The Digital Connection

Digital Information and Communication Technology and Systems

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The Digital Connection

Digital Information and Communication Technology and Systems

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Dedication

To my wife whose inspiring support made this work possible.

To my students who created the need for such a book.

There was time when a digital divide existed.

Now is the time for a digital connection to be made .

Foreword

This book is based on lectures given to the senior class at Cairo University. The course amalgamates the principles of digital communication, digital electronics and information theory as a background and focuses on modern applications such as multimedia communication, mobile, GPS etc.

This book tries to strike a balance between theory, techniques and actual systems. As such, it should be viewed as a survey book on selected topics in digital communication systems and information technology. In fact, it draws from communication theory only as much as needed to explain techniques used in selected communication systems.

This book is more application - oriented than an academic comprehensive treatise. It emphasizes the role of communication not as a mere vehicle for transport of information but as a fully integrated operation.

In the end, it attempts to fill in the gap between theory and application and give the communication jargon practical substance.

M. Sameh Said

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CHAPTER 1
Digitization:
Transition from Analog to Digital

1.1 Why Digital?

Electrical communication rests on the principle of converting non-electrical signals - such as sound, images etc into electrical signals - through a device called transducer (microphone, camera etc). The electrical signals may then be sent directly (baseband) or after modulation using a carrier. They can also be converted to electromagnetic signals through the transmitter. The channel which is the medium where the signals are transmitted may be a wire (for electrical signals) or an optical fiber (for optical signals) or may be a waveguide or a coaxial cable (for microwave signals). All such channels are fit for what is called guided transmission, so called because the signals are contained in conduits carrying the information Alternatively, the signals (in electromagnetic form) may be sent through the atmosphere or space. This is called unguided transmission. In all cases as signals propagate through the channel, they pick up unwanted signals called noise, since the channel is non ideal. Noise is an unwanted random signal which mars the original signal. At the receiver, the electrical signal (plus noise) is retrieved and handed over to a transducer to be perceived by the user. This transducer could be a loudspeaker, TV, etc.

Fig. (1.1) shows a block diagram of this analog communication system. The word analog stands for all continuously varying electrical signals (voltage or current), as the instantaneous value of the signal can vary at will.

One major disadvantage with analog communication is that the noise picked up in the channel may cause much deterioration in the received signal. Sometimes this deterioration reaches the point where the signal can no longer be extracted from the noise background, and we say then that the signal is buried in the noise.

The idea of digital communication rests on the principle that instead of sending the signal as an absolute value of voltage or current, we send a binary code which represents, the value of the signal in terms of 0s and 1s. The advantage is now clear. For even if the transmitted code picks up noise, the user at the receiving end should still be able to extract back the coded information despite the presence of noise as long as he can distinguish 0 from 1. Thus, information is almost unaffected by noise, since the user is not interested in the absolute value of the signal. What counts is his ability to decipher the code which means he should be able to tell whether the bit (unit of binary code) is 1 or 0. It does not matter that the pulse representing either 1 or 0 has noise on top of it so long as the user will be able to differentiate "1" from "0".

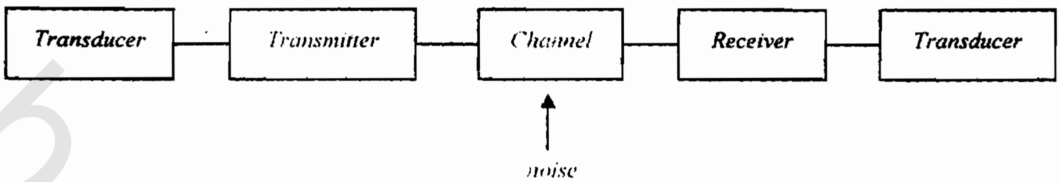


Fig. (1.1) Analog communication system

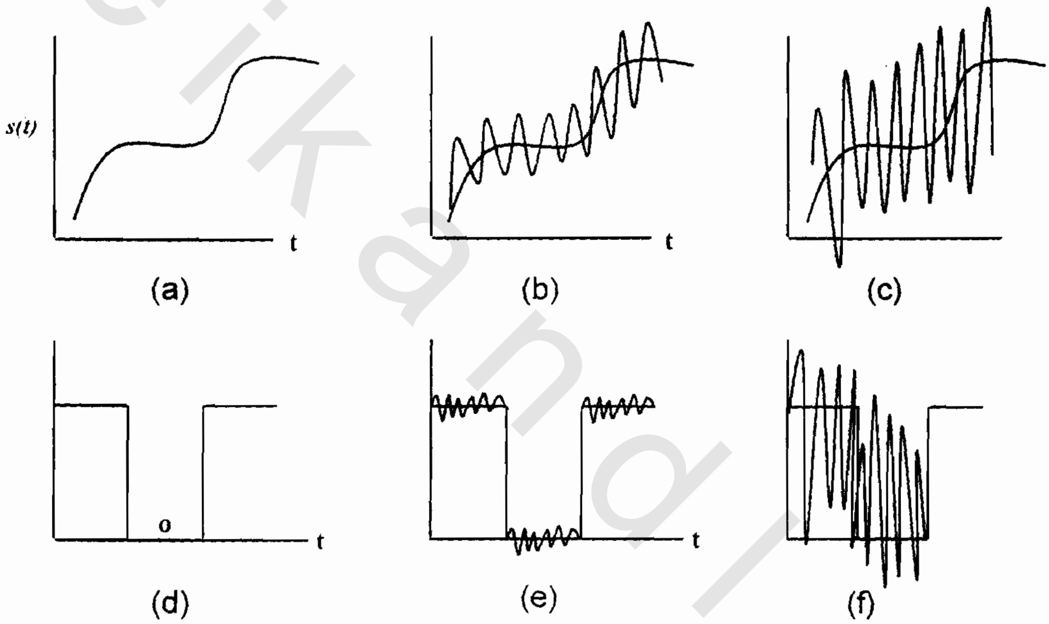


Fig. (1.2) Effect of noise

a) analog signal | *b) analog signal plus noise*
c) analog signal buried in noise | *d) digital signal (binary pulses of 0.1)*
e) digital signal plus noise | *f) digital signal buried in noise (noise exceeding limit of discernment of 0 from1)*

This does not mean that digital communication is perfect without noise at all, but surely there is still noise from other causes, but the direct addition of noise on the binary pulse is greatly curtailed (Fig. 1.2). Of course, the mutilation of the signal by noise must be within limits, otherwise the user will not ever be able to tell "0" from "1". The role of distortion is similar to noise in degrading the quality of the signal. Once again, digital communication proves to be more able to withstand distortion. That is why digital systems are HIFI (high fidelity).

As the distance the signal has to travel increases there is more noise and more distortion. In analog systems, repeaters (amplifiers) are usually used to strengthen the signal which is weakened by long distance transmission. Amplifiers actually do not help remove noise or distortion. In fact, amplifiers add more noise and more distortion. Why then use them in the first place? We use them only when we have to, i.e., when the signal gets too weak to be observed. Then, we use amplifiers to bring the signal up to a level which we can perceive. Of course, this is done at the expense of the quality of the signal, i.e. by adding on more noise and distortion contributed by the amplifier itself. This effect is cumulative as we use many repeaters.

In a digital system, the repeater will be able to reshape the binary signal, so as to regenerate the pulse in a fresh form, clean from noise picked up before the repeater. Hence, the effect of noise is not as cumulative as before.

One more advantage of digital systems is the ability to interface with computers which are all but based on digital technology. Also, digital systems can deal with different digital signals at the same time (multiplexing).

Digital signals can be easily stored in computer-based systems. The cost of digital hardware continues to decline thanks to the fast growing field of electronics and the continually improving technology leading to an increase in capacity, speed and efficiency

1.2 TDM and PCM:

We start with an analog signal which represents for example a voice message. The device that transforms this analog signal into a digital signal is called source encoder (coder). We suppose that the voice signal is limited to a frequency range of 100 Hz to 4 kHz as is usually demanded by telephone standards. The electrical signal representing the voice enters a device called a sampler, which selects values of the signal at selected instants of time. The idea here-as we will prove shortly - makes digital communication a real asset in that we do not have to transmit a specific signal all the time. Only selected samples of such a signal are required to be transmitted to the receiver. We may be able to retrieve the entire signal as if the channel were dedicated completely to this subscriber. Thus, we serve many subscribers at the same time. We take a sample from each and transform it into a digital signal, then we switch to the next sample from another subscriber and so on. The transmitting medium is now carrying a stream of bits belonging to all sampled subscribers.

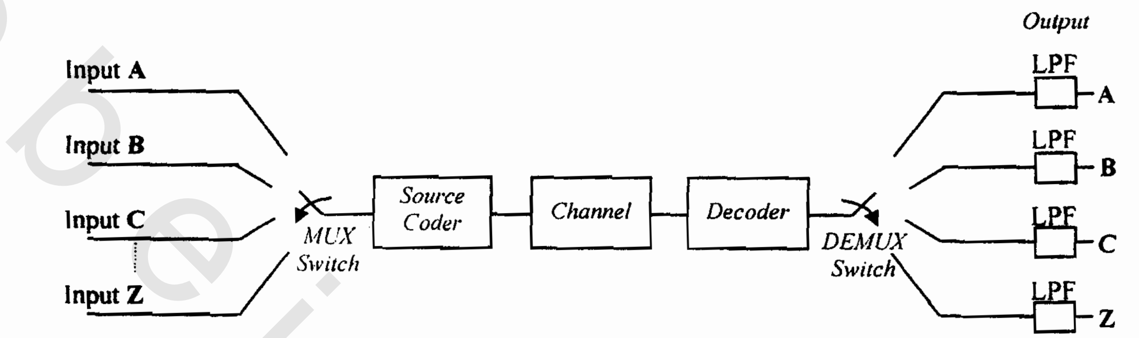


Fig. (1.3) TDM Link

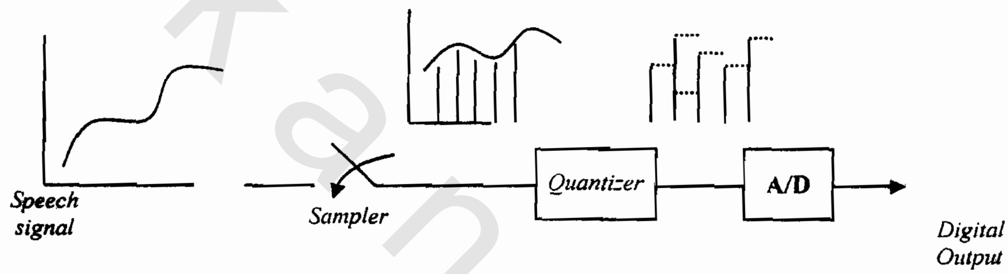


Fig. (1.4) Source coder

At the receiver, we must be able to discern samples from different subscribers. The first process of combining different digital samples from different subscribers into the transmission channel is called multiplexing, while the process of recombining the bit steam into various outputs is called demultiplexing. Multiplexing (MUX) and demultiplexing (DEMUX) must somehow be performed by synchronized switches. The decoded signals can thus be channeled to the corresponding outputs after filtering (Fig. 1.3). This is called Time Division Multiplexing (TDM).

The advantage of TDM is obvious. We can use the same hardware and transmission channel for many subscribers without extra cost. It is based on the principle of time sharing since dedicating a channel to each subscriber will not give us any new information. This is a marvelous advantage of going digital. There is, however, a condition on the speed of the MUX switch, namely, that the frequency of sampling each of the inputs must be equal or greater than twice the maximum frequency component in the signal (bandwidth). In other words,

$$f_s \geq 2B \qquad (1-1)$$

where f_s is the sampling frequency and B is the bandwidth of the signal. This is called Nyquist sampling theorem. We will prove this theorem shortly. So if the bandwidth is 4 kHz, the sampling rate must be 8 k samples/s (or 8 kHz).

The source coder (Fig. 1.4) consists of a sampler, which takes samples of the signal at the specific instants dictated by Nyquist sampling theorem. Suppose now that we are working with 8 bit words, i.e., each sample value is to be represented by a byte, then, we divide the full operating range of the analog signal to 2^8 levels = 256. The quantizer assigns the sampled value to the nearest one of these levels. The rounding off of the actual value to the nearest quantized value entails some error. But this is something we have to live with. If we want to reduce such error (called quantization error) we have to increase the word length i.e., increase the number of quantum levels.

After the quantizer, the signal is inputted to an analog to digital converter (ADC or A/D). It may also called symbol to bit mapper. Here, each one of the nearest assigned quantum levels is converted to its corresponding binary code. Thus, we have transformed the sample of the analog signal into a digital code. We must note that this digital code is processed and transmitted before the next sample from the next subscriber comes in. This sets the limit on the speed of the MUX and DEMUX switches. The technique of transforming a sample into a digital code is called Pulse Code Modulation (PCM).

1.3 The Sampling Function:

Given an analog input $x(t)$. The sampler multiplies this function by the ideal sampling function which is an impulse train $g_S(t)$ of period T_S :

$$g_S(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s) \qquad (1-2)$$

The output of the sampler is

$$x_s(t) = x(t) g_S(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s) \qquad (1-3)$$

This multiplication is shown (Fig. 1.5). The output is made up of impulses at times $k\,T_s$ of height $x(k\,T_s)$, i.e.,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(n\,T_s)\delta(t - n\,T_s) \qquad (1-4)$$

For a periodic function $g_s(t)$ with period $T_s = 2\pi / \omega_o$, we may write its Fourier series as

$$g_S(t) = \sum_{n=-\infty}^{\infty} a_n\, e^{jn\omega_o t} \qquad (1-5)$$

Taking Fourier transform of both sides

$$G_S(\omega) = \mathcal{F}\left[\sum_{n=-\infty}^{\infty} a_n\, e^{jn\omega_o t} \right] ..$$

$$= \sum_{n=-\infty}^{\infty} a_n\, \mathcal{F}\left[e^{jn\omega_o t} \right]$$

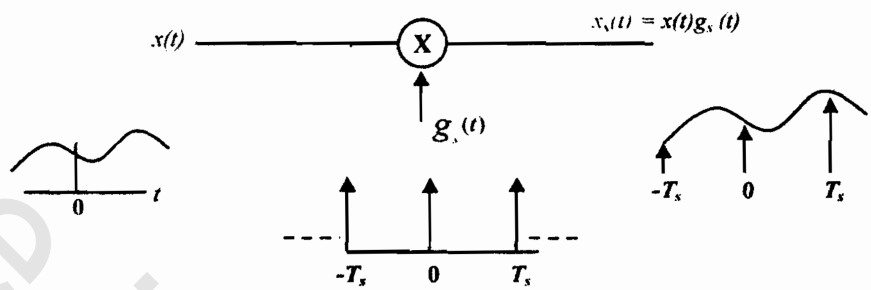


Fig. (1.5) Ideal sampling

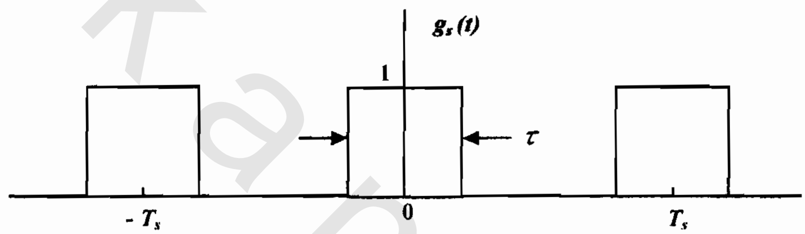


Fig. (1.6) Rectangular sampling function

$$= 2\pi \sum_{n=-\infty}^{\infty} a_n \, \delta(\omega - n\omega_o) \qquad (1-6)$$

Now a_n can be expressed in terms of the Fourier transform of $g_{T_s}(t)$, the waveform over one period, with,

$$a_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} g_{T_s}(t) e^{-jn\omega_o t} dt$$

$$= \frac{1}{T_s} \int_{-\infty}^{\infty} g_{T_s}(t) e^{-jn\omega_o t} dt$$

$$= \frac{1}{T_s} G_{T_s}(jn\omega_o) \qquad (1-7)$$

Thus,

$$G(\omega) = \omega_o \sum_{n=-\infty}^{\infty} G_{T_s}(jn\omega_o) \, \delta(\omega - n\omega_o) \qquad (1-8)$$

$$g_{T_s}(t) \Leftrightarrow \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} G_{T_s}\left(j\frac{2\pi n}{T_s}\right) \delta\left(\omega - \frac{2\pi n}{T_s}\right) \qquad (1-9)$$

In terms of frequency, we have

$$G(f) = \sum_{n=-\infty}^{\infty} a_n\, \delta(f - nf_o) \qquad (1-10)$$

$$g_{T_s}(t) \Leftrightarrow \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G_{T_s}\left(j\frac{2\pi n}{T_s}\right) \delta\left(f - \frac{n}{T_s}\right) \qquad (1-11)$$

The Fourier transform is, thus, seen to be a series of impulses at the harmonics of the repetition period with strengths determined by the shape of the waveform in one period

Ex 1.1

Consider the sampling function shown. Obtain the Fourier transform of this sampling function Then show what happens as this function becomes a delta function

Solution

The Fourier transform of the truncated function g_{T_s} (Fig.1-7) is given by

$$g_{T_s}(t) = g_s(t) \ , \quad \frac{-T_s}{2} < t < \frac{T_s}{2} \qquad (1-12)$$

$$= 0 \qquad \text{elsewhere}$$

$$G_{Ts_s}(j\omega) = \tau \frac{\sin \omega\tau / 2}{\omega\tau / 2} \qquad (1-13)$$

From eqns (1 – 8), (1 – 10),

$$G_{T_s}(j\omega) = \frac{2\pi\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin \pi n\tau/T_s}{\pi n\tau/T_s} \delta\left(\omega - \frac{2\pi n}{T_s}\right) \qquad (1-14)$$

$$G_{T_s}(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin \pi n\tau/T_s}{\pi n\tau/T_s} \delta\left(f - \frac{n}{T_s}\right) \qquad (1-15)$$

When the sampling function becomes a delta function we have

$$g_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \qquad (1-16)$$

Expanding by Fourier series with $\omega_o = \dfrac{2\pi}{T_s}$

$$a_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-jn\omega_o t} dt \qquad (1-18)$$

$$= \frac{1}{T_s} \qquad (1-19)$$

Thus,

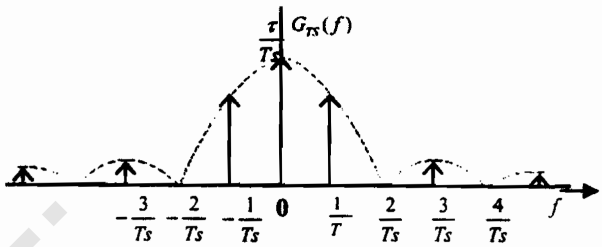


Fig. (1.7) Amplitude spectrum of Fourier transform of a rectangular sampling function for $\tau / T_s = 1/2$

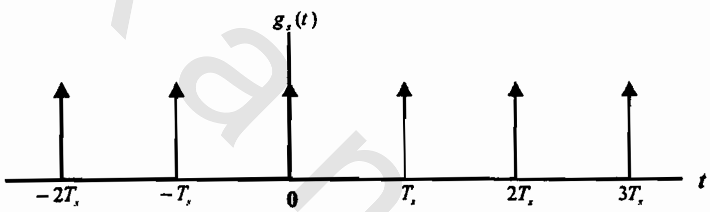


Fig. (1.8) Delta sampling function

$$g_s(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\omega_o t} \qquad (1-20)$$

Taking Fourier transform

$$G_s(j\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \mathcal{F}\left(e^{jn\omega_o t}\right) \qquad (1-21)$$

Thus,

$$= \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_o) \qquad (1-22)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \Leftrightarrow \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T_s}\right) \qquad (1-23)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \Leftrightarrow \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \qquad (1-24)$$

$$f_s = 1/T_s \qquad (1-25)$$

Thus, the impulse train in the time domain has its Fourier transform an impulse train in the frequency domain

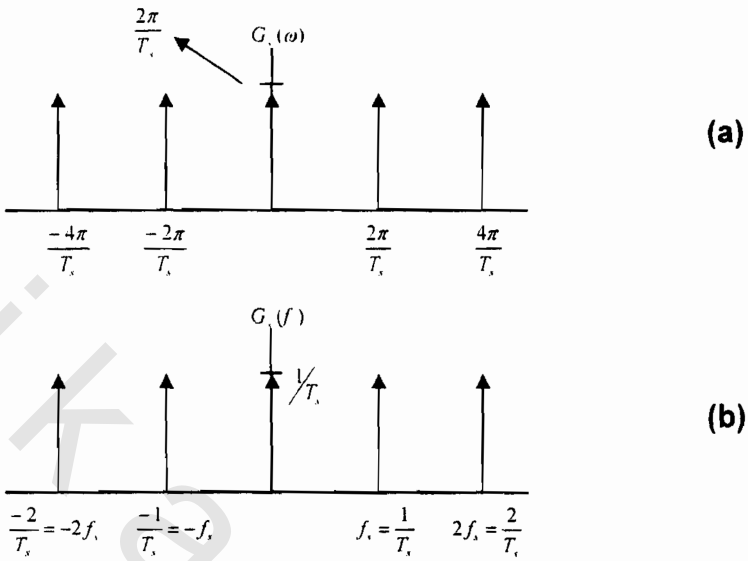


Fig. (1.9) Amplitude Spectrum for delta sampling function
a) $G(j\omega)$ *b)* $G(f)$

1.4 Sampling Theorem:

Back to the sampling situation depicted in Fig (1.5). From eqn (1 – 24) we have

$$G_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_s) \qquad (1-26)$$

Now let us evaluate $X_s(f)$, the Fourier transform of $x_s(t)$ as in (Fig. 1.5),

$$X_s(f) = \mathcal{F}\{x_s(t)\} = \mathcal{F}\{x(t)\,g_s(t)\} \qquad (1-27)$$

From the properties of Fourier transform, multiplication in the time domain corresponds to convolution in the frequency domain

Thus $$X_s(f) = X(f) * G_s(f) \qquad (1-28)$$

where $*$ stands for convolution . From eqn. (1-26),

$$X_s(f) = X(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_s) \qquad (1-29)$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f) * \delta(f-nf_s) \qquad (1-30)$$

From the shifting property of the delta function

$$X(f)*\delta(f-nf_s)=\int_{-\infty}^{\infty} X(\lambda)\delta(f-nf_s-\lambda)d\lambda \quad (1-31)$$

$$=X(f-nf_s) \quad (1-32)$$

Thus, eqn (1 – 30) reduces to

$$X_s(f)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f-nf_s) \quad (1-33)$$

If $X(f)$ as given in (Fig. 1.10a) is limited to a bandwidth of maximum frequency component f_M, then $X_s(f)$ is seen from eqn (1 – 33) to be a repetition of $X(f)$ around f_o and its multiples (Fig. 1.10b).

Note in Fig. (1.10b) the $X(f)$ term does not overlap with its neighbors as long as point $f_1 < f_2$ and point $f_4 < f_3$. This simply means, for the condition

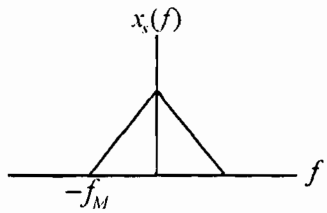
$$f_1 = f_M \le f_2 = f_s - f_M \quad (1-34)$$

then the $X(f)$ term is preserved without overlap. This can be stated as

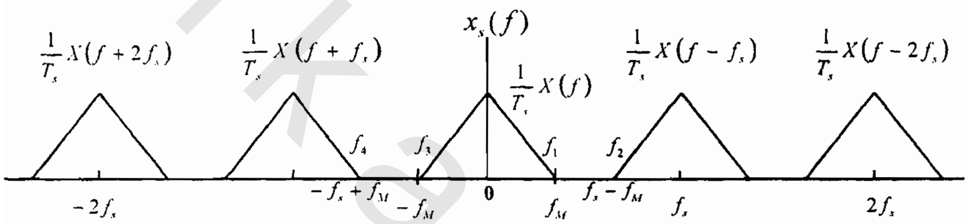
$$f_s \ge 2f_M \quad (1-35)$$

which is the condition for proper sampling. When $f_s = 2f_M$, f_s is called Nyquist rate. It is the smallest sampling rate that can be used while the original signal can be properly retrieved from the samples. It means that the sampling rate must be at least twice the maximum frequency component (or twice the bandwidth) in order to retrieve the original signal. The retrieval process can easily be made using a LPF (Fig. 1.11). The cut off frequency of the filter must be $f_m < f_c < f_c - f_M$. In the limit when $f_s = 2f_M$, $f_c = f_s/2 = f_M$, all the information of $x(t)$ of the incoming signal is preserved and nothing is lost despite the fact that the signal was not transmitted in full but sent at discrete instants, provided that the rate of sampling is at least twice the bandwidth. This theory - called Nyquist theory - has tremendous impact in communication.

Now if the sampling rate is below the limit given by eqn. (1– 35), overlapping takes place. It becomes impossible to retrieve the actual signal correctly. This is called under-sampling, and the distortion produced is called aliasing (Fig. 1.12). We see from (Fig. 1.12), that in the case of under-sampling the $X(f)$ component and the $X(f+f_s)$ component overlap. As a result, the original signal $X(f)$ is no longer preserved.



(a)



(b)

Fig. (1.10) Fourier transform in the sampling process

a) Fourier transform of the input signal $x(t)$.

b) Fourier transform of the ideal sampling output $x_s(t)$

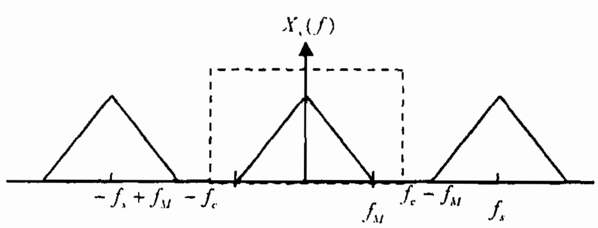


Fig. (1.11) Use of a LPF to recover $X(f)$ from $X_s(f)$

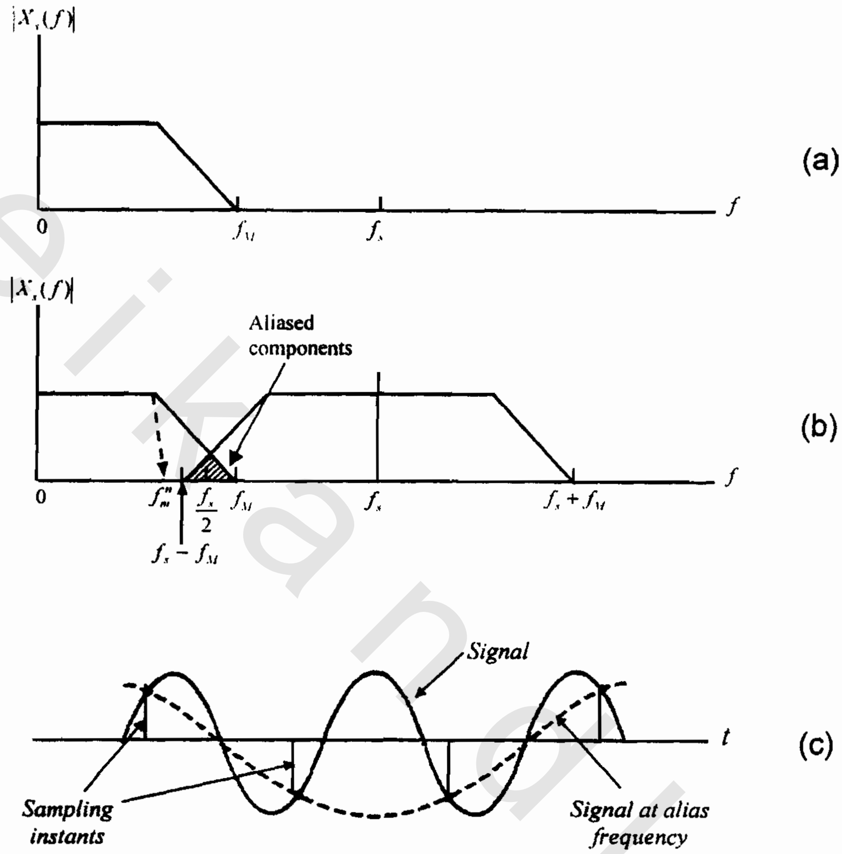


Fig. (1.12) Aliasing due to under-sampling $f_s < 2f_M$

a) signal spectrum *b) overlapping* *c) alias frequency*

1.5 Sample and Hold:

This is also called zero order hold sampling (Fig. 1.13). After we take samples of the signal, we need to keep the values of the signal at the instants of sampling for a while to give the circuitry enough time to convert such values one at a time to a digital code.

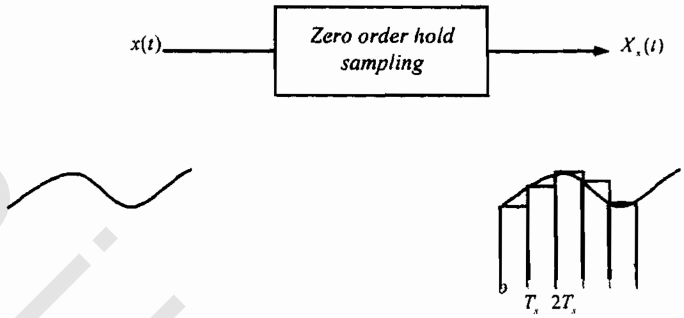


Fig. (1.13) Input and output of zero order hold sampling

Fig. (1.14) shows the incoming signal $x(t)$ first going into an ideal sampler. Here, it gets multiplied by a pulse train $g_s(t)$. This leads to impulses of height $x_s(nT_s)$ once every nT_s seconds. This leads to an output in the form

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \qquad (1-36)$$

Next, the output of the ideal sampler $x_s(t)$ enters into a linear time invariant (LTI) system, which is described by the impulse response $h(t)$. This leads to an output for each incoming sample of height $x(nT_s)$, which corresponds to holding on to the values $x(nT_s)$ for a duration of T_s. The total output $x_{sh}(t)$ is shown (Fig. 1.14). It is a signal with height $x_s(nT_s)$ in each time interval $[nT_s\ ,\ (n+1)T_s]$.

We want to check now that the zero order hold sampling still contains the same information in the original signal. From Fig. (1.14), we see that $X(f)$ can be obtained back from $x_{sh}(t)$ if we use an LTI system with response $h^{-1}(t)$, then we get back the sampled function.

We saw before that information can be retrieved from a sampled function if we use a LPF with cut off frequency $f_c = f_s/2 = f_M$ and with gain T_s. Such a system is shown (Fig. 1.15)

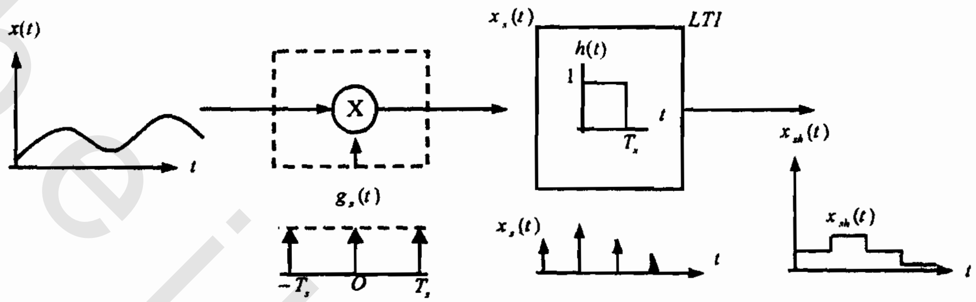


Fig. (1.14) Zero order hold sampling process

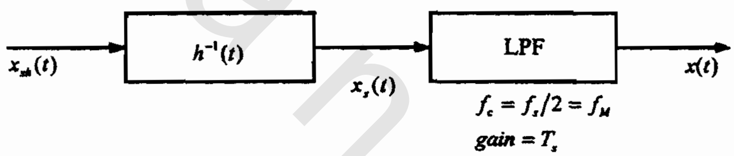


Fig. (1.15) System undoing the effects of zero order hold sampling

Ex 1.2

Obtain the output of zero order hold sampling for an input signal $x(t) = \dfrac{\sin \pi t}{\pi t}$

Solution

The output of the ideal sampler for sample times 0, $0.5s$, $15s$, $1.5s$ is shown (Fig. 1.16a). The output of the holding circuit is shown (Fig. 1.16b).

Ex 1.3

Show that the information can be retrieved from sampling using rectangular pulses instead of delta functions with the same Nyquist criterion.

Solution

A practical sampling system using a rectangular pulse train is shown in Fig.(1.17)

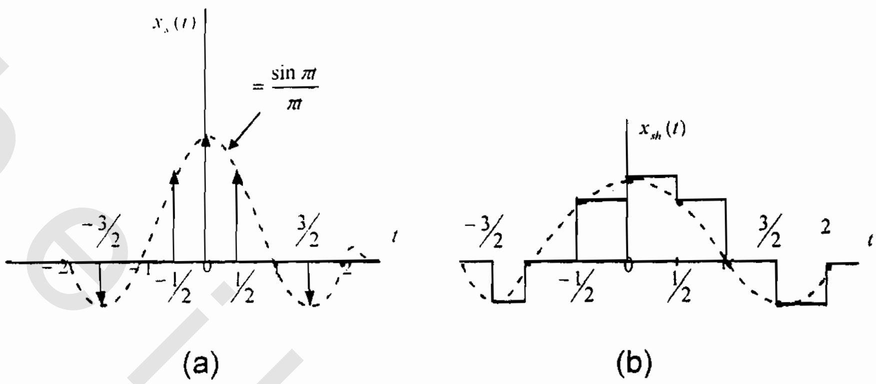


Fig. (1.16) Sampling the function $(\sin \pi t / \pi t)$

a) the output of the sampler *b) the output of the holding circuit*

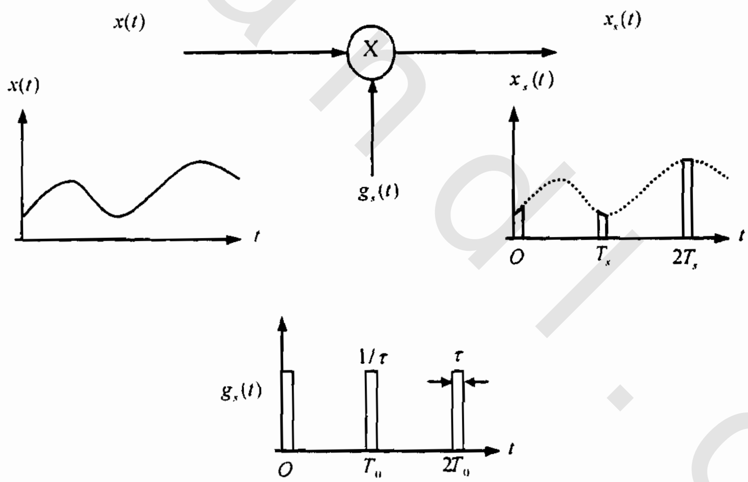


Fig. (1.17) Sampling with a narrow rectangular pulse train

The rectangular pulse has a height $\frac{1}{\tau}$ and width τ. The pulses are spaced at T_s seconds apart. Assume $x(t)$ as in Fig. (1.10a).

$$x_s(t) = x(t)\ g_s(t) \qquad (1-37)$$

$$X_s(f) = \mathcal{F}\{x(t)\ g_s(t)\} \qquad (1-38)$$

Since $g_s(t)$ is a periodic function

$$g_s(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f_s t} \tag{1 – 39}$$

Where C_n, the Fourier series coefficient is given by

$$C_n = \frac{1}{T_s} \operatorname{sinc}\left(\frac{n\tau}{T_s}\right) = \frac{1}{T_s} \frac{\sin \pi n \, \tau/T_s}{\pi n \, \tau/T_s} \tag{1 – 40}$$

Thus,

$$X_s(f) = \mathcal{F}\left\{x(t) \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f_s t}\right\} \tag{1 – 41}$$

$$= \sum_{n=-\infty}^{\infty} C_n \mathcal{F}\left\{x(t) \, e^{j2\pi n f_s t}\right\} \tag{1 – 42}$$

From Fourier transform shifting property

$$X_s(f) = \sum_{n=-\infty}^{\infty} C_n X(f - nf_s) \tag{1 – 43}$$

Thus, $X_s(f)$ consists of replicas of $X(f)$ added together. Each n^{th} replica is shifted by $n f_s$ and multiplied by C_n. Fig. (1.18) shows the result.

1.6 Quantizer:

The quantizer is device which takes the amplitude of the samples and changes it to the nearest level out of N allowed levels. For example, if the absolute value of the sample $x = 7.32\ V$, this must be changed in the quantizer to the output $\hat{x}$ (pronounced x hat) $\hat{x} = 7\,V$. (Fig 1-19).

We can see the importance of the quantizer because we cannot encode all the numbers in the world with decimal fractions. So we limit ourselves to a finite set of levels and approximate the absolute value of the sample to the closest level. Quantizers may be classified to one of four types:

a) Mid tread, if it has 0 as one of its allowed output amplitudes or of its code words, i.e., it changes amplitudes close to 0 to 0 (Fig. 1.21a).
b) Mid riser, it does not have 0 as one of its code words, the output is approximated to ± 0.5 or ± 1.5 and so on (Fig. 1.21b).
c) Uniform if all its codewords are equally spaced.
d) Non-uniform if the quantizer has codewords that are not equally spaced.

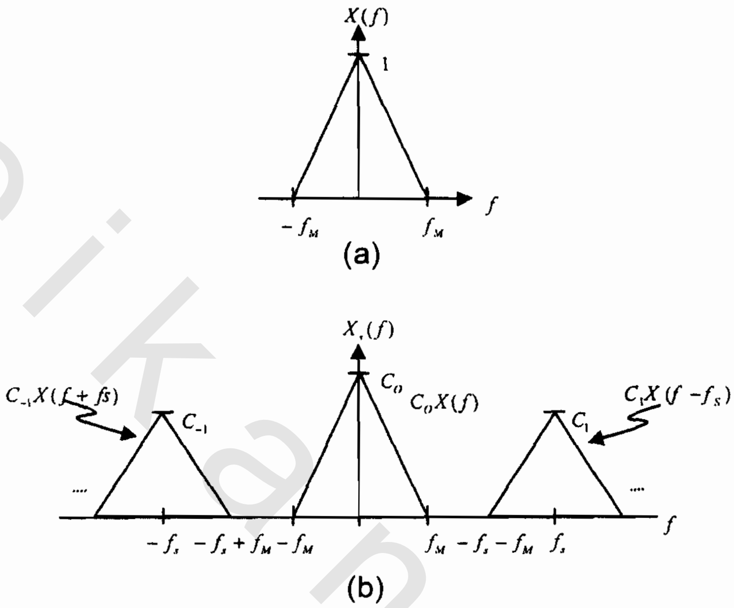


Fig. (1.18). Fourier transform output when a rectangular pulse train is used
a) Fourier transform of input $x(t)$ *b) Fourier transform of output* $x_s(t)$

Ex 1.4 For a quantizer whose characteristic is given in Fig. (1.19) and whose input $x(t)$ is shown in Fig. (1.20a) find the output $\hat{x}(t)$.

Solution
For an $x(t)$ input in Fig. (1.20a) the output $\hat{x}(t)$ is shown in Fig.(1.20b).

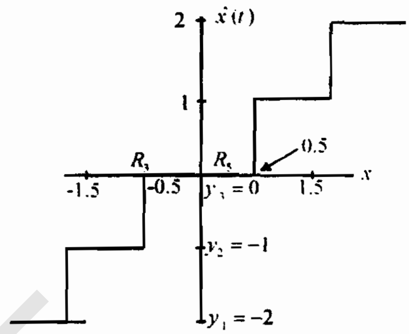
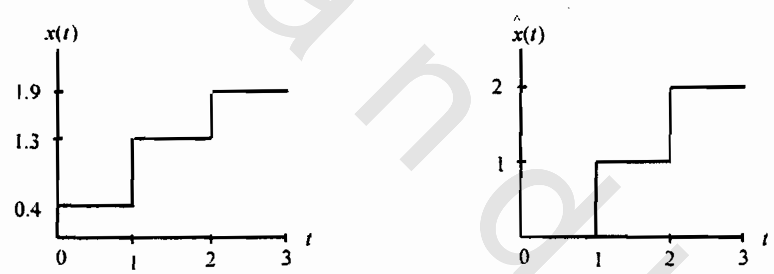


Fig. (1.19) Transfer characteristic of a quantizer



(a) (b)

Fig. (1.20) input and output of a quantizer
a) input *b) output*

1.7 Quantizer Performance:

In the quantizer, we would like to keep the output of the quantizer as close as possible to its input. This way less information is lost. For example, 7.3 is approximated to 7, then 0.3 is lost, whereas if the codewords are 7.1, 7.2, ... 8 then 7.13 is approximated to 7.1, then only .03 is lost. We can tell how well the quantizer is doing its job if we look at the difference between the amplitude of the input to the quantizer x and the amplitude of the output from the quantizer $\hat{x}$. We call this error signal $e(x)$. The quantizer performance is measured by this error signal.

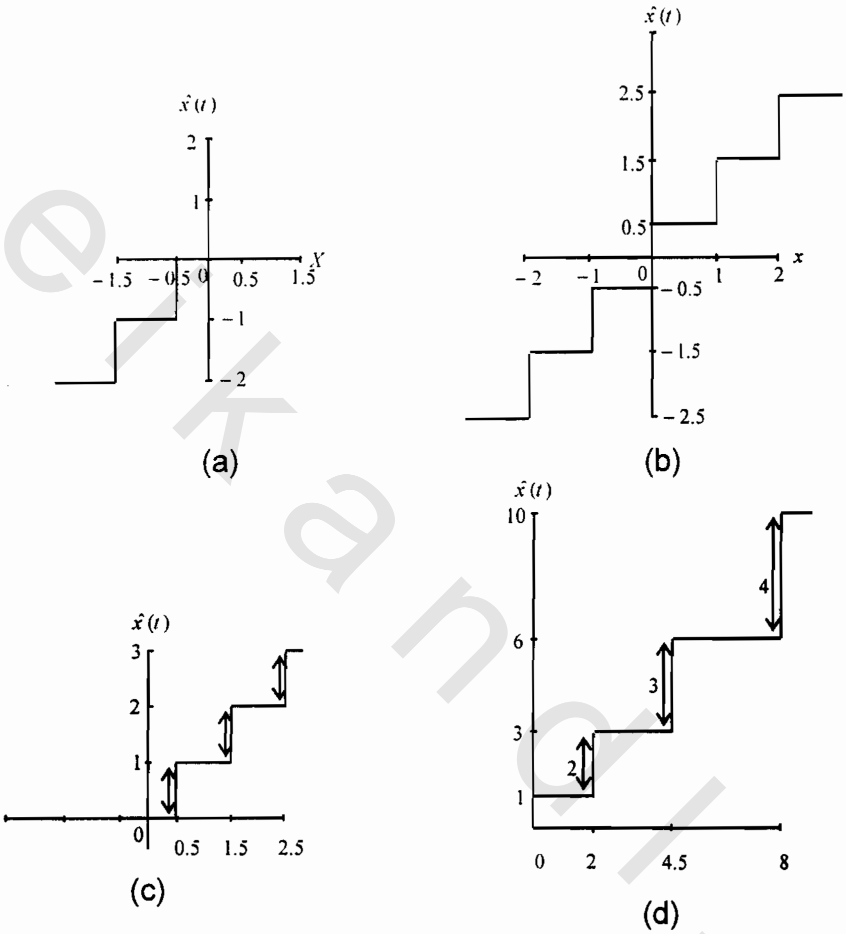


Fig. (1.21) Types of quantizers

a) mid tread *b) mid riser* *c) uniform* *d) non-uniform*

$$e(x) = \left| \hat{x} - x \right| \tag{1 – 44}$$

A good quantizer has a small error signal. To proceed, we must resort to probability theory. We assume we know how likely it is that a particular amplitude comes into the quantizer. Assume that the probability density function of the amplitudes coming into the quantizer is $f_x(x)$, where x is the incoming amplitude An overall measure of quantizer performance is the mean squared error (*mse*)

$$mse = E\left[\left(x-\hat{x}\right)^2\right] = \int_{-\infty}^{\infty}\left(x-\hat{x}\right)^2 f_x(x)dx \qquad (1-45)$$

where E means the average value.

A quantizer with a small *mse* is a good quantizer. A second overall measure of quantizer performance is called signal to quantizer ratio (SQNR). We must remember that quantizer noise is the randomness due to the error signal since the absolute value of the sample is approximated to the nearest allowed level. Thus, SQNR is the ratio of the signal input power to the power of the error (or noise) introduced by the quantizer. Thus

$$SQNR = \frac{P_s}{P_e} = \frac{\int_{-\infty}^{\infty}(x-x_m)^2 f_x(x)dx}{\int_{-\infty}^{\infty}\left(x-\hat{x}\right)^2 f_x(x)dx}$$

$$= \frac{\int_{-\infty}^{\infty}(x-x_m)^2 f_x(x)dx}{mse} \qquad (1-46)$$

where x_m is the average (mean) x value. We see from eqn (1 – 46) that as *mse* gets smaller, SQNR improves and hence we have a better quantizer.

Assuming that x is a uniformly distributed random variable between (a , b), let us calculate *mse* and SQNR for a uniform quantizer such that $f_x(x) = 1/B$,where $B = b - a$, (Fig. 1.22). From eqn (1 – 45)

$$mse = \int_{-\infty}^{\infty}\left(x-\hat{x}\right)^2 f_x(x)dx$$

$$= \frac{1}{B}\int_a^b\left(x-\hat{x}\right)^2 dx \qquad (1-47)$$

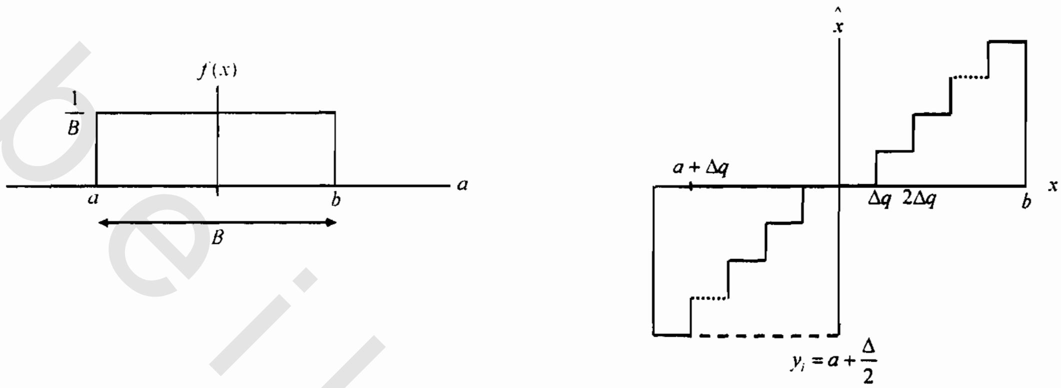
We must break up the integral in eqn. (1 – 47) into the "cells" R_1 , R_2 ... R_N noting that all values of x that fall into the cell R_i have an output from the quantizer $\hat{x}$ corresponding to the value y_i

Thus

$$mse = \frac{1}{B}\int_{R_1}(x-y_1)^2 dx + \dots \int_{R_N}(x-y_n)^2 dx$$

$$= \frac{1}{B}\sum_{i=1}^{N}\int_{R_i}(x-y_i)^2 dx \qquad (1-48)$$

Considering the i^{th} call R_i , the term $x - y_i$ is the error of the quantizer $\in_i$



(a) (b)

Fig. (1.22) Uniform quantizer with uniform input power density function
a) uniform input power density function. *b) transfer function x of a uniform quantizer*

$$\in_i = x - y_i \qquad (1-49)$$

$$mse = \frac{1}{B}\sum_{i=1}^{N}\int_{\in_{i\,min}}^{\in_{i\,max}} \in_i^2 \, d\in_i \qquad (1-50)$$

Where $\in_{i_{min}}$ and $\in_{i_{max}}$ refer to the smallest value of the quantizer error and the largest value of the quantizer error in a cell R_i

Referring to Fig. (1.22b), in the first cell all values between a and $a+\Delta q$ are mapped into the value $y_1 = a + \Delta q/2$. The error is $\pm\Delta q/2$. The largest value of error occurs when the input $x = a - \Delta q/2$ is mapped to the output $y_1 = a$. In this case, the error $\in_{i_{max}} = \Delta q/2$.

Similarly, the smallest value of the error occurs when the input $x = a + \Delta q/2$ is mapped to the output $y_1 = a$. In this case the error value is $-\Delta q/2$ Thus, $\in_{i_{min}} = -\Delta q/2$. The same reasoning applies to any cell. Thus, eqn (1 – 50) becomes

$$mse = \frac{1}{B}\sum_{i=1}^{N}\int_{-\Delta q/2}^{\Delta q/2} \in_i^2 \, d\in_i \qquad (1-51)$$

Since all N integrals are identical, eqn (1 – 51) becomes

$$mse = \frac{N}{B}\int_{-\Delta q/2}^{\Delta q/2} \in_i^2 \; d\in_i \qquad (1-52)$$

We note that

$$N\Delta q = B \qquad (1-53)$$

Thus, eqn (1 – 52) becomes

$$mse = \frac{\Delta q^2}{12} \qquad (1-54)$$

Let us now assume $[a\ ,b]=[-A\ ,A]$. We may now calculate SQNR (eqn 1 – 46), noting $x_m = 0$ and $B = 2A$ and $\Delta q = 2A/N$

$$SQNR = \frac{\frac{1}{2A}\int_{-A}^{A} x^2\, dx}{mse} \qquad (1-55)$$

$$= \frac{A^2/3}{\Delta^2 q/12} = \frac{4A^2}{\left(2A/N\right)^2} = N^2 \qquad (1-56)$$

It is clear from this astounding result that as the number of levels N increases, the signal to noise ratio increases quadratically.

Ex 1.5

An L level linear quantizer has an analog signal with peak to peak range of $V_{pp} = 2V_p$ where V_p is the maximum voltage. Assume quantization levels to be uniformly distributed over the full range (uniform quantizer), find quantizer error variance and S/N.

Solution

The step size between quantum levels $\Delta q = V_{pp}/L$ The maximum error is $\Delta q/2$ in the positive direction and $\Delta q/2$ in the negative direction. Assuming that the quantization error $\in_i$ is uniformly distributed over one interval Δq wide, i.e., the analog input takes on all values with equal probability, the quantizer error variance σ^2 is given by

$$\sigma^2 = \int_{-\Delta q/2}^{\Delta q/2} \in_i^2 f(\in_i)\, d\in_i = \int_{-\Delta q/2}^{\Delta q/2} \in_i^2 \frac{1}{\Delta q}\, d\in_i = \frac{\Delta q^2}{12} \qquad (1-57)$$

where $f(\in_i) = 1/\Delta q$ for uniform probability density function of the quantization error. The variance σ^2 corresponds to the average quantization noise power. The peak power of the analog signal normalized to 1Ω is

$$V_p^2 = \left(\frac{V_{pp}}{2}\right)^2 = \left(\frac{L\Delta q}{2}\right)^2 = \frac{L^2\Delta q^2}{4} \qquad (1-58)$$

Thus

$$S/N = \frac{L^2\Delta q^2/4}{\Delta q^2/12} = 3L^2 \qquad (1-59)$$

With infinite quantization levels there is no quantization noise, we also note that S/N is independent of Δq. From eqn. 1-57 and 1-54 we note that

$$\text{mse} = \sigma^2 \qquad (1-60)$$

1.8 Pulse Code Modulation (PCM):

The source coder which maps an analog signal into a digital signal consists of a sampler, a quantizer and a device called symbol to bit mapper or analog to digital converter (ADC or A/D). In Fig. (1.23), the input is $x(t)$, the output of the sampler is $x_s(t)$ which is inputted to the quantizer. The output of the quantizer is $\hat{x}_s(t)$ which is inputted to the symbol to bit mapper, where the output is the digital signal, which is a set of bits $0,1$. These bits are represented here as short pulses, where 1 is $+5V$ pulse and 0 is $-5V$ pulse.

The symbol to bit mapper associates bits 00 for 0, 01 for 1, 10 for 2, and 11 for 3. The sampling rate is the number of samples per second which the sampler creates. The symbol rate is the number of samples per second that leave the quantizer. Since the quantizer creates one sample out of each sample that comes in, the symbol rate is also the rate of the symbols that come into the quantizer which is equal to the sampling rate. The bit rate indicates how many bits per second come out of the symbol to bit mapper. The bit rate is equal to the symbol rate times the number of bits per symbol. What makes a PCM a good source coder are two parameters:

a) small quantizer error or large SQNR such that the information lost is as small as possible.
b) small bandwidth which requires a low bit rate. (to give the quantizer enough time to do the conversion) .

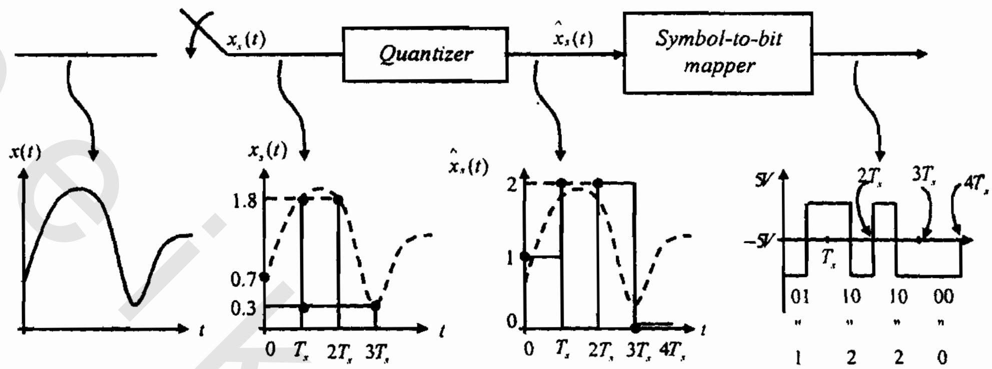
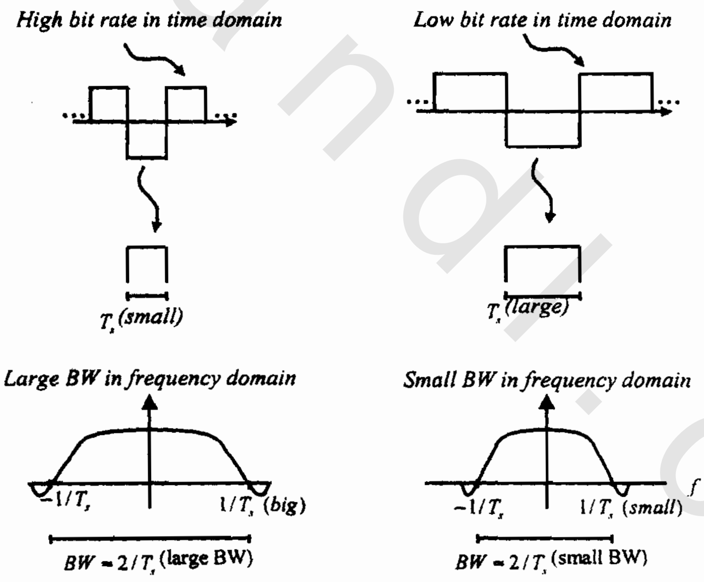


Fig. (1.23) PCM system



(a) (b)

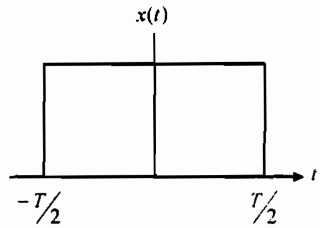
Fig. (1.24) Bit rate in relation to bandwidth

a) high bit rate *b) small bit rate*

We see from Fig. (1.24) that a high bit rate requires a narrow pulse width and hence large bandwidth, whereas a low bit rate results in a small bandwidth.

Problems

1. Consider a given waveform $x(t)$ as shown. Find its Fourier transform. Then find the spectrum of this function when sampled with a unit impulse train. Also find the spectrum of this function when sampled by a pulse train with pulse width τ, amplitude $1/\tau$ and period T_s. Show that in the limit as τ approaches zero the two spectra converge.



2. An input signal is $A\sin\omega_i t$ with period T_i. It is sampled with a sampling function whose period T_s is $3/4T_i$. Show the result of under-sampling on the recovery of the signal. What happens when you sample at $T_s = \frac{1}{4}T_i$, $\frac{1}{2}T_i$, T_i, $2T_i$. Take the first sample at the first peak of the input. What do you conclude?

3. Obtain an expression for the bandwidth of PCM of N levels and signal bandwidth B. Obtain also the information rate of the bit stream.

4. In a CD digital audio system, an analog signal is digitized so that the ratio of peak signal power to the peak quantization noise is at least $96dB$. How many quantization levels are needed? How many bits per sample are needed? What is the bit rate if the sampling rate is $44k$ samples/s?

5. Compare σ^2, mse, SQNR as figures of merit. Apply this comparison to the data of the previous problem.

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