Chapter 2
SEMICONDUCTORS

2.1 Bond Model

Semiconductors are a middle category of materials between conductors and non-conductors (insulators). Silicon is a major semiconductor element in electronics. Its atomic number is 14, which means that electrons are distributed in its shells as 2, 8, 4. Thus, the valence (outer) electrons are 4, hence called tetravalent. The inner electrons do not take part in any interaction, while the outer electrons play a key role in chemical reactions. The silicon atom strives to complete its outer shell to contain 8 electrons for the atom to be stable. Since it is difficult to lose or gain 4 electrons in ionic bonding a different type of bonding, called covalent bonds, prevails.

Silicon exists in the earth's crust and exists in the form of sand (S_iO_2). But the silicon we are talking about is silicon ore. It is handled in a special way to form a pure silicon crystal, usually grown from molten silicon on a seed crystal (Fig. 2.1). In the solid state, silicon atoms are arranged in an orderly arrangement, such that each silicon atom shares its 4 electrons with 4 neighbors (Fig. 2.2). At $T = 0^o K$, all covalent bonds in pure (intrinsic) silicon are intact (unbroken). As a result, there are no free electrons in the crystal $T = 0^o K$. If an external field is applied no current flows and the semiconductor behaves as an insulator.

We may represent a silicon atom as a core of charge $(+4|q|)$, four outer electrons (assuming 10 electrons shield a charge of $(+10|q|)$ on the nucleus (Fig. 2.3).

2.2 Holes

As the temperature is raised, some bonds are broken, i.e., some electrons gain enough energy to leave their parent atoms and break away from the bond and become free electrons. Since the silicon atom is neutral the electron quitting the atom will leave behind a positive charge. We cannot call this atom an ion because soon enough another free electron will fall onto the vacancy created in the bond by the escaping electron. It might also happen that the vacancy captures an electron from a neighboring bond. This way the vacancy moves around in the bonds. We call this vacancy a hole (Fig. 2.4). This hole is moving around randomly, since an electron breaks away somewhere else creating a hole in that bond.

We note that the number of free electrons n_0 must equal the number of holes p_0 in an intrinsic semiconductor. This is called condition of charge neutrality. We should also note that at a certain temperature the number of the electrons and the number of holes are constant at thermal equilibrium. This is true on an average or statistical basis. But not the same electrons are free all the time. After a short lifetime an electron patches up a bond, but an electron somewhere else is liberated. We should also note the number of free electrons or holes (or the number of broken bonds) is on the increase as temperature is raised, i.e., as we pump in free electrons and holes. But we always have for intrinsic semiconductor the concentration of electrons or holes is n_i which increases with temperature.

$$n_0 = p_0 = n_i \qquad (2\text{-}1)$$

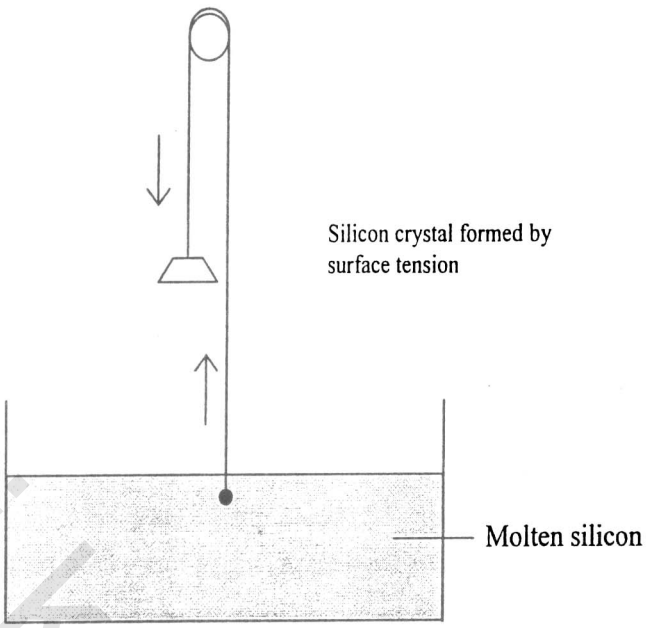

Fig. (2.1) Growing a silicon crystal

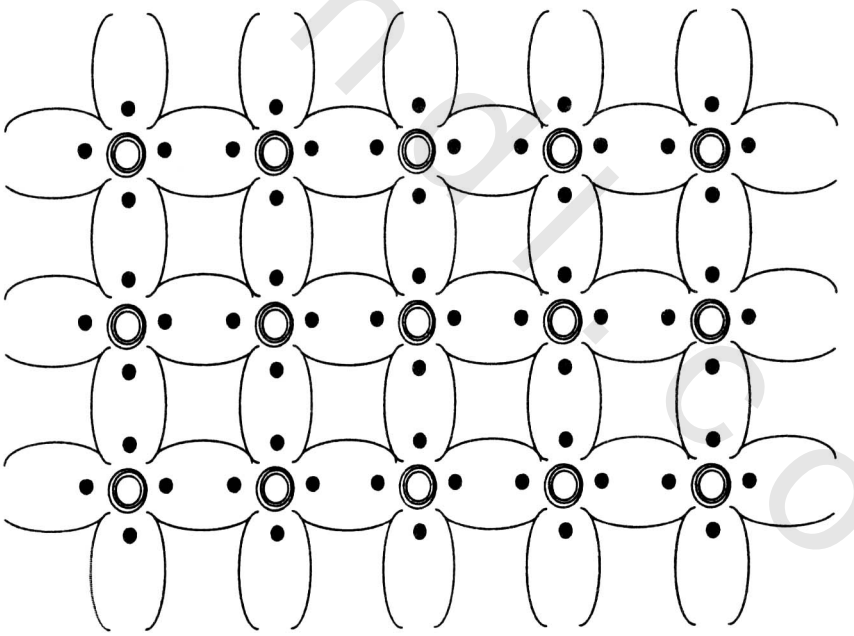

Fig. (2.2) Pure silicon crystal

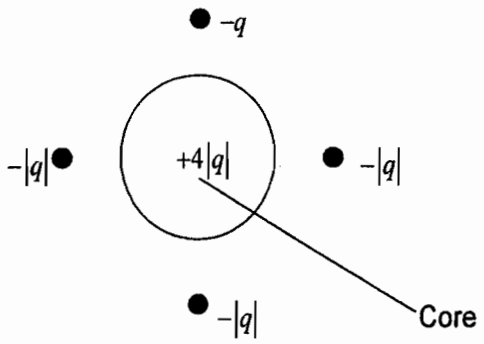

Fig. (2.3) Atomic model of a silicon atom containing a core ($+4|q|$) and 4 outer electrons

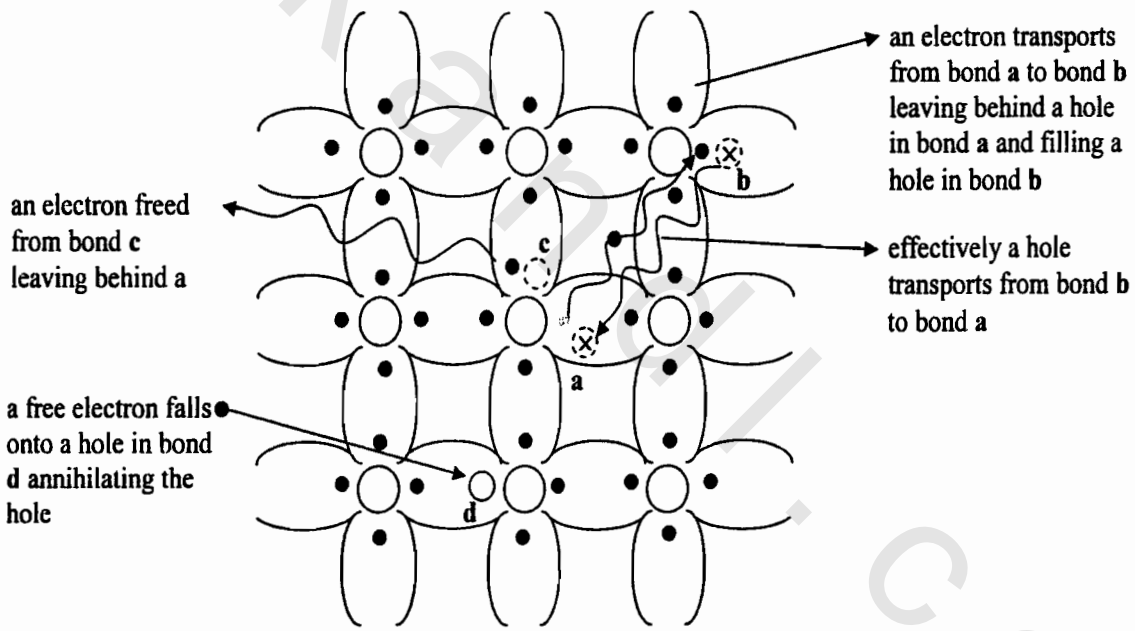

Fig. (2.4) Hole formation and random transport of holes in an intrinsic semiconductor

2.3 Doping

One of the important characteristics of semiconductors is sensitivity to temperature. Also, semiconductors are sensitive to impurities, pressure, humidity, pollution etc. Impurities may be added to semiconductors either intentionally or unintentionally. There are two types of important impurities, atoms from the fifth column of the periodic table (pentavalent atoms) or atoms from the third column (trivalent atoms).

The intentional addition of impurities is called doping, and the material added is called dopant. This process renders the semiconductor extrinsic rather than intrinsic.

Let us consider the addition of small amount of pentavalent element (such as phosphorus P to silicon in the molten state. The phosphorus atom contains $+5|q|$ core and 5 outer electrons (Fig. 2.5). When silicon crystallizes, the phosphorus atom replaces a silicon atom, it tries to perform the same type of bonding as the missing silicon atom (Fig. 2.6). The excess electron being left out from the bond formation finds it easy to leave its parent atom becoming a free electron, leaving behind a positive ion (Fig. 2.7). Now the free electrons come from two sources; broken bonds and phosphorus atoms, hence the P atom is called donor atom. At thermal equilibrium, we must maintain the condition of charge neutrality. The total positive charge is $p_0 + N_D^+$ where N_D^+ is the density of donor ions which must equal the negative charges n_0. The energy required for ionization of the P atom is very small, so it is safe to say that all donor atoms are ionized at room temperature. Thus,

$$p_0 + N_D = n_0 \tag{2-2}$$

It has been found that

$$n_0 p_0 = n_i^2 \tag{2-3}$$

This is called mass action law which shows that as n_0 increases from eqn. (2-2) p_0 decreases. Thus, in the case of using pentavalent atoms as dopant, $n_0 > p_0$. We call the semiconductor in this case n-type.

When we use trivalent atoms (such as aluminum $A\ell$) as dopart, an $A\ell$ atom replaces a silicon atom. But in this case the $A\ell$ atom has a core of $+3|q|$ and 3 outer electrons (Fig. 2.8). This atom tries to form covalent bonds as the missing silicon atom, but it is one electron short. So it snatches an electron from a neighboring bond thus creating a hole there. The $A\ell$ atom becomes a negative ion (Fig. 2.9). Since $A\ell$ atom accepts an electron it is called an acceptor. From the condition of charge neutrality the total negative charge is $(N_A^- + n_0)$, where N_A^- is the density of acceptors with all total positive charge p_0. Again, at room temperature it is safe to assume that all acceptor atoms are ionized

$$n_0 + N_A = p_0 \tag{2-4}$$

The mass action law also holds. Thus, $p_0 > n_0$ and the material is called p-type. Again, holes come from two sources, broken bonds and acceptor atoms. In both cases of n-type and p-type semiconductors it may well be that electrons or holes coming from the dopant exceed those coming from broken bonds. Thus, for an n-type material, we have

$$n_0 = N_D \tag{2-5}$$

$$p_0 = \frac{n_i^2}{N_D} \tag{2-6}$$

and for p-type material, we have

$$p_0 = N_A \tag{2-7}$$

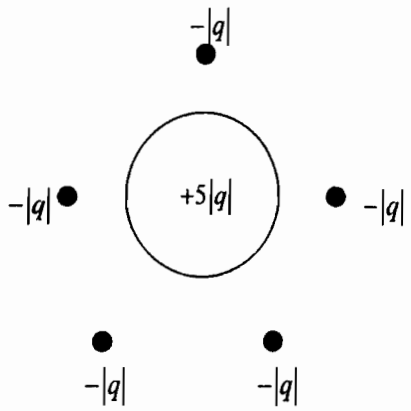

Fig. (2.5) Model of a P atom (donor) as a core $(+5|q|)$ and 5 outer electrons

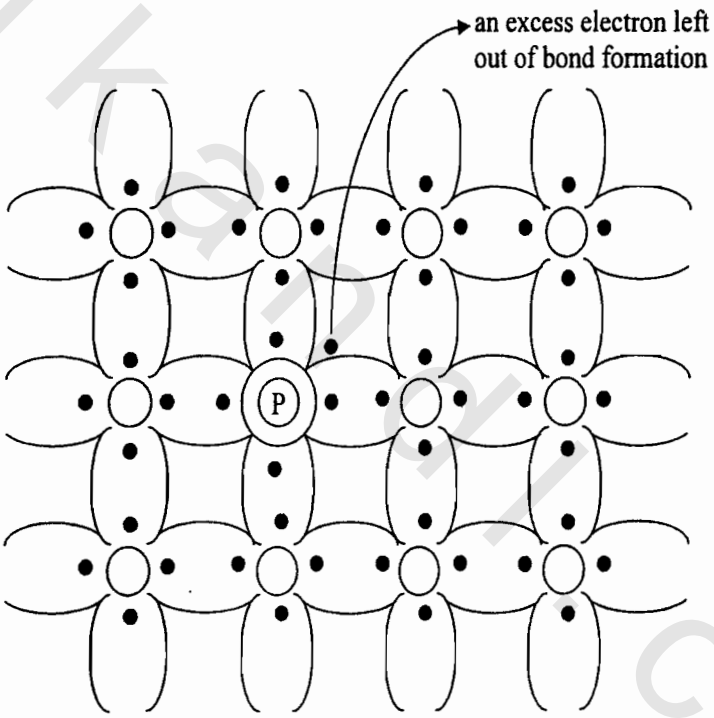

Fig. (2.6) A P atom replaces a Si atom in the crystal

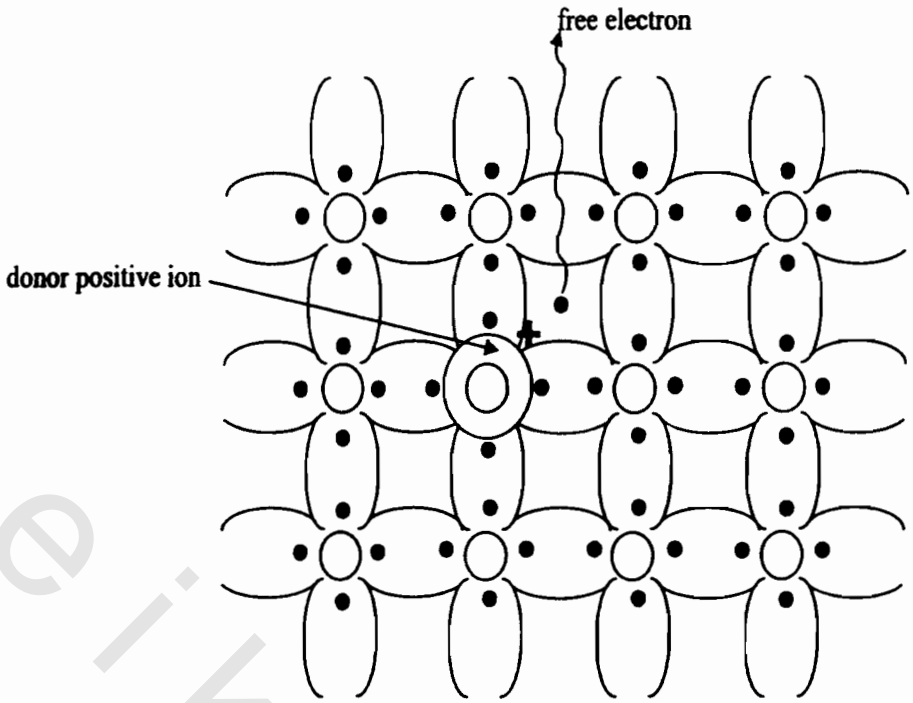

Fig. (2.7) An excess electron escapes a P atom leaving behind a positive ion (donor)

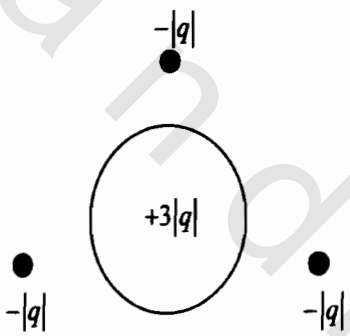

Fig. (2.8) Model of an $A\ell$ atom (acceptor) as a core $(+3|q|)$ and 3 outer electrons

$$n_0 = n_i^2 / N_A \qquad (2\text{-}8)$$

We note here that we are dealing with two types of mobile charge carriers: electrons and holes, whereas in metals we have only electrons. We should also note that unlike metals - where the electron density is constant for a particular metal - the electrons and holes in a semiconductor can vary either by doping or by temperature. We also note that in a semiconductor we can convert from intrinsic to extrinsic by adding a proper dopant. If we wish to return it to intrinsic again there is no way to extract atoms which have already been added, but we might add atoms of the opposite kind. Thus, we can manipulate the type of the semiconductor by adding impurities in controlled amounts at will (Fig. 2.10).

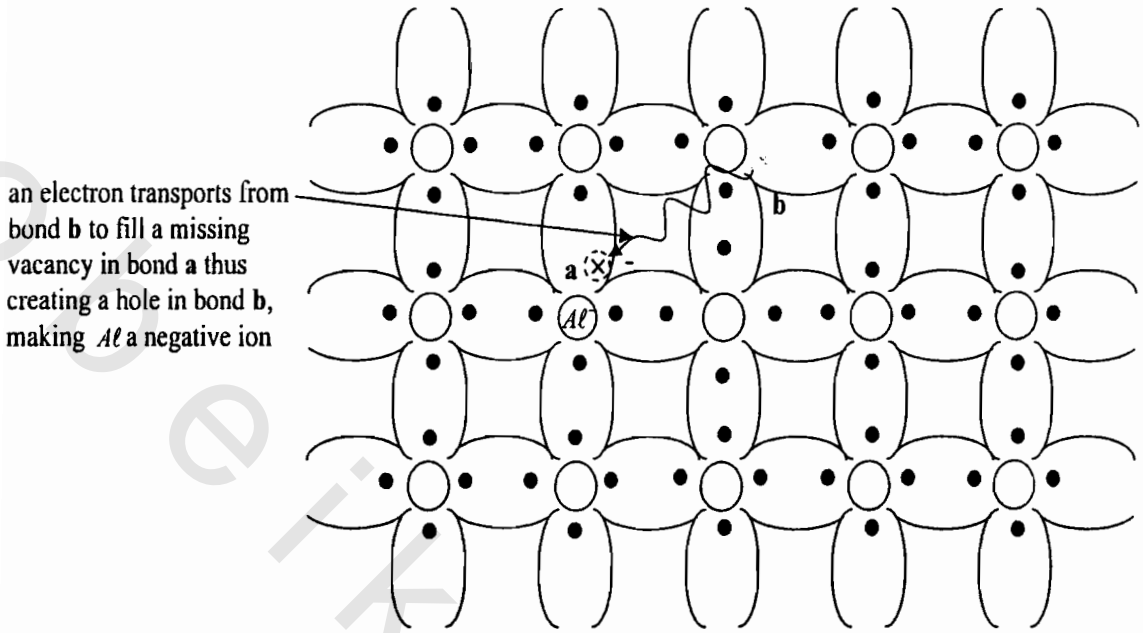

Fig (2.9) An $A\ell$ atom replaces Si atom in the crystal and snatches an electron from a neighboring bond creating a hole there

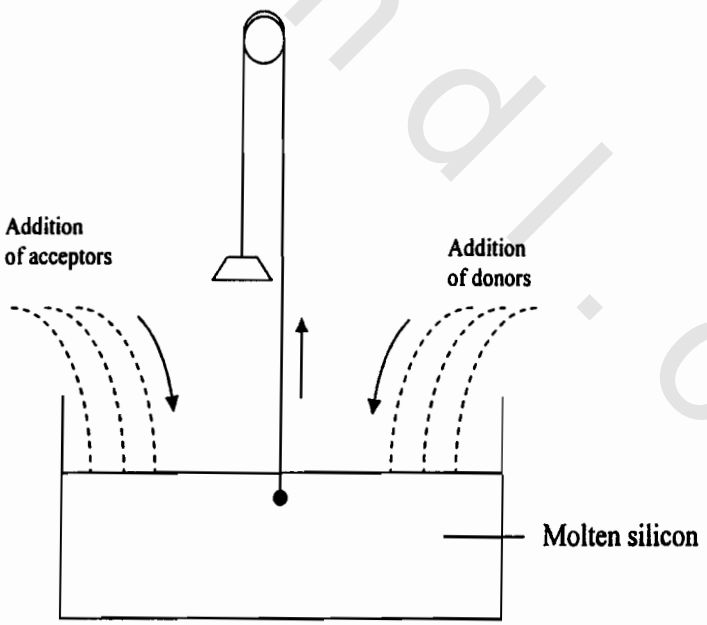

Fig. (2.10) Alternate addition of donors and acceptors changes the type of a semiconductor as desired

We realize that very small additions of impurities can change the properties of a semiconductor dramatically. It can even change its type altogether. This hypersensitivity to impurities is what makes semiconductors unique.

Continuous search for new materials characterizes the ever growing progress in electronics through research work in the fields of material science and solid state physics. Yet, silicon remains at the core of electronics industry. There are different ways to grow and control the doping of silicon. One way is through crystallization from the molten state. Another is growing doped silicon from vapor deposition (epitaxy).

Ex. 2.1

Calculate the concentration of atoms in silicon. If phosphorus is added such that the donor impurity concentration is part per 10^8 silicon atoms, find the change in electron concentration. Calculate the weight of the added phosphorus per 1 gram of silicon. Then, calculate the weight of aluminum to be added to the crystal to restore it to be intrinsic.

Solution

The atomic weight in grams (a gram mole A_w contains Avogadro's number of atoms $N_{av} = 6.02\times10^{23}$. Then the density $d(gram/cm^3)$ contains $C_a(atom/cm^3)$ given for silicon by

$$C = \frac{N_{av}d}{A_w} \qquad (2\text{-}9)$$

$$= 6.02\times10^{23}\frac{atoms}{mole}\times\frac{2.33gram}{cm^3}\times\frac{1}{28.1gram}$$

$$= 4.99\times10^{22}atoms/cm^3$$

If 1 donor atom per 10^8 silicon atoms is added

$$N_D = 4.99\times10^{14}atoms/cm^3$$

$$n_0 \cong N_D$$

$$p_0 \cong \frac{n_i^2}{N_D} = \frac{(1.5\times10^{10})^2}{4.99\times10^{14}} = 4.51\times10^5 holes/cm^3$$

Thus, the addition of 1 donor atom in 10^8 silicon atoms has increased the conduction electron density by a factor of 3.3×10^4. This shows how sensitive the semiconductor is to impurities. For 1 gram of silicon, the volume is $\frac{1}{2.33}cm^3$. The total number of P atoms per gram silicon which is $N_D\times\frac{1}{2.33}$

$$= 4.99\times10^{14}\times\frac{1}{2.33} = 2.14\times10^{14}atoms$$

Again, the atomic weight in grams contains N_{av} atoms. What is the weight of the P atoms? This is given by the weight of per gram silicon

$$= \frac{31\times2.14\times10^{14}}{6.02\times10^{23}} = 1.1\times10^{-8}gram$$

If we want to neutralize the donor impurities we must add acceptor impurities of the same atomic concentration.

The total number of Al atoms per gram silicon

$$= N_A\times\frac{1}{2.33} = 2.14\times10^{14}atoms$$

Since $N_A = N_D = 4.99\times10^{14}\ atoms/cm^3$

The weight of Al per gram silicon

$$= \frac{27\times2.14\times10^{14}}{6.02\times10^{23}} = 2.96\times10^{-8}\ gram$$

2.5 Band Model

The difference in the band model between a metal and a semiconductor - or an insulator for that matter - is that in a metal the outermost electrons roam around the crystal in one continuous band, called conduction band. The innermost electrons are completely isolated from all interfering neighbors. In a semiconductor, we distinguish between 3 classes of electrons. The innermost electrons are attached to their parent atoms and take part in no interactions with neighboring atoms. The second class is electrons which form bonds but can move around from one bond to another, so they are partly mobile but not entirely free. Of course the movement of electrons from one bond to another can be alternatively expressed by the motion of holes. So holes describe the motion of valence electrons due to the vacancies in the covalent bonds created by the missing electrons. We say that these holes move around within the valence band (VB). The third class of electrons are those electrons which break away with their bonds and become free, and behave very much the same way as the electrons in the metal, hence, called conduction electrons, and their band is called conduction band (CB). The CB and VB are separated by a forbidden gap, also called band gap (E_g). It represents the binding energy of the bond, i.e., the minimum energy needed to free one electron from the bond.

We may define CB as the lowest band partially filled containing electrons, while VB is the highest band mostly filled by electrons. In fact, the value of E_g determines whether the material is a semiconductor or an insulator. In a semiconductor, E_g is small enough (1eV for silicon), so that at room temperature many electrons can break away and become free, i.e., raised from VB to CB (Fig. 2.11). In an insulator E_g is so great that at room temperature, it is virtually impossible to find one electron in the CB. (Fig. 2.12).

2.6 Fermi Level in Intrinsic Semiconductors

Without knowing apriori the location of Fermi level, we can refer to eqn. (1-21),

$$n_o = \int_{E_c}^{\infty} N(E)dE = \int_{E_c}^{\infty} \frac{C\sqrt{E}dE}{1+e^{(E_c-E_F)/kT}} \qquad (2\text{-}10)$$

Where E_c is the conduction band edge. Tthe same statistics for conduction electrons in a metal is used for conduction electrons in a semiconductor. The above integration can be alternatively expressed as

$$n_o = N_c f_e(E_c) = N_c \frac{1}{1+e^{(E_c-E_F)/kT}} \qquad (2\text{-}11)$$

where N_c is called the effective density of states (per unit volume) at CB edge, which is a constant that depends on temperature, and $f_e(E_c)$ is the probability of occupation for an electron at the CB edge. What eqn. (2-11) actually says is that the total number of electrons in CB per unit volume can be calculated by multiplying the number of levels (states) of CB as if they coalesce at E_c times the probability of finding an electron at energy E_c

Similarly, we may express the density of holes in the VB as

$$p_o = N_V f_p(E_V) \tag{2-12}$$

where N_V is the affective density of states at the valence band edge E_V, and f (E_V) is the probability of finding a hole at E_V. This can be found from

$$f_p(E) = 1 - f_n(E) \tag{2-13}$$

$$= 1 - \frac{1}{1+e^{(E-E_F)/kT}}$$

$$= \frac{1}{1+e^{-(E-E_F)/kT}} \tag{2-14}$$

Thus, eqn. (2-12) reduces to

$$p_0 = N_v \frac{1}{1+e^{-(E-E_F)/kT}} \tag{2-15}$$

Between the bond model and band model the correspondence is one to one. An electron leaving the bond is equivalent to an electron excited from VB to CB, given enough energy. This is called electron - hole pair generation. Soon enough an electron in the CB falls onto a vacancy in the bond which is equivalent to electron - hole pair recombination (annihilation), which is a accompanied by release of energy. This takes place after certain time called lifetime. There is a dynamic balance between the two processes. But on the average n_o and p_o remain constant for constant temperature and doping conditions, at thermal equilibrium.

Proceeding with eqns. (2-11) and (2-15), assuming symmetry i.e., $N_c = N_v = N_e$, which is called effective density of states for an intrinsic semiconductor $n_v = p_0$ requires

$$1 + e^{(E_c - E_{F_i})/kT} = 1 + e^{(E_v - E_F)/kT}$$

$$(E_c - E_{F_i}) = (E_v - E_{F_i})$$

$$2E_{F_i} = E_c + E_v$$

$$E_{F_i} = \frac{1}{2}(E_c + E_v) \tag{2-16}$$

where E_{F_i} is Fermi level for an intrinsic semiconductor. This means that Fermi level for an intrinsic semiconductor lies in the middle of the band gap, a provocative result! If Fermi level exists in the middle of the forbidden gap how can we reconcile this fact with our understanding that such a gap is forbidden? In fact, Fermi level is not a real level, it is a fictious level. It is only an indicator. It represents the highest energy level an electron can have in a semiconductor at $T = 0^o K$. If for some reason there is an allowed level embedded within the spanning from E_c to E_F it will be occupied, since $f_n(E) = 1$ at $T = 0^o K$ (Fig. 2.13). Otherwise, the highest level is E_v at $T = 0^o K$.

As temperature is increased, the Fermi-Dirac probability function becomes

$$f_n(E) = \frac{1}{1+e^{(E-E_{F_i})/kT}} \tag{2-17}$$

If $(E - E_{F_i})/kT \gg 1$, i.e., E is well into the CB

$$f_n(E) \cong e^{-(E-E_{F_i})/kT} = e^{E_{F_i}/kT} e^{-E/kT}$$

$$= \text{const}\, e^{-E/kT} \tag{2-18}$$

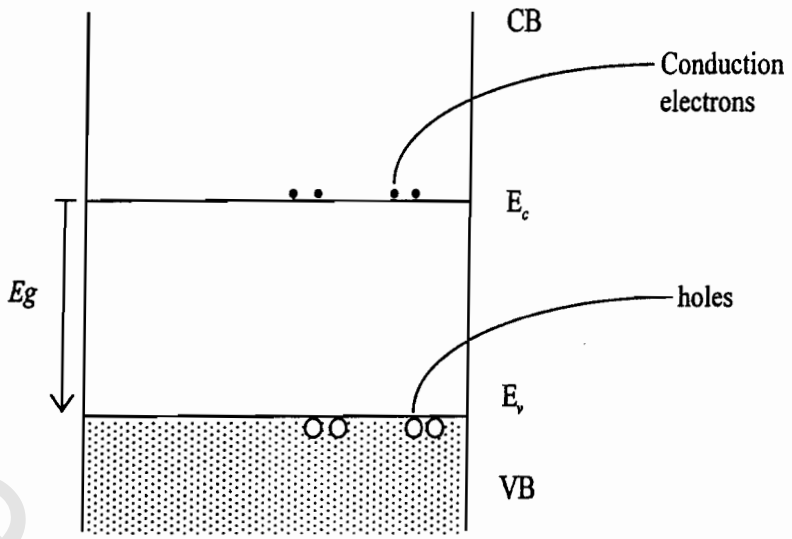

Fig. (2.11) Energy band diagram in a semiconductor

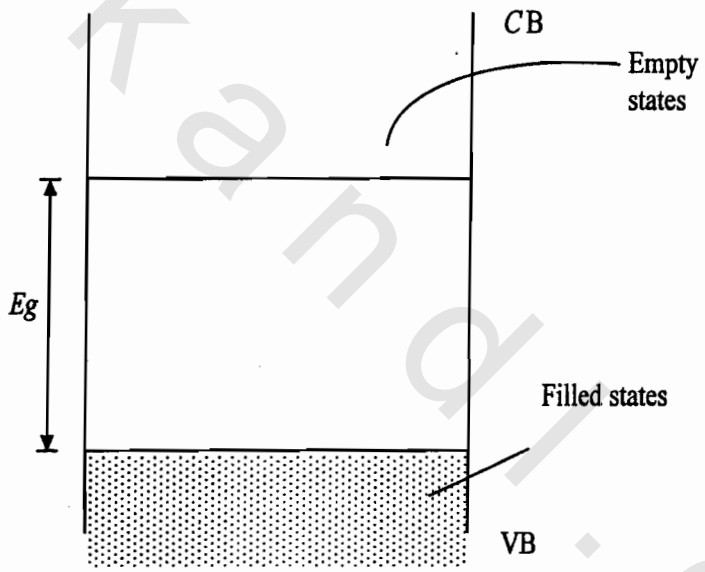

Fig. (2.12) Energy band diagram in an insulator

which is the same as Maxwell-Boltzmann probability function (1-13). Thus, a Maxwellian tail extends into CB (Fig. 2.14).

Invoking the calculations of the electron density functions $N(E)$ of eqn. (1-18), where $S(E)$ is still proportional to $\sqrt{E}$ for both electrons in CB and holes in VB, we obtain $N_n(E)$ for electrons in CB and $N_p(E)$ for holes in VB by multiplying $S(E)$ in both CB and VB by the corresponding Maxwellian tails (Fig.2.15), which leads to symmetric distribution profiles for CB and VB in intrinsic semiconductors

(Fig.2.16). In intrinsic semiconductors, the total areas under the distribution curves in both CB and VB are equal. i.e:

$$n_0 = \int_{E_c}^{\infty} N_n(E)dE = n_i \qquad (2\text{-}19)$$

$$p_0 = \int_{E_c}^{\infty} N_P(E)dE = n_i \qquad (2\text{-}20)$$

Rewriting eqns. (2-11) and (2-12) assuming $N_C = N_v = N_e$

$$n_0 = n_i = N_e \frac{1}{1+e^{(E_c - E_{F_i})/kT}} \qquad (2\text{-}21)$$

For $\frac{E_c - E_F}{kT} \gg 1$

$$n_0 = n_i = N_e e^{-(E_c - E_{F_i})/kT} \qquad (2\text{-}22)$$

Since for an intrinsic semiconductor E_{F_i} lies in the band gap,

$$E_c - E_{F_i} = \frac{1}{2}E_g$$

$$n_0 = n_i = N_e e^{-E_g/2kT} \qquad (2\text{-}23)$$

Similarly, for holes,

$$p_0 = n_i = N_e^2 e^{-E_g/2kT} \qquad (2\text{-}24)$$

Thus,

$$n_0 p_0 = n_i^2 = N_e^2 e^{-E_g/kT} \qquad (2\text{-}25)$$

It is found that N_e^2 is proportional to T^3

Thus,

$$n_i = A_0 T^{3/2} e^{-E_g/2kT} \qquad (2\text{-}26)$$

Where A is a constant for

$$N_e^2 = A_0^2 T^3 \qquad (2\text{-}27)$$

2.7 Fermi Level in Extrinsic Semiconductors

Invoking eqn. (2-21), for extrinsic semiconductors we may write

$$n_0 = N_e \frac{1}{1+e^{(E_c - E_F)/kT}} \qquad (2\text{-}28)$$

For $\frac{E_c - E_F}{kT} \gg 1$

$$n_0 = N_e e^{-(E_c - E_F)/kT} \qquad (2\text{-}29)$$

From eqn. (2-22)

$$n_0 = n_i = N_e e^{-(E_c - E_{F_i})/kT}$$

$$\frac{n_0}{n_i} = e^{-(E_F - E_{F_i})/kT}$$

$$n_0 = n_i e^{(E_F - E_{F_i})/kT} \qquad (2\text{-}30)$$

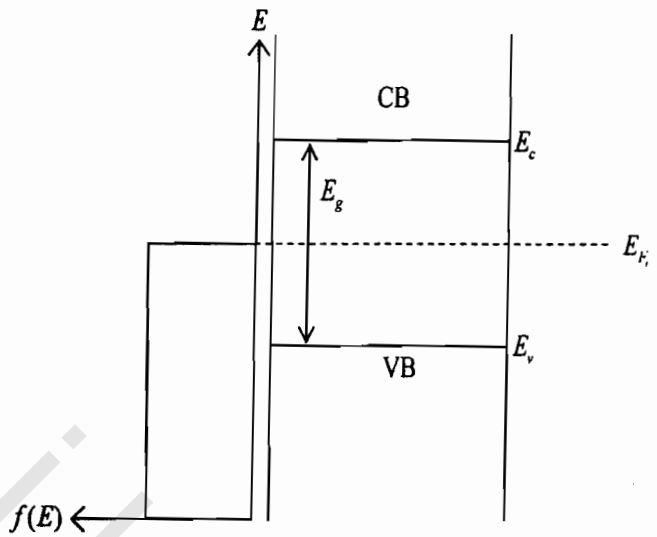

Fig. (2.13) Fermi level lies in the middle of the band gap in an intrinsic semiconductor at $T = 0^o K$

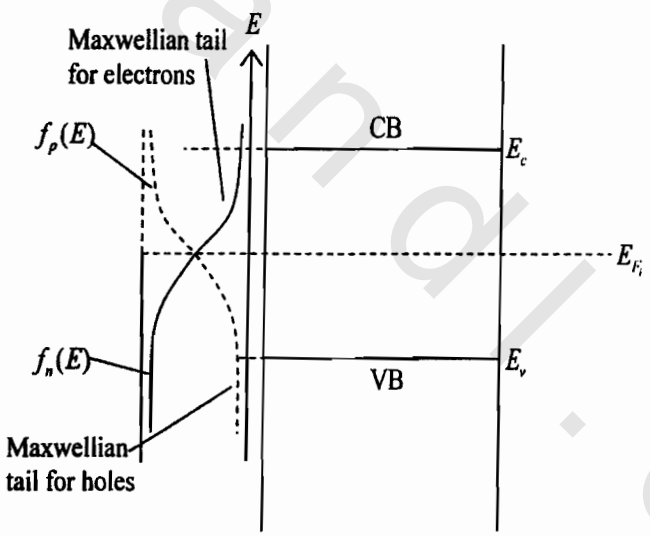

Fig. (2.14) Fermi level in an intrinsic semiconductor and Maxwellian tails for $T > 0^o K$

Similarly

$$p_0 = n_i e^{(E_F - E_{F_i})/kT} \tag{2-31}$$

Noting from eqns. (2-30), (2-31) and (2-26)

$$n_0 p_0 = n_i^2 = A^2 T^3 e^{E_g/kT} \tag{2-32}$$

If $E_F > E_{F_i}$ then $n_0 > n_i > p_0$ the material is n-type

If $E_F < E_{F_i}$ $p_0 > n_i > n_0$ the material is p-type

Thus, Fermi level is a good indicator for the material type. If Fermi level is closer to CB, then the material is n-type, and if it is closer to the VB it is p-type. The distribution profiles are no longer symmetric (Fig. 2.17, Fig. 2.18). Since the energy required for ionization of a P atom in silicon crystal is very small because the excess electron does not take part in bonding, it is customary to represent this ionization energy for n-type material by a shallow level below E_c by an amount ΔE_D which is the ionization energy of phosphorus.

Similarly, for p-type material, we may represent the ionization of acceptor atoms such as Al by a shallow level ΔE_A above E_v. It might be a bit challenging to acknowledge the presence of such shallow levels in an otherwise forbidden gap. The answer is that the forbidden gap is clean only in pure semiconductors. Impurities introduce allowed levels in the band gap due to disturbances which these impurities produce in the band structure. Sometimes these levels are shallow and sometimes they are deep, depending on the distance on the energy scale from the band edges.

2.8 Probability Dilemma

We now calculate the probability of ionization of P atoms. If N_D^0 represents the unionized (neutral) donor atoms density and N_D represents the total density of donor atoms, then $\frac{N_D^0}{N_D}$ represents the probability of finding an electron at the donor level E_D i.e., the probability that the donor atom is neutral (keeping its electron)

$$f_n(E_D) = \frac{N_D^0}{N_D} = \frac{1}{1 + e^{(E_D - E_F)/kT}} \tag{2-33}$$

For $\frac{E_D - E_F}{kT} \gg 1$

$$f_n(E_D) \cong e^{-(E_D - E_F)/kT} \tag{2-34}$$

which is nearly zero, i.e., no electrons exist at E_D, which means that the ionized donor atoms density N_D^+ is nearly N_D. The challenging question here is this. If $f_n(E_D)$ is presumably zero and $f_n(E_c) < f_n(E_D)$, then how could it be that the electron density at E_D is taken to be zero while the conduction electron density n_0 is not? The answer here is that to have an electron at a certain level we require two conditions at the same time, namely, the existence of a density of allowed states function and a high probability for an electron to exist at such states. For shallow donor levels, the density of states is quite small and the probability of finding an electron there is also very small. Thus, the product is vanishingly small. While for CB electrons, the density of states is much higher. Therefore even for the small probability the product is finite.

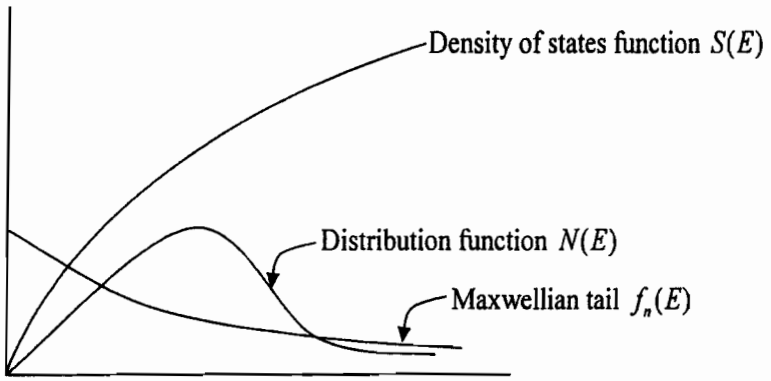

Fig. (2.15) Distribution function $N(E)$ is the product of the density of states function $S(E)$ and the Maxwellian tail

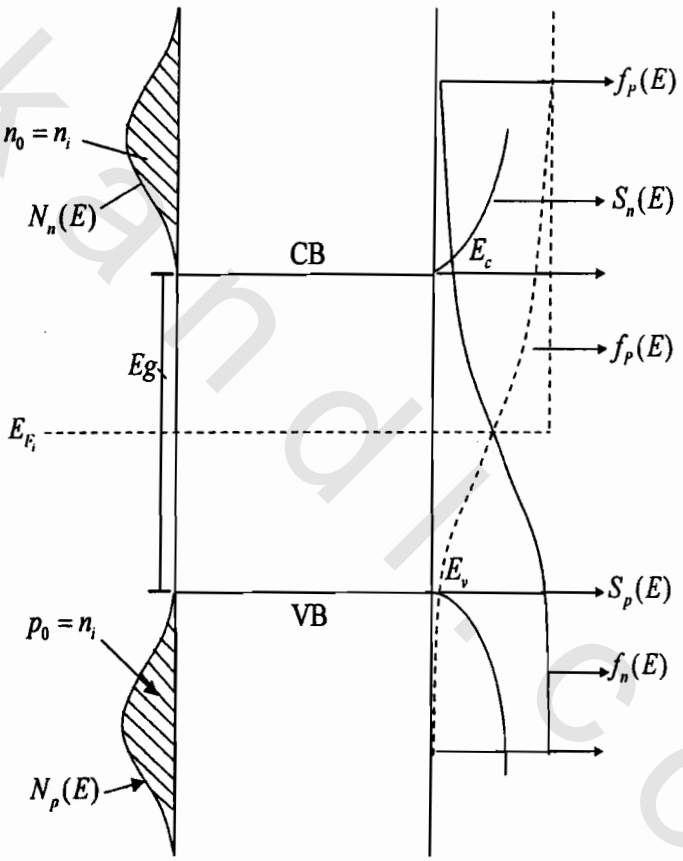

Fig. (2.16) Profile for the distribution function in CB and VB in an intrinsic semiconductor

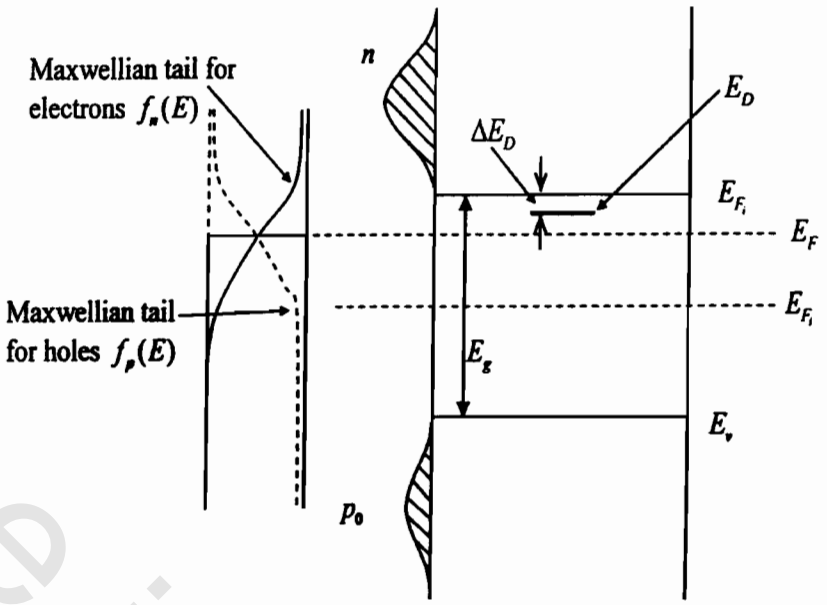

Fig. (2.17) Distribution profile in an n-type material $(n_0 > p_0)$

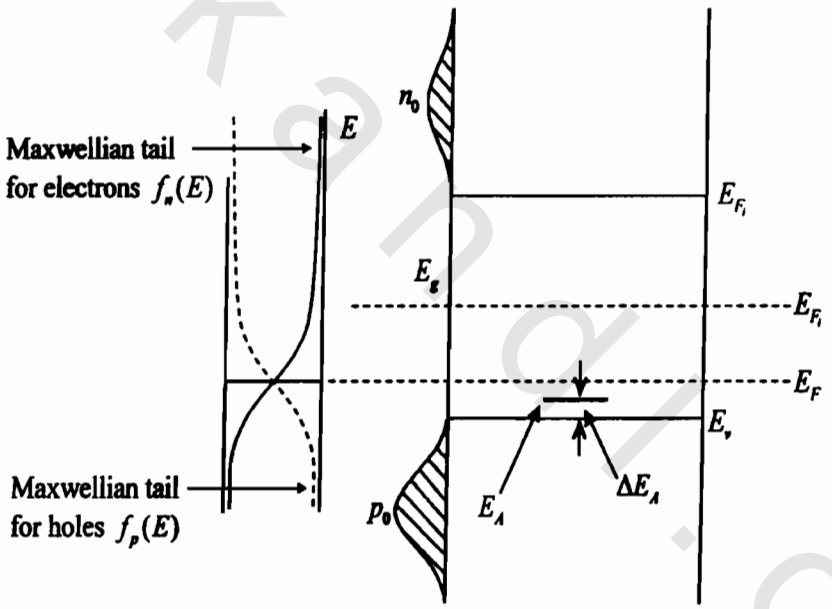

Fig. (2.18) Distribution profile in a p-type material $(p_0 > n_0)$

We should also note that Fermi level-being a fictious level - gives us the clue, namely that if an allowed level exists in the gap below E_F then due to the high probability value this level will be occupied. Whereas for a level below E_F such a level will be empty. If we decrease the temperature toward $0^\circ K$ the CB will be empty and the donor atoms will be neutral. Therefore, E_F will be midway between E_c and E_D. Whereas if we increase the temperature excessively the density of the broken bonds will

outnumber the donor atoms density $(n_i > n_0)$ and the material turns intrinsic. This is why electronics may suffer in warfare under conditions of excessive heat caused by nearby blasts, since device materials then lose their type identity (Fig. 2.19), and hence their features of performance.

Ex. 2.2

An n-type silicon sample has 1.5×10^{14} phosphorus $atoms/cm^3$. Verify that the donor atoms are all ionized if the donor level is $0.05\ eV$ below E_c, $E_g = 1.1\ eV$, $kT = 0.025\ eV$

Solution

$$E_{F_i} = 0.55\ eV$$

From eqn. (2-30) and $\frac{n_0}{n_i} = 10^4$

$$E_F - E_{F_i} = kT\ell n\frac{n_0}{n_i} = 0.025\ell n10^4$$

$$= 0.025\times4\times2.3 = 0.23\ eV$$

Thus, measured from E_c, we have a donor level at E_D, $0.05\ eV$ below E_c and E_F at $0.32\ eV$ below E_c which is well below E_D. From eqn. (2-33) and noting

$\frac{E_D - E_F}{kT} = \frac{0.27}{0.025} = 11$, we have $\frac{N_D^{\ 0}}{N_D} = e^{-11} = 10^{-4.8}$, which is very small.

Then nearly all P atoms are ionized at room temperature.

2.9 Conduction in Semiconductors

We follow here the same lines as in sections (1.23), (1.24). The only difference here is that we have in semiconductors two current carriers: conduction electrons and valence holes. We take the direction of hole motion as the direction of the current. Due to the negative charge of the electrons, we consider the electron current in the direction opposite to the direction of electron movement. Thus, the overall currents due to both electrons and holes are in the same direction, which is that of the field. We can thus express the total current density as

$$J = J_n + J_p \qquad (2\text{-}35)$$

$$= |q| n_0 v_n + |q| p_0 v_p$$

$$= qn_0\mu_n\varepsilon + qp_0\mu_p\varepsilon \qquad (2\text{-}36)$$

$$= \sigma_n\varepsilon + \sigma_p\varepsilon$$

$$= \sigma\varepsilon \qquad (2\text{-}37)$$

$$\sigma = |q|\left(n_0\mu_n + p_0\mu_p\right) \qquad (2\text{-}38)$$

where μ_n is the electron mobility and μ_p is the hole mobility.

At absolute zero all bonds are intact. There are no free electrons or free holes. At room temperature, there are some broken bonds. When an electric field is applied, electrons - free or bound - move due to Newton's force opposite to the direction of the field. Electrons in bonds may be pulled in to fill in vacancies. The motion of electrons from bond to bond creates transport of holes in an opposite direction to electron flow, i.e., in the direction of the field. It is this composite motion that characterizes conductivity in semiconductors from conductivity in metals (Fig. 2.20).

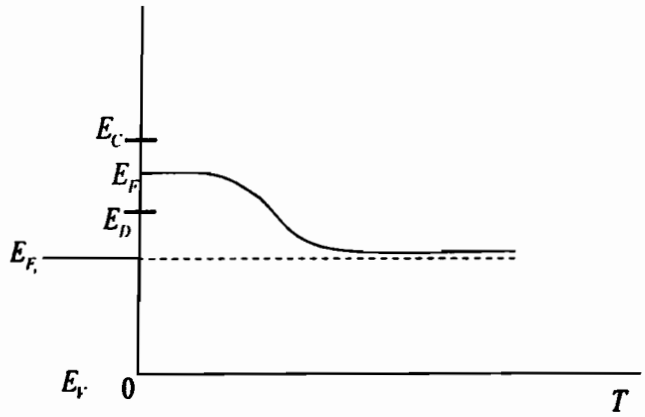

Fig. (2.19) Variation of Fermi level with temperature for an n-type semiconductor

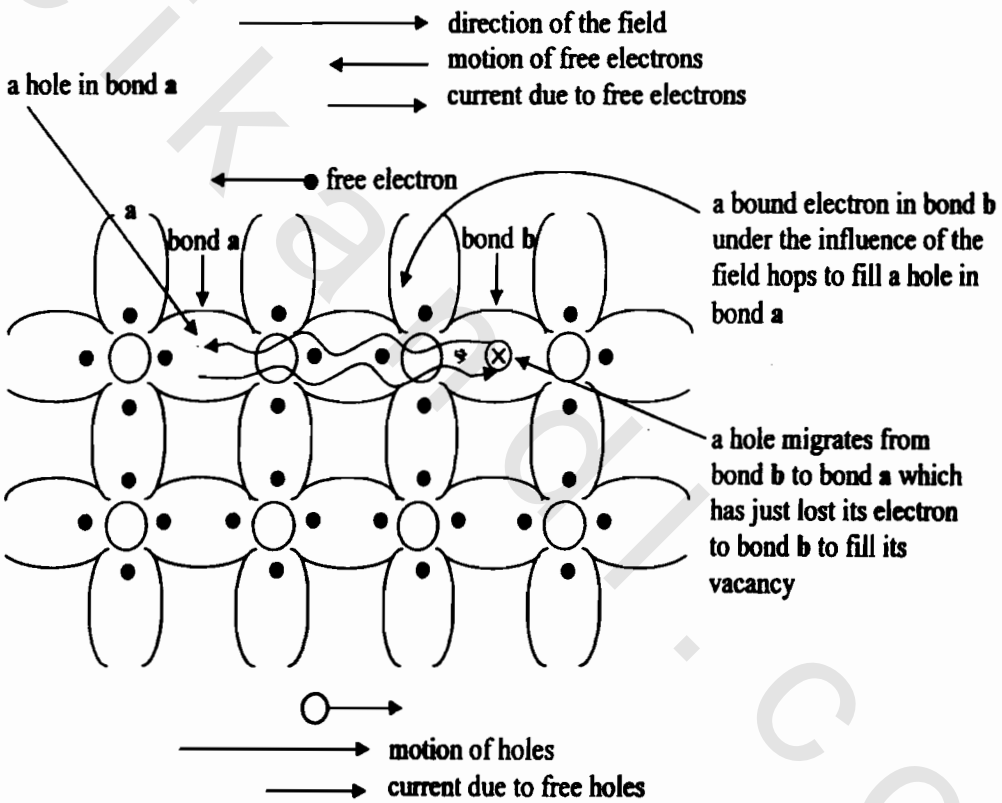

Fig. (2.20) Two current carriers in a semiconductor, a hole in bond is filled by an electron from bond b creating a hole in bond b

2.10 Band Tilt in a Semiconductor

Energy band diagram tilts the same way as in sections (1.26), (1.27), (1.28) for metals. For a semiconductor shown (Fig. 2.21) $V_a > V_b$, $PE_a < PE_b$

$PE_a = |-q|V_a$. Thus, $E_{c_b} > E_{c_a}, E_{v_b} > E_{v_a}$ and the energy band tilts as shown. It is the same tilt for all levels including E_F. From eqn. (2-30)

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

Since E_F and E_{F_i} tilt the same way, we gurantee that n_0 is constant, which is characteristic of drift current. The slope of the band edge and Fermi levels is given by eqn. (1-41)

$$\varepsilon = \frac{1}{|q|}\frac{dE_F}{dx} = \frac{1}{|q|}\frac{dE_{F_i}}{dx} = \frac{1}{|q|}\frac{dE_c}{dx} = \frac{1}{|q|}\frac{dE_v}{dx} \qquad (2\text{-}39)$$

The total shift in Fermi level is given by

$$\Delta E_F = |q|V_{BB} \qquad (2\text{-}40)$$

We can again explain the energy exchange mechanism in a semiconductor under an external electric field as we did for metals (Fig. 2.22). There are two differences, however, in a semiconductor we have two current carriers while in metals we have only one. Secondly, in metals the entire electron population drifts because of the empty levels in the CB. In a semiconductor current is limited only by the Maxwellian tails of electrons and holes (Prob. 2.16).

The power loss P is the energy lost per electron multiplied by the number of electrons crossing per second.

$$P = |q|V_{BB} \times \frac{I}{|q|} = IV_{BB} \qquad (2\text{-}41)$$

Using eqn. (1-37)

$$P = I^2 R \qquad (2\text{-}42)$$

From eqn. (1-35)

$$R = \frac{\ell}{\sigma A} \qquad (2\text{-}43)$$

We should note that the energy band diagram is drawn from the point of view of electrons. That is why electrons slide from point b to point a. if we wish to see that holes slide from a to b as they do, we must reverse the direction of the band tilt as we draw the energy band diagram from the point of view of holes (Prob. 2.11). In the electron band diagram, note that in the VB a hole climbs up from point a to b like a floating bubble, as valence electrons are drawn out by the metal from point a.

Ex. 2.3

Two semiconductor specimens are connected in series. One is n-type with $N_D = 10^{14} cm^{-3}$ length $1cm$ and area $0.1\ cm^2$, and the other is p-type with $N_A = 10^{15} cm^{-3}$ length $4cm$ and area $0.05\ cm^2$.

A battery of 9V is applied across the combination. Calculate the current components in each specimen as well as the total current. Discuss how the power loss is divided between the two specimens. Compare the current density and the rate of carrier supply in all parts of the circuit, and sketch the band diagram. Take $\mu_n = 1300 cm^2/Vs$, $\mu_p = 500 cm^2/Vs$ and $n_i = 1.5 \times 10^{10} cm^{-3}$

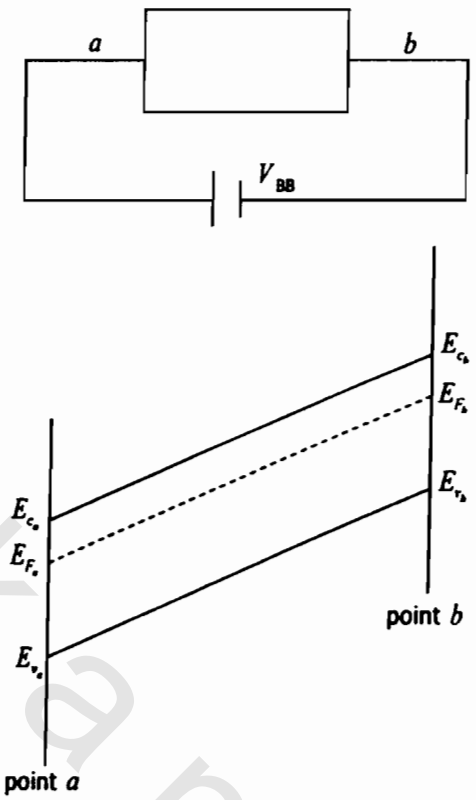

Fig. (2.21) Band tilt in a semiconductor

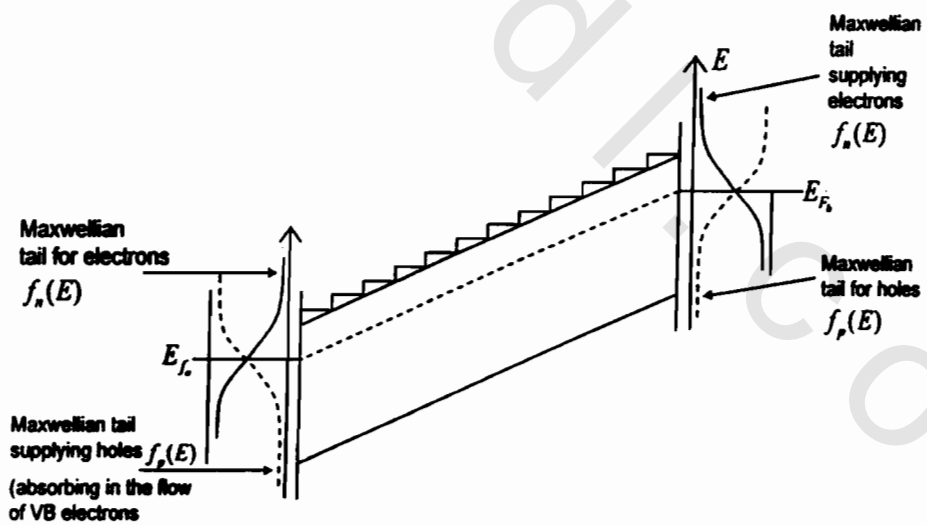

Fig. (2.22) Energy exchange and supply of carriers in an n-type semiconductor under external electric field

Solution

$$\sigma_{1n} = |q|\mu_n N_D = 1.6\times10^{-19}\times1300\times10^{14}$$
$$= 0.02\ \text{A/V cm}$$
$$\sigma_{2p} = |q|\mu_p N_A = 1.6\times10^{-19}\times500\times10^{15}$$
$$= 0.08\ \text{A/V cm}$$

These conductivities are based on majority carriers (electrons in an n-type semiconductor and holes in a p-type semiconductor). Now we look into minority carrier contributions.

$$\sigma_{1p} = |q|\mu_p \frac{n_i^2}{N_D} = 1.6\times10^{-19}\times500\times\frac{\left(1.5\times10^{10}\right)^2}{10^{14}}$$
$$= 1.8\times10^{-10}\ \text{A/V cm}$$

$$\sigma_{2n} = |q|\mu_n \frac{n_i^2}{N_A} = 1.6\times10^{-19}\times1300\times\frac{\left(1.5\times10^{10}\right)^2}{10^{15}}$$
$$= 4.68\times10^{-11}\ A/V\ cm$$

We see that we can neglect minority carrier contributions
Thus,

$$\sigma_1 = \sigma_{1e} \text{ and } \sigma_2 = \sigma_{2h}$$

We note $I_1 = J_1 A_1$ and $I_2 = J_2 A$. In a series circuit $I = I_1 = I_2$
We must then have

$$\frac{J_1}{J_2} = \frac{A_2}{A_1} = \frac{\sigma_1 \varepsilon_1}{\sigma_2 \varepsilon_2}$$

Since $\varepsilon = V/\ell$

$$\frac{J_1}{J_2} = \frac{A_2}{A_1} = \frac{\sigma_1}{\sigma_2}\frac{V_1}{V_2}\frac{\ell_2}{\ell_1}$$
$$\frac{V_1}{V_2} = \frac{\ell_1}{\ell_2}\frac{\sigma_2 A_2}{\sigma_1 A_1}$$
$$= \frac{R_1}{R_2}$$

Using eqn. (2-43), noting that in a series circuit the voltage is divided up in the ratio of resistances

$$R_1 = \frac{\ell_1}{\sigma_1 A_1} = \frac{1}{0.02\times0.1} = 500\Omega$$
$$R_2 = \frac{\ell_2}{\sigma_2 A_2} = \frac{1}{0.08\times0.05} = 1000\Omega$$

The 9 volts will be then divided up between the two resistors R_1 and R_2 in the ratio of their resistances, i.e. $V_1 = 3V$ and $V_2 = 6V$. From Ohm's law

$$I = \frac{9}{1500} = 6\times10^{-3} = 6mA$$
$$J_1 = \frac{I}{A_1} = \frac{6}{0.1} = 60\,mA/cm^2$$

$$J_2 = \frac{I}{A_2} = \frac{6}{0.05} = 120\, mA/cm^2$$

The band diagram is shown (Fig. Ex. 2.3) Fermi levels tilt such that the total potential difference is 9V. In the wire we assume σ to be infinite so $\varepsilon = 0$ there, and J is infinite for finite current I, since $J = \frac{I}{A_m}$, where A_m is the cross sectional area of the metallic wire. We note that the rate of carrier supply is $\frac{I}{|q|}$ which includes an electron supply in the wire and the n-type resistor as well as the hole supply in the p-type resistor which is the outward electron supply in the wire from the p-type semiconductor. Remember that hole movement is nothing more than electron movement is the valence band in the opposite direction.

2.11 Thermistor as a Bulk Device

Bulk devices are devices which depend on properties of the interior not the surface of a material. A resistor may be called a bulk device. Bulk devices depend in their operation on majority carriers.

A thermistor is a bulk device in which conductivity is a function of temperature. This makes it a good candidate for temperature sensing, and hence; temperature control. We take the thermistor to be an intrinsic semiconductor. Assuming

$$\mu_n = \text{constant}\ T^{-3/2} \tag{2-44}$$

$$\mu_p = \text{constant}\ T^{-3/2} \tag{2-45}$$

Using eqn. (2-38)

$$\sigma_i = q\left(\mu_n + \mu_p\right) n_i$$

$$= |q| \times \text{constant}\ T^{-3/2} n_i \tag{2-46}$$

From eqn. (2-24)

$$\sigma_i = |q| \times \text{constant}\ A_0 e^{-E_g/2kT}$$

Taking ℓn of both sides

$$\ell n \sigma_i = D - \frac{E_g}{2k}\frac{1}{T} \tag{2-47}$$

where

$$D = \ell n\left[|q| \times \text{constant}\ A_0\right]$$

Calling $\ell n \sigma_i = y$, $\frac{1}{T} = x$ and $\frac{E_g}{2k} = \gamma_T$

$$y = D - \gamma_T x \tag{2-48}$$

This is the equation of a straight line whose slope is $-\gamma_T$

From eqn. (2-43)

$$\frac{\Delta R}{R} = -\frac{\Delta \sigma_i}{\sigma_i} = -\frac{-\gamma_T}{T^2}\Delta T \tag{2-49}$$

$$= -\alpha_T \Delta T \tag{2-50}$$

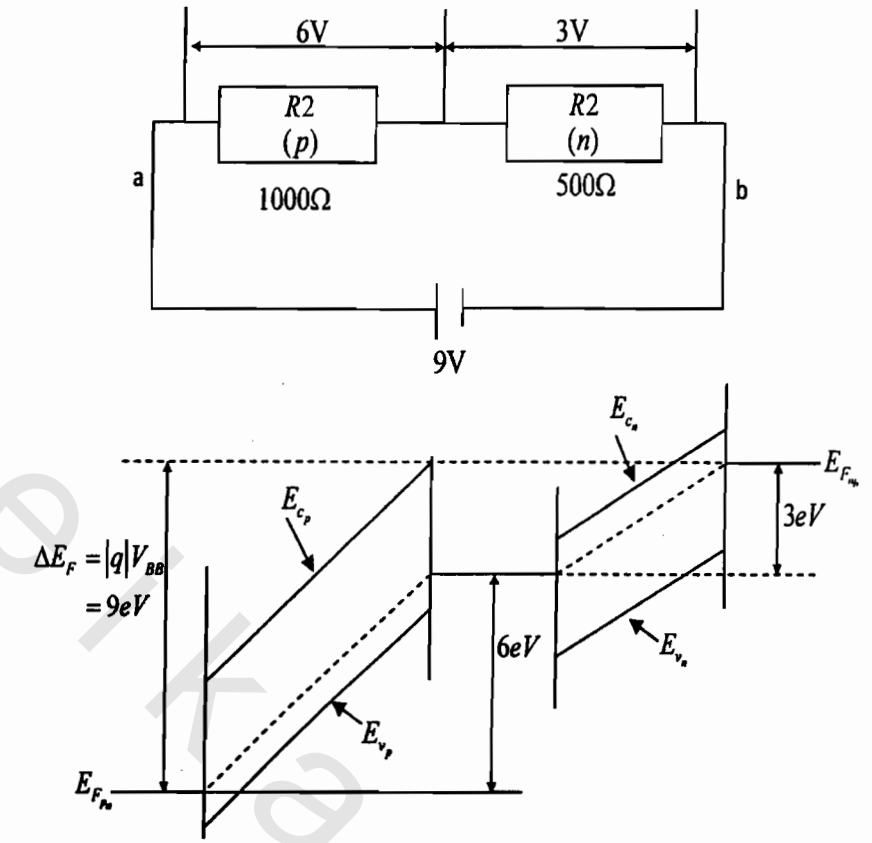

Fig. (Ex. 2.3) Band diagram for two semiconductor resistors in series

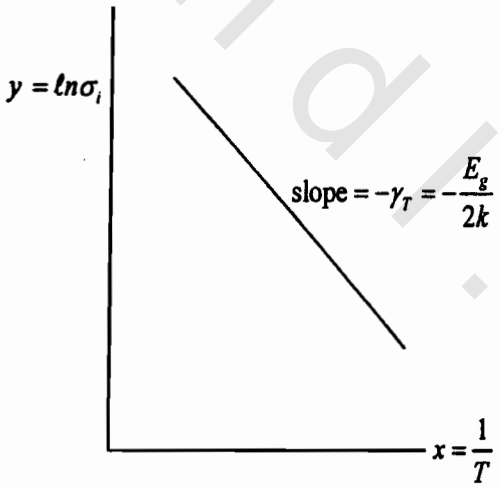

Fig. (2.23) Thermistor characteristic

where α_T is defined as the thermal coefficient of resistance. It is clear that we may use the thermistor to measure temperature (Prob. 2.7). Thermistors can also be used to protect a device (Prob. 2.8). The slope of the characteristic of Fig. (2.23) can be used to determine E_g of the material.

Problems

1- a- Calculate the concentration of silicon atoms?
b- Calculate the conductivity of intrinsic silicon at 300°K?
c- If a donor type impurity is added 0.01 parts per million (ppm), i.e., one part in 10^8, calculate the new conductivity and the percentage change in conductivity?
d- Calculate the percentage of broken bonds to the total bonds at 300°K in intrinsic silicon and the percentage of electrons from donor atoms to those from broken bonds?
(Atomic number of silicon=14, atomic weight=28.1, density=2.33 gram/cm^3,
$n_i(300^o k) = 1.5\times10^{10}/cm^{-3}$, $\mu_n = 1300 cm^3/Vs$, $\mu_p = 500 cm^3/Vs$,
Avogadro's number=6.02×10^{23} atoms/mole)

2- An n-type specimen of silicon is 2 cm long and has 1 x 1 mm cross section. Calculate n_0 and drift velocity if 1 volt is applied, $N_D = 10^{14} cm^{-3}$ with the constants given in the problem above.
a- Calculate the number of scattering events along the transit path of an electron?
b- Show that the total power loss is IV_{BB}

3- a- A silicon crystal is to be grown and it is desired that the ingot contain 10^{16} phosphorus atoms/cm^3. The initial load of silicon is 5 kg. How many grams of phosphorus should be added? The atomic weight of phosphorus is 31.
b- Calculate the position of Fermi level?
c- Calculate the actual electron and hole concentrations. Compare with n_i. What do you calculate?

4- a- Calculate the weight of aluminum to be added in the above problem so that the final silicon crystal becomes intrinsic. The atomic weight of Al is 27.
b- If an error in the weight of the added aluminum occurs in the range of 1%, inspect the effect of this error.

5- Compare the probability function at E_c and at E_D. Then compare the effective density of states at E_c with the density of donor states. What do you conclude?

6- Using reasonable approximations, calculate and sketch the electron and hole distribution profiles for the above problem?

7- Sketch the energy band diagram for holes in a p-type semiconductor. Discuss the band tilt and energy exchange.

8- Calculate the supply of electrons and holes for each specimen in Ex. 2-3. Show how Maxwellian tail can explain the current flow for both electrons and holes in that example?

9- Redo Ex. 2-3 for the two resistors connected in parallel?

10- Redo the above problem if the metal wire has total resistance of 1Ω?

11- Propose a circuit to measure temperature and use Fig. (2.23) to calibrate it for $E_g = 1\,eV$?

12- Propose a circuit for protecting a device using a thermistor.

13- a- Calculate the thermal coefficient of resistance for Si. Is it positive or negative?
b- Calculate the percentage change in resistance for 1% increase in temperature.

14- For a silicon resistor of length $1cm$ and area $0.1cm^2$, $N_D = 1.8\times10^{13} atoms/cm^3$ and $N_A = 3.8\times10^{13} atoms/cm^3$, show that the sample is essentially intrinsic at $500^o K$?

15- Find the position of Fermi level in the above problem if N_D is increased in steps 25-50-100 times.

16- Show how currents in a semiconductor are limited by Maxwellian tails, whereas in metals they are not. Then explain why conduction ceases in semiconductors at $T = 0^o K$ while in metals it does not.

17- Find an expression for the variation of Fermi level with temperature for an n-type material using reasonable approximations?

18- Find the density of ionized phosphorus atoms if $E_D = 0.05$eV below E_c if $n_i = 10^{14} cm^{-3}$?

19- Calculate the probability of occupation of an electron at E_D and at E_c ?

20- Using the density of states function and donor concentrations in the above problem verify that we may neglect the presence of electrons in the donor atoms while we cannot neglect the electron density at room temperature in the CB. Consider $n_i = 10^{10} cm^{-3}$? Make any reasonable assumptions.

References

1- "Solid State Electronic Devices" 6th edition, B. Streetman, Pearson-Prentice Hall, Upper Saddle River, N.J., 2006.

2- "Principles of Electronic Materials and Devices", S.O. Kasap, 2nd edition, McGraw Hill, N.Y., 2002

3- "Physical Foundations of Solid State and Electronic Devices" A.M. Ferendeci, McGraw- Hill, N.Y., 1991.

4- "Semiconductor Devices", M. Zambuto, McGraw- Hill, N.Y., 1989.

5- "Integrated Circuits: Materials, Devices and Fabrication", W. Till, J. Luxon, Prentice Hall, Englewood Cliffs, N.J, 1982.