *[0, 90]*		σ_1	E	0
				σ_2	F	F
				σ_6	0	0
		[±45]		σ_1	$(1/2)(E+F)$	$(1/2)F$
				σ_2	$(1/2)(E+F)$	$(1/2)F$
				σ_6	$(1/2)(E-F)$	$-(1/2)F$

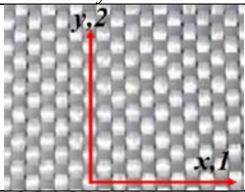
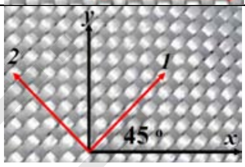
3.3.5 The local stresses for combined completely reversed bending plus internal pressure (open or closed cylinder)

For combined completely reversed bending plus internal pressure (open or closed cylinder), according to equation (3.5), we can get:

$$\left.\begin{aligned} \sigma_x &= E + A \\ \sigma_y &= F \\ \tau_{xy} &= 0 \end{aligned}\right\} \qquad (3.15)$$

And substituting into equations (3.10) and (3.11), the local stresses for fiber orientations, *[0,90°]* and *[±45°]* are as given in Table 3.4.

Table 3.4: The local stresses for combined completely reversed bending plus internal pressure (open and closed cylinder)

<table>
<tr><td>load</td><td colspan="7">The stress equations</td></tr>
<tr><td></td><td colspan="5"></td><td>closed cylinder</td><td>open cylinder</td></tr>
<tr><td rowspan="9">combined completely reversed bending plus internal pressure</td><td rowspan="3">Global</td><td colspan="4">σ_x</td><td>$E + A$</td><td>A</td></tr>
<tr><td colspan="4">σ_y</td><td>F</td><td>F</td></tr>
<tr><td colspan="4">τ_{xy}</td><td>0</td><td>0</td></tr>
<tr><td rowspan="6">local</td><td rowspan="6">Fiber orientation</td><td rowspan="3">[0, 90]</td><td rowspan="3">
</td><td>σ_1</td><td>$E + A$</td><td>A</td></tr>
<tr><td>σ_2</td><td>F</td><td>F</td></tr>
<tr><td>σ_6</td><td>0</td><td>0</td></tr>
<tr><td rowspan="3">[±45]</td><td rowspan="3">
</td><td>σ_1</td><td>$(1/2)(E + A + F)$</td><td>$(1/2)(A + F)$</td></tr>
<tr><td>σ_2</td><td>$(1/2)(E + A + F)$</td><td>$(1/2)(A + F)$</td></tr>
<tr><td>σ_6</td><td>$(1/2)(E + A - F)$</td><td>$(1/2)(A - F)$</td></tr>
</table>

3.4 Stress distribution in multilayer thick-walled tube

The geometrical model of the three layer thick tube in this study is made of woven-roving glass fiber reinforced epoxy (GFRE) with two different fiber orientations $[0,90°]_{3s}$ and $[\pm45°]_{3s}$, and two different manufacturing methods will be applied to prepare the test specimens. In the first method (old method) M_1, the test specimens will be prepared by molding all layers around the mandrel in one step then the epoxy will be poured-on-it, then leave it to cure, and the new method M_2, the test specimens will be prepared by molding first layer only around the mandrel then the epoxy resin will be poured -on-it, then leave to cure, and repeat previous steps for the second layer and so on (i.e. as compound tubes). The two different manufacturing methods will be discussed clearly in the next chapter, the inner surface with radius r_i is subjected to internal pressure P_i, the geometrical model and the stress distribution is represented in Figure 3.3.

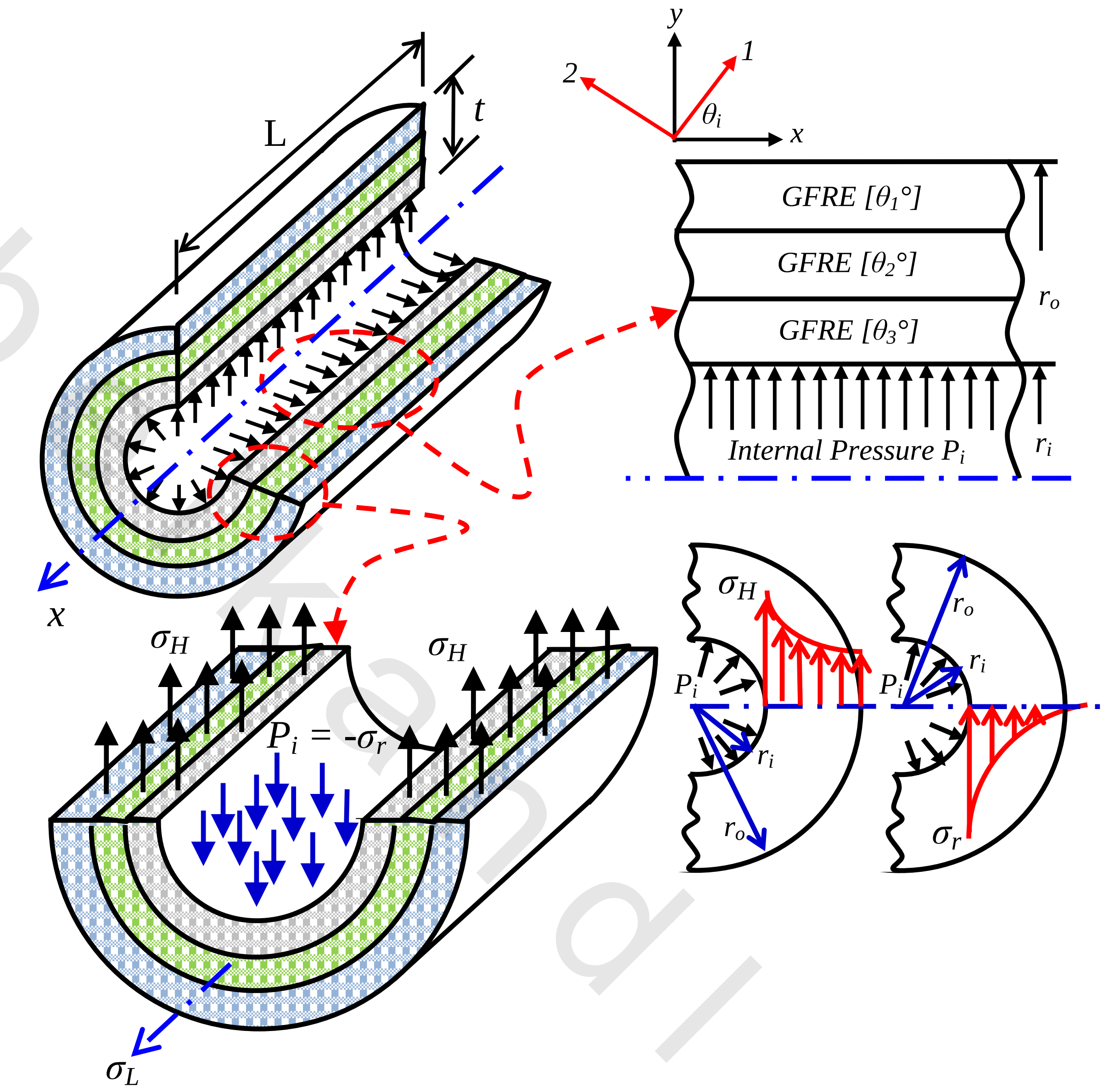


Figure 3.3 Geometrical model and the stress distribution in a thick-walled tube subjected to internal pressure.

3.4.1 Stress Calculations of the First Method of Manufacturing

Consider a three-layer tube mould with the first method of manufacturing, M_1 under internal hydrostatic pressure, P_i, Figure 3.4. The internal radius of the tube is designated by r_i and the external radius of the tube is designated by r_o. The material of each layer is assumed to be linearly elastic and isotropic. The elasticity material parameters of all layers are designated by modulus of elasticity E and Poisson's ratio v respectively.

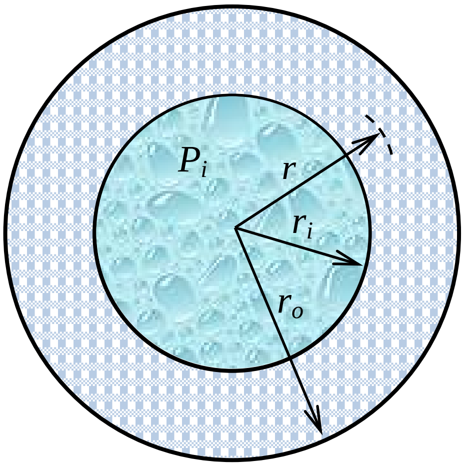


Figure 3.4.A three-layer tube mould with the first method of manufacturing M_1

The stress conditions present in the tubes will be discussed based on loading conditions and linear elastic response. The hoop stress σ_H is not constant through the thickness in these tubes. It has a maximum tensile value at the inside surface of the tube and a minimum value on the outside surface. In addition, the radial stress σ_r, varies through the thickness with the maximum compressive stress on the inside surface equal to the internal pressure and the minimum value equal to zero on the outside surface. The longitudinal stress σ_L does not appear in tubes condition (i.e. open cylinder). It has to be determined according to theory of thick-walled cylinders, see appendix (4).

The final elastic stresses σ_H, σ_r and σ_L are given in the following equations:

$$\sigma_H = \frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2}\left(1 + \frac{{r_o}^2}{r^2}\right), \tag{3.16}$$

$$\sigma_r = \frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2}\left(1 - \frac{{r_o}^2}{r^2}\right), \tag{3.17}$$

$$\sigma_L = \frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2} \qquad \textit{for closed cylinder} \tag{3.18}$$

$$\sigma_L = 0 \qquad \textit{for open cylinder} \tag{3.19}$$

$$\delta = \frac{(1-\upsilon)}{E}\left(\frac{P_i {r_i}^2}{{r_o}^2 - {r_i}^2}\right) r + \frac{(1+\upsilon)}{E}\left(\frac{P_i {r_i}^2 {r_o}^2}{r({r_o}^2 - {r_i}^2)}\right) \tag{3.20}$$

3.4.2 Stress Calculations of the Second Method of Manufacturing

Consider a three-layer tube made of three bonded layers (mould with the first method of manufacturing, M_2) under internal hydrostatic pressure, P_i, Figure 3.5. The internal radius of consequence layers are designated by a, b, and c respectively, while the external radius of the external layer is designated by *d*. The material of each layer is assumed to be linearly elastic and isotropic. The elastic parameters of the internal, the middle and the external layers are designated by E_1, v_1 , E_2, v_2 and E_3, v_3, respectively.

Where:

E_i is the modulus of elasticity,

v_i is Poisson's ratio ,

i = 1,2,3.

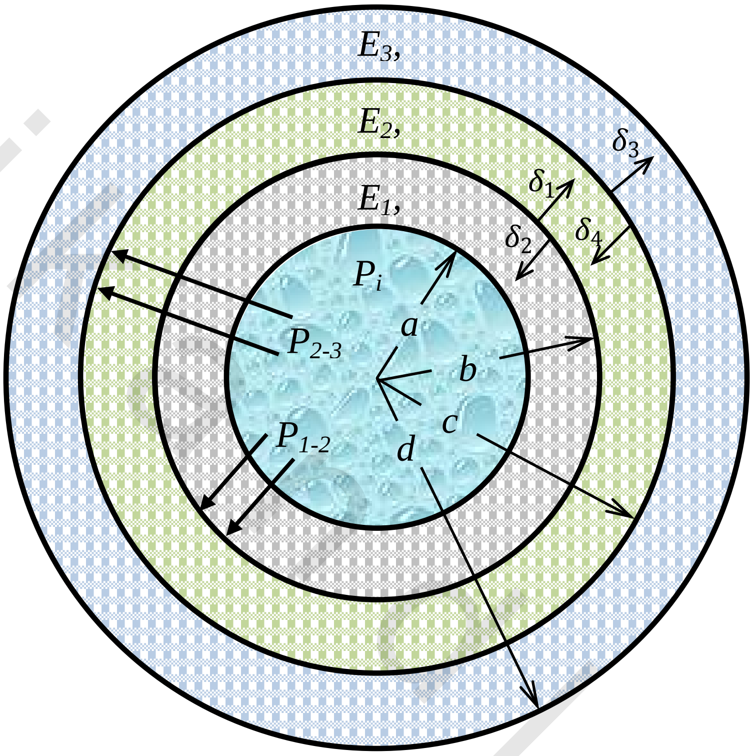


Figure 3.5.A three-layer tube made of three bonded layers.

The outside radius *b* of the internal layer was larger than the inside radius of the middle layer by an amount of $\delta_{1\text{-}2}$, creating the interface pressure, $P_{1\text{-}2}$, between the internal layer and the middle layer after assembly. Also the outside radius *c* of the middle layer was larger than the inside radius of the external layer by an amount $\delta_{2\text{-}3}$, creating the interface pressure $P_{2\text{-}3}$ between the middle layer and the external layer after assembly.

Due to the action of the internal pressure P_i, interaction of the three layers an interface radial stresses $P_{1\text{-}2}$ and $P_{2\text{-}3}$ are produced. Using the classical elasticity theory of multilayer thick-walled cylinders, see appendix (4), the interface pressure can be obtained by the following relations:

For the same material of layers $E_1 = E_2 = E_3 = E$ and $\upsilon_1 = \upsilon_2 = \upsilon_3 = \upsilon$

$$P_{1-2} = \frac{\frac{2P}{(R_1{}^2-1)}}{\left[\frac{[(1-\upsilon)+R_2{}^2(1+\upsilon)]}{(R_2{}^2-1)}+\frac{[(1+\upsilon)+R_1{}^2(1-\upsilon)]}{(R_1{}^2-1)}-\frac{4R_2{}^2}{(R_2{}^2-1)^2\left[\frac{(R_3{}^2+1)}{(R_3{}^2-1)}+\frac{(R_2{}^2+1)}{(R_2{}^2-1)}\right]}\right]} \tag{3.21}$$

$$P_{2-3} = \frac{\frac{4P}{(R_1{}^2-1)(R_2{}^2-1)}}{\left(\frac{(R_3{}^2+1)}{(R_3{}^2-1)}+\frac{(R_2{}^2+1)}{(R_2{}^2-1)}\right)\left(\frac{[(1-\upsilon)+R_2{}^2(1+\upsilon)]}{(R_2{}^2-1)}+\frac{[(1+\upsilon)+R_1{}^2(1-\upsilon)]}{(R_1{}^2-1)}\right)-\frac{4R_2{}^2}{(R_2{}^2-1)^2}} \tag{3.22}$$

Where: $R_1 = \frac{b}{a}$, $R_2 = \frac{C}{b}$ and $R_3 = \frac{d}{C}$

The hoop stresses in the internal, middle and external layers can be obtained by the following relations:

$$\sigma_{H1} = \frac{P}{R_1{}^2-1}\left(1+\frac{b^2}{r^2}\right) - \frac{P_{1-2}R_1{}^2}{R_1{}^2-1}\left(1+\frac{a^2}{r^2}\right) \tag{3.23}$$

$$\sigma_{H2} = \frac{P_{1-2}}{R_2{}^2-1}\left(1+\frac{c^2}{r^2}\right) - \frac{P_{2-3}R_2{}^2}{R_2{}^2-1}\left(1+\frac{b^2}{r^2}\right) \tag{3.24}$$

$$\sigma_{H3} = \frac{P_{2-3}}{R_3{}^2-1}\left(1+\frac{d^2}{r^2}\right) \tag{3.25}$$

Where: σ_{H1}, σ_{H2} and σ_{H3} are hoop stresses in the internal, middle and external layers respectively.