HOOK PRODUCT DECOMPOSITION OF YOUNG PATTERNS

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ABSTRACT

The Braunschweig-Hecht decomposition of houng patterns into linear combinations of either antisymmetric factor patterns (column outer products) or symmetric factor patterns (row outer products) is generalized to a decomposition into linear combinations of single hook outer products. The decomposition matrices, with their inverses, are tabulated for $n \leq 10$ and checked using the known outer product dimension formula. Applications to multiple hook young operator expansions are being considered.

INTRODUCTION

Brauns shweig and Hecht 1978 introduced decmposition of Young patterns into their completely symmetric or antisymmetric components.

In this paper the decomposition of Young patterns into linear combinations of either symmetric or an tisymmetric factor patterns is generalised to a decomposition into linear combinations of single hook outer products.

The decomposition matrices, with their inverses, are tabulated for n 10 and checked using the known outer product dimension formula.

Applications to multiple hook Young operator expansions are being considered.

In the following tables the decomposition matrices with their inverses are presented.

Table 1 A

$f_{p'} = f_p$	1
$p' = p$	[1]

$f_{H'} = f_H$	$H' = H$	
1	H 1	1

Table 1 B

$f_{H'} = f_H$	1
$H' = H$	H1

$f_{p'} = f_p$	$p' = p$	
1	1	1

Table 2 A

$f_{p'} = f_p$	1
p	[2]
p'	1^2

$f_{H'} = f_H$	H'	H	
1	$H1^2$	H2	1

Table 2 B

$f_{H'} = f_H$	1
H	H2
H1	$H1^2$

$f_{p'} = f_p$	p'	p	
1	$[1^2]$	[2]	1

Table 3 A

		$f_{p'} = f_p$	1	2
		p	[3]	[21]
$f_{H'} = f_H$	H'	H \ p'	$[1^3]$	
1	$H1^3$	H3	1	
2	H21			1

Table 3 B

		$f_{H'} = f_H$	1	2
		H	H3	H21
$f_{p'} = f_p$	p'	p \ H'	$H1^3$	
1	1^3	3	1	
2	[21]			1

REFERENCES

1. D. Braunschweig and K.T. Hecht, J. Math. Phys., Vol. 19, No. 3, March (1978).

2. N. G. El-Sharkaway and A.H. Jahn, J. Phys. A : Math. Gen., Vol. 10, No. 5, (1977).

Table 4 A

$f_{H`} = f_H$	$H`$	H \ $p`$ / p	$[1^4]$ / $[4]$ ($f_{p`} = f_p$ = 1)	$[21^2]$ / $[31]$ ($f_{p`} = f_p$ = 3)	$[2^2]$ ($f_{p`} = f_p$ = 2)
1	$H1^4$	$H4$	1		
3	$H21^2$	$H31$		1	
8	$H2^2$			1 1	1

Table 4 B

$f_{p`} = f_p$	$p`$	$H`$ / H \ p	$H1^4$ / $H4$ ($f_{H`} = f_H$ = 1)	$H21^2$ / $H31$ ($f_{H`} = f_H$ = 3)	$H2^2$ ($f_{H`} = f_H$ = 8)
1	$[1^4]$	$[4]$	1		
3	$[21^2]$	$[31]$		1	
2	$[2^2]$			-1 -1	1

Table 5 A

$f_{H^1} = f_H$	H^1	ρ / H \ ρ'	$f_{\rho^1} = f_\rho$: 1 ρ: (5) ρ': (1^5)	4 (41) (21^3)	5 (32) (2^21)	6 (31^2)
1	$H1^5$	H5	1			
4	$H21^3$	H41		1		
15	$H2^21$	H32		1	1	1
6	$H31^2$					1

Table 5 B

$f_{\rho^1} = f_\rho$	ρ'	H / ρ \ H^1	$f_{H^1} = f_H$: 1 H: H5 H^1: $H1^5$	4 H41 $H21^3$	15 H32 $H2^21$	6 $H31^2$
1	(1^5)	(5)	1			
4	(21^3)	(41)		1		
5	(2^21)	(32)		-1	1	-1
6	(31^2)					1

Table 6 A

$f_H^1 = f_H$	H^1	H \ ρ (ρ')	(6) (1^6)	(51) (21^4)	(42) (2^21^2)	(41^2) (31^3)	(3^2) (2^3)	(321)
		$f_\rho = f_{\rho}^1$	1	5	9	10	5	16
1	$H1^6$	$H6$	1					
5	$H21^4$	$H51$		1				
24	$H2^21^2$	$H42$		1	1	1		
10	$H31^3$	$H41^2$				1		
45	$H2^3$	$H3^2$		1	1	1	1	1
36	$H321$					1 1		1

Table 6 B

$f_\rho^1 = f_\rho$	ρ'	ρ \ H (H^1)	$H6$ ($H1^6$)	$H51$ ($H21^4$)	$H42$ ($H2^21^2$)	$H41^2$ ($H31^3$)	$H3^2$ ($H2^3$)	$H321$
		$f_H^1 = f_H$	1	5	24	10	45	36
1	(1^6)	(6)	1					
5	(21^4)	(51)		1				
9	(2^21^2)	(42)		-1	1	-1		
10	(31^3)	(41^2)				1		
5	(2^3)	(3^2)			-1	1 1	1	-1
16	(321)					-1 -1		1

Table 7 A

$f_{H^1} = f_H$	H^1	$H \backslash P$	(7)	(61)	(52)	(51^2)	(43)	(421)	$(3^2 1)$	(41^3)
		P (of H^1)	(1^7)	(21^5)	$(2^2 1^3)$	(31^4)	$(2^3 1)$	(321^2)	(32^2)	
		$f_P = f_{P^1}$	1	6	14	15	14	35	21	20
1	$H1^7$	H7	1							
6	$H21^5$	H61		1						
35	$H2^2 1^3$	H52		1	1	1				
15	$H31^4$	$H51^2$				1				
84	$H2^3 1$	H43		1	1	1	1	1		
70	$H321^2$	H421				1		1		1
126	$H32^2$	$H3^2 1$				1		1 1	1	1
20	$H41^3$									1

Table 7 B

		$f_H 1 = f_H$	1	6	35	15	84	70	126	
		H	$H7$	$H61$	$H52$	$H51^2$	$H43$	$H421$	$H3^21$	
$f_\rho 1 = f_\rho$	ρ	ρ \ H	$H1^7$	$H21^5$	$H2^21^3$	$H31^4$	$H2^31$	$H321^2$	$H32^2$	
1	(1^7)	(7)	1							
6	(21^5)	(61)		1						
14	(2^21^3)	(52)		-1	1	-1				
15	(31^4)	(51^2)				1				
14	(2^31)	(43)			-1	1	1	1		1
35	(321^2)	(421)				-1		1		
21	(32^2)	(3^21)						-1	1	1
20	(41^3)									1

Table 8 A

		$f_P^1 = f_P$	1	7	20	21	28	64	35	14	70	56	90	42
		P	(8)	(71)	(62)	(61^2)	(53)	(521)	(51^3)	(4^2)	(431)	(422)	(421^2)	(3^22)
$f_H^1 = f_H$	H'	H \ P'	(1^8)	(21^6)	(2^21^4)	(31^5)	(2^31^2)	(321^3)	(41^4)	(2^4)	(3^221)	(3^21^2)		
1	$H1^8$	$H8$	1											
7	$H21^6$	$H71$		1										
48	$H2^21^4$	$H62$		1	1	1								
21	$H31^5$	$H61^2$				1								
140	$H2^31^2$	$H53$		1	1	1	1	1						
120	$H321^3$	$H521$				1		1	1					
35	$H41^4$	$H51^3$							1					
224	$H2^4$	$H4^2$		1	1	1	1	1		1	1			
280	$H32^21$	$H431$				1		1	1		1		1	
280	$H3^21^2$	$H42^2$						1	$\frac{1}{1}$		1		1	
160	$H421^3$								$\frac{1}{1}$				1	
72	$H3^22$							$\frac{1}{1}$	$\frac{1}{1}$		$\frac{1}{1}$	$\frac{1}{1}$	2	1

Table 8 B

$f_\rho = f$	ρ'	$f_H = f_{H^1}$	1	7	48	21	140	120	35	224	280	280	160	672
		H	$H8$	$H71$	$H62$	$H61^2$	$H53$	$H521$	$H51^3$	$H4^2$	$H431$	$H42^2$	$H421^2$	$H3^22$
		ρ \ H^1	$H1^8$	$H21^6$	$H2^21^4$	$H31^5$	$H2^31^2$	$H3213$	$H41^4$	$H2^4$	$H32^21$	$H3^21^2$		
1	(1^8)	(8)	1											
7.	$(21\ )$	(71)		1										
20	(2^21^4)	(62)		-1	1	-1								
21	(31^5)	(61^2)				1								
28	(2^31^2)	(53)			-1	1	1	-1	1					
64	(321^3)	(521)				-1		1	-1					
35	(41^4)	(51^3)							1					
14	(2^4)	(4^2)					-1	1	-1 -1	1	-1		1	
90	(32^21)	(431)						-1	1 1		1		-1	
56	(3^21^2)	(42^2)				1		-1	1			1	-1	
90	(421^2)								-1 -1				1	
42	(3^22)							1 1	-1 -1		-1 -1	-1 -1	2	1