CHAPTER 10
Phase Locked Loop

10.1 The Loop Action:

The phase locked loop (PLL) consists of the following units (Fig. 10.1): phase detector (PD), loop filter (LF), amplifier (A) and a voltage controlled oscillator (VCO).

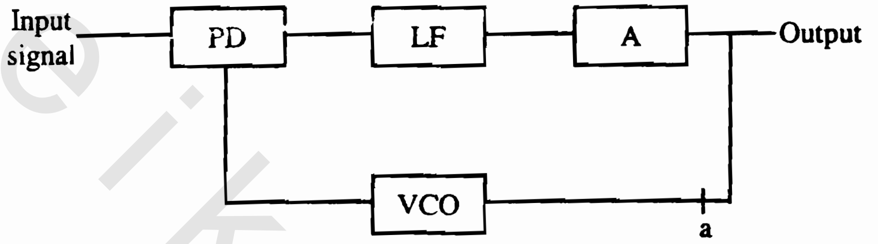


Fig. 10.1 PLL block diagram

Nowadays, this loop is constructed in ICs, and has a variety of applications in communication. If there is no external voltage applied on the VCO, the oscillator will run at its free running angular frequency ω_o. When there is an applied voltage, the oscillator angular frequency ω_{osc} will vary according to this applied voltage as

$$\omega_{osc} = \omega_o + k_o V_o, \qquad (10-1)$$

where k_o is called the oscillator gain (rad/Vs) and V_o is the output voltage of the loop, which is the input to the VCO. Thus, the shift in the oscillator angular frequency from the free running value $\Delta\omega_{osc}$ is given by:

$$\Delta\omega_{osc} = \omega_{osc} - \omega_o = k_o V_o \qquad (10-2)$$

This means that we increase or decrease ω_{osc} according to to the polarity and magnitude of V_o. The phase detector compares the output of the VCO with the input to the PLL. If they both have the same frequency, then the PD produces an average or dc voltage, which is a function of the phase difference ϕ, either linearly or as $\cos\phi$, as we have seen before. The loop filter filters out this output voltage, and the amplifier amplifies it.

When a loop is locked onto an incoming periodic signal, the VCO frequency is exactly equal to the frequency of the incoming signal. The phase detector produces a dc or low frequency signal, which is a function of that phase difference. This phase sensitive signal is then applied to the loop filter and the amplifier, and then to the control input of the VCO.

We must distinguish between several cases. If the frequency of the incoming signal is equal to ω_O (the free running angular frequency of the VCO), we must expect that the output of the PD must be zero, so that the voltage at the input of the VCO is zero to let VCO run at ω_O. We call this case locking onto the free running frequency of the VCO, and it is characterized by $V_O = 0$.

The second case is when ω_i does not equal ω_O, but is close to it. In this case, an output voltage V_O will be developed such as to force the VCO to change its frequency according to eqn. (9-1), so that $\omega_{OSC} = \omega_i$. The phase detector - in this case - compares two waveforms of the same frequency. The output voltage from the PD produces ultimately V_O. A phase shift is forced to exist between the two waveforms sufficient to produce V_O, required for forcing the frequency shift needed for the VCO to follow (track) the incoming frequency. We call this process locking onto ω_i ($\omega_i \neq \omega_O$). The process by which the VCO captures the incoming frequency is called the capture (lock-in) process. The third case is when the loop is already locked, and the input frequency is varied to the extent that the loop can no longer follow the input frequency, and the loop falls out of lock. We call the range over which an already locked loop can maintain lock, the lock-in (hold-in) range. We call the range of frequencies over which an unlocked loop may be locked the capture (lock-in) range. When the loop is unlocked, the VCO runs at ω_o, and $V_O = 0$. Unfortunately, the same is true in the case when the loop is locked at ω_o (Prob. 10.1).

10.2 The Capture Transient Response:

If the frequency of the incoming signal shifts slightly, the phase difference between the VCO signal and the incoming signal will begin to increase with time. This will change the control voltage of the VCO in such a way as to bring the VCO frequency back to the new value of the incoming signal. This would require a voltage V_O to be developed proportional to the shift that occurred in the input frequency. To understand the mechanism of the capture process, let us assume a PLL running at the center frequency ω_o, corresponding to $V_O = 0$. A periodic function is applied at the input with frequency ω_i, slightly less than ω_o. Assume that the loop is open at point a (Fig. 10.1). We can view the PD as a simple analog multiplier. Thus, the PD will contain the components. $\omega_O + \omega_i$ and $\omega_O - \omega_i = \Delta\omega$.

Assume that the sum frequency component is sufficiently high that it is filtered out by the LPF. The output of the LPF is a sinusoid with a frequency equal to the difference between the VCO free running frequency, and the incoming signal frequency, i.e., $\cos(\Delta\omega t + \varphi_O)$.

Now, assume that the loop is suddenly closed. The difference frequency sinusoid is now applied to the VCO input. Thus, $V_O = C \cos(\Delta\omega t + \varphi_O)$. This will cause the VCO frequency ω_{OSC} to become a sinusoidal function of time according to eqn. (10.1)

$$\omega_{OSC} = \omega_O + k_O C \cos(\Delta\omega t + \varphi_O) \quad (10-3)$$

$$\Delta\omega_{OSC} = \omega_{OSC} - \omega_O = k_O C \cos(\Delta\omega t + \varphi_O) \quad (10-4)$$

The VCO frequency ω'_{OSC} moves closer to the incoming frequency ω_i and further away.

$$\omega'_{OSC} = \omega_i + \Delta\omega + k_O C \cos(\Delta\omega t + \varphi_O) \quad (10-4)$$

$$\Delta\omega'_{OSC} = \omega_{OSC} - \omega_i = \Delta\omega + k_O C \cos(\Delta\omega t + \varphi_O) \quad (10-5)$$

Thus, the output of the PD is near sinusoid, whose frequency is the difference between the VCO frequency and the input frequency, i.e., $\Delta\omega'_{OSC}$. When the VCO frequency moves away from the incoming frequency, this sinusoid moves to a higher frequency. When the VCO frequency moves closer to the incoming frequency the sinusoid moves to a lower frequency, as seen from eqn. (10.5). The frequency of this sinusoidal difference-frequency waveform is reduced when its incremental amplitude is negative, and increased when its incremental amplitude is positive. This causes the PD output to have an asymmetrical waveform during capture. This asymmetry in the waveform introduces a dc component in the PD output that shifts the average VCO frequency toward the incoming signal frequency, so that the difference frequency gradually decreases, and $\Delta\omega'_{OSC}$ reaches a steady state

$$\Delta\omega'_{OSC} = \Delta\omega = \omega_0 - \omega_i .$$

Thus, we may modify eqn. (10.5) to be

$$\Delta\omega'_{OSC} = \Delta\omega + k_O C \cos(\Delta\omega t + \varphi_O)e^{-\alpha t}, \quad (10-6)$$

where the exponential term $e^{-\alpha t}$ accounts for the decay due to the asymmetrical shift in frequency, and the eventual growth of a steady state dc voltage.

$$V_0 = \frac{\Delta\omega}{k_O} \quad (10-7)$$

When the system is locked, $\omega_{OSC} = \omega_i$. In this case, the voltage that develops at the amplifier output V_O is negative, since $\omega_i < \omega_O$. This process is shown in Fig. 10.2

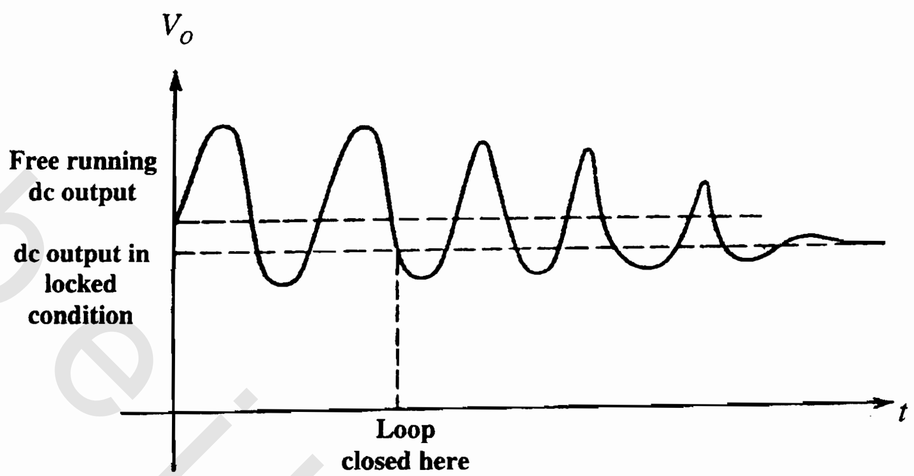


Fig. 10.2 PD output during capture transients

During the capture process, we may define the pull-in time as the time required for the loop to capture a signal. Both the capture range and pull-in time are parameters which depend on the amount of gain in the loop itself, and the bandwidth of the loop filter. We have noted that the loop filter acts to filter out the sum frequency components and spurious signals far from the center frequency. It also provides a memory for the loop shift, in case lock is momentarily lost due to a large interfering signal. Reducing the loop filter bandwidth improves the rejection of out - of - band signals, but at the same time the capture range is decreased, and the pull-in time becomes longer. On the other hand, when the BW is increased, the loop may respond to interfering transients more readily, the capture range is increased, the pull-in time becomes shorter. The loop may then respond to fast variations in the input frequency. Usually, a compromise is made.

10.3 Phase Detector Characteristics:

In general, we may consider the output of PD, (υ_D) as the average of the product of the two inputs. If the two inputs are $x(t)$ and $y(t)$, then υ_D is given by

$$\upsilon_D(\tau) = \frac{1}{T}\int_{o}^{T} x(t)\ y(t+\tau)dt = R_{xy}(\tau)\,, \qquad (10-7)$$

where $R_{xy}(\tau)$ is the crosscorrelation function of $x(t)$ and $y(t)$, T is the period of the input waveform, and τ is the time delay between the two input waveforms. If both inputs are sinusoids

$$x(t) = A_1 \cos \omega_{OSC} t$$

$$y(t) = A_2 \cos \omega_{OSC} t$$

Then, $R_{xy}(\tau) = \frac{1}{2} A_1 \; A_2 \cos \omega_{OSC} \tau$,

or $$R_{xy}(\phi) = \frac{1}{2} A_1 \; A_2 \cos \phi \qquad (10-8)$$

This result applies to the case when both inputs are sinusoidal, and to the case when one input is sinusoidal, while the other is a square wave. When both inputs are square waves (or amplitude limited sinusoids), the PD characteristic becomes triangular (Fig. 10.3).

From the PD characteristics, we observe that there are points at which υ_D is zero. These lie at odd multiples of $\pm T/4$. These points are called the null points, which are the operating points when the loop is locked onto the free running angular frequency ω_O, or when it is not locked at all, and hence, running at ω_O. The loop action aims at correcting the VCO to keep the loop locked at angular frequency equal to ω_i. This would require a voltage υ_D or V_O to produce this desired shift in the VCO frequency. We note that the PD output goes through a null point with a positive slope for $\tau=-T/4 \pm T$ (or $\phi=-\pi/2 \pm 2\pi$), and with a negative slope for $\tau=-T/4 \pm T$ (or $\phi=\pi/2 \pm 2\pi$).

To understand how the loop action works, assume that the operating point is at the null point $\tau = T/4$. Suddenly, the input frequency is reduced below ω_O. This means that the input is delayed by $\Delta\tau$, and the phase shift increases by $\Delta\phi$. This produces a negative voltage on υ_D (Fig. 10.3b). Assume that the amplifier in the loop is noninverting. Thus, the negative voltage on υ_D will be transmitted to the input of the VCO, which will tend to reduce the VCO frequency from ω_O to match ω_i. Thus, the loop action tends to keep the operating point close to the null point. On the other hand, if the amplifier is inverting, the loop will produce an incremental delay which will add to $\Delta\tau$, and will drive the loop away from the operating point, rendering the loop unstable. Thus, this null point is a stable operating point for a noninverting amplifier, and an unstable operating point for an inverting amplifier. Conversely, for the null point at $\tau = -T/4$, the above statement is reversed. We note that when the loop is locked onto an incoming signal with $\omega_i = \omega_O$, or when the loop is unlocked, there is a phase shift between the incoming signal and the VCO output equal to $\pi/2$.

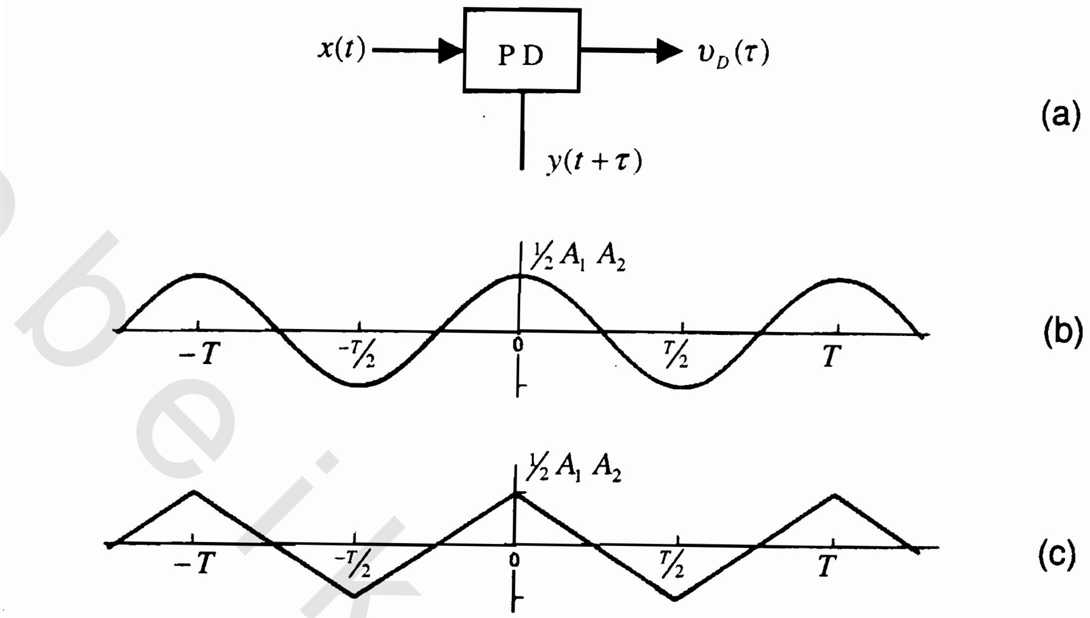


Fig. 10.3 PD characteristics

a) crosscorrelator
b) two sinusoidal inputs one sinusoidal and one square wave inputs
c) two square wave inputs

This phase shift is forced by the loop action. When ω_i varies around ω_O, a phase shift occurs in the vicinity of $\pi/2$ (lead or lag). Hence, a voltage $V_O(t)$ develops (positive or negative) to maintain this phase shift. It is convenient to call the total phase shift $\phi = \frac{\pi}{2} + \theta$, where θ represents variations of phase around the null point. At the null point, the incoming signal and the VCO output are at $\frac{\pi}{2}$ phase difference, whereas around the null point, we have:

$$|\upsilon_D(t)| = k_D\,|\cos\phi| \qquad (10-9)$$

$$= k_D\left|\cos\left(\frac{\pi}{2} + \theta(t)\right)\right|$$

$$= k_D\,|\sin\theta(t)|, \qquad (10-10)$$

where k_D is the PD gain (V). For small values of $\theta(t)$,
eqn. (10.10) reduces to

$$|\upsilon_D|(t) = k_D\;\theta(t) \qquad (10-11)$$

This equation is restricted to small deviations from the null point, or to the range $-T/4$ to $+T/4$ for square wave inputs. It is referred to as the case of a linearized PLL.

10.4 PLL Performance:

From eqn. (10-2), we note that the voltage developed at the output of the PLL, $V_O(t)$, is given by

$$V_O(t) = \frac{1}{k_O}\,\Delta\omega_{OSC}(t) = \frac{1}{k_O}\left[\omega_{OSC}(t) - \omega_O\right]$$

$$= \frac{1}{k_O}\left[\omega_i(t) - \omega_O\right] = \frac{1}{k_O}\,\Delta\omega_i(t) \qquad (10-12)$$

Thus, the output voltage is proportional to the instantaneous variation of the input frequency, provided that locking has occurred and has been maintained throughout the time variation of $\omega_i(t)$. Thus, the PLL acts in this mode as a perfect FM demodulator, where

$$\omega_i(t) = \omega_c + \Delta\hat{\omega}\cos\omega_m t \qquad (10-13)$$

Thus, for $\omega_O = \omega_c$,

$$V_O(t) = \frac{1}{k_O}\,\Delta\hat{\omega}\,\cos\,\omega_m t \qquad (10-14)$$

This result is conditional on the validity of eqn. (10.12).

From the PD relations (10.10) and (10.11), we have incremental characteristics of sinusoidal PD and triangular (linear) PD, Fig. (10.4).

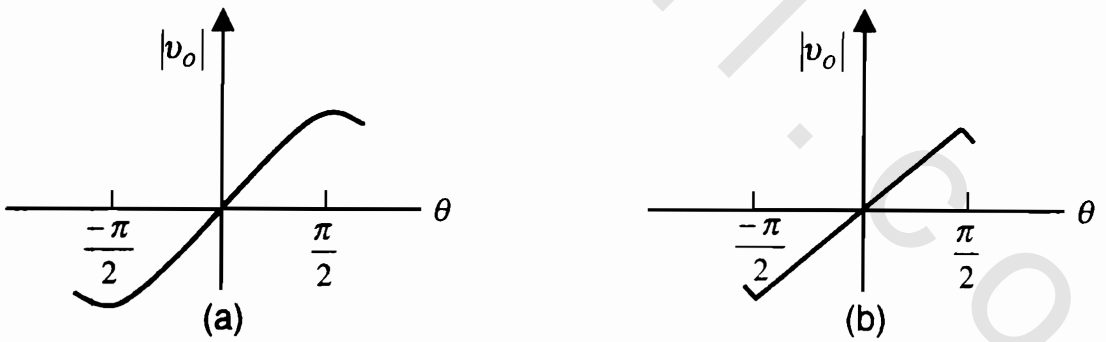


Fig. 10.4 Incremental PD characteristics
a) sinusoidal PD b) triangular PD

From eqn. (10-10),

$$\sin\theta = v_D / k_D \qquad (10-15)$$

From eqn. (10-12),

$$\Delta\omega_{OSC} = k_O \; V_O \tag{10 - 16}$$

We define the open circuit gain k_V as

$$k_V = k_D \; A \; k_O \tag{10 - 17}$$

From eqns. (10.-5) and (10-16), eqn. (10-17) becomes

$$k_V = \frac{\upsilon_D}{\sin\theta} \frac{V_O}{\upsilon_D} \frac{\Delta\omega_{OSC}}{V_O} \tag{10 - 18}$$

$$= \frac{\Delta\omega_{OSC}}{\sin\theta} \tag{10 - 19}$$

or

$$\Delta\omega_{OSC} = k_V \sin\theta \tag{10 - 20}$$

For triangular PD characteristic,

$$\theta = \frac{\upsilon_D}{k_D} \tag{10 - 21}$$

Eqn. (10-18) becomes

$$k_V = \frac{\upsilon_D}{\theta} \frac{V_O}{\upsilon_D} \frac{\Delta\omega_{OSC}}{V_O} = \frac{\Delta\omega_{OSC}}{\theta} \tag{10 - 22}$$

or

$$\Delta\omega_{OSC} = k_V \; \theta \tag{10 - 23}$$

For a sinusoidal PD, the maximum value of $\theta = \pm\pi / 2$, (or $\sin\theta = \pm 1$), corresponding to maximum $\Delta\omega_{OSC}$.

From eqn. (10.20),

$$\Delta\omega_L = \Delta\hat{\omega}_{osc} = \pm k_V \tag{10 - 24}$$

For a triangular PD, the maximum value of $\theta = \pm\frac{\pi}{2}$

From eqn. (10.23),

$$\Delta\omega_L = \Delta\hat{\omega}_{osc} = \pm\frac{\pi}{2} k_V , \tag{10 - 25}$$

where $\Delta\omega_L$ is the lock-in range. Thus, from eqns. (10.24) and (10.25), we have the maximum hold-in range. We see that k_V has a special meaning. It is roughly equal to the lock-in or hold-in range. The higher k_V, the larger the range the PLL can capture and remain locked onto an incoming frequency.

10.5 Steady State Analysis of a Linearized PLL at Locking:

We assume a linear PLL. This can come about either by considering a triangular PD characteristic, or a linearized PD characteristic when $\sin\theta$ is replaced by θ (for small θ).

In the block diagram (Fig. 10.5), each block has its transfer function expressed in its gain constant. Here, k_D represents the PD, $F(s)$ represents the loop filter, A is the amplifier gain, k_O represents the VCO. We note that the integrator block $(1/s)$ is added. Actually, this is not a hardware component. It is simply there, because we want to look into the phase difference not the frequency. We also note that ω is the time rate of φ or θ, i.e., $\omega(t) = \frac{d\theta_i(t)}{dt}$.

We know that in Laplace's terminology, this is equivalent to saying

$$\Delta\omega_i(s) = s\,\theta_i(s) \qquad (10-26)$$

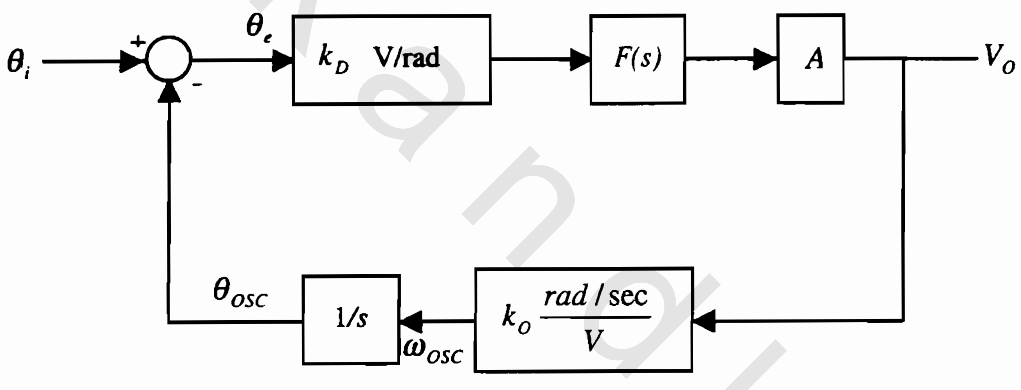


Fig. 10.5 Block diagram for steady state analysis of a linearized PLL

All θ's represent phase deviations from the null point. Thus, θ_i corresponds to the phase shift for the input waveform, θ_{OSC} represents the phase shift for the VCO output, θ_D represents the phase shift for the difference operation inherent in the PD, ($\Delta\omega_i$) refers to the deviation of ω_i form ω_O. As $s = 0$ (dc), $\Delta\omega_i = 0$ and $\omega_i = \omega_0$ or a constant (dc) shift from ω_0.

Thus,

$$\theta_D = \theta_i - \theta_{OSC} \qquad (10-27)$$

$$\dot{\theta}_D = \omega_i - \omega_{OSC} \qquad (10-28)$$

When $\omega_i = \omega_{OSC}$ (locking), θ_D is constant and $\dot{\theta}_D = 0$, while $\dot{\theta}_i = \omega_i$ and $\dot{\theta}_{OSC} = \omega_{OSC}$.

The closed loop transfer function is given by

$$\frac{V_O(s)}{\theta_i\,(s)} = \frac{k_D\ F(s)\,A}{1 + k_D\ F(s)\,A\dfrac{k_O}{s}} \qquad (10-29)$$

We may obtain the transfer function in terms of $\Delta\omega_i$, using eqn. (10.26),

$$\frac{V_O(s)}{\Delta\omega_i(s)} = \frac{1}{s}\ \frac{V_O(s)}{\theta_i(s)} = \frac{k_D\ F(s)\,A}{s + k_D\,k_O\,A\ F(s)} \qquad (10-30)$$

We consider the case when the loop filter is removed, i.e., $F(s)=1$. This is called first order loop. Using eqn. (10.17),

$$\frac{V_O(s)}{\Delta\omega_i(s)} = \frac{1}{k_O}\ \frac{k_V}{s + k_V} \qquad (10-31)$$

This is the transfer function of a LPF. Thus, the loop inherently acts as a LPF, whose dc response is k_V / k_0, and the 3dB (cut off) point or angular bandwidth is k_V.

We should note that eqn. (10.31) expresses the transfer function related to $\Delta\omega_i$, which is the input angular frequency referred to ω_O. In eqn. (10.31), s refers to the complex angular frequency of the transfer function, not the absolute value of $\Delta\omega_i$. In other words, if we substitute $s = j\Omega$, then, Ω is the angular frequency of variation of $\Delta\omega_i(t)$, assuming sinusoidal time dependence for $\Delta\omega_i(\Omega,t)$.

When $\Delta\omega_i$ is a dc angular frequency shift, then Ω=0. In this case,

$$\frac{V_{O\,dc}}{\Delta\omega_{i\,dc}} = \frac{1}{k_O}, \qquad (10-32)$$

which is expected, as $\Delta\omega_i = \Delta\omega_{OSC}$ at locking. This is true, whether locking is at ω_O or at an angular frequency deviated from ω_O, i.e.,

$$\omega_i = \omega_{OSC} = \omega_O + k_O\ V_O \qquad (10-33)$$

$$\Delta\omega_i = \Delta\omega_{OSC} = k_O\ V_O \qquad (10-34)$$

Thus, $\Delta\omega_i$ is constant (dc), and V_O is a dc voltage. In the special case $\Delta\omega_i = O$, $V_O = O$, $V_O / \Delta\omega_i$ is finite from eqn. (10.32). When $\Delta\omega_i$ is a sinusoidal function of time - which is the case of FM with carrier $\omega_c(t) = \omega_i(t)$ - we have

$$\omega_c(t) = \omega_i(t) = \omega_{OSC}(t) = \omega_O + \Delta\hat{\omega}\ \sin\omega_m t \qquad (10-35)$$

$$\Delta\omega_i(t) = \Delta\hat{\omega}\ \sin\omega_m t \qquad (10-36)$$

In this case, $\Omega = \omega_m$. The output $V_O(t)$ can be obtained from eqn. (10.31). However, that equation must be put in a phasor form, i.e., when $s = j\Omega$,

$$\frac{V_O(j\,\Omega)}{\Delta\omega_i\,(j\,\Omega)} = \frac{1}{k_O}\,\frac{k_V}{j\Omega + k_V} \qquad (10-37)$$

We must distinguish here among several cases:

a) $\Omega_m < k_v$, and $\hat{\Delta\omega}$ is within the lock-in range.

In this case, $\frac{V_O(j\,\Omega)}{\Delta\omega_i\,(j\,\Omega)} = \frac{1}{k_O}$ (10 – 38)

$$V_O(t) = \frac{1}{k_O}\,\hat{\Delta\omega}\,\sin\omega_m t = \frac{1}{k_O}\,\hat{\Delta\omega}\,\sin\Omega_m t\,, \qquad (10-39)$$

where $\omega_m = \Omega_m$, which is the modulating signal. Thus, the PLL acts as a perfect FM demodulator.

b) $\Omega_m \geq k_v$, and $\hat{\Delta\omega}$ may not exceed the lock-in range. Thus, eqn. (10.37) prevails. We have

$$V_O(t) = \frac{1}{k_O}\,\frac{k_V}{\sqrt{\Omega^2 + k_V^2}}\,\sin\left(\Omega_m t - \Psi\right) \qquad (10-40)$$

where

$$\Psi = \tan^{-1}\Omega_m / k_V \qquad (10-41)$$

c) $\Omega_m >> k_V$, and $\hat{\Delta\omega}$ lies outside the lock-in range, $V_O(t) = O$ as the PLL loses locking.

Thus, k_V has a special meaning. It is the angular bandwidth for which the PLL transfer function is constant. Beyond k_V, the loop attenuates the output at a rate of 6dB/octave. We should distinguish between the lock-in range and the loop angular bandwidth, although both are governed by k_V. The maximum frequency variation should not exceed the lock-in range to maintain locking. The modulating frequency $\Omega_m = k_V$ should not increase beyond k_V, otherwise the response deteriorates rapidly.

10.6 The Lock-in Feature:

We should not confuse the angular bandwidth required in PLL detection of FM signal - which is basically $\Omega_m \leq k_V$ - and the angular bandwidth dictated by Carson's rule

$$BW_{\omega} = 2\omega_m (1+\beta) \qquad (10-42)$$

In PLL, we do not require the entire bandwidth needed for FM as given by eqn. (10.42). We only require an angular bandwidth equal to ω_m. Thus, the reduction of the bandwidth needed for FM demodulation is at the ratio of $2(1+\beta)$. This is a major advantage for the use of PLL as an FM detector.

We see now that the choice of k_V must be a compromise between several factors. Increasing k_V increases noise and increases interference. However, it increases the capture range, making it easier for the capture process to take place. It also reduces the response time, making the PLL better suited to follow fast variations in frequency, hence, better able to keep track of fast changing frequencies.

The main power of PLL lies in its ability to lock onto an incoming signal, and the fact that it does not require a lot of bandwidth for this. This is particularly obvious when the modulating frequency is much less than the carrier frequency. In this case, all what is needed is a PLL with small k_V, just to account for the modulation - if any - of a near pure carrier transmission. With almost zero bandwidth, the noise is minimal. This is particularly useful in the use of laser radars and in the use of lock-in amplifiers, where the name of the game is to extract a weak signal buried in noise. The lock-in amplifier picks up the signal with a very small noise contamination. So, the lock-in amplifier is capable of improving S/N, and is used in a variety of applications in research, as well as in the military, e.g., beam riding missiles, smart bombs and laser guided bombs. Lock-in techniques are used to lock in onto the illuminating beam, or the beam reflected from the target. Because of the small bandwidth, both noise and interference effects are minimal. Jamming is only possible if the enemy knows exactly the carrier frequency used, which can be controllably changed by changing ω_O.

Ex. 10.1:

A PLL has $k_O = 10 kHz/V$, $k_V = 200\ Hz$, $f_O = 1\ kHz$. Find the output in the following cases:

a) $f_i = 250\ Hz$

b) $f_i(t) = 1\ kHz$

c) $f_i(t) = 2\pi(1+0.1\sin 2\pi \times 100\, t)kHz$

d) $f_i(t) = 2\pi(1+0.1\sin 2\pi \times 1000\, t)kHz$

e) $f_i(t) = 2\pi(1+0.5\sin 2\pi \times 100\, t)kHz$

Solution:

a) From eqn. (10.32),

$$V_{odc} = \frac{1}{k_O}\ \Delta\omega_{idc} \ = \frac{-2\pi}{10}\times 0.75 = -0.075\times 2\pi \ \ V$$

b) $V_{odc} = \dfrac{2\pi}{10}\times 0.5 + 0.05\times 2\pi \ \ V$

c) $f_c = f_i = 2\pi \ \ kHz$, $\quad \Delta\hat{f} = 2\pi\times 0.1$, $\quad f_m = 2\pi\times 0.1 \ \ kHz$

Noting that both $\Delta\hat{f}$ and f_m are within the lock-in range and bandwidth,

$$V_O(t) = \frac{2\pi}{10}\times 0.1\sin 2\pi\times 100\ t$$

d) $f_m = 1\ kHz >> k_V / 2\pi$

$V_O = O$

e) $\Delta\hat{f} = 0.5\,kHz > k_V / 2\pi$

The PLL is out of lock, $V_O = O$

10.7 Phase Compensation:

Operating the loop with no loop filter is undesirable. Since the PD is a multiplier, the sum frequency component and the out - of - band interfering signals will appear at the output. Most common configurations for PLL contain a loop filter. Suppose $F(s)$ is a single- pole LPF (Fig. 10.6) expressed by,

$$F(s) = \frac{1}{1+\dfrac{s}{\omega_1}} \qquad (10-43)$$

Substituting in eqn. (10.30),

$$\frac{V_O(s)}{\Delta\omega_i(s)} = \frac{1}{k_O}\,\frac{k_V\left(\dfrac{1}{1+\dfrac{1}{\omega_1}}\right)}{s + k_V\left(\dfrac{1}{1+\dfrac{s}{\omega_1}}\right)}$$

$$= \frac{1}{k_O}\,\frac{1}{1+\dfrac{s}{k_V}+\dfrac{s^2}{\omega_1 k_V}} \qquad (10-44)$$

$$= \frac{1}{k_o} \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1}, \qquad (10-45)$$

where

$$\omega_n = \sqrt{k_v\ \omega_1} \qquad (10-46)$$

$$\xi = \frac{1}{2}\sqrt{\frac{\omega_1}{k_V}}, \qquad (10-47)$$

where ξ is the damping factor.

The roots of the transfer function are

$$s_{1,2} = -\xi\,\omega_n \pm \sqrt{(\xi\,\omega_n)^2 - \omega_n^2} \qquad (10-48)$$

$$= \frac{-\omega_1}{2}\left[1 \pm \sqrt{1 - \frac{4k_V}{\omega_1}}\right] \qquad (10-49)$$

The root locus for this feedback system as k_v varies is shown along with the frequency response (Fig. 10.6), (see Appendix K).

The basic factor setting up the bandwidth is k_V, as in the first order loop. As ω_1 decreases, the damping factor decreases, resulting in more peaking in the maximum (Fig. 10.7). The peaking is of concern, because it distorts the demodulator FM output. It also introduces transient ringing for a poorly damped oscillating response.

Thus, ω_1 can be made as low as possible, without causing an unacceptable amount of peaking in the frequency response. A good compromise is using a maximally flat LPF configuration, in which the poles are placed as radials angled 45° from the negative real axis.

ξ in this case = $\frac{1}{\sqrt{2}}$.

Thus, from eqn. (10-47),

$$\frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{\frac{\omega_1}{k_V}} \qquad (10-50)$$

$$\omega_1 = 2\,k_V$$

Thus,

$$\omega_{3dB} = \omega_n = \sqrt{k_V\,\omega_1} = \sqrt{2}\,k_V \qquad (10-51)$$

This can be proven (Prob. 10-8). Also, the rise time t_r is given by (prob. 10-2);

$$t_r = \frac{2.2}{\omega_{3dB}} \qquad (10-52)$$

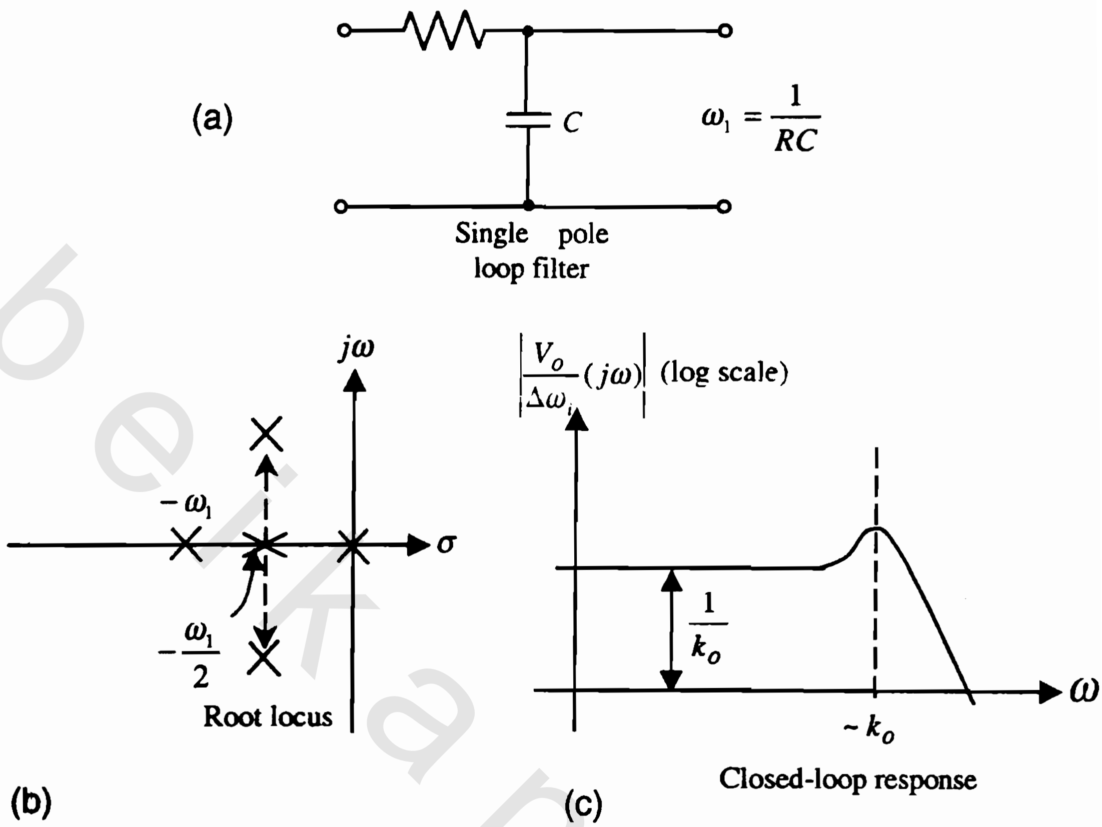


Fig. 10.6 Response of second order PLL
a) circuit b) root locus c) frequency response

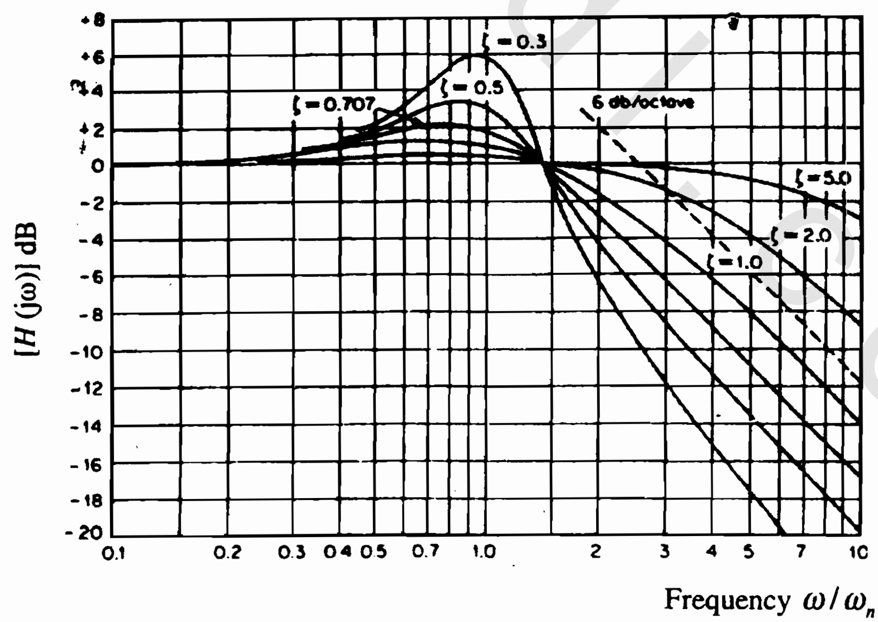


Fig. 10.7. Frequency response to a second order loop for several damping ratios

A disadvantage of the second order loop is that the 3dB bandwidth of the loop is basically determined by k_v. But k_v sets the lock-in range. In communication systems, we need a wide lock range for tracking large signal variations, yet a narrow loop bandwidth for rejecting out - of - band signals. Using very small ω_1 would accomplish this. However, as ω_1 becomes small, an underdamped loop response results. What we need is to add a zero to the loop filter, so that the loop filter pole can be made small, while still maintaining a good loop damping.

To understand what is going to happen due to the added zero, let us first look at the open loop response with no loop filter. Because of the integration inherent in the loop, the response has 20 dB/ decade slope, and crosses unity at k_V (Fig. 10.8a). In Fig 10.8b, a loop filter in which $\omega_1 << k_v$ has been added. As a result, the loop phase shift is very nearly 180^{O} at the crossover frequency. By adding a zero in the loop filter at ω_2 (Fig. 10.8c), the loop margin can be greatly improved.

The loop bandwidth - which is equal to the crossover frequency - is $<< k_V$. This ability to set up loop bandwidth and k_V independently is an advantage of this type of loop filter. To realize this scheme, an RC circuit is shown together with the root locus and the closed loop response (Fig. 10.9).

In this case, $\omega_{3dB} \cong k_V(\omega_1/\omega_2)$ (10 – 52)

10.8 The Effect on Capture Range:

We can now investigate the capture range. The lock-in range is based on the assumption that the loop was initially locked. This remains unchanged. When the PLL is not initially locked onto the signal, the frequency of the VCO is f_O. The phase angle difference between the signal and the VCO is $\Delta\phi = (\omega_i - \omega_O)t + \Delta\theta$, and thus, is not constant but changes with time at a rate $\frac{d\theta}{dt} = \omega_i - \omega_0$. The PD output is an ac voltage with a triangular waveform of peak amplitude $k_D\frac{\pi}{2}$ and a fundamental angular frequency of $\omega_i - \omega_0$. If the LPF is a simple RC network, it has a transfer function

$$H(\omega) = \frac{1}{1+j\omega\tau} = \frac{1}{1+j\dfrac{\omega}{\omega_1}}, \qquad (10-50)$$

When $\tau = RC$, $\omega_1 = \frac{1}{RC} = \frac{1}{\tau}$

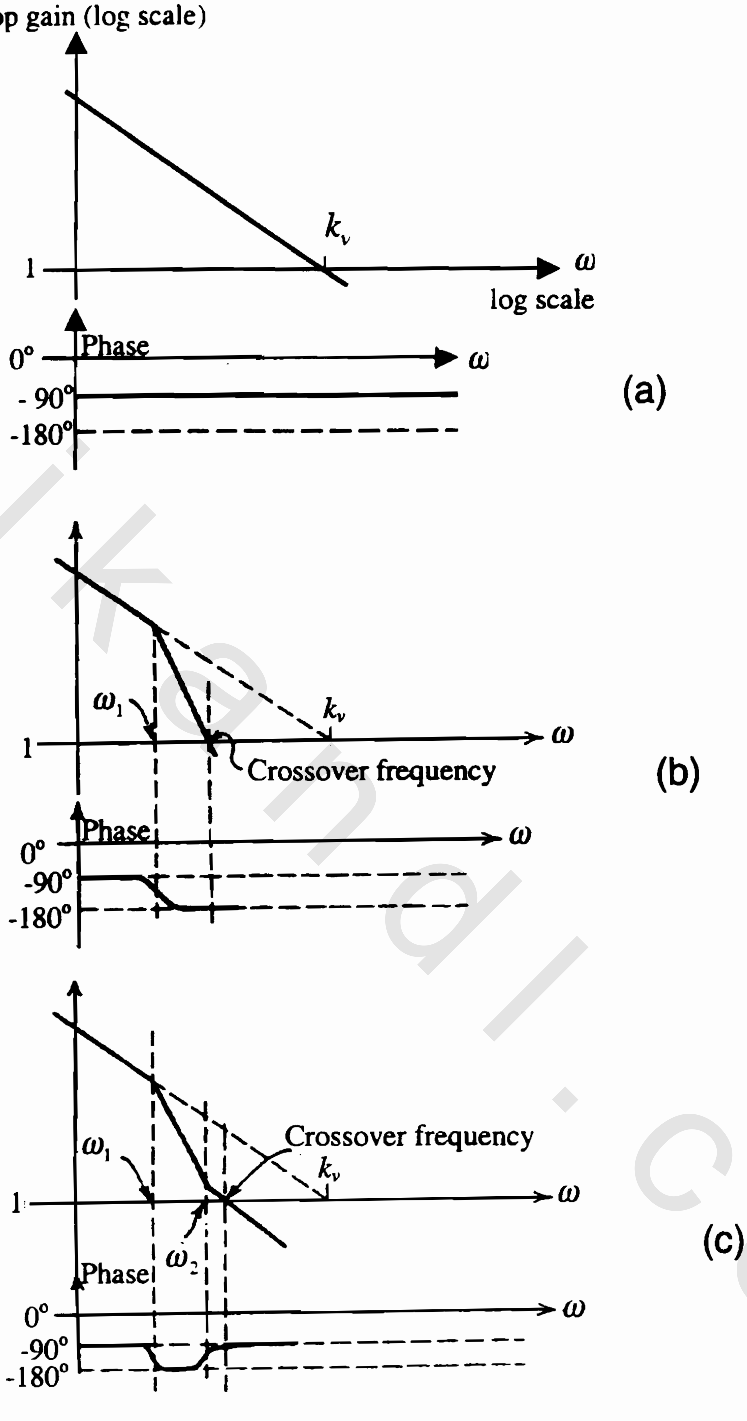
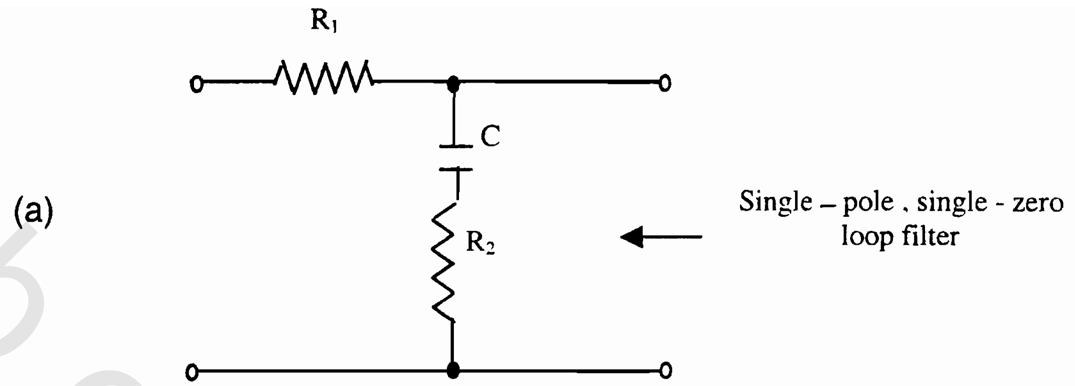


Fig. 10.8 Open loop response with a zero added in the loop

a) with no loop filter b) with a single-pole filter, and $\omega_1 << k_v$

c) with a zero added in loop filter at $s = -\omega_2$



$$F(s)=\left(\frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}}\right)\quad,\quad \omega_2=\frac{1}{CR_2}\quad,\quad \omega_1=\frac{1}{C(R_1+R_2)}$$

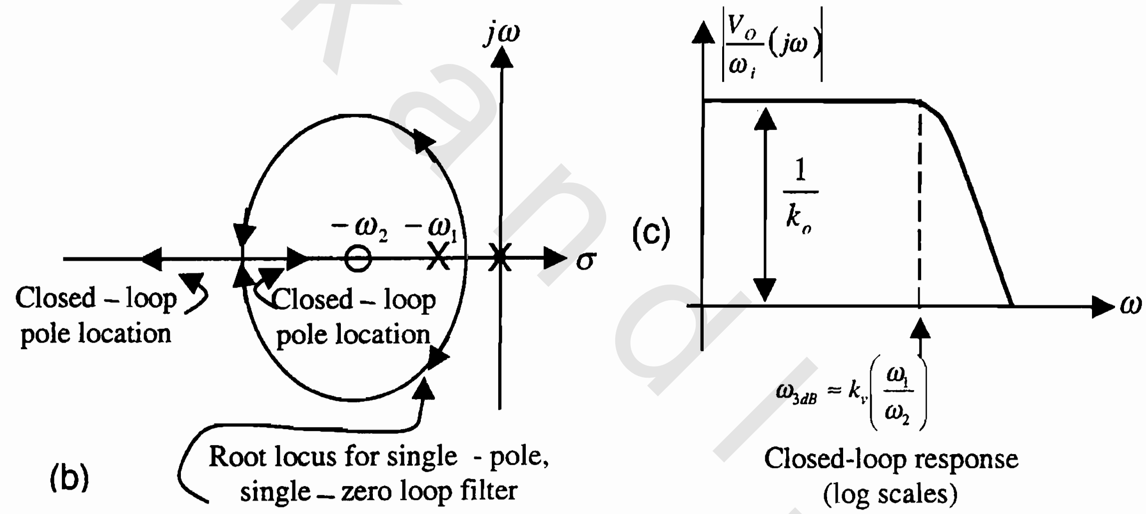


Fig. 10.9 Single-pole single-zero loop filter
a) circuit b) root locus c) closed loop response

For $\left(\frac{\omega}{\omega_1}\right) >> 1$,

$$H(\omega) \approx \frac{\omega_1}{j\omega} \tag{10-51}$$

The fundamental input angular frequency term supplied to LPF by the phase detector is at $\Delta\omega = \omega_i - \omega_0$. If $\Delta\omega > 3\omega_1$, from eqn. (10.51),

$$|H(\Delta\omega)| = \omega_1 / \Delta\omega \tag{10-52}$$

The voltage available as VCO input is

$$V_{OSC} = \upsilon_D H(\omega) A, \qquad (10-53)$$

which has a maximum value

$$\hat{V}_{OSC} = \pm k_D \frac{\pi}{2} \frac{\omega_1 A}{\Delta\omega} \qquad (10-54)$$

The corresponding value of the maximum VCO angular frequency shift is given by

$$(\omega - \omega_O)_{\max} = k_O \hat{V}_{OSC} \qquad (10-55)$$

$$= \pm k_O k_D \frac{\pi}{2} A \frac{\omega_1}{\Delta\omega} \qquad (10-56)$$

For the acquisition of the signal frequency, we must have $f = f_i$, so the maximum angular frequency range that can be acquired by the PLL ($\Delta\omega_c$) is given by

$$\Delta\omega_c = (\omega_i - \omega_O)_{\max} = \pm k_V \frac{\pi}{2} \frac{\omega_1}{\Delta\omega_c} \qquad (10-57)$$

$$(\Delta\omega_c)^2 = \omega_1 \Delta\omega_L, \qquad (10-58)$$

where $\Delta\omega_L$ is the lock-in range given by eqn. (10.25).

Thus, $$\Delta\omega_c = \sqrt{\omega_1 \Delta\omega_L} \qquad (10-59)$$

The full capture range is $2\sqrt{\omega_1 \Delta\omega_L}$, for the normal case of $\Delta\omega_L >> \omega_1$. We see that a PLL cannot acquire a signal outside the capture range, but once the PLL captures the signal, it will hold onto it, unless the input frequency goes beyond the limits of the lock-in range.

When the PLL is locked in onto an FM signal, we have $\omega_i = \omega_O + k_O V_O$. If

$\omega_i = \omega_c + \Delta\hat{\omega} \sin \omega_m t$

Thus,

$$V_O(t) = \frac{\omega_i(t) - \omega_O}{k_O} = \frac{\omega_c - \omega_O + \Delta\hat{\omega} \sin \omega_m t}{k_O} \qquad (10-60)$$

We should set $\omega_c = \omega_O$. The maximum VCO control voltage that will be available to drive the VCO is

$$\hat{V}_{OSC} = \pm k_D \frac{\pi}{2} A |H(\omega_m)|$$

$$= \pm k_D \frac{\pi}{2} A \frac{\omega_1}{\omega_m}, \quad \omega_m > \omega_1 \qquad (10-61)$$

The control voltage supplied to the VCO input is

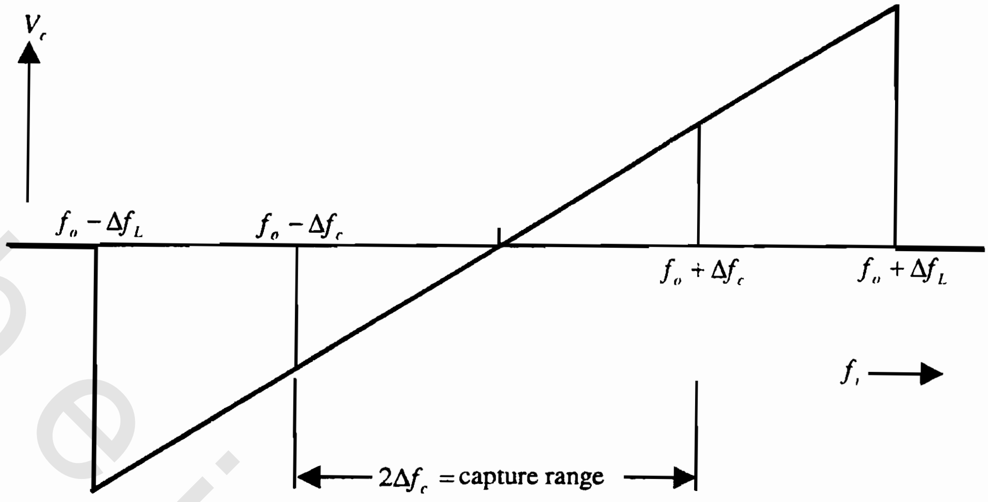


Fig. (10.10). Lock-in range and capture range

$$v_{OSC}(t) = \frac{\Delta\hat{\omega}}{k_O}\sin\omega_m t \qquad (10-62)$$

From eqns. (10.61) and (10.62),

$$\hat{V}_{OSC} = \frac{\Delta\hat{\omega}}{k_O} = k_D\frac{\pi}{2}A\frac{\omega_1}{\omega_m} \qquad (10-63)$$

So

$$\Delta\hat{\omega} = k_O\ k_D\frac{\pi}{2}A\frac{\omega_1}{\omega_m} \qquad (10-64)$$

Thus,

$$\omega_m\ \Delta\hat{\omega} = k_O\ k_D\frac{\pi}{2}A\omega_1 \ = \Delta\omega_L\ \omega_1 \qquad (10-65)$$

From eqn. (10.58), eqn. (10.65) becomes

$$\omega_m\ \Delta\hat{\omega} = (\Delta\omega_c)^2 \qquad (10-66)$$

If the product of the modulation angular frequency ω_m and the angular frequency deviation $\Delta\hat{\omega}$ exceeds this value, the VCO will not be able to follow the instantaneous angular frequency variations of the FM signal and distortion results.

10.9 Frequency Shift Keying:

Frequency shift keying (FSK) is a type of frequency modulation in which the frequency modulation varies between two constant values of carrier frequencies. This is suitable in many digital systems in which f_1 represents 0, and f_2 represents 1. The shift from 0 to 1, corresponds to a shift in frequency from f_1 to f_2. A PLL can be used as an FSK demodulator (Fig. 10.11). If both f_1 and f_2 are within the capture range, then the shift in frequency from f_1 to f_2 corresponds to a shift in VCO control voltage from $V_{01} = \omega_1 - \omega_0 / k_0$ to $V_{02} = \omega_2 - \omega_0 / k_0$, and $\Delta V = \omega_2 - \omega_1 / k_0$

A reference voltage is obtained by passing the VCO control voltage through LPF_2 which has a long time constant compared to the pulse period, so as to develop a dc voltage midway between V_{01} and V_{02}. The same loop that is used for the demodulation of the FSK signal can also be used for the production of an FSK signal. This is done by using the VCO portion and driving it by a binary input. The PLL can, thus, serve as a modem (modulator – demodulator) for FSK data communication over telephone lines.

10.10 Slewing Rate:

The response time of the PLL to a step function of frequency change is an important parameter. Assuming that both ω_1 and ω_2 fall within the PLL capture range, the VCO control voltage swing is $\Delta V_0 = \Delta\omega / k_0$.

The VCO control voltage is obtained from across the capacitor of the RC LPF. The time Δt required for the voltage across the capacitor to change by an amount ΔV_o is given by

$$\Delta V_O = \frac{\Delta Q}{C} = \frac{I\ \Delta t}{C} \qquad (10-67)$$

The voltage swing from PD applied to LPF corresponds to a step function.

$$\Delta V_{step} = k_D \frac{\pi}{2} A \qquad (10-68)$$

This corresponds to the positive side of the lock-in range. The capacitor charges as

$$\upsilon_c(t) = \Delta V_{step}\left[1 - e^{-t/CR}\right] \qquad (10-69)$$

The initial charging current is

$$\frac{C\ d\upsilon_c(t)}{dt} = \frac{\Delta V_{Step}}{R} \qquad (10-70)$$

If we assume this current to be constant within Δt_1, then for a voltage change ΔV_o across the capacitor in time Δt ,we have

$$\Delta V_0 = \frac{\Delta V_{step}}{R} \frac{\Delta t}{C} \qquad (10-71)$$

$$= \frac{k_D \frac{\pi}{2} A}{R} \frac{\Delta t}{C} \qquad (10-72)$$

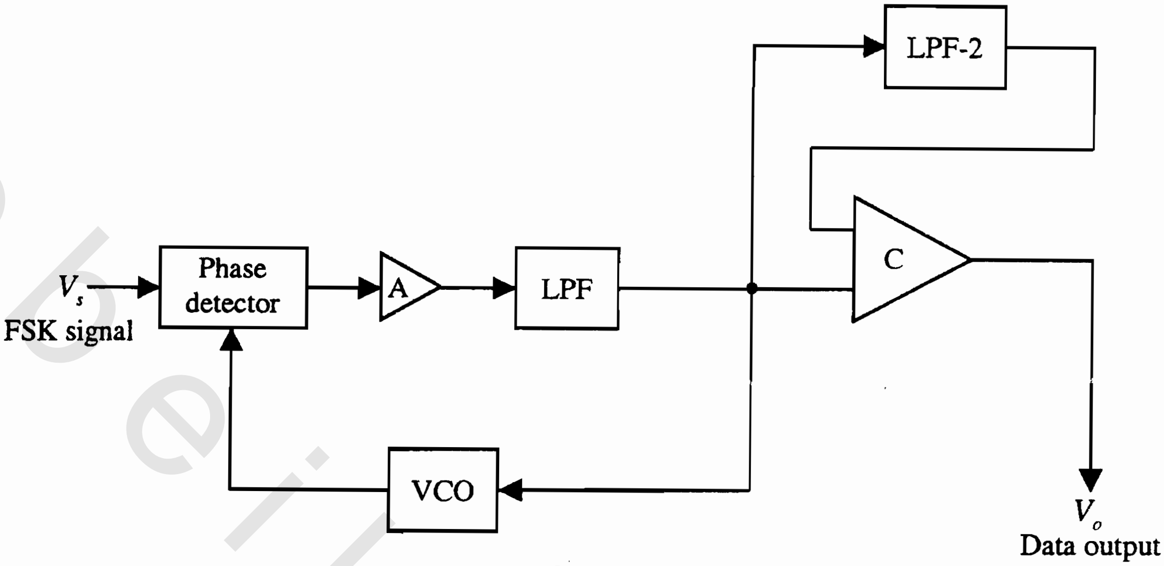


Fig. 10.11 FSK demodulator

Thus,

$$\Delta t = \frac{RC\ \Delta V_O}{k_D \frac{\pi}{2} A} = \frac{RC\,\Delta\omega_{osc}}{k_O\ k_D\ A\frac{\pi}{2}} \qquad (10-73)$$

Since

$$\Delta\omega_L = \pm\frac{\pi}{2}\,k_V \qquad (10-74)$$

Then, eqn. (10.73) becomes

$$\Delta t = RC\frac{\Delta\omega_{osc}}{\Delta\omega_L} = \tau\frac{\Delta\omega_{osc}}{\Delta\omega_L} \qquad (10-75)$$

Since $\omega_1 = \frac{1}{CR}$, and from eqn. (10.58),

$$\Delta t = \frac{\Delta\omega_{osc}}{\omega_1\ \Delta\omega_L} = \frac{\Delta\omega_{osc}}{\Delta\omega_c^2} \qquad (10-76)$$

Therefore,

$$\frac{\Delta\omega_{osc}}{\Delta t} = \Delta\omega_c^2 = \omega_1\ \Delta\omega_L \qquad (10-77)$$

This can be considered the maximum rate at which the VCO frequency can change with time. It is called the PLL slewing rate. It depends on RC of the LPF and the loop gain k_V.

Problems:

1- Show that the output $\upsilon_o(t)$ is zero when PLL is locked onto ω_o.

2- Show that the response time $t_r = 0.35/BW$. Hence, discuss the effect of increasing and decreasing the bandwidth on the speed of the response. If the input frequency varies as a square waveform, show how the output V_o varies with time. Sketch V_o if the square wave has a frequency of 10 kHz and BW is

a) 100 Hz b) 1 kHz c) 10 kHz

3- Show that, with a loop filter in place, the capture range is less than that given by eqn. (10.24) or eqn. (10.25).

4- A PLL has

a) $k_0 = 2\pi$ rad/V s b) $k_v = 2\pi x\, 500$ rad/s c) $\omega_0 = 2\pi x\, 500$ rad/s

d) $\omega_i(t) = 2\pi\,(500)\,[1 + 0.1\sin 2\pi \times 100\, t]$, find $V_o(t)$.

5- In the above problem, discuss the effects of changing k_v to

a) $2\pi x\, 1000$ rad/s b) $2\pi x\, 100$ rad/s.

6- In Prob. (10.3), f_i switches from 250Hz to 500Hz, then to 1 kHz, and to 250 Hz, in equal intervals of 1 second each. Sketch the output $\upsilon_o(t)$.

7- An input frequency varies as a square wave. It switches from 250 Hz to 500 Hz, $k_v = 2\pi x\, 600$ (rad/s). Choose a suitable carrier and then sketch the output $\upsilon_o(t)$.

8- Analyze the PLL response, if $F(s)$ is included

a) $F(s) = \dfrac{1}{1 + s/\omega_1}$ b) $F(s) = \dfrac{1 + s/\omega_2}{1 + s/\omega_1}$

9- A laser radar has an RF carrier frequency 10 MHz. Propose a PLL-based detection system. Make any reasonable assumptions.

10-A signal $0.01\sin 2\pi\, 100\, t$ mV is buried in a noise background of 10 µv rms/Hz. Show how a lock-in amplifier may be used to detect the signal. Show how S/N may be calculated. Make any reasonable assumptions.

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