ON THE NORMAL IMPACT ON A FLEXIBLE MEMBRANE

By

AHMED GALAL EL-SAKKA

Faculty of Science, Ain Shams University

Abstract

In this work the problem of normal impact of a sphere on a flexible membrane is studied. The method of characteristics for solving the obtained differential equation is used. It is found that the obtained results for a small interval of time agree with those obtained by Pavlinko (1) using an approximate analytical formula for the normal impact on a flexible membrane.

Introduction

The problem of normal impact on a flexible membrane by a cone was studied by many authors (2) with assumption that the velocity of the point of break of membrane b is less than the velocity of propagation of the longitudinal wave. Consequently, two fields of motion will directly appeear, these are the field of pure radial motion and the field of meridional motion. In this work the problem of transverse impact on a flexible plate by a sphere is considered with the assumption that the sphere moves with a constant velocity V_o, where the velocity of the wave of high discontinuity b exceeds the velocity of sound in the material, i.e. meridional motion only appeears first and then radial motion.

Mathematical Procedure

Assume that a flexible plate with infinite extent falls on a sphere, such that the surface of thet sphere remains in contact with the plate after impact. Let u be the distance of an element of the plate in the vicinity of the origin 0, where the velocity of particle is zero and denote the angle between the axis O′X and the tangent to the sphere at the considered point by γ (Fig. 1) which is a known function of time for the impacting body.

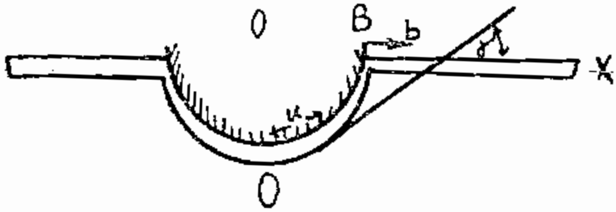

Fig. (1)

Meridional and peripheral deformations of the element of the plate respectively will be : (1)

$$\varepsilon_r = \frac{\partial u}{\partial r} - 1 ,$$

$$\varepsilon_\theta = \frac{x(u)}{r} - ,$$

where r is he lagrangian coordinate of the element. If we consider the equilibrium of the element of the plate under the action of inertia forces, meridional and peripheral stresses σ_r and σ_θ , then we get the following equation of motion (1) :

$$\rho_o \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta \cos \gamma}{r} ,$$

which in the case of linear relation between stresses and strains given by the well known formulae :

$$\sigma_r = \frac{E}{1 - \nu^2} (\varepsilon_r + \nu \varepsilon_\theta),$$

$$\sigma_\theta = \frac{E}{1 - \nu^2} (\varepsilon_\theta + \nu \varepsilon_r),$$

will have the form :

$$\frac{1}{a_o^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{x(u) \cos \nu}{r^2} - \frac{(1 + \nu)(1 - \cos \nu)}{r}$$

where : $a_o^2 = \frac{E}{\rho_o (1 - \nu^2)}$,

Po is the initial density of the plate. The point of break B (Fig. 1) moves with some velocity b (t) along the plate and it represents the first front of the propagating cylindrical wave of high discontinuity. In the environment of this point we have following relation (1) :

$$b = \frac{V_o}{\tan \nu} ,$$

Where γ is defined as before

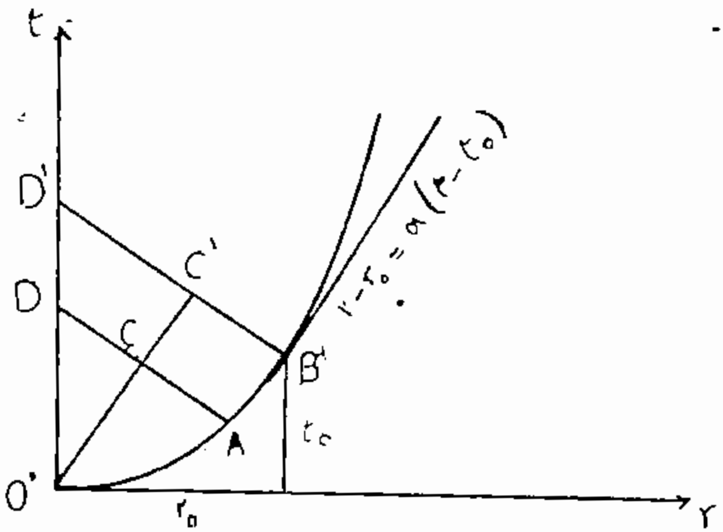

Fig. (2)

Because of the convexity of the impacting body the angle γ increases with time but b decreases, we have $b = \infty$ when $\gamma = 0$. Thus within some time t_0 the velocity of the wave of high discontinuity b remains greater than the velocity of sound a_0 in the material.

If the relation $\gamma = \gamma(t)$ is known, then t_o can be found from the relation (1) :

$$\nu (t_o) = \tan^{-1} \frac{V_o}{a_o} \qquad (2)$$

The picture of motion in the plane r,t is shown on figure (2) where the curve O′B′ representing the wave of high discontinuity depends on the shape of the impacting body. In the case of sphere it will be an ellipse of the form :

$$\frac{\left(t - \frac{R}{V_o} \right)^2}{\frac{R^2}{V_o^2}} + \frac{r^2}{R^2} = 1$$

whose centre is $(0, \frac{R}{V_o})$ and semi-axes are $\frac{R}{V_o}$ and R, where R is the radius of the sphere.

Using relation (2) for our case we have :

$$t_0 = \frac{R}{V_0} - \frac{R}{V_0} \sqrt{\frac{a_o^2}{a_o^2 + V_o^2}}$$

It is clear that the time delay of pure radial motion is proportional to the radius of the sphere, but it is inversely proportional to the velocity of impact.

Taking into consideration that on the wave of high discontinuity acts the concentrated force Q, we can write the conditions on it :

$$\left.\begin{aligned} b u_r &= \frac{b}{\cos \nu} - u_t \ , \\ \rho_0 \, b \, (V_0 \sin \nu - u_t) &= \sigma_r \\ \rho_0 \, b \, V_0 \cos \nu &= - Q \end{aligned}\right\} \qquad (3)$$

Solving first two equations we find that :

$$u_r = \frac{\cos \nu - \lambda^2}{1 - \lambda^2} \qquad (4)$$

$$u_t = \frac{V_0 \left[\sin^2 \nu - \lambda^2 (1 - \cos \nu) \right]}{\sin \nu (1 - \lambda^2)} \qquad (5)$$

$$u = \int_0^r \frac{dr}{\cos \nu} \ , \quad \lambda^2 = \frac{a_o^2}{b^2}$$

The third equation of (3) can be used to find Q. The condition for the plate to leave the surface of the impacting body is $Q = 0$, but $Q \neq 0$ until the moment when $b > a_0$. If suppose that $Q = 0$ when $b > a_0$, then equation (3) gives that $\cos \gamma = 0$ and then $\gamma = \frac{\pi}{2}$, but this is contrary with the condition that $b > a_0 \neq 0$, since $b = 0$ when $\gamma = \frac{\pi}{2}$.

Consequently, tearing off of the plate from the impacting body may happen after appearance of pure radial motion.

From equation (4) we find that $u \longrightarrow \infty$ when $b \longrightarrow a_0$.

The point A on the curve O' B' which represents the beginning of plastic deformations may be found from the condition :

$$\varepsilon_s = \frac{\cos \nu - 1}{1 - \lambda^2}$$

Thus, the problem of defining motion in the region O'B'C' (O'C' and B'C' are the characteristics of the differential equation) tends to Cauchy's problem with the conditions (4) and (5) on the curve O'B' for equation (1). In the region O'C'D' the compound problem is solved for the boundary conditions :

$u = 0$ and $u_t = 0$ where $r = 0$

Equation (1) was solved by characteristic method on the computing machine after putting it in the form :

$$\frac{\partial^2 \bar{u}}{\partial \tau^2} = \frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\sin 2\bar{u}}{2\bar{r}^2} - \frac{(1+\nu)(1-\cos \bar{u})}{\bar{r}}$$

where :

$$\bar{u} = \frac{u}{R} \quad , \quad \bar{r} = \frac{r}{R} \quad , \quad \tau = \frac{\frac{t}{R}}{a_o}$$

Conditions on the curve O'A in dimensionless form will be :

$$\bar{u} = \sin^{-1} \bar{r} \; ; \; \bar{u}_r = \frac{\cos \bar{u} - \beta^2 \tan^2 \bar{u}}{1 - \beta^3 \tan^2 \bar{u}} \quad ; \quad \beta = \frac{a_o}{V_o}$$

$$\bar{u}_t = \frac{\sin^2 \bar{u} - \beta^2 \tan^2 \bar{u} \,(1 - \cos \bar{u})}{\beta \,(1 - \beta^2 \tan^2 \bar{u}) \sin \bar{u}}$$

Figure (3) shows the relation between $\epsilon\ (0, \tau)$ and τ for different values of β. The material of the taken plate is steel, ($\nu = 0.3$).

For a small interval of time an approximate analytical formula was found (1) :

$$\varepsilon(0, \tau) = 4\, m \frac{\tau}{\beta} \qquad (6)$$

Here $0{,}249 < m < 0{,}249378$

The results of our numerical solution greatly agree with the results according to (6) (See Figure 3).

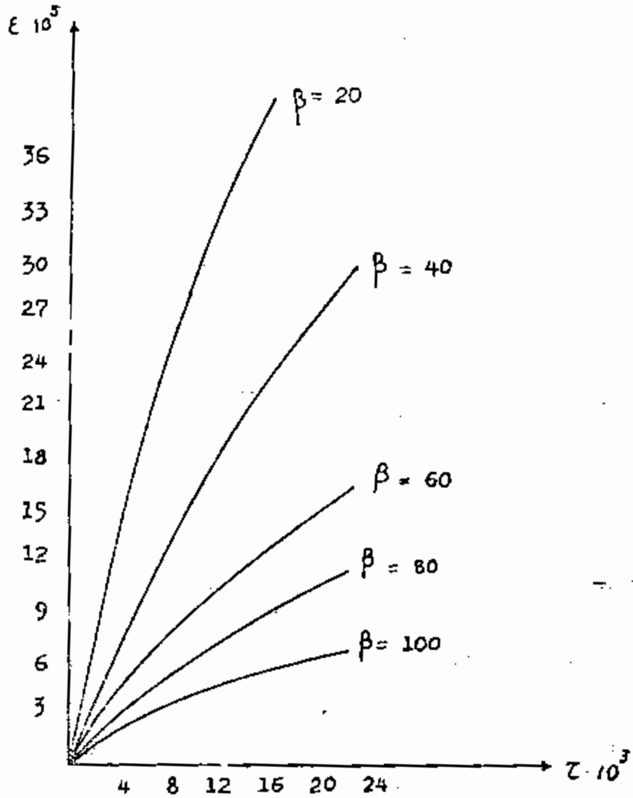

Fig. (8)

References.

1. Pavlinko A.L.
   Normal impact on a flexible membrane by a cone.
   Dissertation for Ph.D.
   Institute of mechanics. Moscow State University 1952.

2. Rakhmatulin K.A. Dimuanov U.A.
   Strength under intensive short-time loads.
   Moscow 1961.

Resume

This paper considers the problem of normal impact against a flexible membrane by a sphere moving with constant velocity. A graph of numerical solution is presented.