

The Digital Connection

**Digital Information and Communication
Technology and Systems**

Copyright © 2010 Shorouk. Intl. Bookshop

All Rights Reserved

Deposit Number 5869/2010

I.S.B.N. 978 - 977- 701- 009- 2

First Edition

2010



7A Farid Semeka, Heliopolis, Cairo, Egypt

Tel. & Fax: 2240 4868 – 2643 2488

2241 5816 – 010 1633 718

E-mail: < shoroukintl@hotmail.com >

< shoroukintl@yahoo.com >

Al-shorouk Review References, and Textbooks Series

The Digital Connection

**Digital Information and Communication
Technology and Systems**

M. Sameh Said, Ph.D.

Professor, Department of
Electronics and Communication
Faculty of Engineering
Cairo University



**Egyptian National Library
Cataloging During Publishing
Prepared by Technical Affairs Department**

Said, Sameh.

The Digital Connection: Digital Information and
Communication Technology and Systems.

Sameh Said.

1st. ed . - Cairo: Shorouk International Bookshop, 2010.

752p. ; 28cm.

Al Shorouk References and Textbooks Series.

I.S.B.N. 978 - 977 - 701 - 009 - 2

621.382 1 - Digital Communication - Engineering.

Dep. No. 5869/2010

I.S.B.N. 978 - 977 - 701 - 009 - 2

Dedication

To my wife whose inspiring support made this work possible.

To my students who created the need for such a book.

There was time when a digital divide existed.

Now is the time for a digital connection to be made .

obeykandi.com

Foreword

This book is based on lectures given to the senior class at Cairo University. The course amalgamates the principles of digital communication, digital electronics and information theory as a background and focuses on modern applications such as multimedia communication, mobile, GPS etc.

This book tries to strike a balance between theory, techniques and actual systems. As such, it should be viewed as a survey book on selected topics in digital communication systems and information technology. In fact, it draws from communication theory only as much as needed to explain techniques used in selected communication systems.

This book is more application - oriented than an academic comprehensive treatise. It emphasizes the role of communication not as a mere vehicle for transport of information but as a fully integrated operation.

In the end, it attempts to fill in the gap between theory and application and give the communication jargon practical substance.

M. Sameh Said

obeykandi.com

Table of Contents

	page
CHAPTER1: Digitization: Transition from Analog to Digital	
1.1 Why Digital?	1
1.2 TDM and PCM	3
1.3 The Sampling Function	5
1.4 Sampling Theorem	9
1.5 Sample and Hold	12
1.6 Quantizer	16
1.7 Quantizer Performance	18
1.8 Pulse Code Modulation (PCM)	23
Problems	25
References	26
CHAPTER2: Baseband Modulation and Demodulation	
2.1 Line Codes	27
2.2 M- ary Pulse Modulation	32
2.3 Channel Bandwidth Limitations	34
2.4 Intersymbol Interference	42
Problems	51
References	53
CHAPTER 3:Random Signals	
3.1 The Real World	54
3.2 Probability Axioms	56
3.3 Distribution Functions	60
3.4 Statistical Averages and Random Variables Transformation	61
3.5 Statistical Moments	62
3.6 The Gaussian Distribution	63
3.7 Several Random Variables	69
Problems	71
References	72
CHAPTER 4:The Power of Correlation	
4.1 Correlation: Making Sense out of Nonsense	73
4.2 Autocorrelation Function	76
4.3 Cross Correlation	82
4.4 Characterization of Randomness	83

	page
4.5 Wiener Khintchine Theorem	86
4.6 Properties of the Power Spectral Density	90
4.7 Cross Spectral Density	96
Problems	100
References	102
CHAPTER 5: Additive White Gaussian Noise (AWGN)	
5.1 The Central Limit Theorem	103
5.2 Gaussian Process	105
5.3 Thermal Noise	108
5.4 Bandlimited Noise	114
5.5 Narrowband Noise	120
5.6 Partition of Noise	128
5.7 Figures of Merit	133
5.8 AM Detection	136
5.9 FM Detection	145
5.10 Preemphasis - Deemphasis	151
Problems	153
References	155
CHAPTER 6: Detection and Decoding Concepts	
6.1 Noise Effects in Baseband Transmission	156
6.2 Signal Representation in Orthogonal Space	158
6.3 Significance of the Signal Space	161
6.4 Gram Schmidt Orthogonalization Procedure	170
6.5 Noise and the Signal Space	174
6.6 Distance Measurement	176
6.7 Correlators' Response to Input Noise	177
6.8 Maximum Likelihood Decoder	183
Problems	188
References	190
CHAPTER 7: Optimum Baseband Receiver	
7.1 Digital Figure of Merit	191
7.2 The Likelihood Ratio Test	193
7.3 Probability of Error	198
7.4 The Matched Filter	205
7.5 Correlator Design	214

	page
7.6 Receiver Performance	224
Problems	234
References	236
CHAPTER 8: Bandpass Modulation	
8.1 Need for Bandpass Modulation	237
8.2 Signal Space	239
8.3 Constellations	245
8.4 Detection of Bandpass Signals	253
8.5 Implementation of Bandpass Demodulators	258
8.6 Performance Measures	261
8.7 BPSK	266
8.8 Carrier Recovery	270
8.9 MPSK	270
8.10 QPSK	279
8.11 QAM	290
8.12 Synchronization	297
8.13 Noncoherent Detector	298
8.14 BFSK	301
8.15 MFSK	318
8.16 Multidimensional Signaling	321
Problems	328
References	330
CHAPTER 9: Spread Spectrum	
9.1 Multiple Access Schemes	331
9.2 Spread Spectrum Modulation	333
9.3 Direct Sequence Binary Phase Shift Keying (DS/BPSK) Spread Spectrum	334
9.4 Comparison between TDMA ,FDMA, and Spread Spectrum	337
9.5 Pseudo Noise Sequence	340
9.6 Maximum Length Sequences	343
9.7 Spread Spectrum Modulation and Demodulation	348
9.8 Bandpass DS-SS	351
9.9 DS-SS Analysis	354
9.10 Probability of Error	357
9.11 Frequency Hopping	359

	page
9.12 Jamming	364
9.13 Synchronization	372
9.14 Code Division Multiplex Access (CDMA)	377
Problems	383
References	386
CHAPTER 10: Multiple Access Systems	
10.1 Communication Resource Conservation	387
10.2 FDMA	388
10.3 TDMA	390
10.4 CDMA	397
Problems	403
References	405
CHAPTER 11: Satellite Communication	
11.1 Types of Satellites	406
11.2 The Transponder	409
11.3 FDM/FM/FDMA System	409
11.4 TDMA/PSK/TDMA System	412
11.5 DAMA	415
11.6 VSAT	417
11.7 Satellite TV Bands	417
11.8 EIRP	418
Problems	421
References	422
CHAPTER 12: Global Positioning System (GPS)	
12.1 Global Surveying	423
12.2 GPS Equations	423
12.3 Solution of User Position	426
12.4 Code Signals	427
12.5 Generation of <i>C/A</i> Code	428
12.6 Acquisition and Tracking	431
Problems	432
References	433

	page
CHAPTER 13:Electronic Warfare	
13.1 Radar	434
13.2 The Range Equation	434
13.3 Pulse Radar	438
13.4 Doppler Radar	439
13.5 Pulse Compression	443
13.6 Laser Radar (LIDAR)	444
13.7 Imaging Radars	444
13.8 Electronic Counter Measures (ECM)	445
Problems	449
References	450
CHAPTER 14 :Information Theory	
14.1 Quantity of Information	451
14.2 Entropy	452
14.3 Redundancy	454
14.4 Conditional Entropy	455
14.5 Source and Channel Coding	458
14.6 Discrete Memoryless Source(DMS)	458
14.7 Source Coding Theorem	460
14.8 Source Coding Theorem (Shannon's First Theorem)	461
14.9 Prefix Coding	461
14.10 Fano Shannon Coding	465
14.11 Matching	466
14.12 Huffman Coding	468
14.13 Information in a Noisy Channel	471
14.14 General Expression for Information Transfer	473
14.15 Equivocation	475
14.16 Channel Capacity	479
14.17 Information in Continuous Signals	480
14.18 Information Capacity of Continuous Signals	484
14.19 Channel Coding Theorem (Shannon's Second Theorem)	487
14.20 Differential Entropy and Mutual Information for Continuous Signals	490
14.21 Information Capacity Theorem: Shannon's Third Theorem	492
14.22 Error Performance	499
Problems	509
References	511

	page
<hr/>	
CHAPTER 15: Channel Coding	
15.1 Repetition Code	512
15.2 Hamming Distance	513
15.3 Error Detection and Correction	514
15.4 Linear Block Codes	518
15.5 Syndrome Decoding	523
15.6 Convolutional Codes	532
15.7 The Trellis Diagram	535
15.8 Channel Decoding	535
15.9 The Viterbi Algorithm	537
Problems	542
References	544
CHAPTER 16: Multimedia	
16.1 What is Multimedia ?	545
16.2 Text Representation	546
16.3 Image Representation	547
16.4 The Monitor	548
16.5 Digital Camera and Scanner	552
16.6 Audio	552
16.7 Synthesized Audio	555
16.8 Video	557
16.9 The Basic Digital Format 4:2:2	559
16.10 Other Formats	561
16.11 PC Video Format	562
Problems	564
References	565
CHAPTER 17: Compression	
17.1 Lossy and Lossless Compression	566
17.2 DCT	569
17.3 JPEG	571
17.4 Audio Compression	583
17.5 MPEG Audio Coder	587
17.6 Video Compression	591
17.7 MPEG Standards	598
Problems	599
References	600

	page
CHAPTER 18: Multimedia Networks	
18.1 Network Types	601
18.2 Telephone Networks	602
18.3 Internet	603
18.4 Broadcast TV Networks	605
18.5 ISDN	607
18.6 Broadband Multiservice Networks	607
18.7 Fax Communication	607
18.8 E-mail	608
18.9 Videoconference	611
18.10 The Internet	616
18.11 Interactive TV	618
18.12 Network Types	618
18.13 Network Performance	625
18.14 DSL	629
18.15 Mobile (Cellular) Telephony	632
18.16 From 2G to 3G	636
18.17 From 3G to 4G	640
18.18 Orthogonal Frequency Division Multiplexing (OFDM)	641
18.19 Multicarrier Spread Spectrum	645
18.20 Wireless Networks :Bluetooth, WiFi and WiMAX	646
18.21 Personal Satellite Communication Systems	649
Problems	651
References	652
 CHAPTER 19: Satellite Networks	
19.1 TV Broadcast Coverage	653
19.2 Satellite TV Bands	656
19.3 Direct Broadcast Satellite (DBS)	657
19.4 The Satellite Receiver	658
19.5 Digital TV	661
19.6 Digital TV Receiver	670
Problems	673
References	674
 CHAPTER 20: Display Devices	
20.1 CRT	675
20.2 Liquid Crystal Cell	677
20.3 Active Matrix LCD (AMLCD)	679

	page
20.4 LCD Light Valve Systems	679
20.5 Emissive Displays	684
20.6 Stereoscopic Imaging	688
20.7 3D Display Types	690
20.8 Virtual Reality (VR)	692
20.9 Tracking	695
20.10 VR Displays	698
Problems	702
References	703

List of Figures

	Page
Chapter 1: Digitization: Transition from Analog to Digital	
Fig. (1.1) Analog communication system	2
Fig. (1.2) Effect of noise	2
Fig. (1.3) TDM link	4
Fig. (1.4) Source coder	4
Fig. (1.5) Ideal sampling	6
Fig. (1.6) Rectangular sampling function	6
Fig. (1.7) Amplitude spectrum of Fourier transform of a rectangular sampling function for $\tau / T_s = 1 / 2$	8
Fig. (1.8) Delta sampling function	8
Fig. (1.9) Amplitude spectrum for delta sampling function	9
Fig. (1.10) Fourier transform in the sampling process	11
Fig. (1.11) Use of a LPF to recover $X(f)$ from $X_s(f)$	11
Fig. (1.12) Aliasing due to under-sampling $f_s < 2f_m$	12
Fig. (1.13) Input and output of zero order hold sampling	13
Fig. (1.14) Zero order hold sampling process	14
Fig. (1.15) System undoing the effects of zero order hold sampling	14
Fig. (1.16) Sampling the function $(\sin \pi t / \pi t)$	15
Fig. (1.17) Sampling with a narrow rectangular pulse train	15
Fig. (1.18) Fourier transform output when a rectangular pulse train is used	17
Fig. (1.19) Transfer characteristic of a quantizer	18
Fig. (1.20) Input and output of a quantizer	18
Fig. (1.21) Types of quantizers	19
Fig. (1.22) Uniform quantizer with uniform input power density function	21
Fig. (1.23) PCM system	24
Fig. (1.24) Bit rate in relation to bandwidth	24
Prob. (1.1)	25
Chapter 2: Baseband Modulation and Demodulation	
Fig. (2.1) Waveform representation of binary digits	28
Fig. (2.2) PCM waveforms (Line codes)	30
Fig. (2.3) RZ AMI detecting an error	30
Fig. (2.4) Bandwidth compression	31
Fig. (2.5) Ideal filter transform function	35

	Page
Fig. (2.6) Impulse response of an ideal LPF	35
Fig. (2.7) RC filter	36
Fig. (2.8) Output characteristics V_S input and circuit characteristics	38
Fig. (2.9) Ideal pulse and its magnitude spectrum	38
Fig. (2.10) Filtering an ideal pulse	39
Fig. (2.11) Comparison of baseband and double side band (DSB) spectra	40
Fig. (2.12) Strictly band limited signal	41
Fig. (2.13) Strictly duration-limited signal	41
Fig. (2.14) Bandpass limited signal	43
Fig. (2.15) ISI equivalent model of a typical baseband band limited system where the input is approximated to impulses separated by T_s	45
Fig. (2.16) Nyquist constraint for zero ISI	45
Fig. (2.17) Raised cosine filter	47
Fig. (2.18) Impulse transfer response and transfer function	49
Fig. (2.19) Modulated PAM received pulse	49
Chapter 3: Random Signals	
Fig. (3.1) Transition probability diagram for a binary symmetric channel	60
Fig. (3.2) <i>pdf</i> and <i>cdf</i> of uniform distribution	62
Fig. (3.3) Gaussian distribution	68
Chapter 4: The Power of Correlation	
Fig. (4.1) Scatter diagram for wealth (Y) vs age (X)	74
Fig. (4.2) $Y - X$ for $\mu_X = \mu_Y = 0$	75
Fig. (4.3) Decorrelation time and time dependence of autocorrelation function	78
Fig. (4.4) $R(\tau)$ of a square wave	80
Fig. (4.5) Random binary wave	81
Fig. (4.6) $R(\tau)$ for random binary wave	81
Fig. (4.7) Random variables, ensemble, and random process	85
Fig. (4.8) Random process inputted to a time invariant filter	87
Fig. (4.9) PSD of the output from narrow BPF	90
Fig. (4.10) PSD of a sinusoidal wave with random phase	93
Fig. (4.11) Random binary wave	94
Fig. (4.12) PSD of the comb filter	96

	Page
Fig. (4.13) Cross spectral density	97
Fig. (4.14) Power relations	99
 Chapter 5: Additive White Gaussian Noise (AWGN)	
Fig. (5.1) Finding $P(z < Z \leq z + dz)$	103
Fig. (5.2) Multiple convolutions of a rectangular pulse	104
Fig. (5.3) Normalized Gaussian distribution	106
Fig. (5.4) Signal sample function of ergodic strict sense Gaussian random process	107
Fig. (5.5) Definition of wide sense Gaussian process	108
Fig. (5.6) Flat noise PSD	109
Fig. (5.7) Noise equivalent circuits	110
Fig. (5.8) PSD of noise under matched condition	111
Fig. (5.9) Rectangular pulse and its Fourier transform	112
Fig. (5.10) Characteristic of PSD and $R(\tau)$ Pair of AWGN	113
Fig. (5.11) Autocorrelation of a periodic function plus noise	115
Fig. (5.12) Cross correlation of random signal plus noise	116
Fig. (5.13) Characteristics of low pass filtered white noise	117
Fig. (5.14) Characteristics of RC filtered white noise	119
Fig. (5.15) Equivalent noise bandwidth	120
Fig. (5.16) Narrowband noise PSD	121
Fig. (5.17) Amplitude modulation as a result of noise passing through a BPF	122
Fig. (5.18) Envelope of narrowband filtered noise	123
Fig. (5.19) Characteristic of ideal BPF	124
Fig. (5.20) Change to polar coordinates for n_I, n_Q	125
Fig. (5.21) Rayleigh distribution	127
Fig. (5.22) Rician distribution	129
Fig. (5.23) In-phase and quadrature noise spectral densities for bandpass noise	131
Fig. (5.24) Extraction and generation of the in-phase and quadrature noise outputs of BPF	133
Fig. (5.25) PSD for $n(t), n_I(t), n_Q(t)$	134
Fig. (5.26) S/N calculation	134
Fig. (5.27) Baseband transmission	136
Fig. (5.28) DSBSC transmission	136
Fig. (5.29) Phasor diagram for envelope detector-Large noise case	141
Fig. (5.30) S/N for AM detection	141
Fig. (5.31) Division of noise PSD into delta functions	144

	Page
Fig. (5.32) FM system	146
Fig. (5.33) Phasor diagram for noise in FM	146
Fig. (5.34) Input – output PSD of noise in FM	147
Fig. (5.35) S/N for wideband FM	149
Fig. (5.36) Preemphasis - Deemphasis	152
Fig. (5.37) S/N improvement using deemphasis	152
 Chapter 6: Detection and Decoding Concepts	
Fig. (6.1) Baseband detection block diagram	157
Fig. (6.2) Conditional <i>pdfs</i>	158
Fig. (6.3) Ex. 6.1	162
Fig. (6.4) Baseband PAM system	163
Fig. (6.5) Quaternary system	164
Fig. (6.6) Digital bandpass transmission system	167
Fig. (6.7) Digital modulation for binary data	168
Fig. (6.8) Components of modulation and demodulation in a general digital transmission link	168
Fig. (6.9) Conceptual digital link	169
Fig. (6.10) Modulator in digital bandpass transmission without noise	169
Fig. (6.11) Detector without noise	169
Fig. (6.12) Detector with noise	170
Fig. (6.13) Coherent and noncoherent detection	170
Fig. (6.14) Schemes for waveform - basis function set transformation	172
Fig. (6.15) Ex 6.2	174
Fig. (6.16) Noise in signal space around two states s_k and s_l	177
Fig. (6.17) Signal space $N = 2$, $M = 3$	179
Fig. (6.18) Noise perturbation in the signal space	184
Fig. (6.19) Partitioning of the observation space into decision regions when $N = 2$, $M = 4$	187
Prob. (6.1)	188
Prob. (6.2)	188
Prob. (6.3)	188
 Chapter 7: Optimum Baseband Receiver	
Fig. (7.1) Components of the decision process of M-ary PAM	194
Fig. (7.2) Conditional <i>pdfs</i> for binary receiver	196
Fig. (7.3) Triangular <i>pdfs</i>	197
Fig. (7.4) <i>pdfs</i> in a binary system	198

	Page
Fig. (7.5) Probability of error	201
Fig. (7.6) Binary system with signal plus noise	206
Fig. (7.7) 4 level baseband signaling	206
Fig. (7.8) Matched filter	207
Fig. (7.9) Equivalence of matched correlator	210
Fig. (7.10) Matched filter output	213
Fig. (7.11) Correlator	213
Fig. (7.12) Correlator configurations	215
Fig. (7.13) Time windowed integrator (moving average filter)	217
Fig. (7.14) Time windowed integrator	220
Fig. (7.15) pdf at filter output	223
Fig. (7.16) probability of error	223
Fig. (7.17) Binary signal vectors	229
Fig. (7.18) Detection of unipolar baseband signaling	229
Fig. (7.19) Detection of bipolar baseband signaling	232
Fig. (7.20) Bit error performance of unipolar and bipolar signaling	233
Prob. (7.3)	234
Prob. (7.4)	234

Chapter 8: Bandpass Modulation

Fig. (8.1) Three basic bandpass modulation techniques	238
Fig. (8.2) ASK modulator	241
Fig. (8.3) BPSK	242
Fig. (8.4) 4-PSK	242
Fig. (8.5) BFSK modulation	245
Fig. (8.6) Bandwidth for BPSK, 4-PSK	246
Fig. (8.7) Signals in a signal space	247
Fig. (8.8) BPSK signals in the signal space	247
Fig. (8.9) Plot of 4-PSK signal in the signal space	249
Fig. (8.10) 8-PSK constellation	250
Fig. (8.11) Signal QAM state	252
Fig. (8.12) 16-QAM constellation	252
Fig. (8.13) Modulator and demodulator	252
Fig. (8.14) Correlator receiver front end	256
Fig. (8.15) Two equivalent implementations	256
Fig. (8.16) 4-PSK representation	256
Fig. (8.17) Complete correlator receiver	259
Fig. (8.18) Matched filter receiver-version	260
Fig. (8.19) Matched filter implementation-alternative version	262

	Page
Fig. (8.20) BPSK demodulator	262
Fig. (8.21) BPSK decision process	263
Fig. (8.22) P_B for BPSK	265
Fig. (8.23) BPSK	268
Fig. (8.24) PRK (band unlimited)	269
Fig. (8.25) BPSK modulator and demodulator	271
Fig. (8.26) Resolution of BPSK signal into PRK plus residual carrier	271
Fig. (8.27) Squaring loop for suppressed carrier recovery	272
Fig. (8.28) Costas loop	272
Fig. (8.29) 8-PSK constellation	274
Fig. (8.30) MPSK quadrature modulator	276
Fig. (8.31) MPSK demodulator	276
Fig. (8.32) Decision regions for MPSK signals	279
Fig. (8.33) P_s for MPSK	280
Fig. (8.34) QPSK (I and Q) equivalent to BPSK	282
Fig. (8.35) QPSK waveforms	283
Fig. (8.36) QPSK modulator and demodulator	283
Fig. (8.37) Ex. 8.3	286
Fig. (8.38) Bandwidth for MPSK (Ex. 8.5)	289
Fig. (8.39) Bandpass MAM modulator and demodulator	293
Fig. (8.40) Gray coded 16 -QAM constellation	295
Fig. (8.41) QAM modulator	295
Fig. (8.42) QAM demodulator	296
Fig. (8.43) Open loop synchronizer	298
Fig. (8.44) Early late gate synchronizer	299
Fig. (8.45) Noncoherent demodulator	300
Fig. (8.46) FSK modulator	303
Fig. (8.47) BFSK	304
Fig. (8.48) FSK signal	306
Fig. (8.49) FSK demodulator	307
Fig. (8.50) P_B of coherent and noncoherently demodulated FSK signal	308
Fig. (8.51) FSK noncoherent demodulator	310
Fig. (8.52) $\rho - T_b$ diagram for BFSK	312
Fig. (8.53) Sunde's FSK	313
Fig. (8.54) FSK transmitter and receiver using VCO	317
Fig. (8.55) MFSK noncoherent modulator and demodulator	320
Fig. (8.56) MFSK coherent modulator and demodulator	321

	Page
Fig. (8.57) MFSK coherent modulator/demodulator	322
Fig. (8.58) P_s and P_b for coherently demodulated equiprobable equal energy and orthogonal MFSK	324
Fig. (8.59) P_s and P_b for noncoherently demodulated equiprobable equal energy and orthogonal MFSK	325
Fig. (8.60) Spectrum of orthogonal MFSK ($M = 2$) as a superposition of OOK signal voltage spectra	326
Fig. (8.61) Orthogonal code set for 8 symbols	326
Chapter 9: Spread Spectrum	
Fig. (9.1) Concept of multiple access	332
Fig. (9.2) Direct sequence BPSK spread spectrum	335
Fig. (9.3) Direct sequence spread spectrum system	336
Fig. (9.4) Codes for users 1,2,3,4	338
Fig. (9.5) Code shape for user k in FH-CDMA	339
Fig. (9.6) Codes for MC - CDMA	341
Fig. (9.7) Frequency spectrum	341
Fig. (9.8) Feedback shift register	342
Fig. (9.9) Maximum length sequence generator for $m = 3$	343
Fig. (9.10) Characteristics of length sequence of Fig. (9.9)	345
Fig. (9.11) Sample function of random binary wave	347
Fig. (9.12) Autocorrelation function of a random binary wave	347
Fig. (9.13) PSD of a random binary wave	347
Fig. (9.14) SS modulation	350
Fig. (9.15) Spread spectrum (SS) baseband systems	350
Fig. (9.16) Coherent DS / BPSK	352
Fig. (9.17) Bandpass DS - SS	353
Fig. (9.18) Model of BPSK / DS-SS link	353
Fig. (9.19) FH-SS system	360
Fig. (9.20) Time bandwidth plane for 8-ary FH - MFSK	362
Fig. (9.21) Frequency hopping with diversity $r = 4$	362
Fig. (9.22) Fast versus slow frequency hopping	363
Fig. (9.23) FFH/MFSK Demodulator	364
Fig. (9.24) Jamming options	367
Fig. (9.25) FH Jammer	371
Fig. (9.26) Direct sequence parallel search acquisition	374
Fig. (9.27) Serial search acquisition	375
Fig. (9.28) Frequency hopping acquisition scheme	375
Fig. (9.29) DLL for tracking DS signals	376

	Page
Fig. (9.30) Tau dither tracking loop	377
Fig. (9.31) CDMA (Code division multiple access)	380
Fig. (9.32) CDMA (signal detection)	380
Fig. (9.33) Interference from DS/BPSK spectrum	380
Fig. (9.34) Detection of signal buried in noise	382
Prob. (9.8)	384
Prob. (9.9)	384
Prob. (9.10)	385
 Chapter 10: Multiple Access System	
Fig. (10.1) TDMA	388
Fig. (10.2) FDMA	389
Fig. (10.3) Modulation plan for FDM	389
Fig. (10.4) Fixed assignment TDMA	391
Fig. (10.5) Packet switching	391
Fig. (10.6) Combined TDMA / FDMA channelization	393
Fig. (10.7) FDMA / TDMA equivalence	393
Fig. (10.8) Channelization	396
Fig. (10.9) FH CDMA	398
Fig. (10.10) Multi destination FDM/FM carriers	399
Fig. (10.11) PCM Multiplex frame structure	400
Fig. (10.12) Burst compression and expansion buffers	400
Fig. (10.13) Intelsat digital transmission European standard	402
Prob. (10.2)	404
 Chapter 11: Satellite Communication	
Fig. (11.1) Geostationary satellite – Clarke’s model	407
Fig. (11.2) Satellite launching	408
Fig. (11.3) Transponder	410
Fig. (11.4) FDM / FM /FDMA transponder satellite network	411
Fig. (11.5) TDMA	411
Fig. (11.6) Block diagram of TDM / QPSK / TDMA earth station	413
Fig. (11.7) TDMA frame structure	414
Fig. (11.8) SSTDMA	418
Fig. (11.9) SSTDMA transponder	418
Fig. (11.10) <i>EIRP</i> calculation transmission	420
 Chapter 12: Global Positioning System (GPS)	
Fig. (12.1) User position in 2D	424
Fig. (12.2) GPS data format	429

	Page
Fig. (12.3) G_1 and G_2 maximum length sequence generators	430
Fig. (12.4) C/A code generator	431
 Chapter 13: Electronic Warfare	
Fig. (13.1) Principle of Radar	435
Fig. (13.2) Echo pulse	435
Fig. (13.3) A simple block diagram of a pulse radar	439
Fig. (13.4) Range of moving target	441
Fig. (13.5) Doppler radar	442
Fig. (13.6) Laser radar receiver	446
 Chapter 14: Information Theory	
Fig. (14.1) Transmission system	452
Fig. (14.2) Entropy for a binary system	454
Fig. (14.3) Digital link with source and channel coding and decoding	460
Fig. (14.4) Source encoder	461
Fig. (14.5) Decoder decision tree	463
Fig. (14.6) Noise and noiseless transmission	472
Fig. (14.7) Meaning of equivocation	477
Fig. (14.8) Venn diagram	478
Fig. (14.9) Binary symmetric channel	479
Fig. (14.10) Variation of information transfer with input probability	481
Fig. (14.11) Variation of capacity with error probability	481
Fig. (14.12) Sawtooth waveform	483
Fig. (14.13) Continuous pdf	483
Fig. (14.14) S/N at output of channel	485
Fig. (14.15) Exchange of bandwidth and S/N	486
Fig. (14.16) Channel coding	488
Fig. (14.17) Channel coding theorem	490
Fig. (14.18) Sphere packing	496
Fig. (14.19) Bandwidth diagram	497
Fig. (14.20) Error rate diagram	499
Fig. (14.21) Bit error probability for several types of binary systems	500
Fig. (14.22) Bit error probability for coherently detected M -ary orthogonal signaling	501
Fig. (14.23) Bit probability for coherently detected multiple phase signaling	502
Fig. (14.24) Ideal P_b versus E_b/η	503
Fig. (14.25) MPSK signal sets $M = 2, 4, 8, 16$	504

	Page
Fig. (14.26) MFSK signal sets $M = 2,3$	504
Fig. (14.27) Comparison of MPSK with the ideal system for $P_s = 10^{-5}$	505
Fig. (14.28) Comparison of MFSK with the ideal system for $P_s = 10^{-5}$	506
Chapter 15: Channel Coding	
Fig. (15.1) Rectangular coder	517
Fig. (15.2) Rectangular decoder	517
Fig. (15.3) Structure of the codeword	520
Fig. (15.4) Hamming distance representation	527
Fig. (15.5) Convolution coder	533
Fig. (15.6) Polynomial representation of convolutional coder	534
Fig. (15.7) Trellis representation	536
Fig. (15.8) Channel coder-decoder	536
Fig. (15.9) Using the trellis to get the right input	538
Fig. (15.10) VA steps for node 0 at time 0	540
Fig. (15.11) VA steps for nodes 1,2,3 at time 1	541
Prob. (15.7)	543
Prob. (15.10)	543
Chapter 16: Multimedia	
Fig. (16.1) Fax	549
Fig. (16.2) Coder models	549
Fig. (16.3) Raster scan display system	551
Fig. (16.4) Screen resolutions	553
Fig. (16.5) Image Capture	554
Fig. (16.6) RGB signal generation	554
Fig. (16.7) Audio encoding and companding	556
Fig. (16.8) Audio / sound synthesizer	557
Fig. (16.9) Sample positions in 4 : 4 : 2 format	558
Fig. (16.10) Sample positions in 4 : 2 : 0	560
Fig. (16.11) Sample position for SIF and CIF	562
Chapter 17: Compression	
Fig. (17.1) Compression and decompression	567
Fig. (17.2) Types of compression	567
Fig. (17.3) Transform coding	570
Fig. (17.4) JPEG encoder block diagram	572
Fig. (17.5) Image/Block preparation	573
Fig. (17.6) DC and AC coefficients	575

	Page
Fig. (17.7) Quantized DCT coefficients	577
Fig. (17.8) Zigzag scan	578
Fig. (17.9) JPEG encoder output bit stream format	581
Fig. (17.10) JPEG decoder	582
Fig. (17.11) DPCM	584
Fig. (17.12) Subband coding	586
Fig. (17.13) Linear predictive coding (LPC)	588
Fig. (17.14) Sensitivity of the ear	589
Fig. (17.15) Effect of frequency on masking effect	590
Fig. (17.16) Temporal masking	591
Fig. (17.17) MPEG perceptual coder	592
Fig. (17.18) Perceptual coder / Basic Dolby system	593
Fig. (17.19) Frame types	594
Fig. (17.20) <i>P</i> frame encoding	596
Fig. (17.21) <i>B</i> frame encoding	597

Chapter 18: Multimedia Networks

Fig. (18.1) Schematic telephone network	602
Fig. (18.2) Digital transmission in analog network	602
Fig. (18.3) HS modems	603
Fig. (18.4) Internet backbone	604
Fig. (18.5) Interactive satellite TV systems	605
Fig. (18.6) Broadcast cable network	606
Fig. (18.7) ISDN networks	606
Fig. (18.8) ATM networks	608
Fig. (18.9) Interpersonal telephony networks	609
Fig. (18.10) Telephony over the internet	610
Fig. (18.11) Fax communication	610
Fig. (18.12) E-mail network	611
Fig. (18.13) Video telephony	612
Fig. (18.14) Videoconference using MCU	612
Fig. (18.15) Multicasting	612
Fig. (18.16) Videoconference	613
Fig. (18.17) Centralized multipoint conference	615
Fig. (18.18) Decentralized multipoint conference	615
Fig. (18.19) Hybrid multipoint conference	615
Fig. (18.20) WWW system	617
Fig. (18.21) Hypertext linkages	617
Fig. (18.22) Interactive TV	619

	Page
Fig. (18.23) Interaction with a video server	620
Fig. (18.24) VOD	621
Fig. (18.25) Communication modes	622
Fig. (18.26) Transmission for constant bit rate stream over a packet switched network	628
Fig. (18.27) ADSL connections	631
Fig. (18.28) DMT operation	632
Fig. (18.29) Cellular telephone system	634
Fig. (18.30) Different multiple access schemes	639
Fig. (18.31) MC Modulation with $N_c = 4$	644
Fig. (18.32) OFDM spectrum with 16 subcarriers	644
Fig. (18.33) MC system employing OFDM	645
Fig. (18.34) Principle of DS - CDMA	647
Fig. (18.35) Principle of MC – CDMA and MC – DS – CDMA systems	647
Fig. (18.36) Typical range for wireless networks	649
 Chapter 19: Satellite Networks	
Fig. (19.1) Geostationary satellite	654
Fig. (19.2) Satellite components	655
Fig. (19.3) Noise temperature and main attenuation as a function of frequency	658
Fig. (19.4) Satellite TV transmission system	660
Fig. (19.5) Source and LNB	661
Fig. (19.6) Block diagram of analog satellite receiver	662
Fig. (19.7) TV program multiplexing	663
Fig. (19.8) Satellite TV interface	665
Fig. (19.9) COFDM system	667
Fig. (19.10) Frame format	669
Fig. (19.11) Linear polarization in digital satellite TV	669
Fig. (19.12) Complete DVB transmission / reception	671
Fig. (19.13) IRD block diagram	671
 Chapter 20: Display Devices	
Fig. (20.1) Basic CRT	676
Fig. (20.2) Shadow mask	676
Fig. (20.3) Molecules in a liquid crystal	678
Fig. (20.4) LC Cell	678
Fig. (20.5) Types of LC cells	678
Fig. (20.6) <i>TFT</i> addressing (operation)	680

	Page
Fig. (20.7) <i>TFT</i> addressing ($I - V$ characteristic)	680
Fig. (20.8) Polarization of light	680
Fig. (20.9) TN cell	682
Fig. (20.10) The cell in action	682
Fig. (20.11) PMLCD	683
Fig. (20.12) AMLCD	683
Fig. (20.13) Video wall	685
Fig. (20.14) LCD display based on magnification	685
Fig. (20.15) A simple LCD projector	686
Fig. (20.16) 3 Panel LCD projector	686
Fig. (20.17) Single LCD with microlens arrays	687
Fig. (20.18) DMD operation	689
Fig. (20.19) ACTFEL structure	689
Fig. (20.20) IV characteristic of a plasma cell	691
Fig. (20.21) Stereoscopic imaging	691
Fig. (20.22) Stereoscopic display	692
Fig. (20.23) Occluding shutter display	693
Fig. (20.24) Polarization rotating display	693
Fig. (20.25) Color anaglyph display	693
Fig. (20.26) Auto stereoscopic lenticular display	694
Fig. (20.27) VR components	694
Fig. (20.28) Motion capture	697
Fig. (20.29) Face tracker	699
Fig. (20.30) Headmount tracker	700
Fig. (20.31) Beacon tracker	700
Fig. (20.32) VR gloves	701
Fig. (20.33) HMD	701
Fig. (A-1-1) The complementary error function and its bounds	706

List of Tables

	Page
Table (7.1) Influence of E_b/η on the probability of error for bit rate 10^5 b/s	192
Table (8.1) B-ASK	240
Table (8.2) 4-ASK/8-ASK	243
Table (8.3) BPSK	243
Table (8.4) 4-PSK (QPSK)	243
Table (8.5) 8-PSK	244
Table (8.6) BFSK/4-FSK	244
Table (8.7) 4-PSK (QPSK) as sums of cosines and sines	248
Table (8.8) Output waveforms represented on orthonormal basis	248
Table (8.9) 8-PSK waveforms as sums of sines and cosines	249
Table (8.10) BASK/4-ASK/8-ASK	251
Table (8.11) BW for M-PSK	290
Table (8.12) P_s in different schemes	314
Table (8.13) Relative power efficiencies in different schemes	314
Table (8.14) Relative spectral efficiencies of binary bandpass modulation schemes	315
Table (8.15) Comparison of several PSK modulation schemes	315
Table (8.16) Bandwidth efficiency of MPSK and MFSK	319
Table (9.1) Truth table for BPSK/DS-SS	351
Table (13.1) Reader frequencies	440
Table (14.1) Probabilities of English alphabet	454
Table (14.2) Probability of error repetition code	489
Table (15.1) Effect of r on $p(e)$ for 5 digit codeword	513
Table (15.2) (Ex 15.2)	516
Table (15.3) Hamming code (7,4)	518
Table (15.4) Codewords of a (7,4) Hamming code	531
Table (15.5) Decoding table for a (7,4) Hamming code	531
Table (15.6) Error and cosets	532
Table (15.7) Error and syndrome	532
Table (15.8) Convolutional coder	533
Table (16.1) Example display resolutions and memory requirements	551
Table (16.2) PC video digitization formats	563
Table (17.1) JPEG encoding	577

	Page
Table (17.2) Default Huffman codewords	578
Table (17.3) Generalized default Huffman codewords	579
Table (18.1) Parameters of 2G to 3G mobile radio systems	637
Table (18.2) WLAN systems	638
Table (18.3) Comparison of different multiple access schemes	639
Table (18.4) Wireless systems using OFDM	641
Table (18.5) Broadcasting standards DAB and DVB-T	643
Table (18.6) Wireless local area network WLAN standards	643
Table (18.7) Wireless local loop (WLL) standards	644
Table (18.8) Characteristics of MC-SS	646

obeykandi.com

CHAPTER 1

Digitization: Transition from Analog to Digital

1.1 Why Digital?

Electrical communication rests on the principle of converting non-electrical signals - such as sound, images etc into electrical signals - through a device called transducer (microphone, camera etc). The electrical signals may then be sent directly (baseband) or after modulation using a carrier. They can also be converted to electromagnetic signals through the transmitter. The channel which is the medium where the signals are transmitted may be a wire (for electrical signals) or an optical fiber (for optical signals) or may be a waveguide or a coaxial cable (for microwave signals). All such channels are fit for what is called guided transmission, so called because the signals are contained in conduits carrying the information. Alternatively, the signals (in electromagnetic form) may be sent through the atmosphere or space. This is called unguided transmission. In all cases as signals propagate through the channel, they pick up unwanted signals called noise, since the channel is non ideal. Noise is an unwanted random signal which mars the original signal. At the receiver, the electrical signal (plus noise) is retrieved and handed over to a transducer to be perceived by the user. This transducer could be a loudspeaker, TV, etc.

Fig. (1.1) shows a block diagram of this analog communication system. The word analog stands for all continuously varying electrical signals (voltage or current), as the instantaneous value of the signal can vary at will.

One major disadvantage with analog communication is that the noise picked up in the channel may cause much deterioration in the received signal. Sometimes this deterioration reaches the point where the signal can no longer be extracted from the noise background, and we say then that the signal is buried in the noise.

The idea of digital communication rests on the principle that instead of sending the signal as an absolute value of voltage or current, we send a binary code which represents, the value of the signal in terms of 0s and 1s. The advantage is now clear. For even if the transmitted code picks up noise, the user at the receiving end should still be able to extract back the coded information despite the presence of noise as long as he can distinguish 0 from 1. Thus, information is almost unaffected by noise, since the user is not interested in the absolute value of the signal. What counts is his ability to decipher the code which means he should be able to tell whether the bit (unit of binary code) is 1 or 0. It does not matter that the pulse representing either 1 or 0 has noise on top of it so long as the user will be able to differentiate "1" from "0".

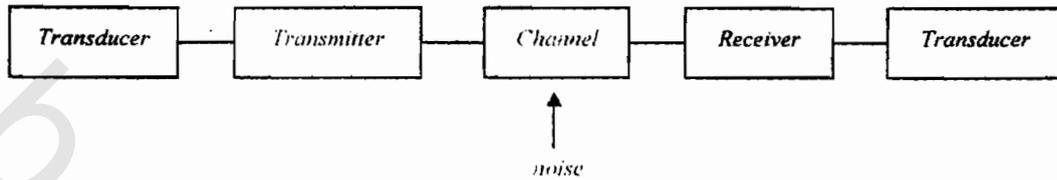


Fig. (1.1) Analog communication system

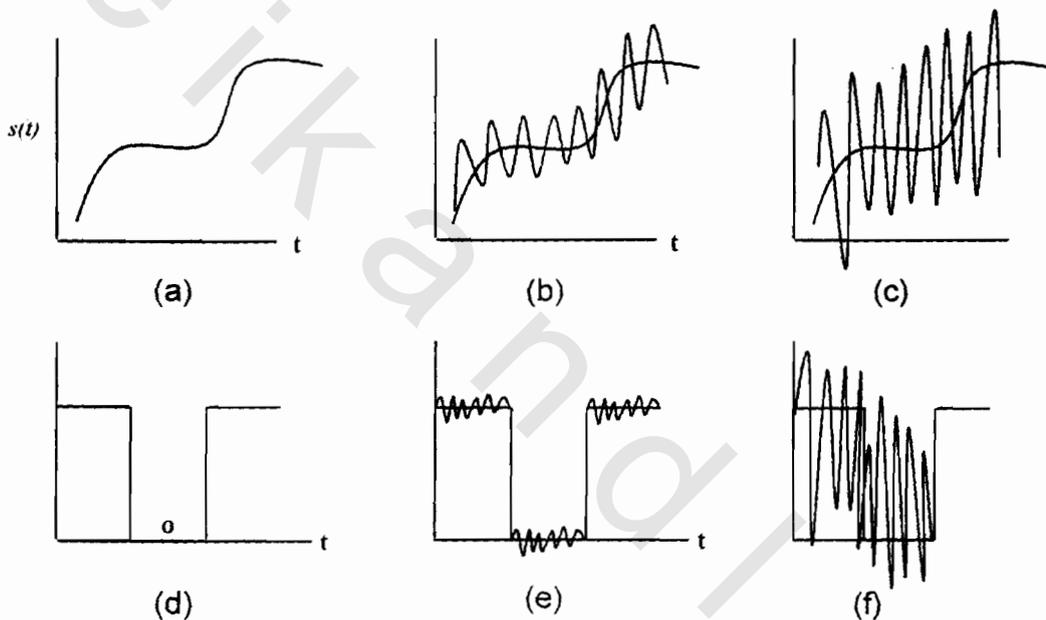


Fig. (1.2) Effect of noise

a) analog signal

b) analog signal plus noise

c) analog signal buried in noise

d) digital signal (binary pulses of 0.1)

e) digital signal plus noise

f) digital signal buried in noise (noise exceeding limit of discernment of 0 from 1)

This does not mean that digital communication is perfect without noise at all, but surely there is still noise from other causes, but the direct addition of noise on the binary pulse is greatly curtailed (Fig. 1.2). Of course, the mutilation of the signal by noise must be within limits, otherwise the user will not ever be able to tell "0" from "1". The role of distortion is similar to noise in degrading the quality of the signal. Once again, digital communication proves to be more able to withstand distortion. That is why digital systems are HIFI (high fidelity).

As the distance the signal has to travel increases there is more noise and more distortion. In analog systems, repeaters (amplifiers) are usually used to strengthen the signal which is weakened by long distance transmission. Amplifiers actually do not help remove noise or distortion. In fact, amplifiers add more noise and more distortion. Why then use them in the first place? We use them only when we have to, i.e., when the signal gets too weak to be observed. Then, we use amplifiers to bring the signal up to a level which we can perceive. Of course, this is done at the expense of the quality of the signal, i.e. by adding on more noise and distortion contributed by the amplifier itself. This effect is cumulative as we use many repeaters.

In a digital system, the repeater will be able to reshape the binary signal, so as to regenerate the pulse in a fresh form, clean from noise picked up before the repeater. Hence, the effect of noise is not as cumulative as before.

One more advantage of digital systems is the ability to interface with computers which are all but based on digital technology. Also, digital systems can deal with different digital signals at the same time (multiplexing).

Digital signals can be easily stored in computer-based systems. The cost of digital hardware continues to decline thanks to the fast growing field of electronics and the continually improving technology leading to an increase in capacity, speed and efficiency

1.2 TDM and PCM:

We start with an analog signal which represents for example a voice message. The device that transforms this analog signal into a digital signal is called source encoder (coder). We suppose that the voice signal is limited to a frequency range of 100 Hz to 4 kHz as is usually demanded by telephone standards. The electrical signal representing the voice enters a device called a sampler, which selects values of the signal at selected instants of time. The idea here-as we will prove shortly - makes digital communication a real asset in that we do not have to transmit a specific signal all the time. Only selected samples of such a signal are required to be transmitted to the receiver. We may be able to retrieve the entire signal as if the channel were dedicated completely to this subscriber. Thus, we serve many subscribers at the same time. We take a sample from each and transform it into a digital signal, then we switch to the next sample from another subscriber and so on. The transmitting medium is now carrying a stream of bits belonging to all sampled subscribers.

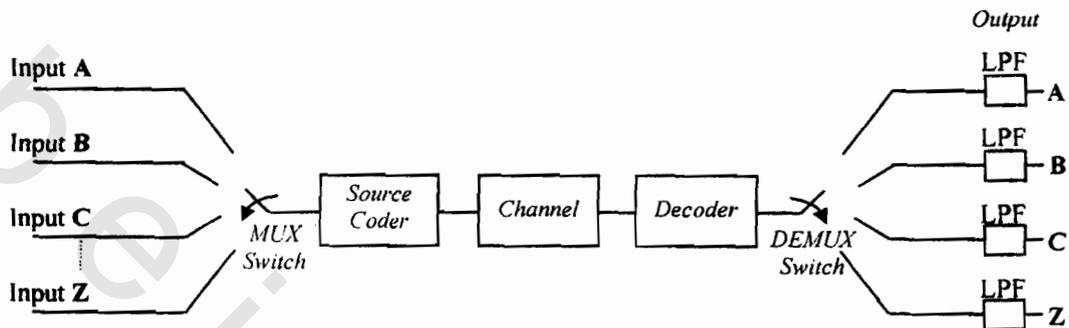


Fig. (1.3) TDM Link

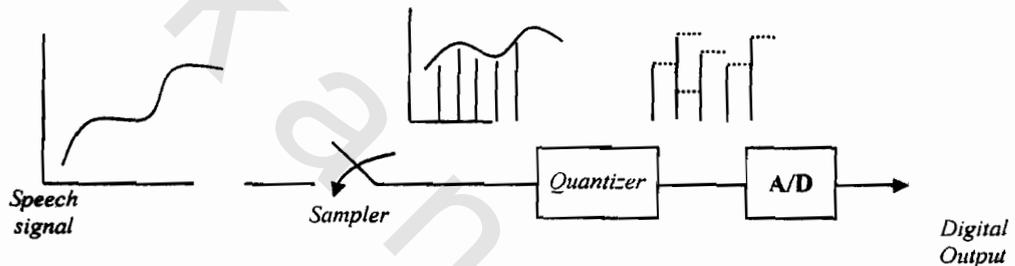


Fig. (1.4) Source coder

At the receiver, we must be able to discern samples from different subscribers. The first process of combining different digital samples from different subscribers into the transmission channel is called multiplexing, while the process of recombining the bit stream into various outputs is called demultiplexing. Multiplexing (MUX) and demultiplexing (DEMUX) must somehow be performed by synchronized switches. The decoded signals can thus be channeled to the corresponding outputs after filtering (Fig. 1.3). This is called Time Division Multiplexing (TDM).

The advantage of TDM is obvious. We can use the same hardware and transmission channel for many subscribers without extra cost. It is based on the principle of time sharing since dedicating a channel to each subscriber will not give us any new information. This is a marvelous advantage of going digital. There is, however, a condition on the speed of the MUX switch, namely, that the frequency of sampling each of the inputs must be equal or greater than twice the maximum frequency component in the signal (bandwidth). In other words,

$$f_s \geq 2B \quad (1 - 1)$$

where f_s is the sampling frequency and B is the bandwidth of the signal. This is called Nyquist sampling theorem. We will prove this theorem shortly. So if the bandwidth is 4 kHz, the sampling rate must be 8 k samples/s (or 8 kHz).

The source coder (Fig. 1.4) consists of a sampler, which takes samples of the signal at the specific instants dictated by Nyquist sampling theorem. Suppose now that we are working with 8 bit words, i.e., each sample value is to be represented by a byte, then, we divide the full operating range of the analog signal to 2^8 levels = 256. The quantizer assigns the sampled value to the nearest one of these levels. The rounding off of the actual value to the nearest quantized value entails some error. But this is something we have to live with. If we want to reduce such error (called quantization error) we have to increase the word length i.e., increase the number of quantum levels.

After the quantizer, the signal is inputted to an analog to digital converter (ADC or A/D). It may also called symbol to bit mapper. Here, each one of the nearest assigned quantum levels is converted to its corresponding binary code. Thus, we have transformed the sample of the analog signal into a digital code. We must note that this digital code is processed and transmitted before the next sample from the next subscriber comes in. This sets the limit on the speed of the MUX and DEMUX switches. The technique of transforming a sample into a digital code is called Pulse Code Modulation (PCM).

1.3 The Sampling Function:

Given an analog input $x(t)$. The sampler multiplies this function by the ideal sampling function which is an impulse train $g_s(t)$ of period T_s :

$$g_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - n T_s) \quad (1 - 2)$$

The output of the sampler is

$$x_s(t) = x(t)g_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - n T_s) \quad (1 - 3)$$

This multiplication is shown (Fig. 1.5). The output is made up of impulses at times $k T_s$ of height $x(k T_s)$, i.e.,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(n T_s) \delta(t - n T_s) \quad (1 - 4)$$

For a periodic function $g_s(t)$ with period $T_s = 2\pi / \omega_s$, we may write its Fourier series as

$$g_s(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_s t} \quad (1 - 5)$$

Taking Fourier transform of both sides

$$\begin{aligned} G_s(\omega) &= \mathcal{F} \left[\sum_{n=-\infty}^{\infty} a_n e^{jn\omega_s t} \right] .. \\ &= \sum_{n=-\infty}^{\infty} a_n \mathcal{F} [e^{jn\omega_s t}] \end{aligned}$$

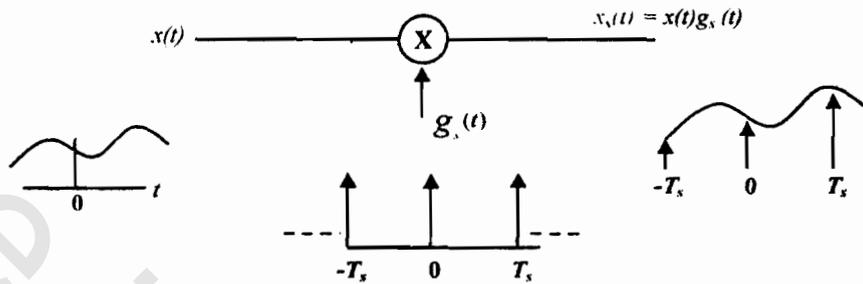


Fig. (1.5) Ideal sampling

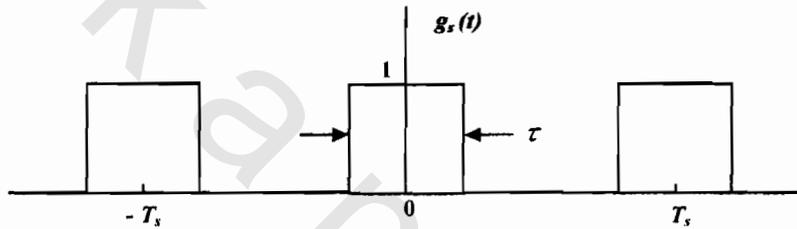


Fig. (1.6) Rectangular sampling function

$$= 2\pi \sum_{n=-\infty}^{\infty} a_n \delta(\omega - n\omega_0) \quad (1-6)$$

Now a_n can be expressed in terms of the Fourier transform of $g_{T_s}(t)$, the waveform over one period, with,

$$\begin{aligned} a_n &= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} g_{T_s}(t) e^{-jn\omega_0 t} dt \\ &= \frac{1}{T_s} \int_{-\infty}^{\infty} g_{T_s}(t) e^{-jn\omega_0 t} dt \\ &= \frac{1}{T_s} G_{T_s}(jn\omega_0) \end{aligned} \quad (1-7)$$

Thus,
$$G(\omega) = \omega_0 \sum_{n=-\infty}^{\infty} G_{T_s}(jn\omega_0) \delta(\omega - n\omega_0) \quad (1-8)$$

$$g_{T_s}(t) \Leftrightarrow \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} G_{T_s}\left(j \frac{2\pi n}{T_s}\right) \delta\left(\omega - \frac{2\pi n}{T_s}\right) \quad (1-9)$$

In terms of frequency, we have

$$G(f) = \sum_{n=-\infty}^{\infty} a_n \delta(f - nf_0) \quad (1 - 10)$$

$$g_{T_s}(t) \Leftrightarrow \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G_{T_s} \left(j \frac{2\pi n}{T_s} \right) \delta \left(f - \frac{n}{T_s} \right) \quad (1 - 11)$$

The Fourier transform is, thus, seen to be a series of impulses at the harmonics of the repetition period with strengths determined by the shape of the waveform in one period

Ex 1.1

Consider the sampling function shown. Obtain the Fourier transform of this sampling function Then show what happens as this function becomes a delta function

Solution

The Fourier transform of the truncated function g_{T_s} (Fig.1-7) is given by

$$g_{T_s}(t) = g_s(t), \quad \frac{-T_s}{2} < t < \frac{T_s}{2} \quad (1 - 12)$$

$$= 0 \quad \text{elsewhere}$$

$$G_{T_s}(j\omega) = \tau \frac{\sin \omega\tau / 2}{\omega\tau / 2} \quad (1 - 13)$$

From eqns (1 - 8), (1 - 10),

$$G_{T_s}(j\omega) = \frac{2\pi\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin \pi n \tau / T_s}{\pi n \tau / T_s} \delta \left(\omega - \frac{2\pi n}{T_s} \right) \quad (1 - 14)$$

$$G_{T_s}(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin \pi n \tau / T_s}{\pi n \tau / T_s} \delta \left(f - \frac{n}{T_s} \right) \quad (1 - 15)$$

When the sampling function becomes a delta function we have

$$g_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad (1 - 16)$$

Expanding by Fourier series with $\omega_0 = \frac{2\pi}{T_s}$

$$a_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-jn\omega_0 t} dt \quad (1 - 18)$$

$$= \frac{1}{T_s} \quad (1 - 19)$$

Thus,

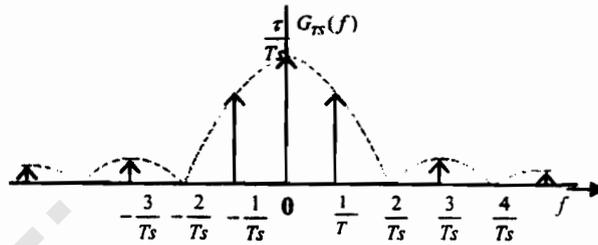


Fig. (1.7) Amplitude spectrum of Fourier transform of a rectangular sampling function for $\tau/T_s = 1/2$

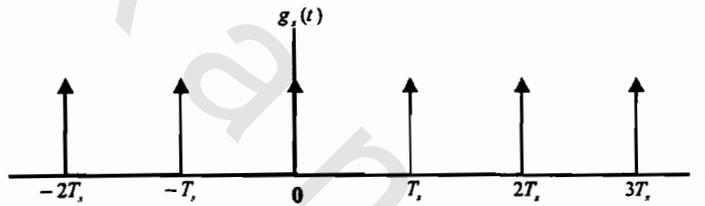


Fig. (1.8) Delta sampling function

$$g_s(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t} \quad (1-20)$$

Taking Fourier transform

$$G_s(j\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \mathcal{F}(e^{jn\omega_s t}) \quad (1-21)$$

Thus,

$$= \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) \quad (1-22)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \Leftrightarrow \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T_s}\right) \quad (1-23)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \Leftrightarrow \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad (1-24)$$

$$f_s = 1/T_s \quad (1-25)$$

Thus, the impulse train in the time domain has its Fourier transform an impulse train in the frequency domain

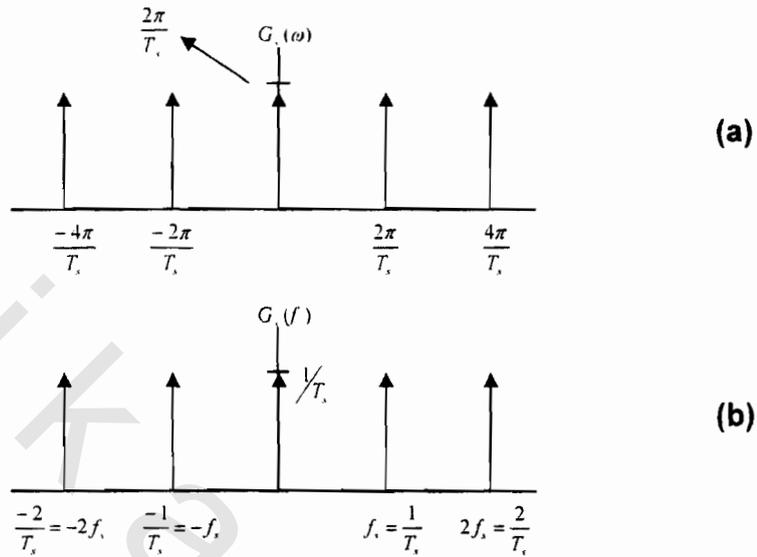


Fig. (1.9) Amplitude Spectrum for delta sampling function
a) $G(j\omega)$ b) $G(f)$

1.4 Sampling Theorem:

Back to the sampling situation depicted in Fig (1.5). From eqn (1 – 24) we have

$$G_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad (1 - 26)$$

Now let us evaluate $X_s(f)$, the Fourier transform of $x_s(t)$ as in (Fig. 1.5),

$$X_s(f) = \mathcal{F}\{x_s(t)\} = \mathcal{F}\{x(t)g_s(t)\} \quad (1 - 27)$$

From the properties of Fourier transform, multiplication in the time domain corresponds to convolution in the frequency domain

Thus $X_s(f) = X(f) * G_s(f) \quad (1 - 28)$

where * stands for convolution . From eqn. (1-26),

$$X_s(f) = X(f) * \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad (1 - 29)$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s) \quad (1 - 30)$$

From the shifting property of the delta function

$$X(f) * \delta(f - nf_s) = \int_{-\infty}^{\infty} X(\lambda) \delta(f - nf_s - \lambda) d\lambda \quad (1 - 31)$$

$$= X(f - nf_s) \quad (1 - 32)$$

Thus, eqn (1 – 30) reduces to

$$X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad (1 - 33)$$

If $X(f)$ as given in (Fig. 1.10a) is limited to a bandwidth of maximum frequency component f_M , then $X_s(f)$ is seen from eqn (1 – 33) to be a repetition of $X(f)$ around f_0 and its multiples (Fig. 1.10b).

Note in Fig. (1.10b) the $X(f)$ term does not overlap with its neighbors as long as point $f_1 < f_2$ and point $f_4 < f_3$. This simply means, for the condition

$$f_1 = f_M \leq f_2 = f_s - f_M \quad (1 - 34)$$

then the $X(f)$ term is preserved without overlap. This can be stated as

$$f_s \geq 2f_M \quad (1 - 35)$$

which is the condition for proper sampling. When $f_s = 2f_M$, f_s is called Nyquist rate. It is the smallest sampling rate that can be used while the original signal can be properly retrieved from the samples. It means that the sampling rate must be at least twice the maximum frequency component (or twice the bandwidth) in order to retrieve the original signal. The retrieval process can easily be made using a LPF (Fig. 1.11). The cut off frequency of the filter must be $f_m < f_c < f_c - f_M$. In the limit when $f_s = 2f_M$, $f_c = f_s / 2 = f_M$, all the information of $x(t)$ of the incoming signal is preserved and nothing is lost despite the fact that the signal was not transmitted in full but sent at discrete instants, provided that the rate of sampling is at least twice the bandwidth. This theory - called Nyquist theory - has tremendous impact in communication.

Now if the sampling rate is below the limit given by eqn. (1– 35), overlapping takes place. It becomes impossible to retrieve the actual signal correctly. This is called under-sampling, and the distortion produced is called aliasing (Fig. 1.12). We see from (Fig. 1.12), that in the case of under-sampling the $X(f)$ component and the $X(f + f_s)$ component overlap. As a result, the original signal $X(f)$ is no longer preserved.

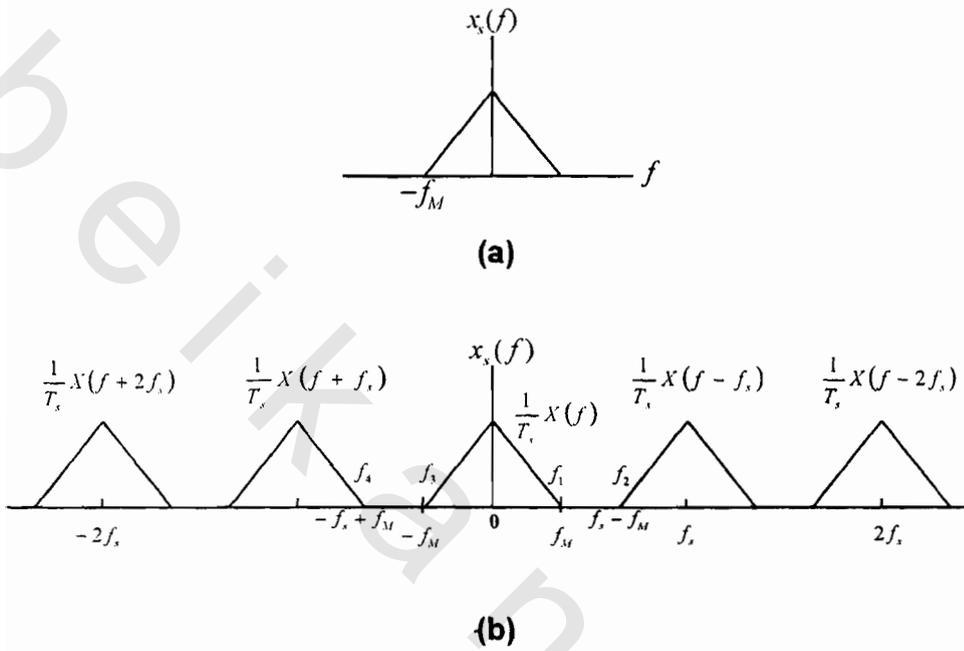


Fig. (1.10) Fourier transform in the sampling process
 a) Fourier transform of the input signal $x(t)$.
 b) Fourier transform of the ideal sampling output $x_s(t)$

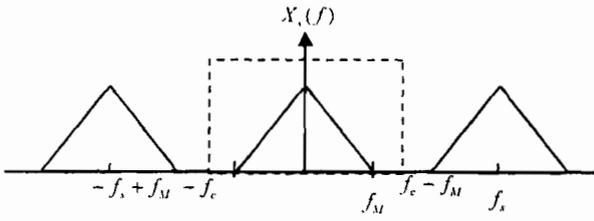


Fig. (1.11) Use of a LPF to recover $X(f)$ from $X_s(f)$

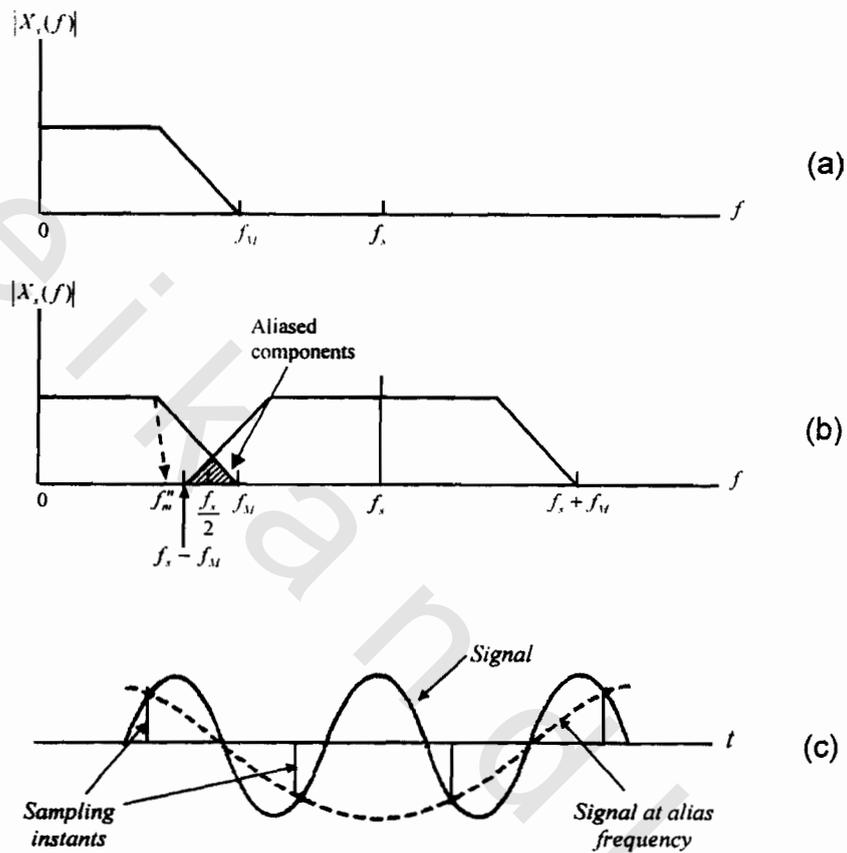


Fig. (1.12) Aliasing due to under-sampling $f_s < 2f_M$
 a) signal spectrum b) overlapping c) alias frequency

1.5 Sample and Hold:

This is also called zero order hold sampling (Fig. 1.13). After we take samples of the signal, we need to keep the values of the signal at the instants of sampling for a while to give the circuitry enough time to convert such values one at a time to a digital code.

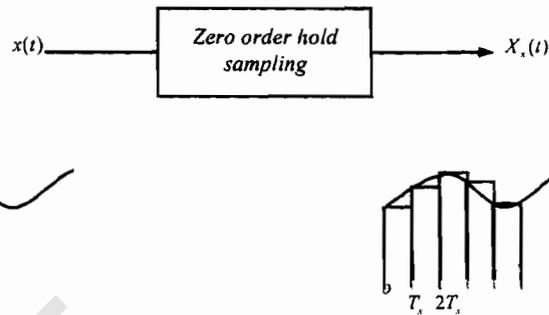


Fig. (1.13) Input and output of zero order hold sampling

Fig. (1.14) shows the incoming signal $x(t)$ first going into an ideal sampler. Here, it gets multiplied by a pulse train $g_s(t)$. This leads to impulses of height $x_s(nT_s)$ once every nT_s seconds. This leads to an output in the form

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \delta(t - nT_s) \quad (1 - 36)$$

Next, the output of the ideal sampler $x_s(t)$ enters into a linear time invariant (LTI) system, which is described by the impulse response $h(t)$. This leads to an output for each incoming sample of height $x(nT_s)$, which corresponds to holding on to the values $x(nT_s)$ for a duration of T_s . The total output $x_{sh}(t)$ is shown (Fig. 1.14). It is a signal with height $x_s(nT_s)$ in each time interval $[nT_s, (n+1)T_s]$.

We want to check now that the zero order hold sampling still contains the same information in the original signal. From Fig. (1.14), we see that $X(f)$ can be obtained back from $x_{sh}(t)$ if we use an LTI system with response $h^{-1}(t)$, then we get back the sampled function.

We saw before that information can be retrieved from a sampled function if we use a LPF with cut off frequency $f_c = f_s/2 = f_M$ and with gain T_s . Such a system is shown (Fig. 1.15)

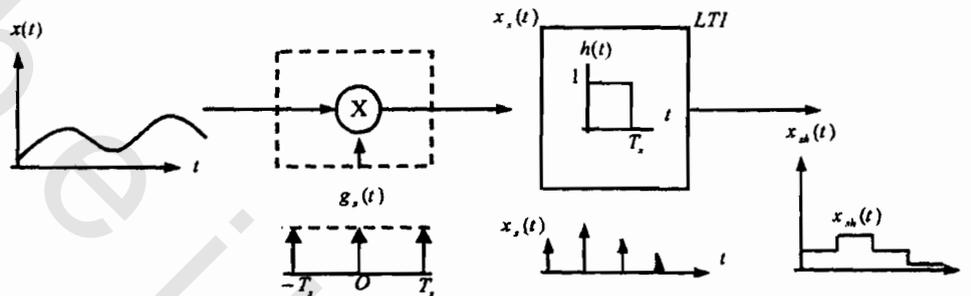


Fig. (1.14) Zero order hold sampling process

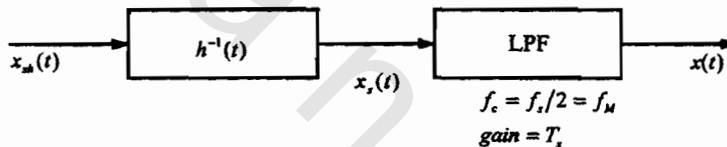


Fig. (1.15) System undoing the effects of zero order hold sampling

Ex 1.2

Obtain the output of zero order hold sampling for an input signal $x(t) = \frac{\sin \pi t}{\pi t}$

Solution

The output of the ideal sampler for sample times 0, 0.5s, 1s, 1.5s is shown (Fig. 1.16a). The output of the holding circuit is shown (Fig. 1.16b).

Ex 1.3

Show that the information can be retrieved from sampling using rectangular pulses instead of delta functions with the same Nyquist criterion.

Solution

A practical sampling system using a rectangular pulse train is shown in Fig.(1.17)

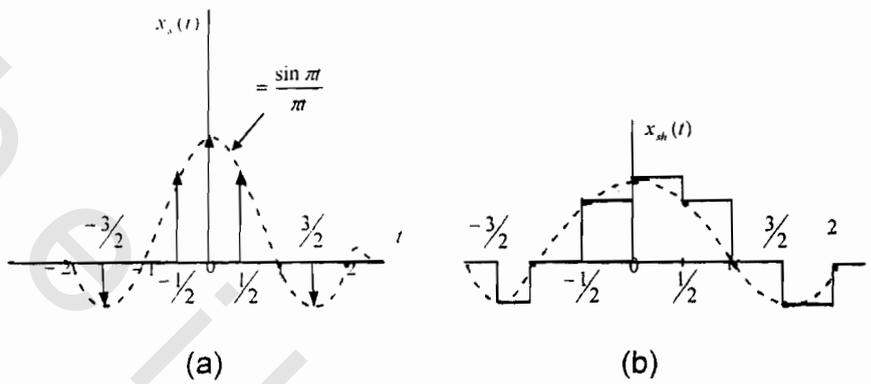


Fig. (1.16) Sampling the function $(\sin \pi t / \pi t)$
 a) the output of the sampler b) the output of the holding circuit

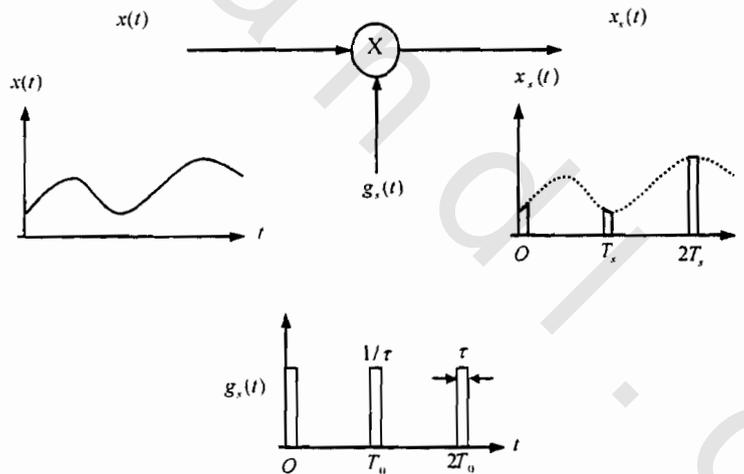


Fig. (1.17) Sampling with a narrow rectangular pulse train

The rectangular pulse has a height $\frac{1}{\tau}$ and width τ . The pulses are spaced at T_s seconds apart. Assume $x(t)$ as in Fig. (1.10a).

$$x_s(t) = x(t) g_s(t) \quad (1 - 37)$$

$$X_s(f) = \mathcal{F}\{x(t) g_s(t)\} \quad (1 - 38)$$

Since $g_s(t)$ is a periodic function

$$g_s(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_s t} \quad (1 - 39)$$

Where C_n , the Fourier series coefficient is given by

$$C_n = \frac{1}{T_s} \operatorname{sinc} \left(\frac{n\tau}{T_s} \right) = \frac{1}{T_s} \frac{\sin \pi n \tau / T_s}{\pi n \tau / T_s} \quad (1 - 40)$$

Thus,
$$X_s(f) = \mathcal{F} \left\{ x(t) \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_s t} \right\} \quad (1 - 41)$$

$$= \sum_{n=-\infty}^{\infty} C_n \mathcal{F} \left\{ x(t) e^{j2\pi n f_s t} \right\} \quad (1 - 42)$$

From Fourier transform shifting property

$$X_s(f) = \sum_{n=-\infty}^{\infty} C_n X(f - n f_s) \quad (1 - 43)$$

Thus, $X_s(f)$ consists of replicas of $X(f)$ added together. Each n^{th} replica is shifted by $n f_s$ and multiplied by C_n . Fig. (1.18) shows the result.

1.6 Quantizer:

The quantizer is device which takes the amplitude of the samples and changes it to the nearest level out of N allowed levels. For example, if the absolute value of the sample $x = 7.32 V$, this must be changed in the quantizer to the output \hat{x} (pronounced x hat) $\hat{x} = 7 V$. (Fig 1-19).

We can see the importance of the quantizer because we cannot encode all the numbers in the world with decimal fractions. So we limit ourselves to a finite set of levels and approximate the absolute value of the sample to the closest level.

Quantizers may be classified to one of four types:

- a) Mid tread, if it has 0 as one of its allowed output amplitudes or of its code words, i.e., it changes amplitudes close to 0 to 0 (Fig. 1.21a).
- b) Mid riser, it does not have 0 as one of its code words, the output is approximated to ± 0.5 or ± 1.5 and so on (Fig. 1.21b).
- c) Uniform if all its codewords are equally spaced.
- d) Non-uniform if the quantizer has codewords that are not equally spaced.

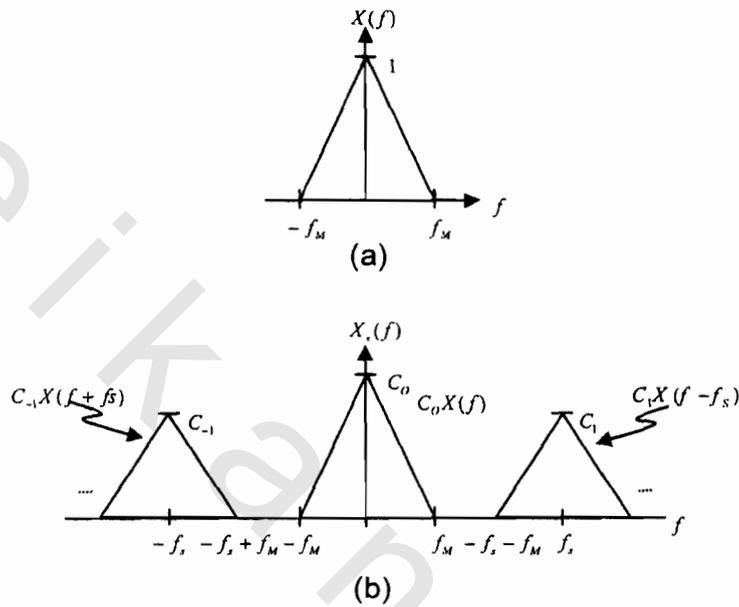


Fig. (1.18). Fourier transform output when a rectangular pulse train is used
 a) Fourier transform of input $x(t)$ b) Fourier transform of output $x_s(t)$

Ex 1.4 For a quantizer whose characteristic is given in Fig. (1.19) and whose input $x(t)$ is shown in Fig. (1.20a) find the output $\hat{x}(t)$.

Solution

For an $x(t)$ input in Fig. (1.20a) the output $\hat{x}(t)$ is shown in Fig.(1.20b).

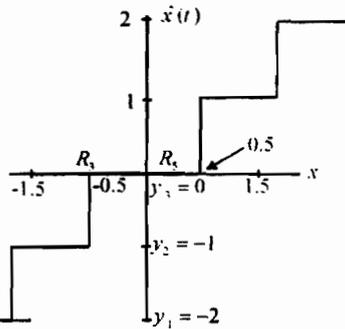


Fig. (1.19) Transfer characteristic of a quantizer

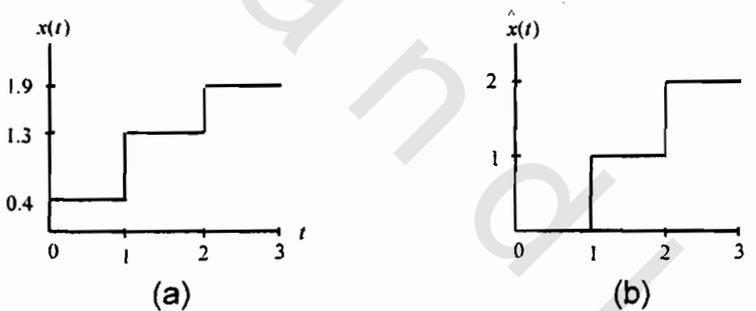


Fig. (1.20) input and output of a quantizer
a) input *b) output*

1.7 Quantizer Performance:

In the quantizer, we would like to keep the output of the quantizer as close as possible to its input. This way less information is lost. For example, 7.3 is approximated to 7, then 0.3 is lost, whereas if the codewords are 7.1, 7.2, ... 8 then 7.13 is approximated to 7.1, then only .03 is lost. We can tell how well the quantizer is doing its job if we look at the difference between the amplitude of the input to the quantizer x and the amplitude of the output from the quantizer \hat{x} . We call this error signal $e(x)$. The quantizer performance is measured by this error signal.

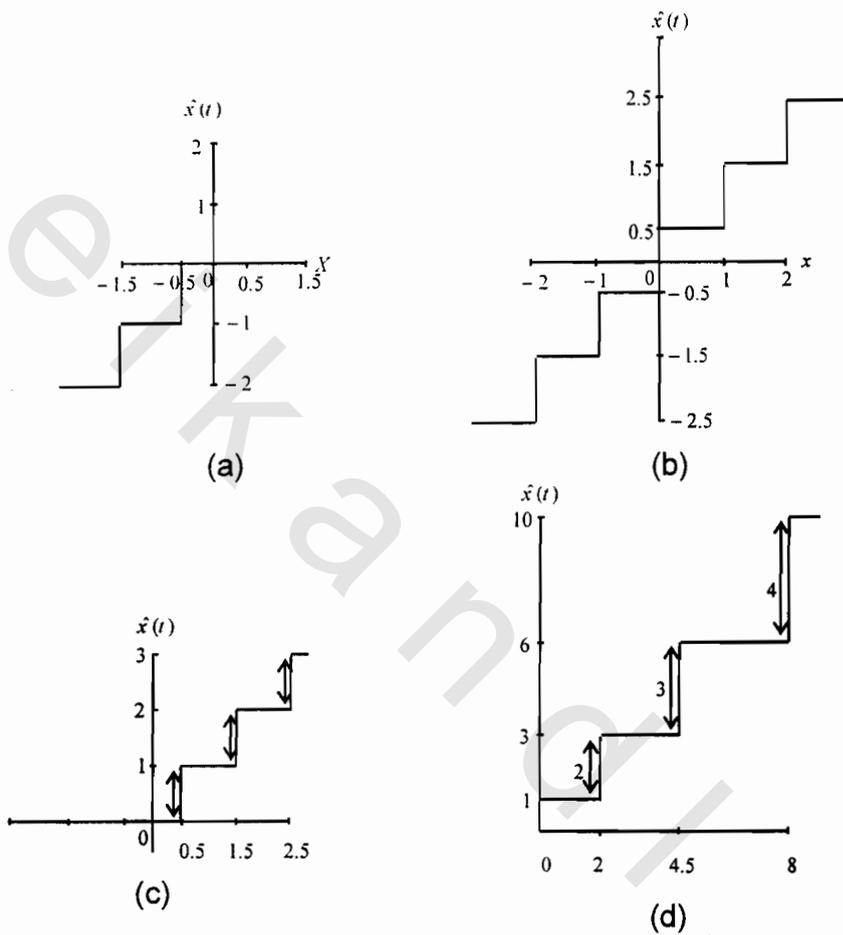


Fig. (1.21) Types of quantizers

a) mid tread

b) mid riser

c) uniform

d) non-uniform

$$e(x) = \left| \hat{x} - x \right|$$

(1 - 44)

A good quantizer has a small error signal. To proceed, we must resort to probability theory. We assume we know how likely it is that a particular amplitude comes into the quantizer. Assume that the probability density function of the amplitudes coming into the quantizer is $f_x(x)$, where x is the incoming amplitude. An overall measure of quantizer performance is the mean squared error (*mse*)

$$mse = E\left[\left(x - \hat{x}\right)^2\right] = \int_{-\infty}^{\infty} \left(x - \hat{x}\right)^2 f_x(x) dx \quad (1 - 45)$$

where E means the average value.

A quantizer with a small mse is a good quantizer. A second overall measure of quantizer performance is called signal to quantizer ratio (SQNR). We must remember that quantizer noise is the randomness due to the error signal since the absolute value of the sample is approximated to the nearest allowed level. Thus, SQNR is the ratio of the signal input power to the power of the error (or noise) introduced by the quantizer. Thus

$$\begin{aligned} SQNR &= \frac{P_s}{P_e} = \frac{\int_{-\infty}^{\infty} (x - x_m)^2 f_x(x) dx}{\int_{-\infty}^{\infty} \left(x - \hat{x}\right)^2 f_x(x) dx} \\ &= \frac{\int_{-\infty}^{\infty} (x - x_m)^2 f_x(x) dx}{mse} \end{aligned} \quad (1 - 46)$$

where x_m is the average (mean) x value. We see from eqn (1 - 46) that as mse gets smaller, SQNR improves and hence we have a better quantizer.

Assuming that x is a uniformly distributed random variable between (a, b) , let us calculate mse and SQNR for a uniform quantizer such that $f_x(x) = 1/B$, where $B = b - a$, (Fig. 1.22). From eqn (1 - 45)

$$\begin{aligned} mse &= \int_{-\infty}^{\infty} \left(x - \hat{x}\right)^2 f_x(x) dx \\ &= \frac{1}{B} \int_a^b \left(x - \hat{x}\right)^2 dx \end{aligned} \quad (1 - 47)$$

We must break up the integral in eqn. (1 - 47) into the "cells" $R_1, R_2 \dots R_N$ noting that all values of x that fall into the cell R_i have an output from the quantizer \hat{x} corresponding to the value y_i

$$\begin{aligned} \text{Thus } mse &= \frac{1}{B} \int_{R_1} (x - y_1)^2 dx + \dots + \int_{R_N} (x - y_n)^2 dx \\ &= \frac{1}{B} \sum_{i=1}^N \int_{R_i} (x - y_i)^2 dx \end{aligned} \quad (1 - 48)$$

Considering the i^{th} cell R_i , the term $x - y_i$ is the error of the quantizer ϵ_i

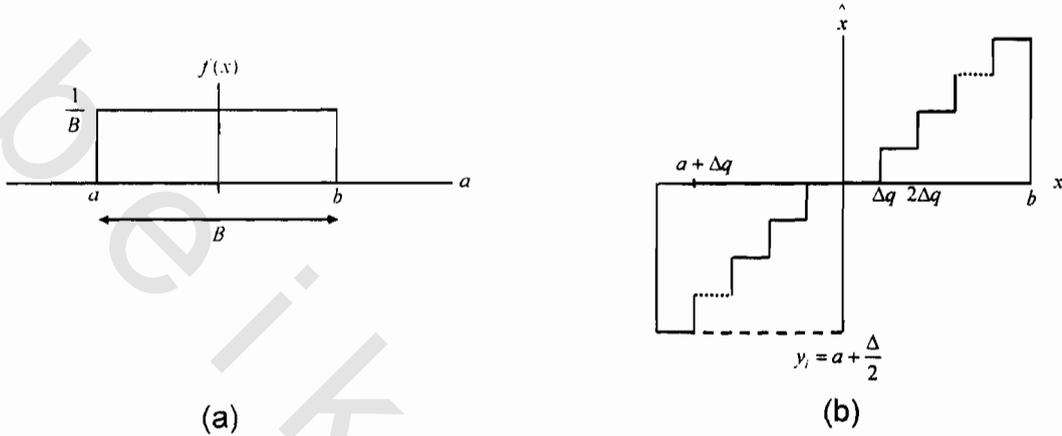


Fig. (1.22) Uniform quantizer with uniform input power density function
 a) uniform input power density function. b) transfer function x of a uniform quantizer

$$\epsilon_i = x - y_i \tag{1 - 49}$$

$$mse = \frac{1}{B} \sum_{i=1}^N \int_{\epsilon_{i, \min}}^{\epsilon_{i, \max}} \epsilon_i^2 d \epsilon_i \tag{1 - 50}$$

Where $\epsilon_{i, \min}$ and $\epsilon_{i, \max}$ refer to the smallest value of the quantizer error and the largest value of the quantizer error in a cell R_i .

Referring to Fig. (1.22b), in the first cell all values between a and $a + \Delta q$ are mapped into the value $y_1 = a + \Delta q / 2$. The error is $\pm \Delta q / 2$. The largest value of error occurs when the input $x = a - \Delta q / 2$ is mapped to the output $y_1 = a$. In this case, the error $\epsilon_{i, \max} = \Delta q / 2$.

Similarly, the smallest value of the error occurs when the input $x = a + \Delta q / 2$ is mapped to the output $y_1 = a$. In this case the error value is $-\Delta q / 2$. Thus, $\epsilon_{i, \min} = -\Delta q / 2$. The same reasoning applies to any cell. Thus, eqn (1 - 50) becomes

$$mse = \frac{1}{B} \sum_{i=1}^N \int_{-\Delta q / 2}^{\Delta q / 2} \epsilon_i^2 d \epsilon_i \tag{1 - 51}$$

Since all N integrals are identical, eqn (1 - 51) becomes

$$mse = \frac{N}{B} \int_{-\Delta q/2}^{\Delta q/2} \epsilon_i^2 d\epsilon_i \quad (1 - 52)$$

We note that

$$N\Delta q = B \quad (1 - 53)$$

Thus, eqn (1 - 52) becomes

$$mse = \frac{\Delta q^2}{12} \quad (1 - 54)$$

Let us now assume $[a, b] = [-A, A]$. We may now calculate SQNR (eqn 1 - 46), noting $x_m = 0$ and $B = 2A$ and $\Delta q = 2A/N$

$$SQNR = \frac{\frac{1}{2A} \int_{-A}^A x^2 dx}{mse} \quad (1 - 55)$$

$$= \frac{A^2/3}{\Delta^2 q / 12} = \frac{4A^2}{(2A/N)^2} = N^2 \quad (1 - 56)$$

It is clear from this astounding result that as the number of levels N increases, the signal to noise ratio increases quadratically.

Ex 1.5

An L level linear quantizer has an analog signal with peak to peak range of $V_{pp} = 2V_p$ where V_p is the maximum voltage. Assume quantization levels to be uniformly distributed over the full range (uniform quantizer), find quantizer error variance and S/N .

Solution

The step size between quantum levels $\Delta q = V_{pp} / L$. The maximum error is $\Delta q/2$ in the positive direction and $\Delta q/2$ in the negative direction. Assuming that the quantization error ϵ_i is uniformly distributed over one interval Δq wide, i.e., the analog input takes on all values with equal probability, the quantizer error variance σ^2 is given by

$$\sigma^2 = \int_{-\Delta q/2}^{\Delta q/2} \epsilon_i^2 f(\epsilon_i) d\epsilon_i = \int_{-\Delta q/2}^{\Delta q/2} \epsilon_i^2 \frac{1}{\Delta q} d\epsilon_i = \frac{\Delta q^2}{12} \quad (1 - 57)$$

where $f(\epsilon_i) = 1/\Delta q$ for uniform probability density function of the quantization error. The variance σ^2 corresponds to the average quantization noise power. The peak power of the analog signal normalized to 1Ω is

$$V_p^2 = \left(\frac{V_{pp}}{2}\right)^2 = \left(\frac{L\Delta q}{2}\right)^2 = \frac{L^2\Delta q^2}{4} \quad (1-58)$$

Thus
$$S/N = \frac{L^2\Delta q^2/4}{\Delta q^2/12} = 3L^2 \quad (1-59)$$

With infinite quantization levels there is no quantization noise, we also note that S/N is independent of Δq . From eqn. 1-57 and 1-54 we note that

$$\text{mse} = \sigma^2 \quad (1-60)$$

1.8 Pulse Code Modulation (PCM):

The source coder which maps an analog signal into a digital signal consists of a sampler, a quantizer and a device called symbol to bit mapper or analog to digital converter (ADC or A/D). In Fig. (1.23), the input is $x(t)$, the output of the sampler is $x_s(t)$ which is inputted to the quantizer. The output of the quantizer is $\hat{x}_s(t)$ which is inputted to the symbol to bit mapper, where the output is the digital signal, which is a set of bits 0,1. These bits are represented here as short pulses, where 1 is $+5V$ pulse and 0 is $-5V$ pulse.

The symbol to bit mapper associates bits 00 for 0, 01 for 1, 10 for 2, and 11 for 3. The sampling rate is the number of samples per second which the sampler creates. The symbol rate is the number of samples per second that leave the quantizer. Since the quantizer creates one sample out of each sample that comes in, the symbol rate is also the rate of the symbols that come into the quantizer which is equal to the sampling rate. The bit rate indicates how many bits per second come out of the symbol to bit mapper. The bit rate is equal to the symbol rate times the number of bits per symbol. What makes a PCM a good source coder are two parameters:

- a) small quantizer error or large SQNR such that the information lost is as small as possible.
- b) small bandwidth which requires a low bit rate. (to give the quantizer enough time to do the conversion) .

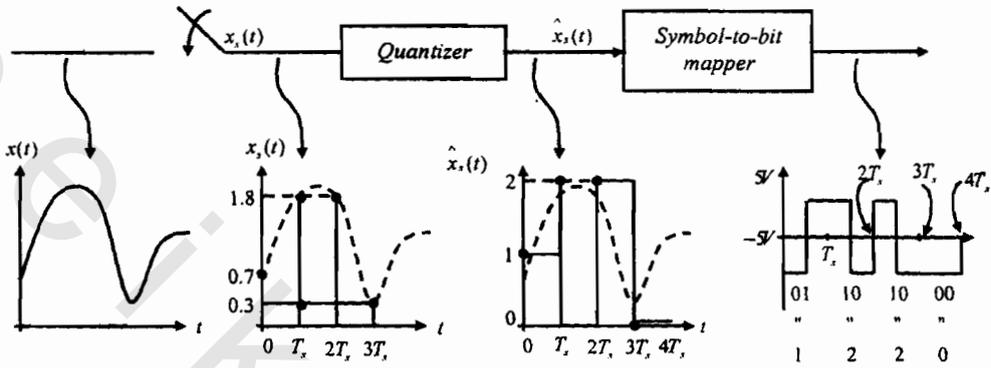


Fig. (1.23) PCM system

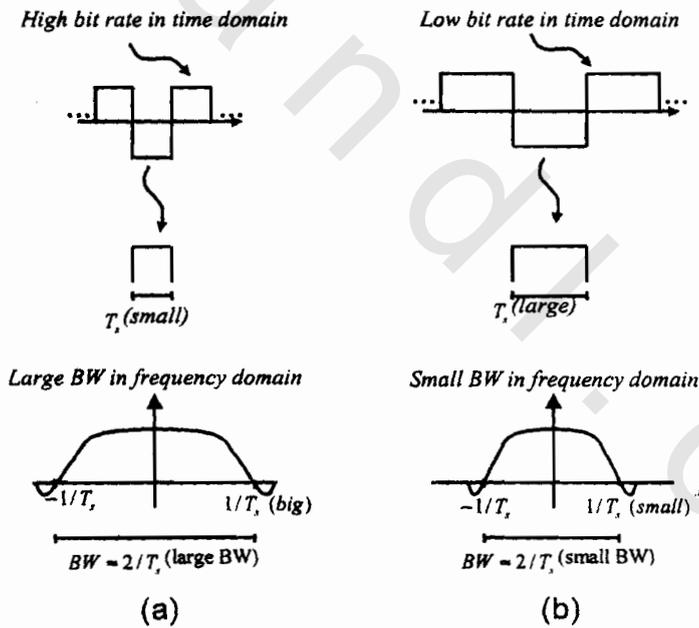
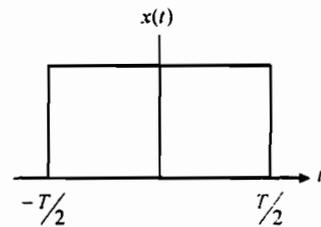


Fig. (1.24) Bit rate in relation to bandwidth
 a) high bit rate b) small bit rate

We see from Fig. (1.24) that a high bit rate requires a narrow pulse width and hence large bandwidth, whereas a low bit rate results in a small bandwidth.

Problems

1. Consider a given waveform $x(t)$ as shown. Find its Fourier transform. Then find the spectrum of this function when sampled with a unit impulse train. Also find the spectrum of this function when sampled by a pulse train with pulse width τ , amplitude $1/\tau$ and period T_s . Show that in the limit as τ approaches zero the two spectra converge.



2. An input signal is $A \sin \omega_i t$ with period T_i . It is sampled with a sampling function whose period T_s is $3/4 T_i$. Show the result of under-sampling on the recovery of the signal. What happens when you sample at $T_s = \frac{1}{4} T_i, \frac{1}{2} T_i, T_i, 2 T_i$. Take the first sample at the first peak of the input. What do you conclude?
3. Obtain an expression for the bandwidth of PCM of N levels and signal bandwidth B . Obtain also the information rate of the bit stream.
4. In a CD digital audio system, an analog signal is digitized so that the ratio of peak signal power to the peak quantization noise is at least 96 dB . How many quantization levels are needed? How many bits per sample are needed? What is the bit rate if the sampling rate is 44 k samples/s ?
5. Compare σ^2 , mse, SQNR as figures of merit. Apply this comparison to the data of the previous problem.

References

1. "Telecommunications Demystified", C.R. Nassar, LLH Technology Publishing, Eagle Rock Va, 2001.
2. "Digital Communications", B. Sklar, 2nd ed, Prentice Hall PTR, Upper Saddle River N.J. 2001.
3. "Digital and Analog Communication Systems", L. Couch, 6th ed., Prentice Hall, Upper Saddle River, N.J. 2001.
4. "Communication Systems", B. Carlson, P. Crilly, J. Rutledge, 4th ed., McGraw Hill, N.Y. 2002.
5. "Introduction to Communication Systems", F. Stremler, 2nd ed., Addison. Wesley, Reading Ma, 1982.
6. "Modern Digital and Analog Communication Systems", B. Lathi, 3rd ed., Oxford, N.Y 1998.
7. "Digital Communications", J, Proakis 4th ed., McGraw Hill, N.Y, 2001.
8. "Communication Engineering Principles", I. Otung, Palgrave, N.Y. 2001.