CHAPTER 9
Spread Spectrum

9.1 Multiple Access Schemes:

Multiple access means allowing multiple users to share the same communication channel. There are two examples, time division multiple access (TDMA) and frequency division multiple access (FDMA). In TDMA, samples of different users are multiplexed and processed at Nyquist rate. At the receiver a demultiplexer separates out the combined signals into samples headed toward their intended destinations. Because of the sampling theorem, samples of a given user can be filtered out to produce the original signal. Samples may be sent out as baseband or as line coded or even bandpass modulated signals. In FDMA, multiple users are assigned different carrier frequencies. Thus, the spectrum is divided among the users. Alternatively, we may let users share the whole spectrum, by devising means by which each user is transparent to the other. This is important since time or spectrum are valuable but limited resources in which case allocation of portions for different users should serve as many users as possible.

In general we have two ways for multi user sharing:

1. Multiplexing schemes, where channel sharing depends on assigning portions of the channel to each user by a system controller at a central location. The system controller assigns a fixed portion for each user and controls access to it.
2. Multiple access schemes: where channel sharing depends on assigning portions or all of the channels to each user simultaneously depending on current availability. The system controller updates the sharing mechanism depending on demand and availability.

Let us assume two users k and j using the same communication channel, each using MPSK signal set. During the time interval (qT_S) to $(q+1)T_S$ we have one of the following signals for user k

$$s_{q,1}^{(k)}(t) \quad , \quad s_{q,2}^{(k)}(t) \;..\; s_{q,M}^{(k)}(t)$$

At the same time we have for user j one of the signals

$$s_{q,1}^{(j)}(t) \quad , \quad s_{q,2}^{(j)}(t) \;..\; s_{q,M}^{(j)}(t)$$

The channel thus carries a combination plus noise (Fig. 9.1a) at the receiver front end we have

$$r_d(t) = s_q^{(k)}(t) + s_q^{(j)} + n(t) \qquad (9-1)$$

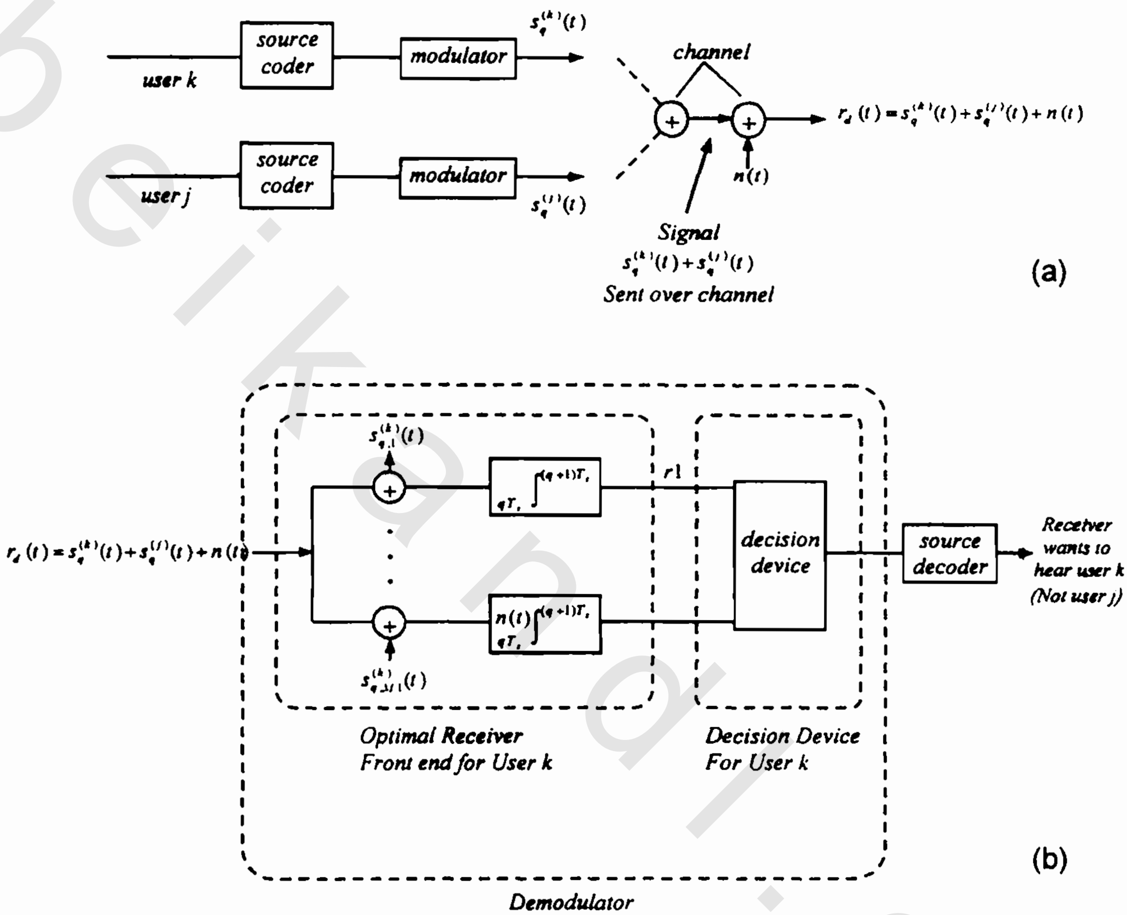


Fig. (9.1) Concept of multiple access

a) user k and user j send their signals over a channel

b) a receiver tries to pick up user k's signal

We need for the demodulator to pick up the signal from user k (Fig. 9.1b). We use the correlator that picks up only $s_q^{(k)}(t)$

The signal coming out of the top branch is

$$r_1 = \int_{qT_s}^{(q+1)T_s} s_{q,1}^{(k)}(t)\, r_d(t)\, dt$$

$$= \int_{qT_s}^{(q+1)T_s} s_{q,1}^{(k)}(t) \left[s_q^{(k)}(t) + s_q^{(j)}(t) + n(t)\right] dt$$

$$= \int_{qT_s}^{(q+1)T_s} s_{q,1}^{(k)}(t)\, s_q^{(k)}(t)\,dt + \int_{qT_s}^{(q+1)T_s} s_{q,1}^{(k)}(t)\, s_q^{(j)}(t)\,dt + \int_{qT_s}^{(q+1)T_s} s_{q,1}^{(k)}(t)\, n(t)\,dt$$

To ensure orthogonality for multiuse operation

$$\int_{qT_s}^{(q+1)T_s} s_{q,i}^{(k)}(t) s_{q,i}^{(j)}(t)dt = 0 \quad , \ i = 1.....M \qquad (9 - 2)$$

In general,

$$\int s_q^{(k)}(t) s_q^{(j)}(t)dt = 0 \qquad (9 - 3)$$

For all possible $s_q^{(k)}$

Occasionally, we may settle for pseudo orthogonal condition where the integral is not quite zero but a small quantity $\in$

$$\int s_q^{(k)}(t)\ s_q^{(j)}(t)dt < \in \qquad (9 - 4)$$

9.2 Spread Spectrum Modulation:

In communication systems, we are concerned about the efficient utilization of bandwidth and power. In some cases, we may sacrifice these merits in favor of secure communication in a hostile environment, such that the transmitted signal may not be easily detected or recognized by unauthorized users. This is done by spread spectrum modulation. It has a particular advantage of rejecting unintentional interference despite the simultaneous sharing of the entire spectrum by multiusers. It can also override the intentional interference by a hostile transmitter attempting to jam the transmission. Spread spectrum modulation is characterized by the following features.

1. Data sequence occupies a bandwidth in excess of the minimum bandwidth needed to transmit it.
2. Spectrum spreading is accomplished before transmission through the use of a code that is independent of the data sequence.

The receiver has the same code which is synchronized with the transmitter and is used to despread the received signal to recover the data sequence. Originally spread spectrum was developed for military purposes to resist interference and jamming and to provide security by disabling unauthorized users to intercept classified messages. Nowadays, spread spectrum modulation is also used in civilian applications such as mobile communication, where multiusers share the same channel bandwidth without need for individual allocation or synchronized access. The main two basic techniques which will be considered here are the direct sequence spread spectrum (DSSS) and frequency hopping spread spectrum (FHSS). In DSSS, the data is used to modulate a wide band code. This transforms the narrowband data sequence into a noise-like wide band code. The resulting wide band signal undergoes a secured modulation using PSK. In FHSS, the spectrum of a data modulated carrier is widened by changing the carrier frequency in a pseudorandom manner. Both of these techniques use a noise-like spreading code called pseudo random or pseudo noise (PN) sequence. It is interesting to note that in spread spectrum, the whole philosophy of communication is reversed. Instead of continually striving to fight noise to keep data from being buried in it, we deliberately bury the data in the noise and use the noise as the carrier or camouflage container for the data, with the ability of retrieving the data from the noise without loss of information through the use of a special code assigned to each user.

9.3 Direct Sequence Binary Phase Shift Keying (DS/BPSK) Spread Spectrum:

Fig. (9.2) shows an example of direct sequence binary phase shift keying (DS / BPSK). Fig. (9.3) shows the transmitter and receiver. In Fig. (9.2), the input data $x(t)$ is first modulated by a carrier wave in a BPSK modulator. This output is then fed to a BPSK code modulator where it is modulated by a code pulse waveform $g(t)$. The output of this modulator is $x(t)g(t)\cos\omega_0 t$ The received signal - neglecting noise for now – is demodulated using the same code, i.e., $x(t)\,g(t)$ is multiplied by $g(t)$ again. Since $[g(t)]^2 = 1$, we obtain $x(t)$ back . Thus, $x(t)$ is obtained back only if the destination is supplied with the code used by the transmitter. Note that in the graph, binary 0 is a positive pulse and binary 1 is a negative pulse. In Fig. (9.3),the data signal $x(t)$ modulates a carrier then the data-modulated signal is again modulated with a high speed (wide band) spreading signal $g(t)$. Consider data modulated carrier having power P and phase modulation $\theta_x(t)$

$$s_x(t) = \sqrt{2P}\,\cos\left[\omega_0 t + \theta_x(t)\right] = \sqrt{2P}\,x(t)\,\cos\,\omega_0 t \qquad (9-5)$$

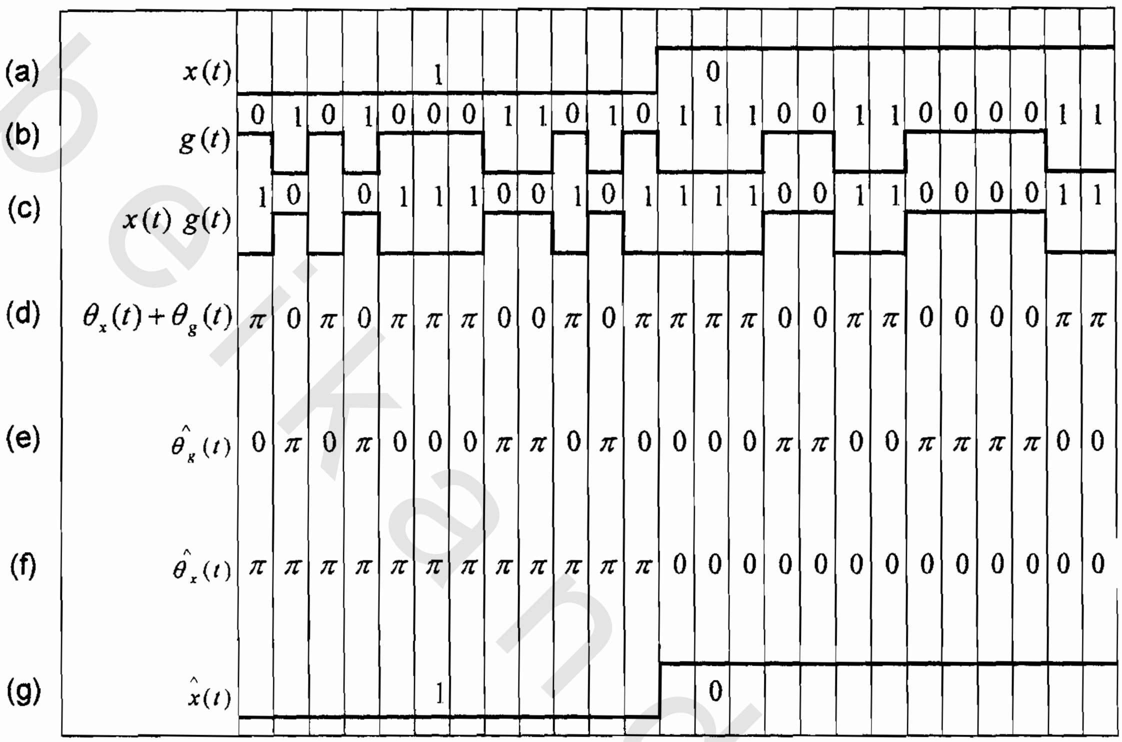


Fig. (9.2) Direct sequence BPSK spread spectrum

a) binary data *b) code sequence*
c) transmitted sequence *d) phase of transmitted carrier*
e) phase shift produced by receiver code
f) phase of received carrier phase shifted (despread) by receiver code
g) demodulated data waveform

Upon further AM modulation by the spreading signal $g(t)$, the transmitted waveform becomes

$$s(t) = \sqrt{2P}\ x(t)\ g(t) \cos \omega_0 t \qquad (9-6)$$

$$s(t) = \sqrt{2P}\ \cos\left[\omega_0 t + \theta_x(t) + \theta_g(t)\right] \qquad (9-7)$$

where the phase of the carrier has two components $\theta_x(t)$ due to the data and $\theta_g(t)$ due to the spreading sequence. The received signal is correlated with a synchronized replica of the spreading signal $g(t - \hat{T}_d)$,

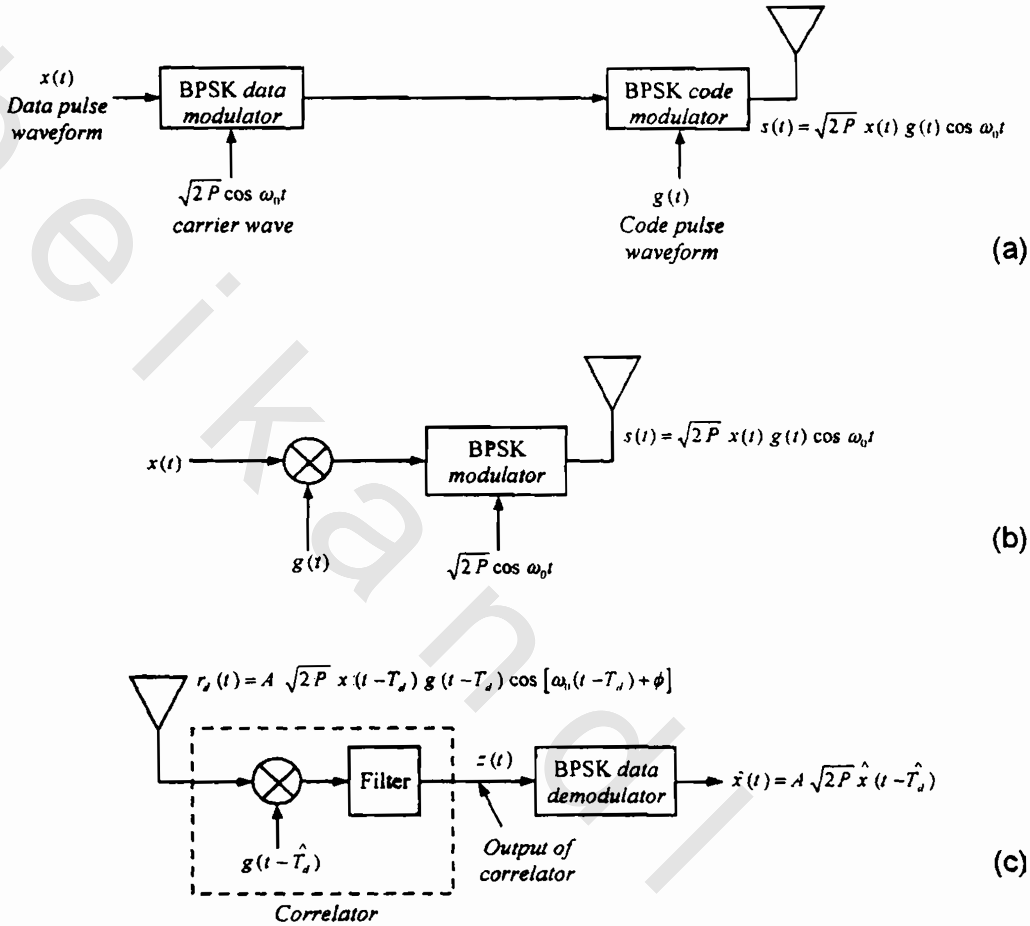


Fig. (9.3) Direct sequence spread spectrum system

a) BPSK direct sequence transmitter

b) simplified BPSK direct sequence transmitter

c) BPSK direct sequence receiver

where $\hat{T}_d$ is the receiver's estimate of the propagation delay T_d from the transmitter to the receiver. The output from the correlator is

$$z(t) = A\sqrt{2P}\, x(t - T_d)\, g(t - \hat{T}_d) \cos\left[\omega_0 (t - T_d) + \theta_0\right] \qquad (9-8)$$

where A is system gain, θ_0 is a random phase angle in the range $(0, 2\pi)$. Since $g(t) = \pm 1$, the product $\left[g(t - T_d) \quad g(t - \hat{T}_d) \right]$ will be unity if $\hat{T}_d = T_d$, i.e., if the code signal at the receiver is exactly synchronized with the code at the transmitter. After despreading a conventional demodulator is used to retrieve the data signal.

9.4 Comparison between TDMA, FDMA, and Spread Spectrum:

In TDMA, the user k has a time slot different from that assigned to user j. In this way the product of the integral (eqn. 9-4) is zero. Thus, users send an entire set of data symbols in their time slots (called bursts). In GSM, this burst is 148 data symbols in one time slot. We can assign the slot time T_{slot}

$$T_{slot} = n\, T_b = n / R_b \qquad (9-9)$$

where n is the number of bits assigned to each user's slot. The sampling rate

$$R_{samp} = \frac{1}{m\, T_{slot}} = \frac{1}{m \;\, n T_b} = \frac{R_b}{m \;\, n} = 2 f_{max} \qquad (9-10)$$

where m is the number of users and f_{max} is the fundamental maximum frequency component, of the analog signal .

In FDMA, the user sends data in one frequency band, and the other users send data in different bands. So user k has zero signal at the frequency where user j is transmitting and vice versa, so that

$$s_q^{(k)}(f)\; s_q^{(j)}(f) = 0 \qquad (9-11)$$

Hence, orthogonality applies

$$\int s_q^{(k)}(f)\; s_q^{(j)}(f)\, dt = 0 \;. \qquad (9-12)$$

Each user has a frequency band assigned to him all the time.

In code division multiple access (CDMA), a code or signature is assigned for user k and another code or signature for user j, so that eqns. (9 – 3) or (9 – 5) applies. This way we can send both user k's signal and user j's signal and not experience any interference. So by giving each user a carefully chosen code, we can send multiple users' signals at the same time and at the same frequency and still be able to separate them. In orthogonal CDMA, users' signals satisfy eqn.(9 – 3).

In pseudo orthogonal CDMA users' signals satisfy eqn. (9 – 5), i.e., a little bit of user j's signal appears in user k's signal. By accepting this concession, we can support much more users than with strictly orthogonal CDMA, and still can distinguish one user from the interfering. remnant signals of the other users.

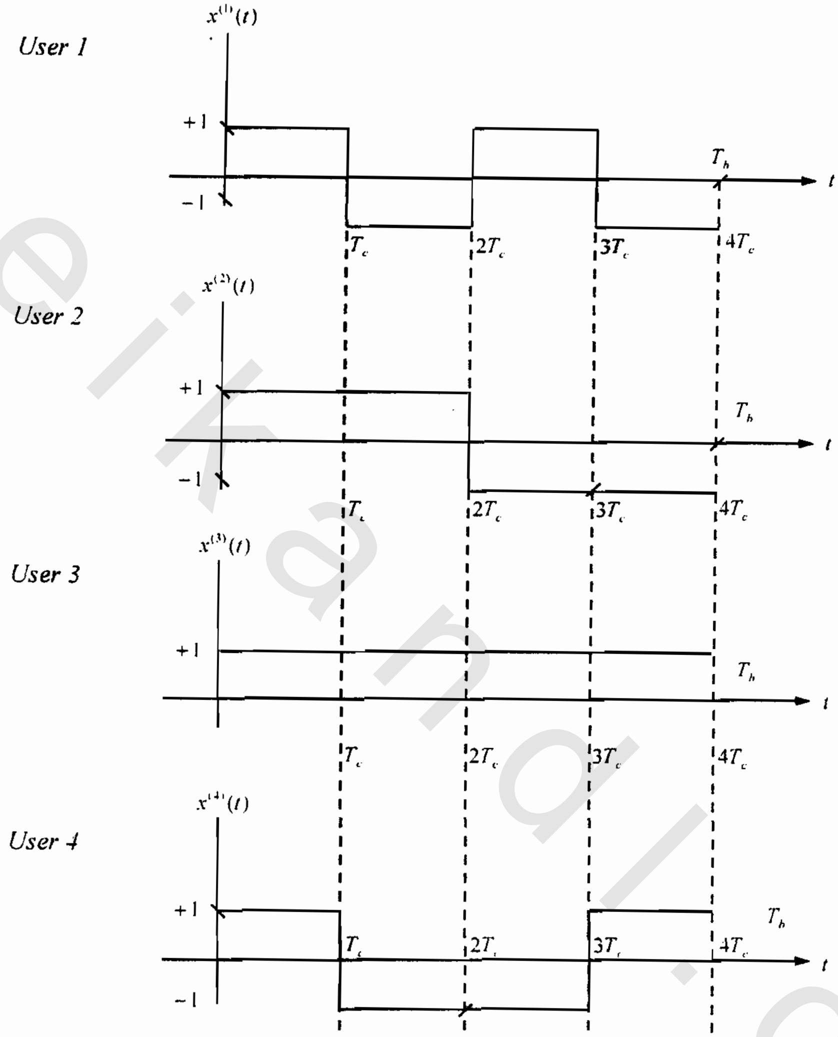


Fig. (9.4) Codes for users 1,2,3,4

There are three different types of CDMA distinguished by the codes given to each user. The first and most popular form of CDMA is direct sequence (DS - CDMA). Each user is given a code in the form of short pulses each of duration T_C. Each short pulse is called a chip of height $+1$ or -1. There are N short pulses that make up the user's code. Each user is given a unique code $x^{(1)}(t)$ for user 1, $x^{(2)}(t)$ for user 2 and so on. For example, Fig. (9.4) shows four codes each one is given to a different user. We can see that

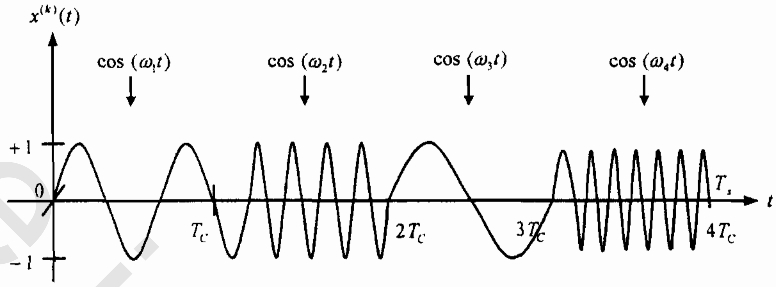


Fig. (9.5) Code shape for user k in FH-CDMA

$$\int_{qT_s}^{(q+1)T_s} x^{(k)}(t)\, x^{(j)}(t)\, dt = 0 \qquad \text{For all } k, j \;,\; k \neq j \qquad (9-12)$$

If the data bit is 1, we multiply $x_q^{(k)}(t)$ by $+1$ to give $s_q^{(k)}(t) = +\, x_q^{(k)}(t)$. If the data bit is zero, we multiply $x_q^{(k)}(t)$ by -1 to give $s_q^{(k)}(t) = -\, x_q^{(k)}(t)$, where q is the order of the bit not the chip. This is an example of orthogonal CDMA. In general, the code in DS-CDMA for user k can be represented by

$$x^{(k)}(t) = \sum_{\nu=0}^{N-1} (-1)^{c_\nu(k)} g_{T_c}(t - \nu T_c) \qquad (9-13)$$

where $c_\nu^{(k)}$ is either 0 or $+1$. Thus, $(-1)^{c_\nu(k)}$ is either $+1$ or -1. Thus, $g_{T_c}(t)$ is the basic chip pulse shape which is assumed rectangular. The pulse train is then modulated on to a carrier.

Another type of CDMA is frequency hopping CDMA (FH-CDMA). In FH-CDMA, the codes are not made up of little pulses as in DS-CDMA, but instead they are made up of little cosine waveforms (Fig. 9.5). The user k using the code $x^{(k)}(t)$ sends either $s_q^{(k)}(t) = +\, x^{(k)}(t)$ to represent bit 1 or $s_q^{(k)}(t) = -\, x^{(q)}(t)$ to represent bit 0.

A signal that is sent at one frequency suddenly jumps (hops) to another frequency and so on. The system must make sure that two users never use the same frequency at the same time and the frequency jumps that one user takes never collides with the frequency jumps that the other users take to avoid interference. In FH-CDMA, the code used by the user may be described as

$$x^{(k)}(t) = \sum_{\nu=0}^{N-1} \cos(\omega_\nu^{(k)} t)\, g_{T_c}(t - \nu T_c) \qquad (9-14)$$

where $\cos\left(\omega_v^{(k)} t\right) g_{T_c}(t)$ represents a cosine waveform which exists over the very short period of time $[0, T_c]$. The value $\omega_c^{(k)}$ indicates the frequency of this cosine waveform chosen from a finite set of possible frequencies.

Another type of CDMA is Multi-carrier CDMA (MC-CDMA). In MC-CDMA, each user is given a code for user k as generated by the block diagram (Fig. 9.6a,b). A pulse of duration T_b is includes many different frequencies. Fig. (9.7) shows the frequency components in the frequency domain. Each bump in frequency represents a pulse of duration T_b in time. The code is $+1$ or -1 applied to each frequency.

User k sends his data bit of 1 by sending the code for user k multiplied by $+1$ and sends the data bit 0 by sending the code for user k multiplied by -1.

User j sends his data the same way. The only difference is that his code uses different $+1$ and -1 (data) values to multiply each frequency. User j may send his signal as $+1$ or -1 multiplied by the generated code. By carefully choosing $+1$ and -1 values for each user's code, we can make sure that their codes and therefore their transmitted signals are orthogonal or pseudo orthogonal satisfying eqns. (9 – 3) or (9 – 5). The MC-CDMA code can be expressed mathematically according to the equation

$$x^{(k)}(t) = \sum_{v=0}^{N-1} (-1)^{C_v^{(k)}} \cos\left[\left(\omega_c + v\ \Delta\omega\right) t\right] b(t) \qquad (9-15)$$

where $C_v^{(k)}$ is either 0 or 1 meaning that $(-1)^{C_v^{(k)}}$ is either $+1$ or -1. This tells as that the frequency component is multiplied by either $+1$ or -1. The $b(t)$ represents the data pulse of duration T_b.

9.5 Pseudo Noise Sequence:

A pseudo noise (PN) sequence is a periodic binary sequence with a noise like waveform that is generated by a feedback shift register (Fig. 9.8). A feedback shift register consists of a shift register made up of m flip-flops and a logic circuit interconnected to form a multiloop feedback circuit. The flip-flops in the shift register are controlled by a single timing clock. With each pulse, the state of the flip-flop is shifted down the register. But the logic circuit computes the new states of the flip-flops and feeds back the result to the input of the registers preventing the shift register from emptying. The PN sequence is determined by the length m of the shift register, the initial state and the feedback logic.

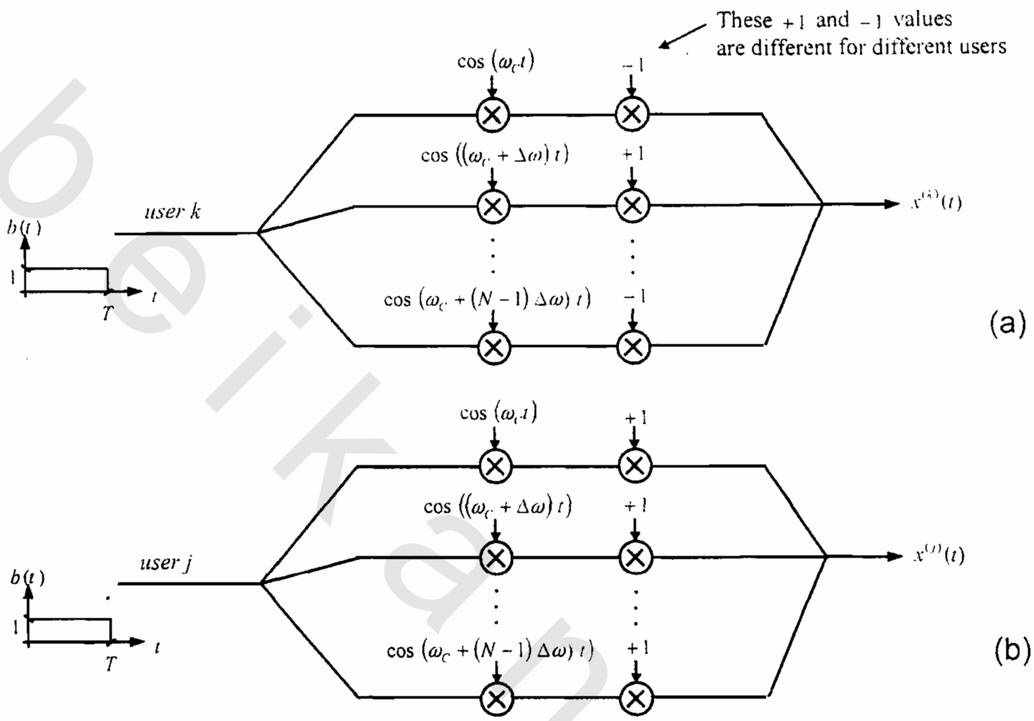


Fig. (9.6) Codes for MC - CDMA

a) code for user k *b) code for user j*

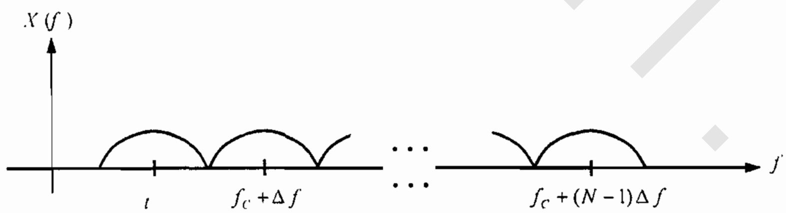


Fig. (9.7) Frequency spectrum

Let $s_j(q)$ be the state of the j^{th} flip-flop after the q^{th} clock pulse. The state of the shift register after the q^{th} clock pulse is then defined by the set $\{s_1(q)\ ,\ s_2(q)\ ..\ s_m(q)\}$, $q \geq 0$. From the definition of the shift register

$$s_j(q+1) = s_{j-1}(q) \qquad \begin{matrix} q \geq 0 \\ 1 \leq j \leq m \end{matrix} \qquad (9-16)$$

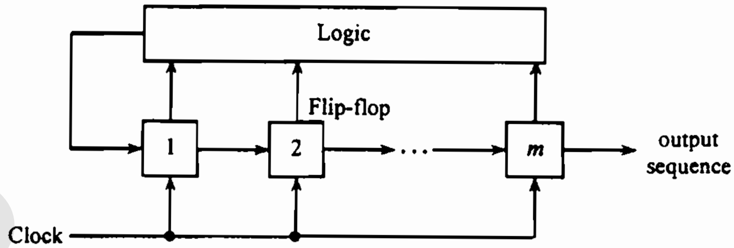


Fig. (9.8) Feedback shift register

and $s_0(q)$ is the input applied to the first flip-flop after the q^{th} clock pulse. This $s_0(q)$ is the Boolean function of the individual states $s_1(q)$, $s_2(q)$ .. $s_m(q)$. For a specified length, this Boolean function uniquely determines the subsequent sequence of states, and therefore, the PN sequence produced at the output of the shift register. With a total number of m flip-flops, the number of possible states of the shift register is 2^m. Thus, the PN sequence generated by the feedback shift register becomes periodic with a period 2^m at most. The feedback shift register is said to be linear when the feedback logic consists entirely of modulo 2 adders. In such a case, the zero state $s_0(q)$ where all flip-flops are 0 is not permitted. Thus, the period is $2^m - 1$.

Ex. 9.1

Consider the linear feedback shift register shown (Fig. 9.9) involving three flip-flips. The input s_0 applied to the first flip-flop is equal to the modulo 2 sum of s_1 and s_3. It is assumed that the initial state of the shift register is 100, find the output sequence

Solution

The succession of the states will be

1 0 0
1 1 0
1 1 1
0 1 1
1 0 1
0 1 0
0 0 1
1 0 0

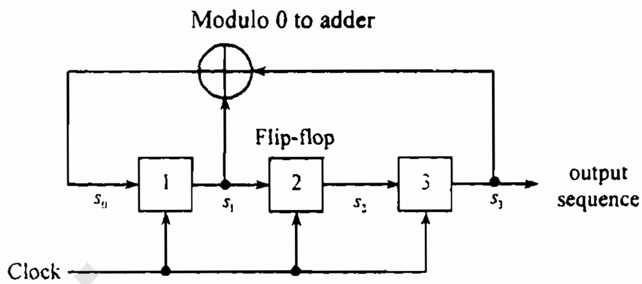


Fig. (9.9) Maximum length sequence generator for $m = 3$

The output sequence will be $0\,0\,1\,1\,1\,0\,1\,0\ldots$ which repeats itself with period $2^3 - 1 = 7$. Note that the choice of the initial state $1\,0\,0$ is arbitrary. If any other state is chosen there will be another cyclic shift in the output

9.6 Maximum Length Sequences:

We define a random binary sequence as one in which 1 or 0 is equally probable. Maximum length sequence is one which has period $2^m - 1$. In such a sequence, we note that the number of 1's is always one more than the number of 0's, noting that the state 000 is forbidden. This property is called the balance property. We define a run as a subsequence of identical symbols within one period. In the previous example, we have 4 runs or in general $(N+1)/2$, where $N = 2^m - 1$. These runs are 00, 111, 0, 1 one half of the runs of each are of length one (0 is a single symbol run and 1 is a single symbol run). One fourth is of length two (00) and one eighth of the runs is of length three (111). This property is called the run property.

We will show that the autocorrelation function of a maximum length sequence is periodic and binary valued. This property is called the correlation property. For a maximum length sequence of period $N = 2^m - 1$, where m is the length of the shift register, let binary symbols 0, 1 be denoted by levels -1 and $+1$. Let $c(t)$ denote the output waveform of the maximum length sequence for length T_p for $N = 7$ (Fig. 9.9). Let

$$T_p = NT_c \qquad (9-17)$$

where T_c is the chip period . The autocorrelation function of a periodic signal of period T_p is given by

$$R_c(\tau) = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} c(t)\, c(t-\tau)\,dt \qquad (9-18)$$

Applying this formula to the maximum length sequence given in Fig. (5.6), we get (Ex. 9.2)

$$R_c(\tau)=\begin{cases}1-\dfrac{N+1}{N\,T_C}|\tau| & , \quad |\tau|\le T_C \\ -\dfrac{1}{N} & \quad \textit{otherwise}\end{cases} \qquad (9-19)$$

This result is plotted in Fig. (9.10) for $m=3$, $(N=7)$. Since periodicity in the time domain is transformed into uniform sampling in the frequency domain, then taking the Fourier transform of eqn. (9 – 19),

$$S_C(f)=\frac{1}{N^2}\delta(f)+\frac{1+N}{N^2}\sum_{n=-\infty}^{\infty}\operatorname{sinc}^2\left(\frac{n}{N}\right)\delta\left(f-\frac{n}{N\,T_C}\right),\ n\neq 0 \quad (9-20)$$

This is shown in Fig. (9.10c), $m=3$, $N=7$. We can visualize the autocorrelation result of eqn. (9–19) by considering the normalized autocorrelation in terms of normalized delay τ (referred to the period or maximum length) as

$$R_x(\tau)=\frac{1}{N}(n_a-n_d) \qquad (9-21)$$

where n_a is the number of agreements and n_d is the number of disagreements in a comparison of one full period of the sequence with a τ shifted sequence. For $\tau=0$, then $c(t)$ and its replica are perfectly matched, $R(\tau)=1$. However, for any cyclic shift between $c(t)$ and $c(t+\tau)$ with $1\le\tau<N$ the autocorrelation function is equal to $-(1/N)$. For large N , the sequences are virtually decorrelated for a shift of a single chip. Let as consider the following sequence $N=7$ undergoing one cyclic shift

0 0 1 1 1 0 1
<u>1 0 0 1 1 1 0</u>
d a d a a d d

From eqn (9.21), with $n_a=3$, $n_d=4$

$$R_x(\tau)=\frac{1}{7}(3-4)=-\frac{1}{7}$$

Let us consider two cyclic shifts

0 0 1 1 1 0 1
<u>0 1 0 0 1 1 1</u>
a d d d a d a

Here we have $n_a=3$, $n_d=4$ .We get

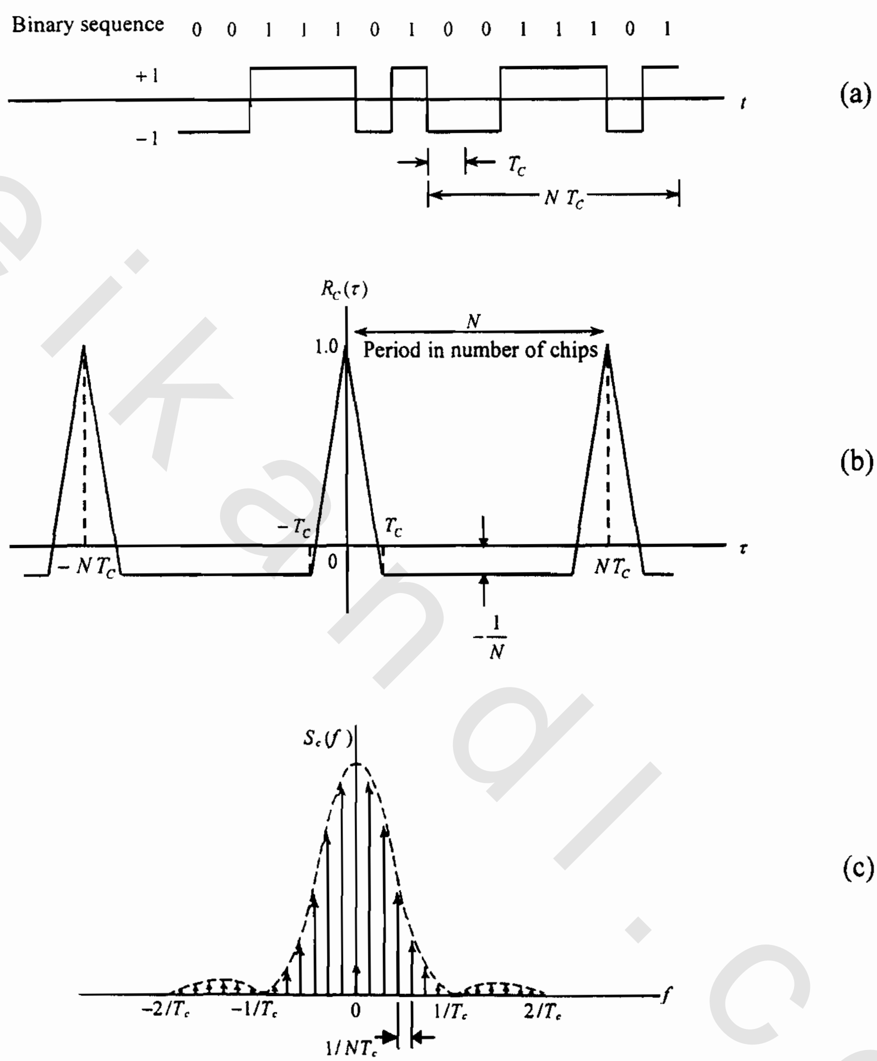


Fig. (9.10) Characteristics of maximum length sequence of Fig. (9.9.)

a) waveform for $m = 3$, $(N = 7)$ *b)* $R_C(\tau)$ *c) PSD*

$$R_x(\tau) = -\frac{1}{7}$$

Thus, any cyclic shift or shifts will produce $-1/N$ within one period. This explains Fig. (9.10b).

Ex. 9.2

Compare the autocorrelation and PSD of maximum length sequence and random binary sequence.

Solution

Let us consider a random sequence of binary symbols $1, 0$ where 1 is represented by amplitude $+A$ and 0 is represented by amplitude $-A$ and duration T seconds. The pulses are not synchronized so that the starting time t_d of the first complete pulse for positive time is equally likely to lie anywhere between zero and T seconds. Thus, t_d is the sample value of a uniformly distributed random variable (Fig. 9.11) with *pdf* given by

$$f(t_d) = \begin{cases} \dfrac{1}{T} & 0 \le t_d \le T \\ 0 & elsewhere \end{cases}$$

To find the autocorrelation function $R(t_1, t_2)$, we must evaluate $E[x(t_1)\, x(t_2)]$ where $x(t_1)$ and $x(t_2)$ are random variables obtained by observing the random process $x(t)$ at times t_1 and t_2 respectively. Consider first the case when $|t_1 - t_2| > T$. In this case, the random variables occur in different pulse intervals and are therefore independent

$$E[x(t_1)\, x(t_2)] = E[x(t_1)]\, E[x(t_2)] = 0, \qquad |t_1 - t_2| > T \qquad (9-22)$$

For the case when $|t_1 - t_2| < T$ and taking $t_1 = 0$ and $t_2 < t_1$, then $x(t_1)$ and $x(t_2)$ occur in the same pulse interval if $t_d < T - |t_1 - t_2|$. Thus,

$$E[x(t_1)\, x(t_2)] = \int_0^{T-|t_1-t_2|} A^2 f(t_d)\, dt_d$$

$$= \int_0^{T-|t_1-t_2|} \frac{A^2}{T}\, dt_d$$

$$= A^2\left(1 - \frac{t_1 - t_2}{T}\right), \qquad |t_1 - t_2| < T \qquad (9-23)$$

Thus, for $\tau = t_1 - t_2$

$$R_x(\tau) = \begin{cases} A^2\left(\dfrac{1-|\tau|}{T}\right), & |\tau| < T \\ 0, & |\tau| \ge T \end{cases} \qquad (9-24)$$

This result is shown in Fig. (9.12).

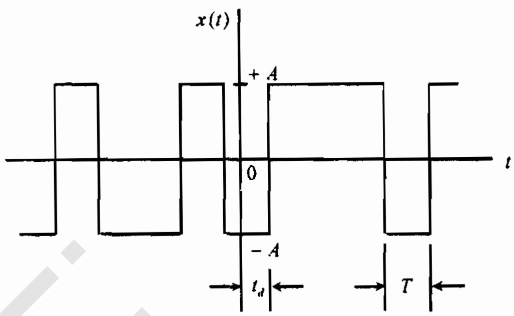


Fig. (9.11) Sample function of random binary wave

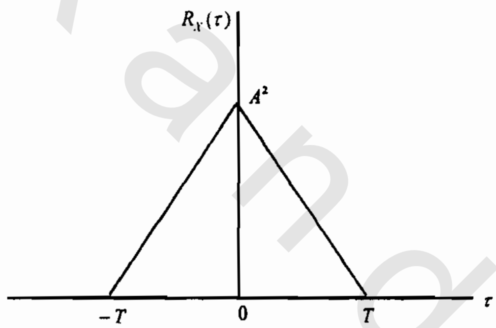


Fig. (9.12) Autocorrelation function for a random binary wave

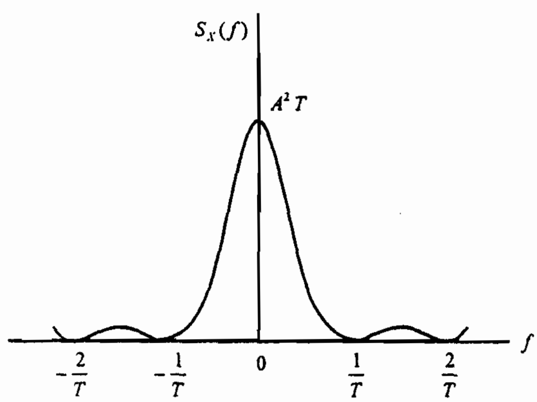


Fig. (9.13) PSD of a random binary wave

The PSD is thus given by

$$S_x(f) = \int_{-T}^{T} A^2 \left(1 - \frac{|\tau|}{T}\right) e^{-j2\pi f \tau}\, d\tau \qquad (9-25)$$

Using Fourier transform of a triangular function

$$S_x(t) = A^2 T \operatorname{sinc}^2(f\,T) \qquad (9-26)$$

This is plotted in Fig. (9.13). Comparing the results of Fig. (9.10) for maximum length sequence with the corresponding results of Fig. (9.13) for a random binary sequence, we note that for a period of the maximum length sequence the autocorrelation function $R_c(\tau)$ is similar to that of random binary wave. They have the same envelope $\operatorname{sinc}^2(f\,T)$ for their PSD. The random binary sequence, however, has a continuous PSD, whereas the maximum length sequence consists of delta functions spaced $1/N\ T_c\ Hz$ apart. As m or N increases the maximum length sequency approaches the random binary sequence. They become identical when N tends to infinity. However this will be at the price of increasing the storage requirement. A compromise is usually made. Thus, a pseudo random generator needed for the code generation in CDMA is achieved using a linear feedback shift register. The number of codes generated can be made arbitrarily large by varying the taps on the shift register and the initial state (Prob. 9.2).

9.7 Spread Spectrum Modulation and Demodulation:

One of the main advantages of spread spectrum is that it can provide protection against externally generated jamming signals with finite power. The jamming signal usually is powerful broad band noise or multi tone waveform that is directed at the receiver to disrupt communication. Protection against jamming is thus provided by purposely making information bearing signal occupy a bandwidth far in excess of the minimum bandwidth needed for transmission. This has the effect of making the information in the form of noise-like waveform blending in the background noise. This way the camouflaged signal will propagate undetected in the channel.

Let $[b_q]$ denote a binary data sequence and $[c_q]$ denote a pseudo noise [PN] sequence. Both $[b_q]$ and $[c_q]$ are NRZ, so they both assume values ± 1. Modulation is achieved by multiplying the data signal $b(t)$ and the PN signal $c(t)$.

We know that multiplication in the time domain corresponds to convolution in the frequency domain. If the message signal $b(t)$ is narrow band and the PN signal $c(t)$ is wideband, the product $m(t)$ will be wideband. Thus, the PN signal is the spreading code and its effect is to spread the data along the spectrum. Each data bit is chopped into a number of small time increments called chips (Fig. 9.14). The baseband product $m(t)$ represents the transmitted signal (Fig. 9.15a)

$$m(t) = c(t)\, b(t) \qquad (9-27)$$

The received signal $r_d(t)$ consist of the transmitted signal $m(t)$ plus an additive interference $j(t)$ (Fig. 9.15c)

$$r_d(t) = m(t) + i(t) \qquad (9-28)$$

$$= c(t)\, b(t) + i(t) \qquad (9-29)$$

To recover the original $b(t)$ the received signal $r_d(t)$ is applied to a demodulator which consists of a multiplier followed by an integrator and a decision device (Fig. 9.15c). The multiplier is supplied with a locally generated PN sequence that is an exact replica of that used in the transmitter and in perfect synchronism with it.
The multiplier output is given by

$$y(t) = c(t)\, r_d(t) \qquad (9-30)$$

$$= c^2(t)\, b(t) + c(t)\, i(t) \qquad (9-31)$$

Noting that $c(t)$ is $+1$ or -1, then

$$c^2(t) = 1 \qquad \text{For all } t \qquad (9-32)$$

$$y(t) = b(t) + c(t)\, i(t) \qquad (9-33)$$

We note that $b(t)$ is retrieved (despread) whereas the interference $i(t)$ is now multiplied by $c(t)$, i.e., the interference is spread. By applying $y(t)$ to a LPF most of the spurious signal $c(t)\, i(t)$ is averaged out. The integration is carried over the bit time T_b providing a sample value v. A decision device decides if $v > 0$, we have 1, and *if* $v < 0$, we have 0 in the interval $0 \le t \le T_b$. As the PN sequence is made long enough, the pseudo random code approaches true noise and the product $c(t)\, i(t)$ approaches zero. For an unauthorized user who does not have the code, it becomes hard for him to detect the information. What we have just described is baseband direct sequence spread spectrum (DS - SS).

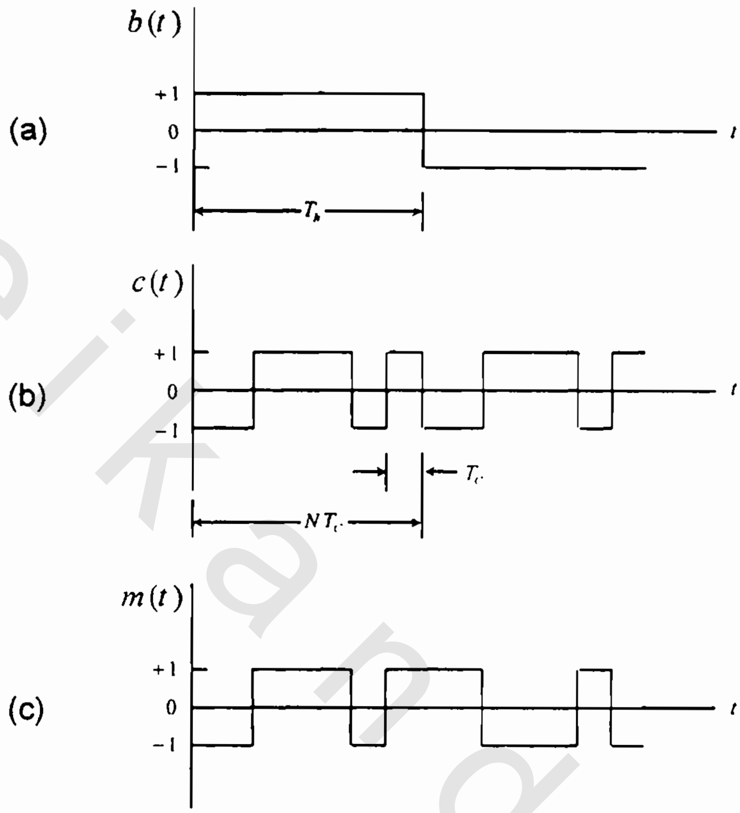


Fig. (9.14) SS modulation
a) data signal $b(t)$ *b) spreading signal* $c(t)$ *c) product signal* $m(t)$

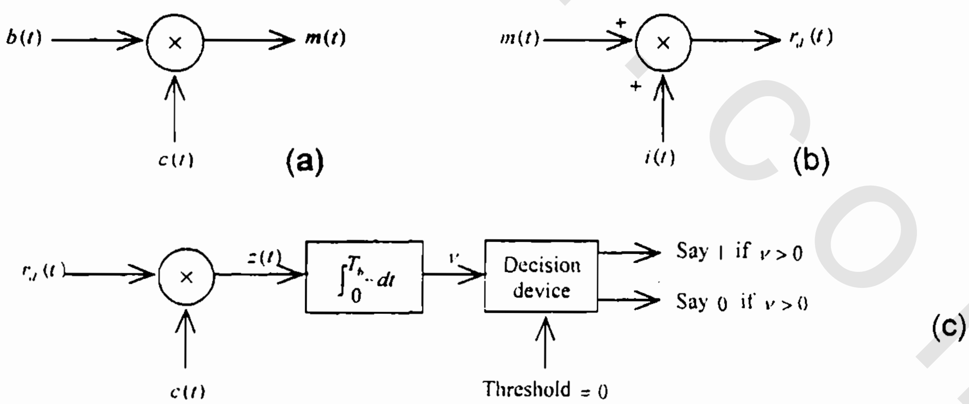


Fig. (9.15) Spread spectrum (SS) baseband systems
a) transmitter *b) channel* *c) receiver*

9.8 Bandpass DS - SS:

To provide for the use of DS - SS in bandpass (passband) transmission for a satellite channel as an example, we may use coherent binary phase shift keying (BPSK) in the transmitter and receiver (Fig. 9.16). The transmitter converts the incoming binary data sequence $\{b_q\}$ into an NRZ waveform $b(t)$ which is followed by two stages of modulation. The first stage consists of a product modulator or multiplier which multiplies $b(t)$ and $c(t)$. The second stage consists of a binary PSK modulator. Thus, the transmitted signal $x(t)$ is direct sequence spread binary phase shift keyed (DS/BPSK) signal. The phase modulation $\vartheta(t)$ of the phase modulated carrier $x(t)$ is one of two values 0 , π according to the truth table (Table 9.1.)

Table 9.1 Truth table for BPSK / DS - SS

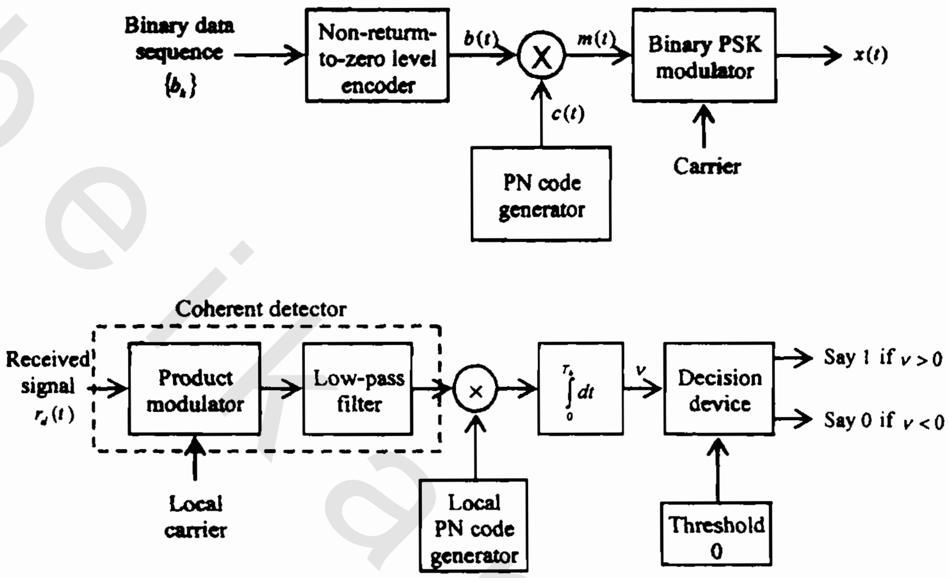
Polarity of PN sequence $c(t)$ at time t	Polarity of data sequence $b(t)$ at time t	
	+	-
+	$\theta(t)=0$	$\theta(t)=\pi$
-	$\theta(t)=\pi$	$\theta(t)=0$

Fig. (9.17) shows the waveform of the second stage of modulation noting that the bit time is equal to one period of the spreading code, i.e,

$$T_b = NT_c \qquad (9-34)$$

Fig. (9.17c) shows the BPSK / DS - SS waveform $s(t)$ resulting from the second stage of modulation. The receiver shown in Fig. (9.16b) consists of two stages of demodulation. In the first stage, the received signal $r_d(t)$ and a locally generated carrier are applied to a product modulator followed by, a LPF whose bandwidth is equal to that of $m(t)$. This stage is the PSK demodulator. The second stage of demodulation performs spectrum despreading by multiplying the LPF output by a locally generated replica of the PN signal $c(t)$ followed by integration over a bit interval $0 \le t \le T_b$, and the decision device decides if the output is 0 or 1. For the sake of analysis, we may reverse the order of spreading and PSK modulation and also PSK demodulation and despreading (Fig. 9.18).
Thus,

$$\begin{aligned} y(t) &= x(t) + i(t) \\ &= c(t)\, s(t) + i(t) \end{aligned} \qquad (9-35)$$



(a)

(b)

Fig. (9.16) Coherent DS / BPSK

a) Transmitter *b) receiver*

where $s(t)$ is the BPSK signal. And $i(t)$ is an interfering or jamming signal.

In the receiver, the received signal $y(t)$ is first multiplied by the PN signal $c(t)$, yielding an output that equals the coherent detector input $u(t)$

$$u(t) = c(t)\ y(t) \tag{9-36}$$

$$= c^2(t)\ s(t) + c(t)\ i(t)$$

$$= s(t) + c(t)\ i(t) \tag{9-37}$$

$$c^2(t) = 1 \text{ for all } t \tag{9-38}$$

Thus, $u(t)$ consists of BPSK signal $s(t)$ embedded in additive code modulated interference $c(t)\ i(t)$. Thus, the interfering (jamming signal is spread out. We must note that a basic requirement for spread spectrum communication is perfect synchronism between the PN sequence used in the receiver and that used in the transmitter.

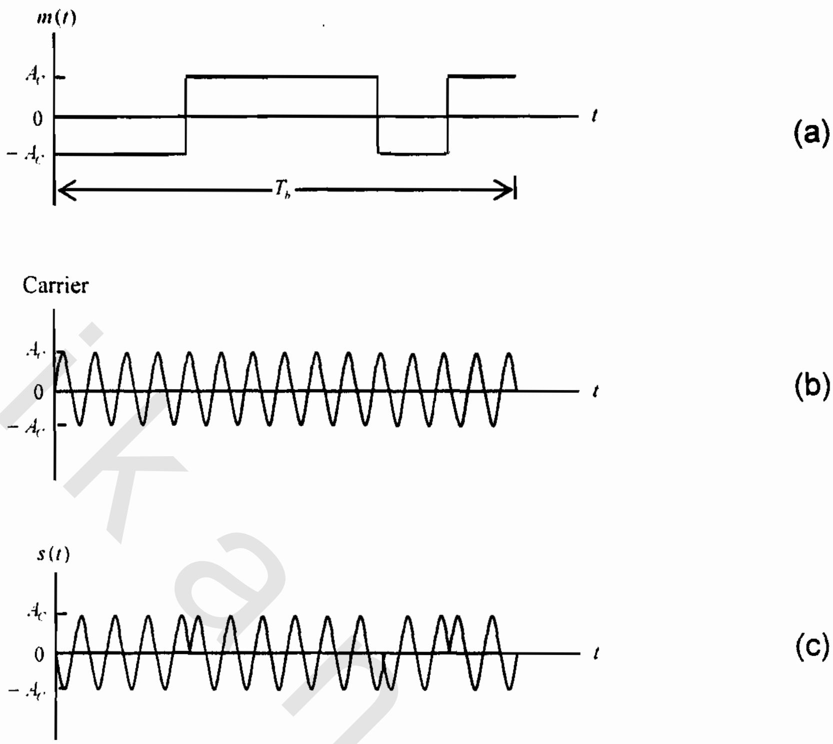


Fig. (9.17) Bandpass DS - SS

a) product signal $m(t)$ *b) sinusoidal carrier* *c) DS / BPSK modulator output*

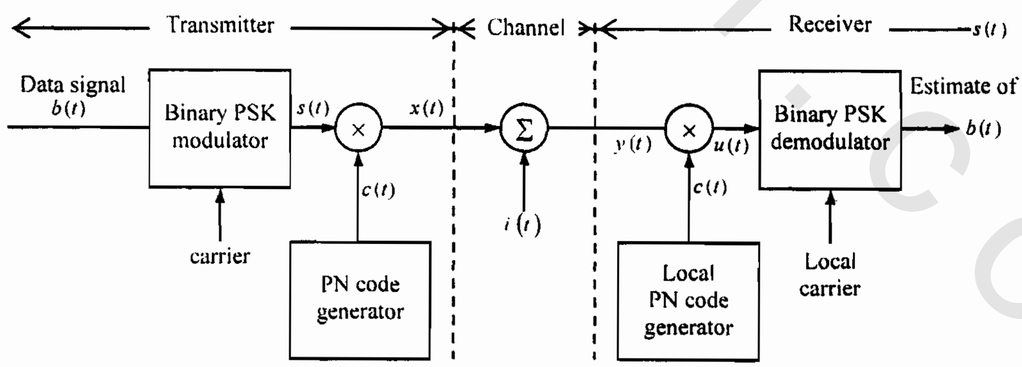


Fig. (9.18) Model of BPSK / DS-SS link

9.9 DS - SS Analysis:

We intend here to develop a signal space representation of the transmitted DS - SS signal based on an model of Fig (9 – 18). Consider the set of orthonormal basis functions

$$\phi_{1q}(t) = \sqrt{\frac{2}{T_c}} \cos(2\pi f_c t) \quad qT_c \le t \le (q+1)T_c \qquad (9-39)$$
$$= 0 \quad \text{otherwise}$$

$$\phi_{2q}(t) = \sqrt{\frac{2}{T_c}} \sin(2\pi f_c t) \quad qT_c \le t \le (q+1)T_c,\ q = 0, 1 \ldots N-1 \qquad (9-40)$$
$$= 0 \quad \text{otherwise}$$

Where T_c is the chip duration and N is the number of chips per bit. The transmitted signal $x(t)$ during one bit becomes

$$x(t) = c(t)\, s(t) \qquad (9-41)$$

$$= \pm\sqrt{\frac{2E_b}{T_b}}\, c(t) \cos(2\pi f_c t) \qquad (9-42)$$

$$= \pm\sqrt{\frac{E_b}{N}} \sum_{q=0}^{N-1} c_q\, \varphi_{1q}(t) \qquad 0 \le t \le T_b \qquad (9-43)$$

where E_b is the signal energy per bit. The plus sign corresponds to 1 and the minus sign to 0. The code sequence $[c_0, c_1 \ldots c_{N-1}]$ is the PN sequence where $c_q = \pm 1$. The transmitted signal $x(t)$ is N dimensional, i.e. it needs N orthonormal functions for its representation.

If for an interfering signal $i(t)$.

$$i(t) = \sum_{q=0}^{N-1} i_{1q}\, \phi_{1q}(t) + \sum_{q=0}^{N-1} i_{2q}\, \phi_{2q}(t) \qquad 0 \le t \le T_b \qquad (9-44)$$

where

$$i_{1q} = \int_0^{T_b} i(t)\, \phi_{1q}(t)\, dt \qquad q = 0, \ldots N-1 \qquad (9-45)$$

$$i_{2q} = \int_0^{T_b} i(t)\, \phi_{2q}(t)\, dt \qquad q = 0 \ldots N-1 \qquad (9-46)$$

The interfering signal $i(t)$ is $2N$ dimensional.

The average power of the interference is

$$I = \frac{1}{T_b} \int_0^{T_b} i^2(t)\, dt \qquad (9-47)$$

$$= \frac{1}{T_b} \sum_{q=0}^{N-1} i_{1q}^2 + \frac{1}{T_b} \sum_{q=0}^{N-1} i_{2q}^2 \qquad (9-48)$$

If there is equal power in the cosine and sine coordinates

$$\sum_{q=0}^{N-1} i_{1q}^2 = \sum_{q=0}^{N-1} i_{2q}^2 \qquad (9-49)$$

Thus,

$$I = \frac{2}{T_b} \sum_{q=0}^{N-1} i_{1q}^2 \qquad (9-50)$$

To find S/N measured at the input and output of the DS / BPSK receiver, we use eqn. (9 – 37) to express the coherent detector output v corresponding to an input $u(t)$

$$v = \sqrt{\frac{2}{T_b}} \int_0^{T_b} u(t) \cos\left(2\pi f_c t\right) dt \qquad (9-51)$$

$$= \sqrt{\frac{2}{T_b}} \int_0^{T_b} \left[s(t) + c(t)\, i(t)\right] \cos\left(2\pi f_c t\right) dt \qquad (9-52)$$

$$= \sqrt{\frac{2}{T_b}} \left[\int_0^{T_b} s(t) \cos\left(2\pi f_c t\right) dt + \int_0^{T_b} c(t)\, i(t) \cos\left(2\pi f_c t\right) dt\right] \qquad (9-53)$$

$$= v_s + v_{ci} \qquad (9-54)$$

where v_s is the output due to despread BPSK signal $s(t)$, and v_{ci} is the spread interference $c(t)\, i(t)$ given by

$$v_s = \sqrt{\frac{2}{T_b}} \int_0^{T_b} s(t) \cos\left(2\pi f_c t\right) dt \qquad (9-55)$$

$$v_{ci} = \sqrt{\frac{2}{T_b}} \int_0^{T_b} c(t)\, i(t) \cos\left(2\pi f_c t\right) dt \qquad (9-56)$$

But

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t\right) \qquad 0 \le t \le T_b \qquad (9-57)$$

If the f_c is an integer multiple of $1/T_b$, we have from eqn. (9 – 55)

$$v_s = \pm\sqrt{E_b} \qquad (9-58)$$

The output due to interference is given by

$$v_{ci} = \sqrt{\frac{2}{T_b}} \sum_{q=0}^{N-1} c_{1q} \int_{qT_c}^{(q+1)T_c} i(t) \cos(2\pi f_c t)\, dt \qquad (9-59)$$

Using eqns. (6 – 39) and (9 – 45)

$$v_{ci} = \sqrt{\frac{T_c}{T_b}} \sum_{q=0}^{N-1} c_{1q} \int_{0}^{T_b} i(t)\, \phi_{1q}(t)\, dt$$

$$= \sqrt{\frac{T_c}{T_b}} \sum_{q=0}^{N-1} c_{1q}\, i_{1q} \qquad (9-60)$$

We note the $c_{1q} = \pm 1$ with almost equal probability. Therefore, by squaring eqn. (9–60) the terms involving c_{1q} will cancel out and $|c_{1q}|^2 = 1$ while the average of v_{ci} is zero. For a fixed vector $\vec{i}$

$$\text{var}\left[v_{ci} \middle| \vec{i}\right] = \frac{1}{N} \sum_{q=0}^{N-1} i_{1q}^2 \qquad (9-61)$$

where $N = \frac{T_b}{T_c}$ is the spreading factor

From eqn. (9 – 50)

$$\text{var}\left[v_{ci} \middle| \vec{i}\right] = \frac{I\, T_c}{2} \qquad (9-62)$$

Thus, the random variable v_{ci} has zero mean and variance $\frac{I\, T_c}{2}$. From eqn. (9–58), the peak instantaneous power of the signal component is E_b. Accordingly, we may define the output S/N as the instantaneous peak power E_b on the coherent detector output by the variance of the equivalent noise power after spreading of the jamming signal

$$S/N\big|_{out} = \frac{2E_b}{I\, T_c} \qquad (9-63)$$

The average signal power at the receiver input equals E_b/T_b. We, thus, define the input S/N as

$$S/N\big|_{in} = \frac{E_b/T_b}{I} \qquad (9-64)$$

From eqns. (9 – 63) and (9 – 64),

$$\frac{S/N|_{out}}{S/N|_{in}} = \frac{2T_b}{T_c} = 2N = 2G \qquad (9-65)$$

$$G = \frac{T_b}{T_c} = \frac{R_c}{R_b} \qquad (9-66)$$

The factor G is called the processing gain $= R_c$ (the chip rate) divided by R_b (the bit rate).

It is seen that using spreading techniques has weakened the interfering signal to the point that S/N after spreading has increased at the output of the detector above that at the input of the detector. Thus, the longer we make the PN sequence, i.e, the smaller the chip time T_c with respect to the bit time T_c the larger the processing gain, and hence, the larger the enhancement of the S/N will be.

9.10 Probability of Error:

The output of the coherent detector is v in the direct sequence spread BPSK system which is a random variable where

$$v = \pm\sqrt{E_b} + v_{ci} \qquad (9-67)$$

where v_{cj} is the noise component produced by external interference and E_b is the transmitted energy per bit. Both bits $0, 1$ are equally likely, so the average probability of error P_B for zero threshold is

$$P_B = P(v > 0 \mid 0\ sent)$$
$$= P\left(v_{ci} > \sqrt{E_b}\right) \qquad (9-68)$$

From eqn. (9 – 61)

$$v_{ci} = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} c_{1q}\ i_{1q} \qquad (9-69)$$

For a set of independently and identically distributed random variables, a random variable v_N may be defined as

$$v_N = \frac{1}{\sqrt{N}} \sum_{q=1}^{N} Y_q \qquad (9-70)$$

where

$$Y_q = \frac{1}{\sigma_x}\left(x_q - \mu_x\right) \qquad q = 1,\dots N \qquad (9-71)$$

$$E\lfloor Y_q \rfloor = 0 \qquad (9-72)$$

$$\text{var}[Y_q] = 1 \qquad (9-73)$$

The central limit theorem states that the *pdf* of v_N approaches a normalized Gaussian distribution in the limit as N approaches infinity.

We see from eqn. (9 – 69) that v_{ci} is the sum of N identically distributed random variables. Hence, from the central limit theorem, we deduce that for large N the random variable v_{ci} assumes a Gaussian distribution. We may therefore conclude that the equivalent noise component v_{ci} contained in the coherent detector output may be approximated as a Gaussian random variable with zero mean and variance $IT_c/2$, where I is the average interference power and T_c is the chip duration. Thus eqn. (9 – 68) becomes

$$P_B = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{I\,T_c}}\right) \qquad (9-74)$$

But for BPSK

$$P_B = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{\eta}}\right) = Q\left(\sqrt{\frac{2\,E_b}{\eta}}\right) \qquad (9-75)$$

We may define a wide band noise of equivalent PSD given by

$$\frac{\eta'}{2} = \frac{I\,T_c}{2} \qquad (9-76)$$

which is in agreement with eqn. (9 – 63). Since the signal energy per bit $E_b = P_s\,T_b$ where P_s is the average signal power and T_b is the bit duration we may express the signal energy per bit to noise spectral density as

$$\frac{E_b}{\eta'} = \left(\frac{T_b}{T_c}\right)\left(\frac{P_s}{I}\right) \qquad (9-77)$$

Using eqn. (9 – 67),

$$\left(\frac{I}{P_s}\right) = \frac{N}{(E_b/\eta')} = \frac{G}{(E_b/\eta)} \qquad (9-78)$$

The ratio (I/P_s) is called the interference resistance margin, (E_b/η') is the minimum value needed to support a prescribed probability of error.

We see from eqns, (9–75) and (9–78) that increasing N increases (I/P_s) for the same $(E_b/\eta)_{req}$ hence P_B. However, for the same $(I/P)_{req}$, increasing N increases $(E_b/\eta)_{req}$, hence reduces the probability of error.

9.11 Frequency Hopping:

The modulation used in this technique is MFSK where $k = \log_2 M$ bits are used to determine one of M frequencies to be transmitted. The position of the M-ary signal set in shifted pseudo randomly by a frequency synthesizer over a hopping bandwidth B_{ss}. In a conventional MFSK system the data symbol modulate a fixed carrier. In an FH/MFSK system the data symbol modulates a carrier whose frequency is pseudo randomly changed. In either case a single tone is transmitted. The FH system (Fig. 9.19) can be thought of as a two step modulation process; data modulation and frequency hopping modulation – even though it can be implemented as a signal step whereby the frequency synthesizer produces a transmission tone depending on the PN code and the data. At each frequency hop time a PN generator feeds the frequency synthesizer a frequency word (a sequence of N chips) to assign one of 2^N symbol set positions. If frequency spacing between consecutive hops is Δf we have

$$\Delta f = \frac{B_{ss}}{N} \quad (9-79)$$

or

$$N = \log_2 \frac{B_{ss}}{\Delta f} \quad (9-80)$$

The bandwidth in one hop is identical to the bandwidth of MFSK which is much less than B_{ss}. But over many hops the entire broadband spectrum is scanned. B_{ss} in FH – SS is much larger than that for DS – SS which makes the processing gain in FH – SS much larger than in DS – SS.

The demodulation process consists of two steps (Fig. 9.19). The received signal is first FH demodulated (dehopped) by mixing it with the same sequence of pseudo randomly selected frequency tones initially used in hopping. The dehopped signal is then applied to a conventional bank of M noncoherent energy detectors to select the most likely symbol. Noncoherent detection is more appropriate since it is difficult to keep track of the phase in a fast randomly hopping scheme. The frequency gain in FH - SS system is given by

$$G = B_{hop} / R_b \quad (9-81)$$

where $B_{hop} = B_{SS}$ is the hopping range

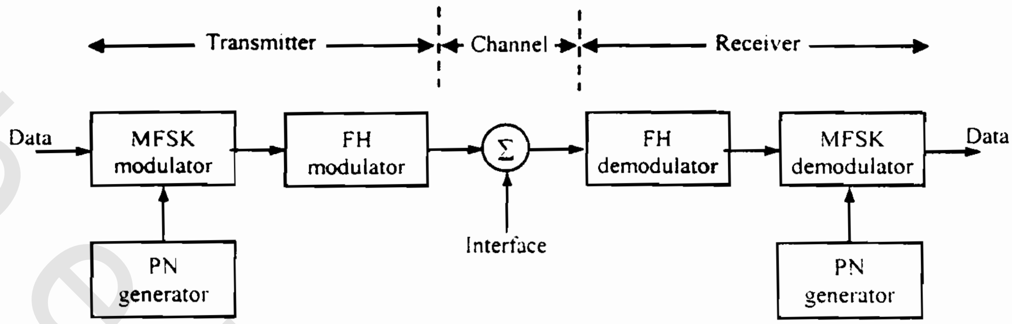


Fig. (9.19) FH-SS system

Ex. 9.3 (case study)

In a FH – SS system the input data rate $R_b = 150\ bits/s$ and the modulation is 8-ary FSK. The frequency is hopped once per symbol at the symbol boundary:

a) Sketch the time bandwidth plane for data sequences 011 110 001.
b) Devise a scheme for robust FH – SS.
c) Propose a demodulator design.

Solution

a) The symbol rate

$$R_s = \frac{R_b}{\log_2 8} = 50\ symbols\ s$$

$$T_s = \frac{1}{50} = 20ms$$

The hopping rate is 50 hops/s. Fig. (9.20) shows the time bandwidth relation where the abscissa represents time and the ordinate represents the hopping bandwidth B_{ss}. We have a set of 8-ary FSK symbol to tone assignments. The tone separation is $1/T_s = 50\,Hz$ which represents the minimum required tone spacing for orthogonal signaling of noncoherent FSK. Since the modulation in 8-ary FSK, the bits are grouped three at a time.

In a conventional 8-ary FSK scheme, a single side band tone (offset from a fixed carrier frequency of the data band) is transmitted according to a tone assignment. In FH/MFSK, the center frequency itself to is not fixed. For each new symbol, f_o hops to a new position in the hop bandwidth. The center frequency is a dashed line and the symbol tone is a solid line (Fig. 9.20).

b) Robustness characterizes a signal's ability to withstand impairments from the channel such as noise, jamming and fading. A signal is replicated into copies where each copy is transmitted on a different carrier, thus, increasing the chance for survival. Multiple transmission at different frequencies spread in time is called diversity.

Each chip is transmitted at a different hopping frequency. In this example the chip repeat factor $r = 4$. During each $20\ ms$ symbol interval there are 4 separate chips to be transmitted for each symbol for the same data sequence of rate $R_b = 150\ b/s$. Thus, each 3bit symbol is transmitted 4 times and for each transmission the center frequency of the data band is hopped to a new region of the hopping band under the control of a PN code generator (Fig. 9.21)

$$T_c = \frac{T_s}{r} = \frac{20}{4} = 5ms$$

The hopping rate R_{hop} is given by

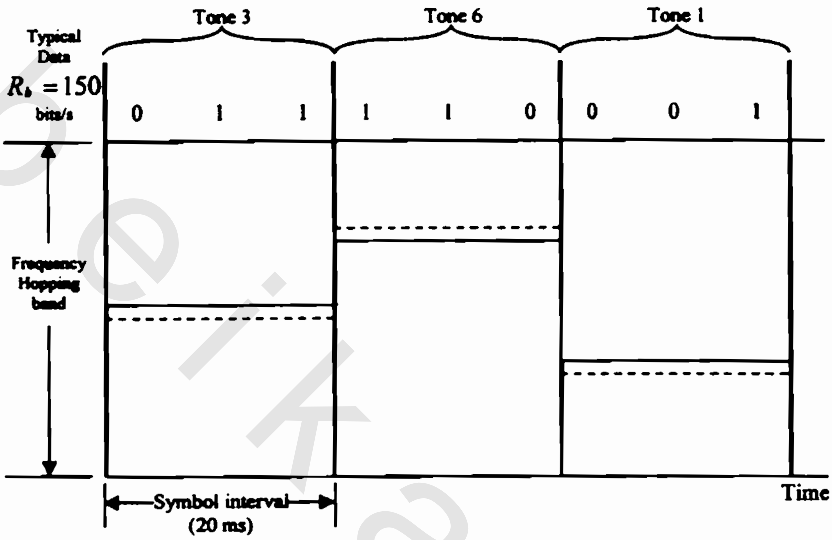
$$R_{hop} = \frac{rR_b}{\log_2 8} = 200\, hops/s$$

Since the duration of each FSK tone is now equal to the chip rate $T_c = T_S / r$, the minimum separation between the tones is $1/T_c = r/T_s = 200\, Hz$.

In the case of DS-SS, the term chip refers to the PN code duration .In FH-SS, the term chip refers to the shortest uninterrupted waveform. We may use slow frequency hopping (SFH) which means there are several modulation symbols per hop. Alternatively, we may use fast frequency hopping (FFH) which means there are several frequency hops per symbol. In SFH, the shortest uninterrupted waveform is that of the data symbol .However, for FFH the shortest uninterrupted waveform is that of the hop.

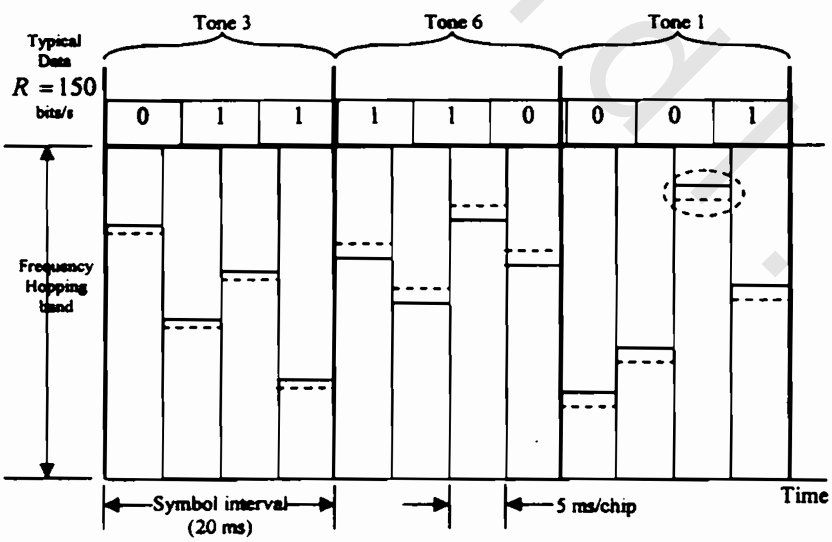
Fig. (9.22a) shows an example of FFH of a binary FSK .There are 4 chips transmitted per bit. For FFH the chip duration is the hop duration. Fig. (9.22b) illustrates an example of an SFH binary FSK. There are 3 bits transmitted during the time duration of a single hop. Then for SFH the chip duration is the bit duration. If the system were implemented as an 8-ary scheme each 3 bits would be transmitted as a single data symbol.

c) Fig. (9.23) illustrates a typical design for FFH/MFSK demodulator. First, the signal is dehopped using a PN generator identical to the one used for hopping. Then after filtering with a LPF that has a bandwidth equal to the data bandwidth the signal is demodulated using a bank of M envelope, (energy) detectors. Each envelope detector is followed by a clipping circuit and an accumulator. The clipping circuit is a safeguard against unpredictable interferences. The demodulator does not make symbol decisions on a chip by chip basis. Instead, the energy from the N chips are accumulated and after the energy for the N^{th} the chip is added to the $N-1$ earlier ones, the demodulator makes a symbol decision by choosing the symbol that corresponds to the accumulator Z_i with the minimum energy, where $i = 1...M$.



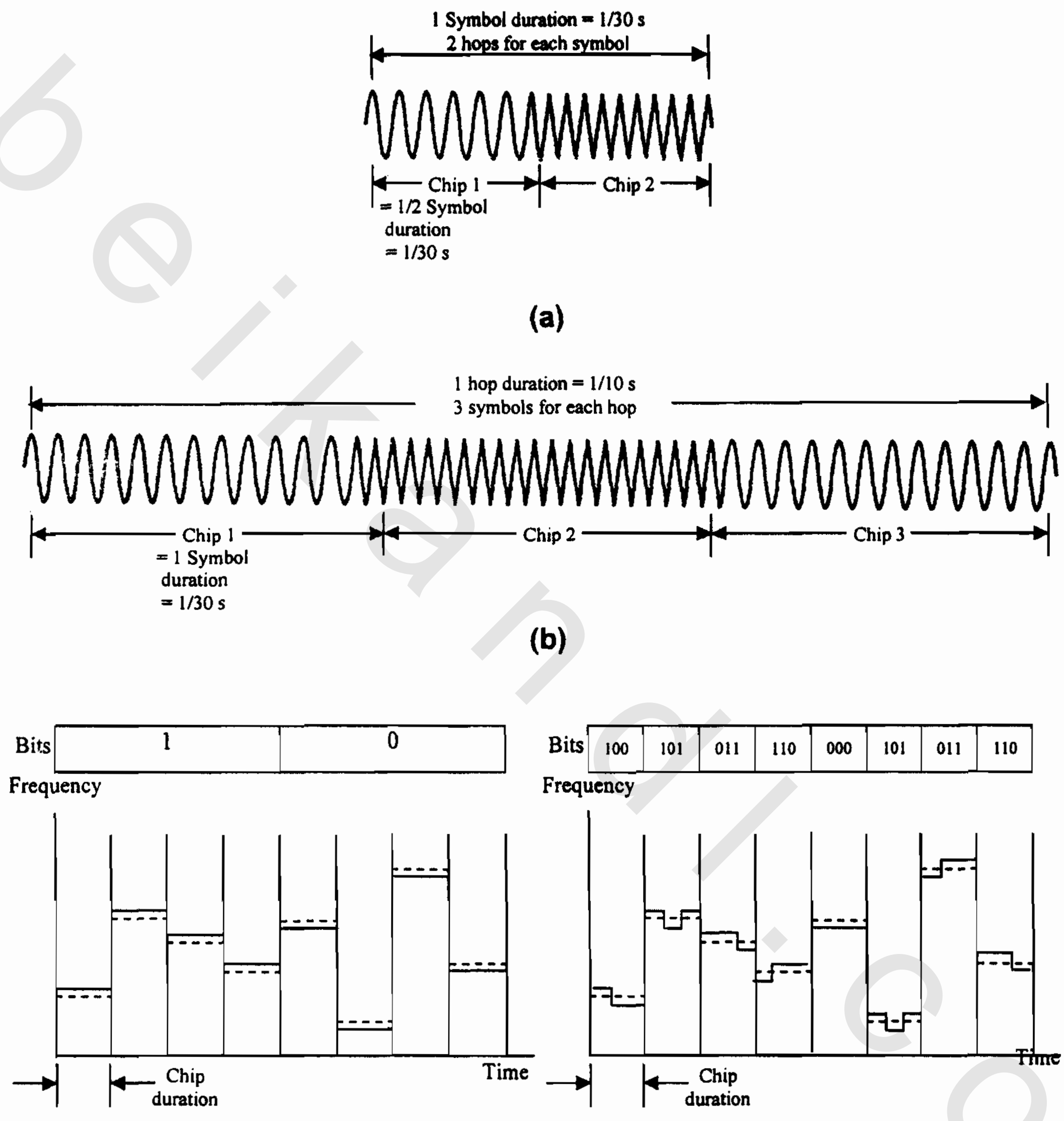
Tone number	Tone	Date symbol
0-	f_0 + 175 Hz	000
1-	f_0 + 125 Hz	001
2-	f_0 + 75 Hz	010
3-	f_0 + 25 Hz	011
4-	f_0 - 25 Hz	100
5-	f_0 - 75 Hz	101
6-	f_0 - 125 Hz	110
7-	f_0 - 175 Hz	111

Fig. (9.20) Time bandwidth plane for 8-ary FH - MFSK



Tone number	Tone	Data symbol
0-	f_0 + 700 Hz	000
1-	f_0 + 500 Hz	001
2-	f_0 + 300 Hz	010
3-	f_0 + 100 Hz	011
4-	f_0 - 100 Hz	100
5-	f_0 - 300 Hz	101
6-	f_0 - 500 Hz	110
7-	f_0 - 700 Hz	111

Fig. (9.21) Frequency hopping with diversity $r = 4$



(c) (d)

Fig. (9.22) Fast versus slow frequency hopping

a) FFH 2hop/symbol (1chip=1hop) *b) SFH 3symbols/hop (1chip=1symbol)*

c) Binary FFH 4hops/bit *d) Binary SFH 3bits/hop*

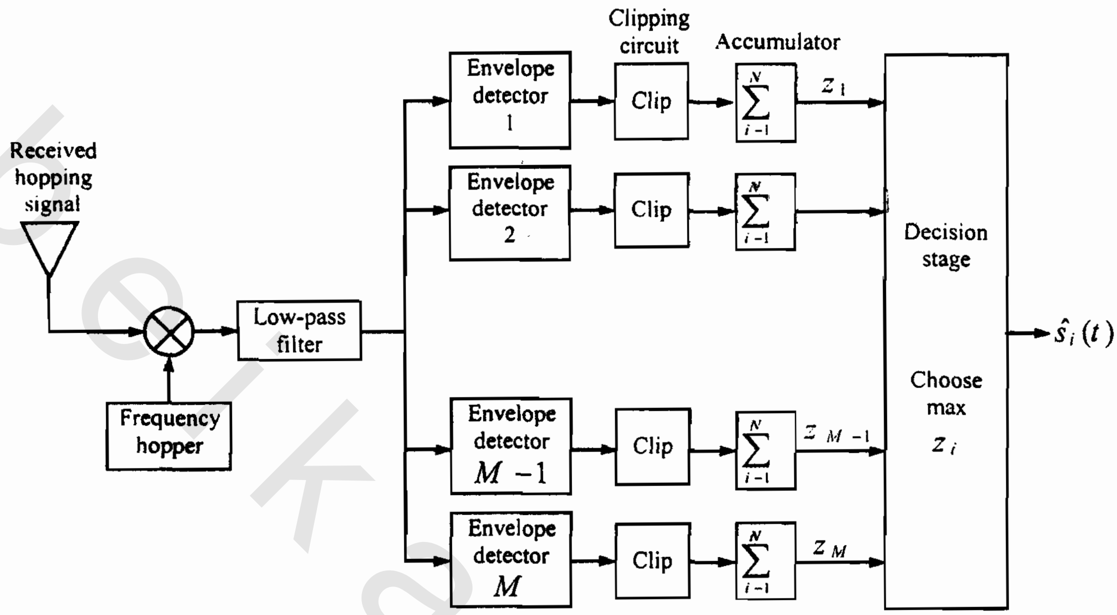


Fig. (9.23) FFH/MFSK Demodulator

9.12 Jamming:

One of the basic advantages of spread spectrum is its resistance to jamming. We have seen that a spread spectrum system fulfills the following requirements.

1) The signal occupies a bandwidth much in excess of the minimum bandwidth required for information.
2) Spreading is accomplished by means of a spreading code which is independent of the data.
3) At the receiver despreading or recovering the original data is accomplished by the correlation of the received spread signal with a synchronized replica of the spreading signal.

We know that effective communication is still possible in the presence of interfering noise of infinite power (infinite bandwidth) because only finite power noise components that are present within the signal space , share the same coordinates as the signal. The balance of the noise is tuned out by the detector. From eqns. (9–44), (9–45), (9–46) an interfering signal (jamming signal in this case) is a $2N$ dimensional signal in the signal space involving a spreading code, or as a signal is time domain there are $2BT$ dimensions .

The jammer has a finite first power and cannot drown all signal coordinates effectively. He must choose between two options,

1. Jam all signal coordinates with equal amount of power resulting in little power in each.
2. Jam a few signal coordinates with increased power.

Fig. (9.23) compares the effect of spreading in the presence of noise with spreading in the presence of an intentional jammer. The PSD of the signal before spreading is $S(f)$ and after spreading is $S_{ss}(f)$. It is seen that the single sided PSD of noise $\eta = kT$ is unchanged as a result of expanding the signal bandwidth from B to B_{ss} (Fig. 9.23a). Note that spreading a signal with a spreading code results in a Fourier transform similar to that of the spreading code, i.e. increased bandwidth. Fig. (9.23b) shows the case of a jammer of finite power J of finite bandwidth B_J. The jammer PSD is $\eta_J = \frac{J}{B_J} = \frac{J}{B}$ *if* $B_J = B$, where B the unspread bandwidth. Once the signal bandwidth is spread the jammer can reduce its PSD to $\eta'_J = \frac{J}{B_{ss}}$ *for* $B'_J = B_{ss}$ (called broadband jammer noise spectral density). Alternatively, the jammer can choose reduction of signal coordinates (reduction of his bandwidth) to concentrate his power on a limited portion of the spectrum so that the jammer PSD is $\eta''_J = \frac{\eta'_J}{\xi}$ *where* $o < \xi < 1$ is the portion of the spread spectrum band B'_J the jammer chooses to jam i.e., $\xi = \frac{B'_J}{B_{ss}}$.

We must note that for a shift register of n stages, the maximum length $N = 2^n - 1$. The N dimensions we are concerned with the code sequence of pulses $q = 0..N-1$ in eqns. (9–39) or (9–40) which repeats itself every N pulses .However the shift register generator can produce sequences that depend on the number of stages, the feedback connections and the initial conditions. So we have a vast collection of codes or a vast group of coordinates that the jammer cannot possibly cover all. Thus, the N coordinates of a prescribed code is but a subset of a much bigger coordinates system.

Ex. 9.4 (case study)

Derive an expression for signal to jammer ratio STR for DS – SS.

Solution

Spread spectrum technique distributes a relatively low dimensional signal in a large dimension signal space, i.e, the signal is hidden in the enlarged signal space. The jammer does not know which coordinates the signal uses. Consider a set of D orthogonal signals $s_p(t)$, where p is the order of the data pulse assuming D pulse within interval T where $1 < p \le D$ in an N dimensional space where $D < N$ where D represents the data sampling pulses within an arbitrary interval T and N represents the chip sampling pulses within the same period sampling pulses . Naturally each of the sampling pulses is orthogonal to any other since it lies in a different time slot. We may now express each data sampling pulse in terms of the N dimensional space as

$$s_p(t) = \sum_{k=1}^{N} a_{ik} \psi_k(t) \qquad p = 1, \ldots D, 0 \le t \le T, D << N \tag{9-82}$$

where T is a prescribed interval covering a prescribed number of data samples or symbols

$$a_{pk} = \int_0^T s_p(t) \psi_k dt \tag{9-83}$$

$$\int_0^T \psi_k(t)\, \psi_\ell(t) dt = 1 \quad k = \ell \qquad = 0 \quad otherwise \tag{9-84}$$

where $\{\psi_k(t)\}$ are linearly independent basis functions that span or characterize the N dimensional space. For every information symbol that is transmitted, a set of coefficients $\{a_{pk}\}$ is chosen independently using a pseudo random spreading code, to hide the D dimensional data symbols in the N dimensional space. The set of random variables $\{a_{pk}\}$ assume the values ± 1 each with equal probability .The receiver has at hand the same set of coefficients to perform correlation despreading. Each symbol uses the same prescribed set. The energy in each symbol is assumed constant

$$E_b = \int_0^T \overline{s_p^{\,2}(t)}\, dt = \sum_{k=1}^{N} \overline{a_{pk}^{\,2}} \qquad p = 1, 2 \ldots D \tag{9-85}$$

where the over bar represents expected value over the ensemble of many symbols (bits) during the interval T . The independent coefficients have zero mean where

$$\overline{a_{pk}\ a_{p\ell}} = \frac{E_b}{N} \quad k \ne \ell \qquad = 0 \quad otherwise \tag{9-86}$$

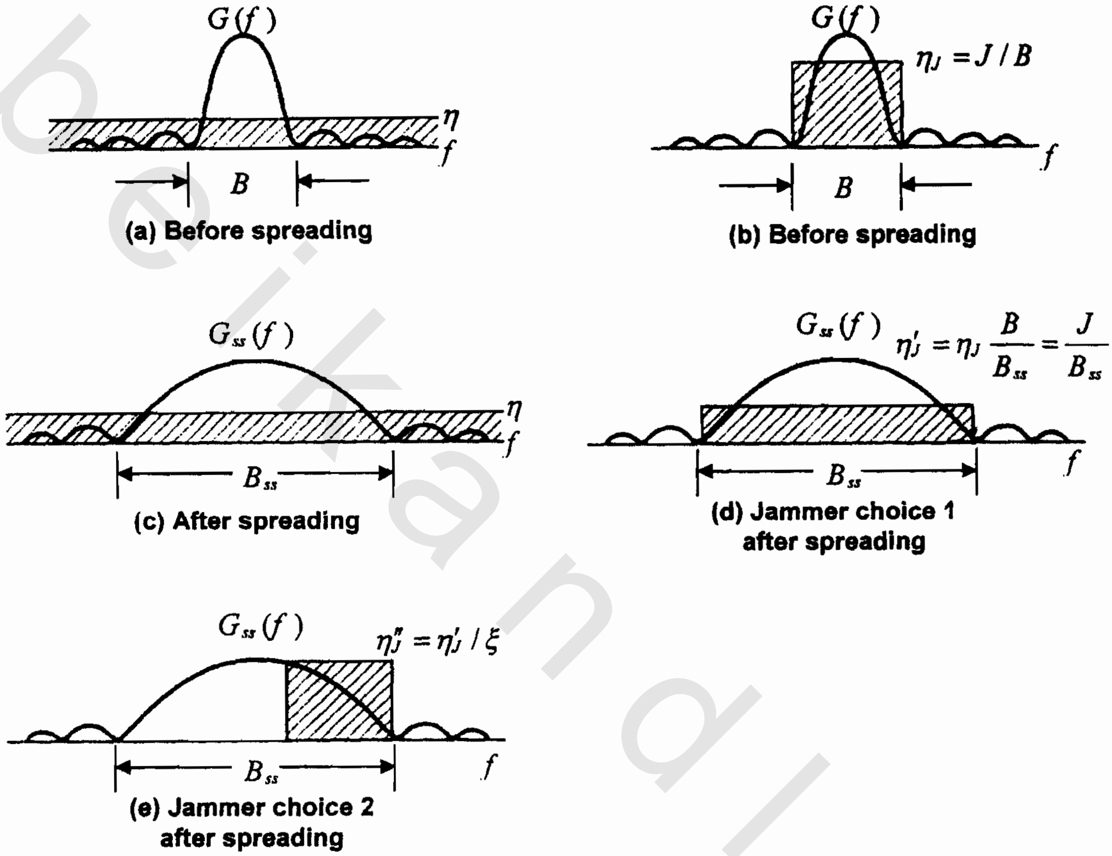


Fig. (9.24) Jamming options

a) white noise before spreading. *b) white noise after spreading.*
c) jamming before spreading $B_J = B$. *d) jamme often spreading for* $B_J' = B_{ss}$.
e) jamming after spreading for $B_J' < B_{ss}$

The jammer has no a prior knowledge of the selection of the signaling coefficients $\{a_{pk}\}$. If the jammer chooses to distribute his power uniformly over the total signal space then

$$j(t) = \sum_{k=1}^{N} b_k \ \psi_k(t) \qquad (9-87)$$

with total energy

$$E_j = \int_0^T j^2(t)\,dt = \sum_{k=1}^{N} b_k^2 \qquad (9-88)$$

The jammer aims at degrading S/N at the receiver output through manipulating the selection of b_k^2. At the receiver, the detector output (neglecting receiver noise) is

$$r_d(t) = s_p(t) + j(t)$$

This is correlated with a set of possible transmitted signals so that the output of the p^{th} correlator is

$$z_p = \int_0^T r_d(t)\,s_p(t)\,dt = \sum_{k=1}^{N}\left(a_{pk}^{\ 2} + b_k a_{pk}\right) \qquad (9-90)$$

The second term averages out to zero over the ensemble of all possible pseudorandom code sequences since the set of random variables $\{a_{pk}\}$ assume values ± 1 each with probably 1/2.

$$E\left(z_p \middle| s_m\right) = \sum_{k=1}^{N} \overline{a_{pk}^{\ 2}}$$

Thus,

$$= E_b \qquad m = p$$

$$= 0 \qquad otherwice \qquad (9-91)$$

Similarly, we compute $\operatorname{var}\left(z_p \middle| s_p\right)$, i.e, the variance at the output of the p^{th} correlator given that the p^{th} signal was transmitted

$$\operatorname{var}\left(z_p \middle| s_p\right) = \sum_{k,\ell} b_k b_\ell\ \overline{a_{pk}\, a_{p\ell}} \qquad (9-92)$$

$$= \sum_{k=1}^{N} b_k^2 \overline{a_{pk}^{\ 2}} \qquad (9-93)$$

$$= \sum_{k=1}^{N} b_k^2 \frac{E_b}{N} \qquad (9-94)$$

$$= \frac{E_j E_b}{N} \qquad (9-95)$$

The signal to jammer ratio at the output of the p^{th} correlator can be defined as

$$SJR = \sum_{m=1}^{D} \frac{\left[E\left(z_p \middle| s_p\right)\right]^2}{\operatorname{var}\left(z_p \middle| s_p\right)} P\left(s_m\right) \qquad (9-96)$$

$$=\frac{E_b^2/D}{E_j E_b/N}=\frac{E_b}{E_j}\frac{N}{D} \qquad (9-97)$$

Since the probability of the m^{th} signal $P(s_m)=1/D$, since the signals are assumed to occur with equal probability, and the signal energy and jammer energy at the p^{th} correlator are denoted $\left[E\left(z_p|s_p\right)\right]^2$ and $\mathrm{var}\left(z_p|s_p\right)$ respectively. The result is independent of the way in which the jammer chooses to distribute his energy. Therefore regardless of how b_k is chosen subject to $\sum_k b_k{}^2=E_j$, the SJR indicates that spreading gives the signal an advantage of a factor of N/D over the jammer. The ratio N/D is the processing gain G.
Since the dimensionality of a signal of bandwidth B and duration T is $2TB$. We can express the processing gain as

$$G=\frac{N}{D}=\frac{2B'_{ss}T}{2B'_{\min}T}=\frac{B'_{ss}}{B'_{\min}}=\frac{B'_{ss}}{R_b} \qquad (9-98)$$

$$=\frac{R_{ch}}{R_b}=N \qquad (9-99)$$

where B'_{ss} is the single sided spread spectrum bandwidth, $B'_{\min}$ is the minimum data single sided bandwidth. R_{ch} is the chip rate, R_b is the data bit rate and N is in length of the code .

Ex. 9.5 (Case study)
Repeat the previous example for FH-SS
Solution

There are many different waveforms that can be used against $FH-SS$ systems. Fig. (9.25) shows examples of jammer waveforms. The three columns represent three instants in time where G_1,G_2 and G_3 symbols are transmitted. In Fig. (9.25a), a low level noise jammer occupies the full spread spectrum bandwidth. In Fig. (9.25b), the jammer puts higher PSD for a limited bandwidth. In Fig. (9.25c), the jammer steps through different regions of the band, at random time. In Fig. (9.25d), the partial jammer uses a group of tones instant of a continuous frequency band. In Fig. (9.25e), a pulse jammer consists of pulse modulated bandwidth noise.

The antijam system aims at dissipating the jammer's resources over a wide frequency band. Antijamming systems aim at evading the jammer by using frequency diversity, time diversity and by spatial discrimination through the use of a narrow beam antenna.

The S/N *is* $E_b/(\eta+\eta_J')$ where η_J' is the *PSD* due to the jammer which is J/B_{ss}. For thermal noise only as received power P we have

$$\frac{E_b}{\eta}=\frac{ST_b}{N/B}=\frac{S/R_b}{N/B} \tag{9 – 100}$$

$$=\frac{S}{N}\left(\frac{B}{R_b}\right)$$

$$=P_r/\eta=(1/R_b)$$

$$\frac{P_r}{\eta}=\frac{E_b}{\eta}\;R_b \tag{9 – 101}$$

We refer to $(E_b/\eta)_o$ as the value of bit energy per noise *PSD* required to yield a specified error probability, while $(E_b/\eta)_r$ is the received (E_b/η). We define link margin or safety margin M_J *as*

$$M_J dB=\left(\frac{E_b}{\eta}\right)_r dB-\left(\frac{E_b}{\eta}\right)_o dB \tag{9 – 102}$$

Similarly, if we assume $\eta_J >> \eta$ we define $(E_b/\eta_J)_0$ as the bit energy per jammer noise *PSD* required for maintaining the link at a specified error probability. Thus, from eqn. (9 – 100)

$$\left(\frac{E_b}{\eta_J}\right)_o=\left(\frac{S/R_b}{J/B_{ss}}\right)_o \tag{9 – 103}$$

$$=\frac{B_{ss}/R_b}{\left(\frac{J}{S}\right)_0}=\frac{G}{(J/S)_o} \tag{9 – 104}$$

Thus,

$$(J/S)_o=\frac{G}{(E_b/\eta_j)_o} \tag{9 – 105}$$

The ratio $(J/S)_o$ is a figure of merit which measures how invulnerable a system is to interference. The larger the $(J/S)_o$ the greater is the systems rejection capability.

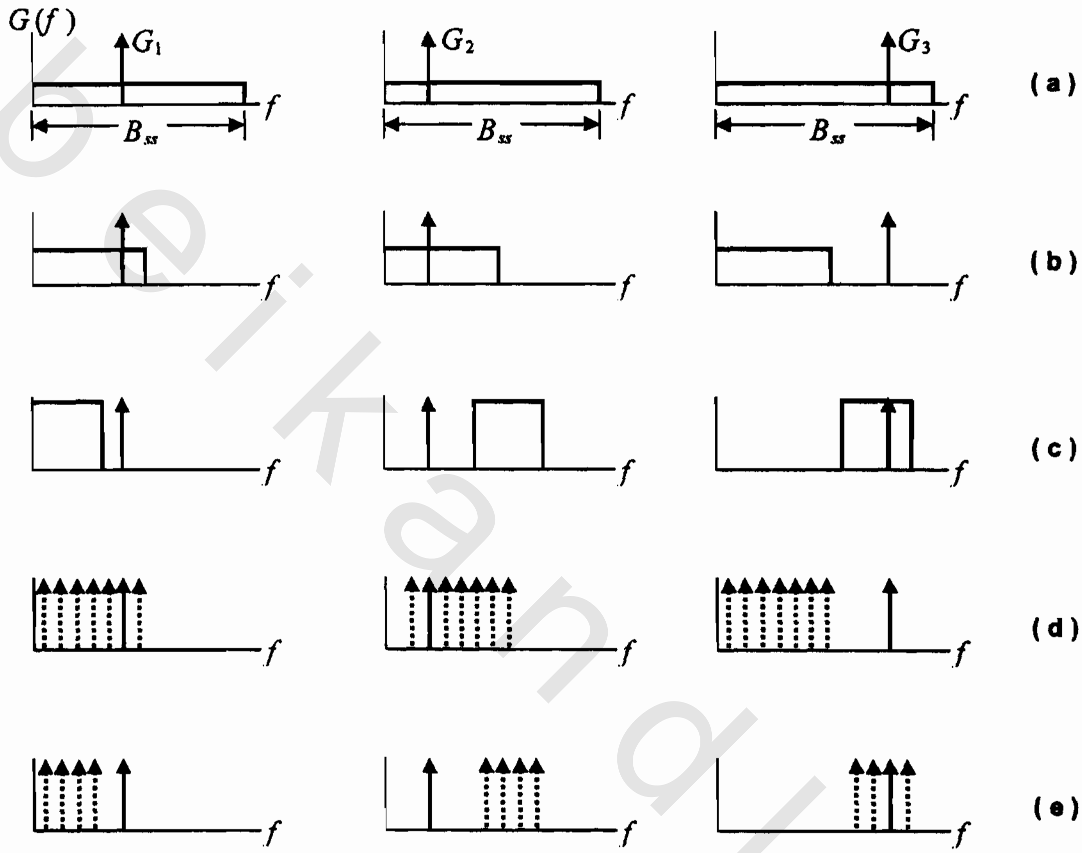


Fig. (9.25) FH Jammer

a) full band noise. *b) partial band noise.*
c) stepped noise. *d) partial band tone.*
e) stepped tones.

This figure of merit describes how much noise power relative to signal power is required to degrade the system's specified error performance. In other words, the jammer tries to force $(E_b/\eta_J')_o$ to be as large as possible, i.e, small $(J/S)_o$. The communicator may increase the processing gain to increase (J/S_o) i.e, make it more difficult for the jammer to degrade the link. This leaves the jammer with the option of broadened and weakened wide band Gaussian noise.

We may define the antijam margin M_{AJ} along the lines of eqn. (9 – 102)

$$M_{AJ}\ dB = \left(\frac{E_b}{\eta_J}\right)_r dB - \left(\frac{E_b}{\eta_J}\right)_0 \qquad (9-106)$$

Following the lines of eq: (9 – 103),

$$\left(\frac{E_b}{\eta_J}\right)_r = \frac{G}{(J/S)_r} \qquad (9-107)$$

Where $(J/S)_r$ is the ratio of the actually received jammer power to signal power.

Thus, eqn.(9 – 105) becomes

$$M_{AJ}\ dB = \frac{G}{(J/S)_r} dB - \frac{G}{(J/S)_o} dB \qquad (9-108)$$

9.13 Synchronization:

In both DS-SS and FH-SS systems, the receiver must have a synchronized replica of the PN code. A locally generated PN code of identical structure used in transmission is to be synchronized with the received code. This is usually done in two steps. The first step is called acquisition, which consists of bringing the two spreading signals into coarse alignment with one another. Once the received spread spectrum signal has been acquired, the second step - called tracking - maintains fine alignment by means of a feedback loop. The whole process involves an acquisition problem meaning searching throughout time and frequency to synchronize the received spread spectrum signal with the locally generated spreading signal. Acquisition schemes can be classified as coherent or non coherent.

Since the carrier phase is unknown most acquisition systems use non coherent detection. The received signal and the locally generated signal are first correlated to produce a measure of similarity between the two.

This measure is then compared with a threshold to decide if the two signals are in synchronism. If they are, the tracking loops take over. Otherwise, the circuit modifies frequency or phase of the locally generated code until synchronism is achieved. In direct sequence, parallel search acquisition system (Fig. 9.26) may be used. The locally generated code $g(t)$ is available with delays that are spaced one half chip apart $T_c/2$. If the uncertainty between the local code and the received code is n_c chips, then $2n_c$ correlators are needed to perform a complete parallel search after a number of chips λ have arrived.

The locally generated code corresponding to the correlator with the largest output is chosen the through the use of maximum likelihood algorithm.

Another strategy for the acquisition of spread spectrum signals is to use a single correlator or a matched filter to serially search for the correct phase of the DS code signal (Fig. 9.27). In such a stepped serial acquisition scheme for a DS system, the locally generated PN signal is correlated with the incoming PN signal.

At fixed examination intervals of λT_c, the output signal is compared to a preset threshold. If the output is below the threshold, the phase of the locally generated code signal is increased by $T_c/2$ and the correlation is reexamined. If the threshold is exceeded, the phase incrementing process stops.

In a similar scheme for FH system (Fig. 9.28), the PN code generator controls the frequency hopper. Acquisition is accomplished when the local hopping is aligned with that of the received signal. The maximum time required for a fully serial DS search is

$$\hat{T}_{acq} = 2N_c\lambda T_c \qquad (9\text{-}109)$$

Once acquisition (or coarse synchronization) is completed, tracking (or fine synchronization) takes over. Tracking code loops may be coherent or non coherent. A coherent loop is one in which the carrier frequency and phase are known. A non coherent loop is one in which the carrier frequency and phase are not known exactly, so a non coherent code loop is used to track the received PN code.

Tracking loops may be classified as full time early late tracking loop - or delay locked loop (DLL) - or as a time shared early late tracking loop referred to as tau dither loop (TDL)

A basic non coherent DLL loop for a DS - SS using BPSK is shown (Fig.9.29). As the data $x(t)$ and code $g(t)$ each modulates the carrier wave using BPSK, the received waveform in the absence of noise is

$$r_d(t) = A\sqrt{2P}\,x(t)g(t)\cos(\omega_o t + \theta) \qquad (9\text{-}110)$$

where the constant A is a system gain parameter and θ is a random phase angle in the range $(0, 2\pi)$. Assume that the locally generated code of the tracking loop is offset in phase from the incoming $g(t)$ by a time τ where $\tau < T_c/2$. The loop provides fine synchronization by first generating two PN sequence $g(t + T_c/2 + \tau)$ and $g(t - T_c/2 + \tau)$ delayed from each other by one chip. The two bandpass filters are designed to pass the data and to average the product of $g(t)$ and the two sequences $g(t \pm T_c/2 + \tau)$. The square law envelope detector eliminates the data since $|x(t)| = 1$.

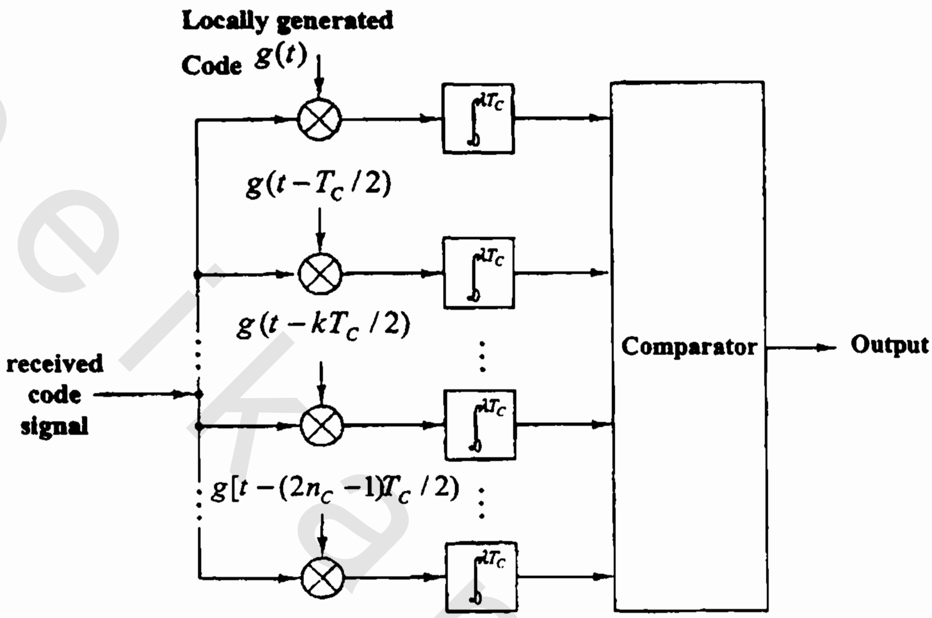


Fig. (9.26) Direct sequence parallel search acquisition.

The output of each envelope detector is given by

$$E_D = E\left\{\left|g(t)g(t\pm\frac{T_c}{2}+\tau\right|\right\}$$

$$=\left|R_g(\tau\pm\frac{T_c}{2})\right| \qquad (9\text{-}111)$$

The feedback circuit is shown (Fig. 9.29a). The autocorrelation function of the PN waveform is shown in Fig. (9.29b).

When τ is positive, the feedback signal $Y(\tau)$ instructs the VCO to increase its frequency, thereby forcing τ to decrease. When τ is negative, $Y(\tau)$ instructs the VCO to decrease its frequency, thereby forcing τ to increase. When τ is a small number approaching zero, $g(t)g(t+\tau)\sim 1$, yielding the despread signal $z(t)$ which is then applied to the input of a conventional data demodulator. The early and late arms must to precisely gain balanced or else the feedback signal $Y(\tau)$ will the offset and will not produce a zero signal when the error is zero. An alternative design using a single correlator called dither tau loop (DTL) is shown (Fig. 9.30).

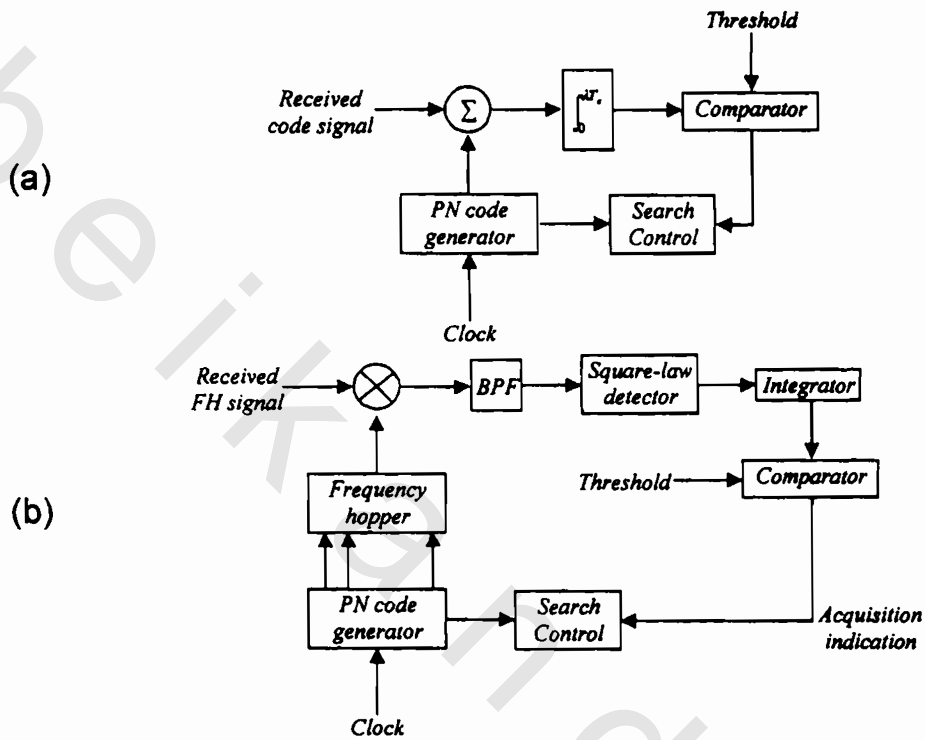


Fig. 9.27 Serial search acquisition
a) DS *b) FH*

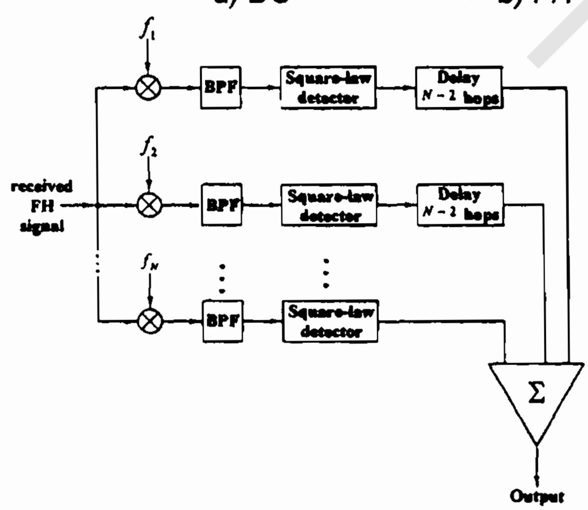


Fig. (9.28) Frequency hopping acquisition scheme

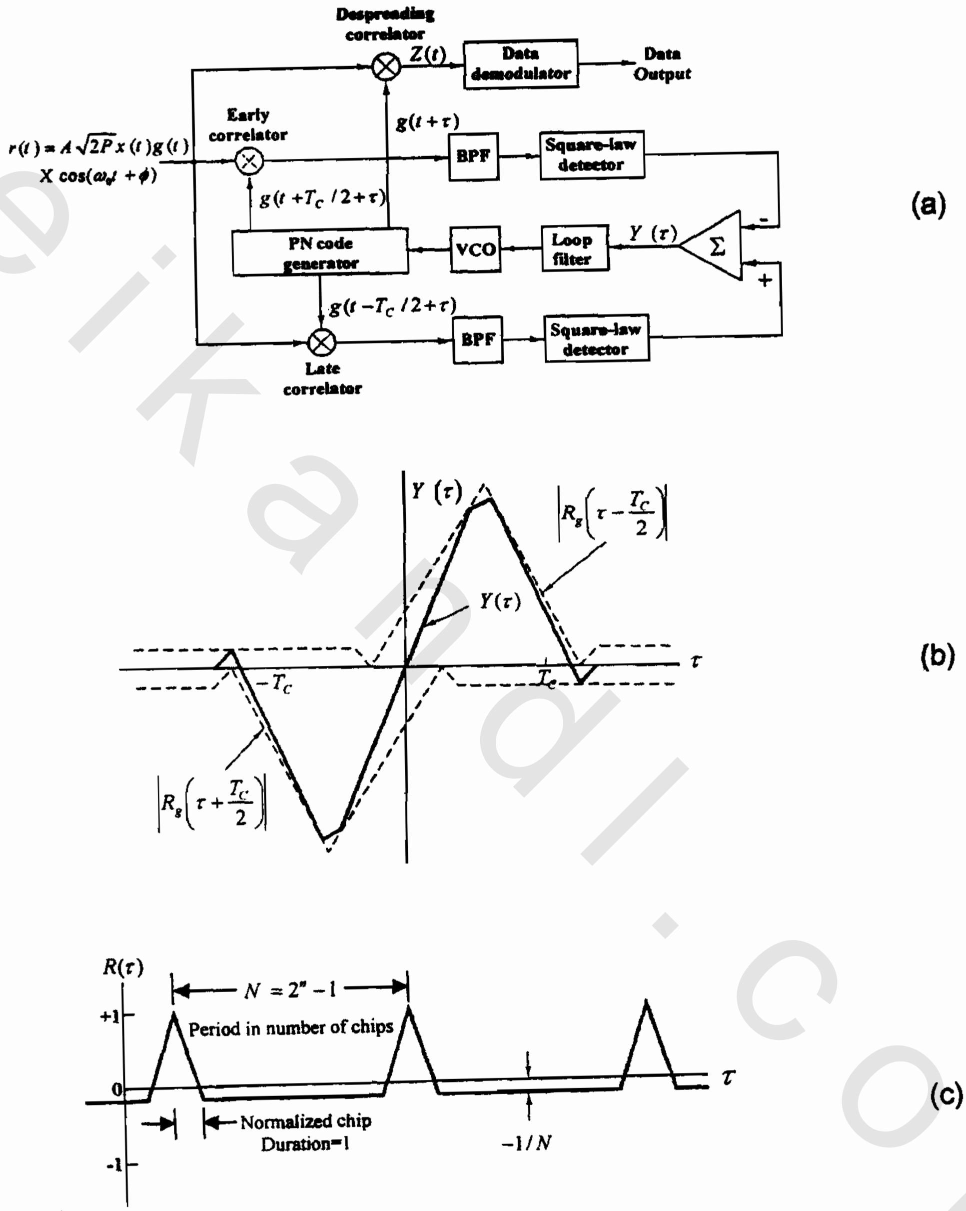


Fig. (9.29) DLL for tracking DS signals

a) Circuit *b) feed back signal* *c) PN autocorrelation function*

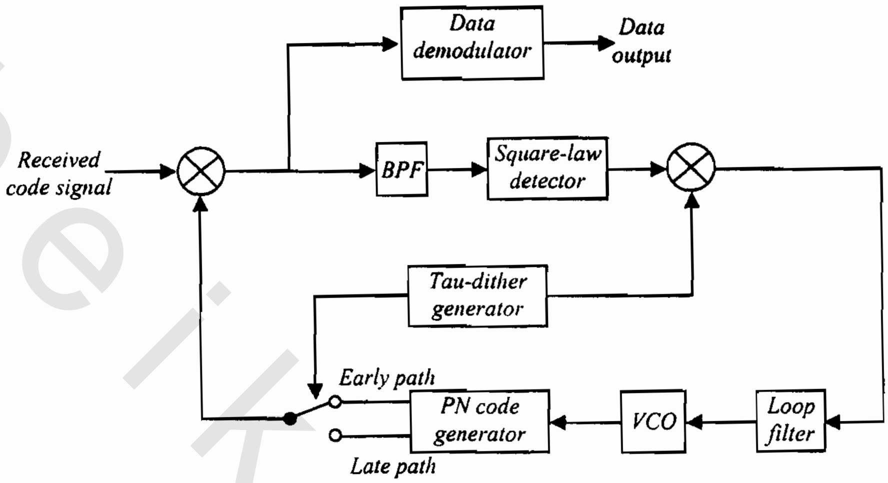


Fig. (9.30) Tau dither tracking loop.

9.14 Code Division Multiplex Access (CDMA):

Spread spectrum allows multiple signals occupying the same RF bandwidth to be transmitted simultaneously without interfering with one another. In CDMA, using direct sequence, each of N users is given a code $g_i(t), i = 1,2,....,N$.

The user codes are approximately orthogonal, so that the cross correlation of two different codes is near zero. Thus, all users share the full spectrum asynchronously. A typical DS/CDMA block diagram is shown (Fig. 9.31). The output of the data modulator corresponding to user 1 is

$$s_1(t) = A_1(t)\cos\left[\omega_o t + \theta_1(t)\right] \tag{9 -112}$$

Next, the data modulated signal is multiplied by the spreading signal $g_1(t)$. The signal present at the receiver is the linear combination of such signals from different users with each code being kept secret except for the authorized user. Thus,

$$r_d(t) = g_1(t)s_1(t) + g_2(t)s_2(t).....g_N(t)s_N(t) \tag{9 -113}$$

Multiplication of $s_1(t)$ by $g_1(t)$ produces a signal whose spectrum is the convolution of the spectrum of $s_1(t)$ with the spectrum of $g_1(t)$. Thus, assuming that the signal $s_1(t)$ is narrowband compared with the code (or spreading signal) $g_1(t)$, the product signal $g_1(t)\, s_1(t)$ will have approximately the bandwidth of $g_1(t)$. At the receiver, the $g_1(t)$ code is generated in perfect synchronism. This is multiplied by $r_i(t)$. We get

$$z(T_s) = \int_0^{T_s} [g_1^2(t)s_1(t) + g_1(t)g_2(t)s_2(t) + g_1(t)g_N(t)s_N(t)]dt \qquad (9\text{-}114)$$

since $g_i^2(t) = 1$

$$z(T_s) = s_i(T_s) + \int_0^{T_s} [g_1(t)g_2(t)s_2(t) ... + g_1(t)g_N(t)s_N(t)]dt \qquad (9\text{-}115)$$

$$s_i(T_s) = \int_0^{T_s} s_1(t)dt \qquad (9\text{-}116)$$

$$s_i(t) = \sum_{j=1}^{N} q_{ij} g_j(t) \qquad (9\text{-}117)$$

$$q_{ij} = \int_0^{T_s} s_i(t) g_j(t)dt \qquad (9\text{-}118)$$

Note

$$\frac{1}{T_s}\int_0^{T_s} g_i(t)g_h(t)dt = \begin{matrix} 1 & j = h \\ 0 & \text{otherwise} \end{matrix}$$

Thus,

$$z(T_s) = s_i(T_s) \qquad (9-119)$$

In practice, the codes are not perfectly orthogonal, hence the cross correlation between user codes introduces performance degradation which limits the maximum number of simultaneous users. Fig. (9.32a) illustrates the wideband input to the receiver. It consists of wanted and unwanted signals each spread by its own code with code rate R_{ch} and with PSD in the form $\text{sinc}^2(f / R_{ch})$.

Receiver thermal noise is shown as having a flat spectrum. Fig. (9.32b) illustrates the spectrum after correlation with the despreading code $g_1(t)$. The desired signal occupying the information bandwidth centered at an IF frequency is then applied to a conventional demodulator with bandwidth just enough to accommodate the despread signal. The undesired signal and noise are now spread by $g_1(t)$ at the receiver. Only that portion of the spectrum of the unwanted signals falling in the information bandwidth of the receiver will cause interference with the desired signal.

Similarly, CDMA has the ability to discriminate against multipath interference. Due to reflections from atmospherics or buildings signals may arrive at the receiver from multiple paths causing interference (Fig. 9.33), so that the received signal for BPSK signal becomes

$$r_d(t) = Ax(t)g(t)\cos\omega_o t + \alpha Ax(t-\tau)g(t-\tau)\cos(\omega_o t + \theta) + n(t) \qquad (9 - 120)$$

where $x(t)$ is the data , $g(t)$ is the code signal, $n(t)$ is the noise, τ is the time delay between the two paths $0 < T < T, \theta$, is a random phase assumed to be uniformly distributed in the range $(0, 2\pi)$, α is the attenuation of the multipath signal relative to the direct path. Since the receiver is synchronized to the direct path signal, the output of the correlator can be written as

$$z(T_s) = \int_0^{T_s} [Ax(t)g^2(t)\cos\omega_o t + \alpha Ax(t-\tau)g(t)g(t-\tau)\cos(\omega_o t + \theta) + + n(t)g(t)] 2\cos\omega_o t dt \qquad (9 - 121)$$

For $\tau > T_c$ and since

$$g^2(t) = 1, \quad \int_0^{T_s} g(t)g(t-\tau)dt = 0$$

For codes of long periods (near random). For $\tau < T_c$

$$z(T_s) = \int_0^{T_s} 2Ax(t)\cos^2\omega_o t + 2n(t)g(t)\cos\omega_o t dt$$

$$= Ax(T_s) + n_o(T_s) \qquad (9 - 122)$$

where $n_o(T_s)$ is a zero mean Gaussian random variable. Thus, spreading eliminates the multipath interference.

In FH-SS the multipath problem is eliminated by rapidly changing the transmitter frequency, thus avoiding the interference by changing the receiver band position before the arrival of the multipath signal. We note that DS - SS renders all multpath signals that are delayed by more than one chip time from the direct signal as invisible to the receiver, while the FH - SS can do the same thing only if the hopping rate is faster than the symbol rate, and if the hopping bandwidth is large enough, which is costly due to the need for high speed frequency synthesizers.

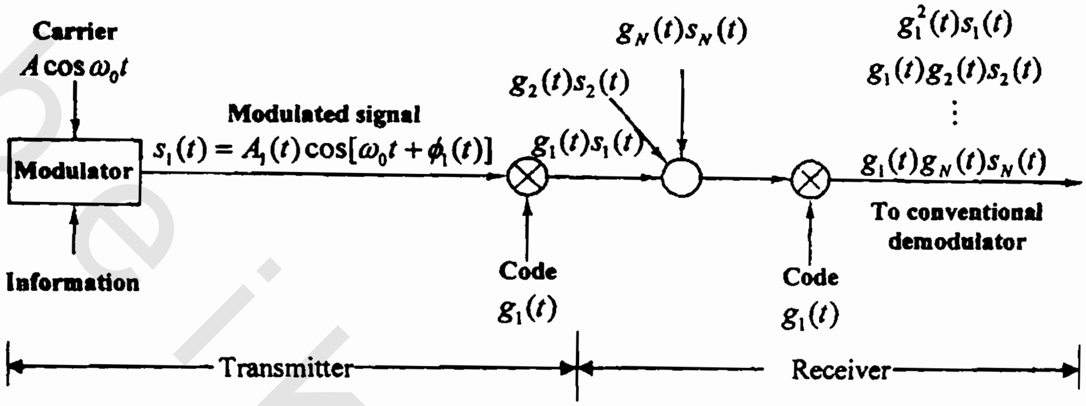


Fig. (9.31) CDMA (Code-division multiple access).

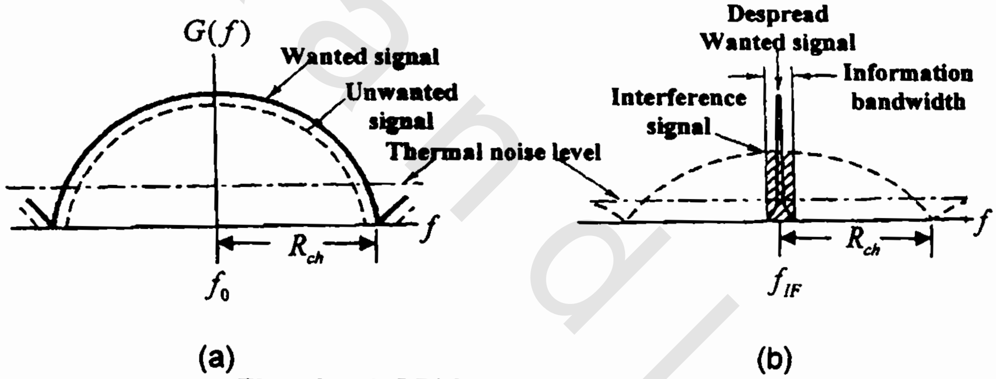


Fig. (9.32) CDMA (signal detection).

a) spectrum at input of the receiver *b) spectrum after correlation*

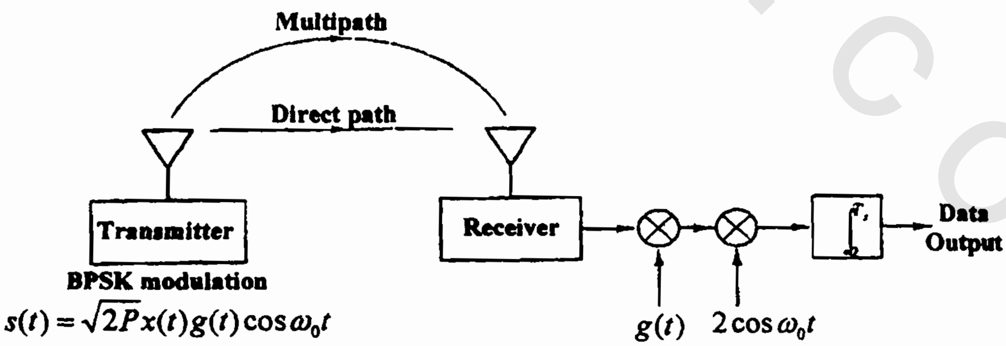


Fig. (9.33) Interference from DS/BPSK spectrum.

Ex. 9.6 (case study)
In this example we show that DS - SS allows for the detection of a signal that has PSD well below noise. Assume PSD of the signal $S_o(f) = 10^{-5}\ W/Hz$ over a bandwidth of 1 MHz and the transmitted data rate is $R_b = 10^6$ bits/s, the noise PSD $\eta = 10^{-6} W/Hz$. Find E_b/η for this narrowband case. Then consider the signal to be spread over a spread spectrum bandwidth of $B_{ss} = 10^8 Hz$. Show that the received E_b/η of the spread spectrum receiver is the same as that for the narrow band case and hence the error performance is unchanged

Solution
Before spreading, the total average power is

$$S = 10^{-5}\ (W/Hz)\, X 10^6\ (Hz) = 10W$$

The total noise power is

$$N_o = 10^{-6}\ (W/Hz)\ X\ 10^6\ (Hz) = 1W$$

$$\frac{E_b}{\eta} = \frac{S/R_b}{\eta} = \frac{10x\,10^{-6}}{10^{-6}} = 10$$

After spreading, the PSD $S_o'(f)$ is reduced by two orders of magnitude as the bandwidth is increased from $10^6 Hz$ to $10^8 Hz$, such that the total power is the same (10W). However, since the noise is AWGN it is not reduced. Hence, the noise power is $N' = 10^{-6} W/Hz x\, 10^8 Hz = 100W$

Then

$$\frac{E_b}{\eta}\Big|_{received} = \frac{S/R}{N'/B_{ss}} = \frac{S}{N_o'}\left(\frac{B_{ss}}{R_b}\right) = \frac{S}{N'}G$$

$$= \frac{10}{100} x100 = 10$$

where $G_s = B_{ss}/R_b = 100$ is the processing gain. Thus, we are able to detect a signal buried in noise (Fig. 9.34).

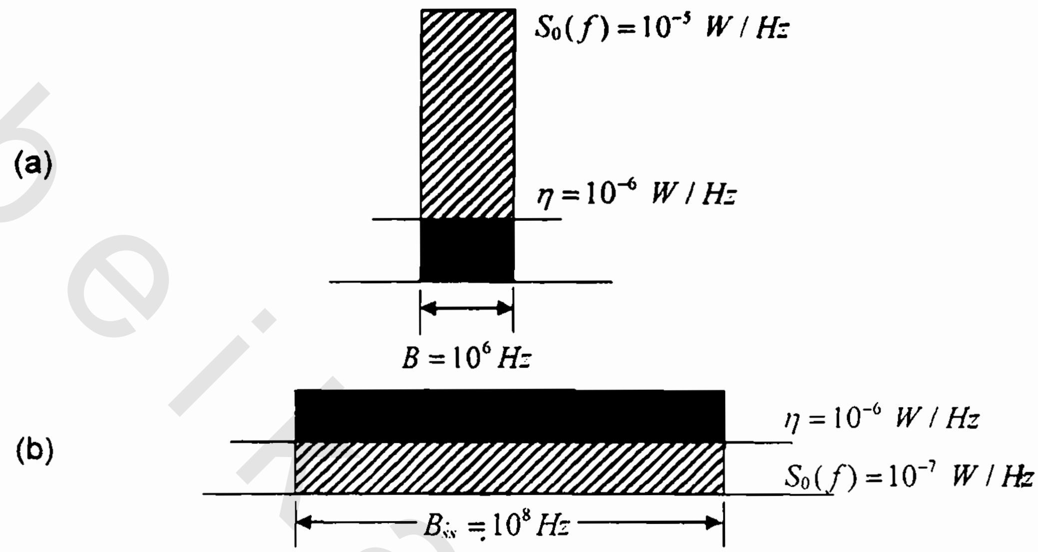


Fig. (9.34) Detection of signal buried in noise
a) before spreading. *b) after spreading.*

Problems

1. Consider a maximum length sequence of length $m = 5$. Compare the output if we select taps $[3, 2]$ and $[2, 1]$ and $[3, 1]$. Take the initial state to be 10000.

2. Obtain the autocorrelation function of the PN code. Then show that the autocorrelation function $R_x(\tau) = (n_a - n_d)/N$, where n_a is the number of agreements in one full period n_d is number of disagreements in one full period of the sequence and τ is position cyclic shift. Then show that for any cyclic shift between $x(t)$ and $x(t + \tau)$ with $t/T_b < \tau/T_b < N$ the autocorrelation function is equal to $-1/N$. Thus the sequences are virtually decorrelated for a shift of one single chip for large N.

3. Verify eqn. (9 – 17) and compare with the above problem.

4. A hopping bandwidth of 400 MHz and a frequency step size of 100 Hz are specified. What is the minimum number of PN chips that are required for each frequency word?

5. A spread spectrum system has $T_b = 4ms$, $T_C = 1\mu s$. Find the processing gain and shift register length and the interference margin for a probability of error $= 10^{-5}$?

6. A hopping bandwidth B_{ss} of 400 MHz and a frequency step size of 100 Hz are specified. What is the minimum number of PN chips that are required for each frequency word? And what is the number of possible hops?

7. Analyze the DLL circuit (Fig. 9.29)

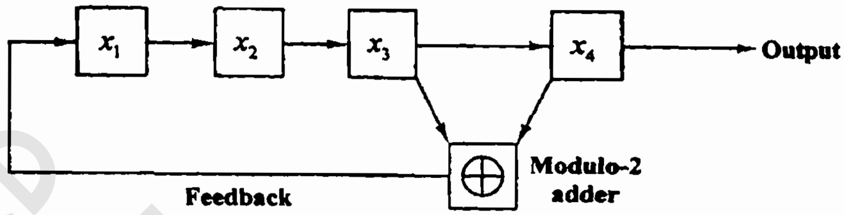
8. Analyze the circuit shown, obtain the code and maximum length

9. Analyze the PN generator circuit shown and obtain the code.

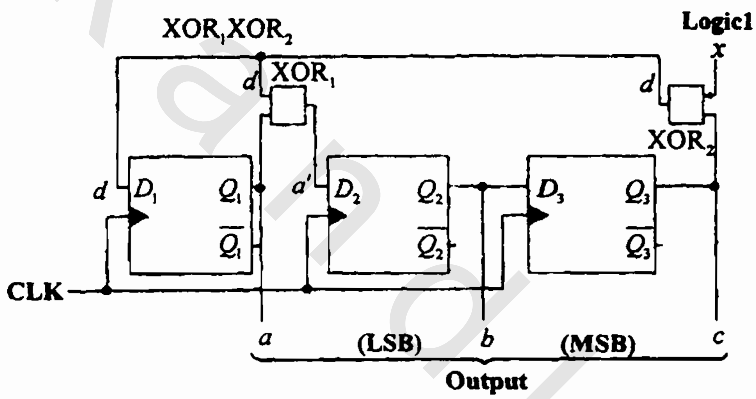
10. Analyze the scrambler and descrambler circuits shown.

11. Analyze the serial search circuits for DS and FH (Fig. 9.27)

12. Analyze the DTL circuit (Fig. 9.30)

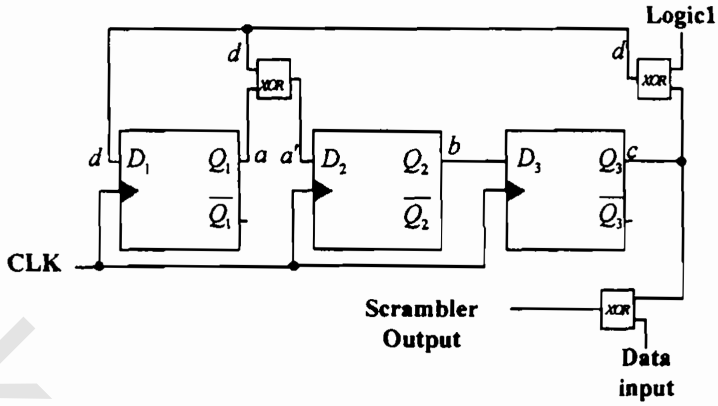


Prob. (9- 8)

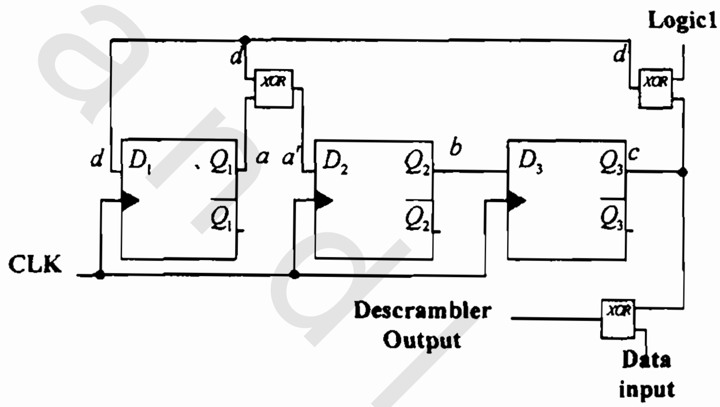


Prob. (9-9)

a	b	c	Data input	Output
0	0	0	1	1
1	1	0	1	1
1	0	1	1	0
0	1	0	1	1
1	1	1	1	0
0	1	1	1	0
0	0	1	1	0
0	0	0	1	1



a	b	c	Data input	Output
0	0	0	1	1
1	1	0	1	1
1	0	1	0	1
0	1	0	1	1
1	1	1	0	1
0	1	1	0	1
0	0	1	0	1
0	0	0	1	1



Prob. (9-10)

References

1. "Digital communications," B. Sklar, 2nd ed., Prentice Hall, Upper Saddle River, N.J. 2001.

2. "Communication Systems", S. Haykin, 4th ed J. Wiley, N.Y, 2001.

3. "Digital Communications", J. Proakis, 4th ed, McGrawHill, N. Y. 2001.

4. "Analog and Digital Communication Techniques", G. Smillie, Arnold, London, 1999.

5. "Telecommunications, Demystified", C. Nassar, LLH Technology, Eagle Rock, Virginia, 2001.